FAMILIES OF FUZZY IMPLICATION OPERATORS WITHIN MEASURE M1 AND THEIR PSEUDO-STRICT MONOTONICITY PROPERTY

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ABSTRACT

In this paper, we try to find some families of fuzzy impli·ations laid between the well known Kleene-Dienes and Lukasiewicz fuzzy implications. It's well known that one can find infinite fuzzy implication operators using t-norms. However, in our approach, we start by locating two fuzzy implication operators and then generalise them giving at least eight families containing an infinite number of fuzzy implication operators.

These families can be seen as sequences of two-place functions. Light will be shed on one interesting property of these sequences, and an example of interval-valued inference is discussed.

1. INTRODUCTION

The Kleene-Dienes and Lukasiewicz fuzzy implication operators, I_{KD} and I_L , were well studied in the literature ([1],[6]). From the Checklist Paradigm ([2],[3]), Bandler and Kohout discovered a new measure "m1" (among others) of implication and proved that its upper and lower limits are I_L and I_{KD} respectively. In interval-valued inference, the inference is computed twice using the two implications above cited, hence producing an interval of result (Yew[8]); empirical experiences showed that, in some cases, this result may overlap the inference band of acceptance (or of rejection) ; in such cases, the present work can be useful, in the sense where one can use an other pair of fuzzy implication operators as a fork in order to remove this overlapping.

In section 2, some of the families of fuzzy implication operators that lay between I_{KD} and I_L are generated ; in section 3, their properties are given and only the pseudo-strict monotonicity property is discussed.

At the end of this paper, section 4, one analyses the behaviour of a particular instance of the inferential structure (namely the SUB-K inference structure) when applied to special cases ; we show that it can infer amazing results that are against a human being's intuition, one of these special cases is detected and the semantic (reason) of such results is explained.

2. GENERATION OF THE FUZZY IMPLICATION FAMILIES LAID BETWEEN I_{KD} AND I_L

It has been shown in [2] that I_{KD} and I_L implication operators are respectively the attainable lower and upper bounds of measure m1.

$$a \xrightarrow{KD} b \leq m1 \leq a \xrightarrow{L} b$$

Moreover, and using once again the Checklist Paradigm, Bandler and Kohout generated a new implication operator worthy of attention, called I_{KDL}, that stands between I_{KD} and I_L, and satisfying the following ordering $I_{KD} \leq I_{KDL} \leq I_L$.

We recall the definitions of the above fuzzy implications.

$\forall$ a,b $\in$ [0,1],

$I_{KD}(a,b) = \mathrm{Max}(b,1-a)$, $I_{KDL}(a,b) = 1-a+ab$ and,

$I_L(a,b) = \min(1, 1-a+b)$

Proceeding by dichotomous division, two special fuzzy implication operators, namely I_X and I_Y, can be generated, and then generalised. I_X stands between I_{KD} and I_{KDL}, while I_Y stands between I_{KDL} and I_L.

$$I_X = \text{Max}\,[\frac{1+a(b-1)+b}{2}, (1-a)+\frac{ab}{2}]$$ and,

$$I_Y = \min\,[1-\frac{a(1-b)}{2}, (1-a)+\frac{b(1+a)}{2}]$$

The previous order can be extended to:

$I_{KD}(a,b) \leq I_X(a,b) \leq I_{KDL}(a,b) \leq I_Y(a,b) \leq I_L(a,b)$, thus one can generate families of fuzzy implications in each of the following intervals.

$[I_{KD}, I_X]$, $[I_X, I_{KDL}]$, $[I_{KDL}, I_Y]$ and $[I_Y, I_L]$.

In all what follows, we assume that $p, q \in N^*$

2.1 Fuzzy Implications Laid Between I_{KD} and I_{KDL}

2.1.1 Between I_{KD} and I_X: One can generate at least two families (sequences) of fuzzy implication operators :

$$I_{KDp}=\text{Max}[\frac{(2^{p-1}-1)[1+a(b-1)]+(2^{p-1}+1)b}{2^p}, (1-a)+\frac{(2^{p-1}-1)ab}{2^p}]$$

$$I'_{KDp} = \text{Max}\,[\frac{1+a(b-1)+(2^p-1)b}{2^p}, (1-a)+\frac{ab}{2^p}]$$

Propositions

1) The sequence (I_{KDp}) is increasing.

$\forall p,q \in N^*, p>q \Rightarrow I_{KDp}(a,b) \geq I_{KDq}(a,b)$

$\lim_{p\to 1} I_{KDp} = I_{KD}$ and $\lim_{p\to\infty} I_{KDp} = I_X$

2) The sequence (I'_{KDp}) is decreasing.

$\forall p,q \in N^*, p>q \Rightarrow I'_{KDp}(a,b) \leq I'_{KDq}(a,b)$

$\lim_{p\to 1} I'_{KDp} = I_X$ and $\lim_{p\to\infty} I'_{KDp} = I_{KD}$

Proof

For the sake of shortness, we consider only (I'_{KDp}) family of fuzzy implications.

Assuming $p>q$, one can check easily that (I'_{KDp}) is decreasing. $(I'_{KDp}(a,b) - I'_{KDq}(a,b) \leq 0)$

Indeed, provided that $(1+a(b-1)-b) = (1-a)(1-b)>0)$, the following is true:

$I'_{KDp}(a,b) - I'_{KDq}(a,b) =$

$\text{Max}[(1+a(b-1)-b)(2^q-2^p)/2^{p+q}, a\,b\,(2^q-2^p)/2^{p+q}] \leq 0$

* $\lim_{p\to 1} I'_{KDp}=\text{Max}[\frac{1+a(b-1)+(2^1-1)b}{2^1}, (1-a)+\frac{ab}{2^1}] = I_X$

* $\lim_{p\to\infty} I'_{KDp} = I_{KD}$, since

$\frac{1+a(b-1)+(2^p-1)b}{2^p} =$

$(1+a(b-1))/2^p + b(2^p/2^p) - b/2^p = b$ (when $p\to\infty$) and,

$(1-a)+\frac{ab}{2^p} = (1-a)$ (when $p\to\infty$)

2.1.2 Between I_X and I_{KDL}: One can generate at least two families (sequences) of fuzzy implication operators.

$$I_{Xp} = \text{Max}\,[\frac{(2^p-1)[1+a(b-1)]+b}{2^p}, (1-a)+\frac{(2^p-1)ab}{2^p}]$$

$$I'_{KDLp} = \text{Max}\,[\frac{(2^{p-1}+1)[1+a(b-1)]+(2^{p-1}-1)b}{2^p}, (1-a)+\frac{(2^{p-1}+1)ab}{2^p}]$$

Propositions

1) The sequence (I_{Xp}) is increasing.

$\forall p,q \in N^*, p>q \Rightarrow I_{Xp}(a,b) \geq I_{Xq}(a,b)$

$\lim_{p\to 1} I_{Xp} = I_X$ and $\lim_{p\to\infty} I_{Xp} = I_{KDL}$

2) The sequence (I'_{KDLp}) is decreasing.

$\forall p,q \in N^*, p>q \Rightarrow I'_{KDLp}(a,b) \leq I'_{KDLq}(a,b)$

$\lim_{p\to 1} I'_{KDLp} = I_{KDL}$ and $\lim_{p\to\infty} I'_{KDp} = I_X$

the proof is similar to **2.1.1**

2.2 Fuzzy Implications Laid Between I_{KDL} and I_L

2.2.1 Between I_{KDL} and I_Y: One can generate at least two families (sequences) of fuzzy implication operators :

$$I_{KDLp}=\min[1-\frac{(2^{p-1}+1)a(1-b)}{2^p}, (1-a)+\frac{b[(2^{p-1}-1)+(2^{p-1}+1)a]}{2^p}]$$

$$I_{Yp} = \min\,[1-\frac{(2^p-1)a(1-b)}{2^p}, (1-a)+\frac{b\,[1+(2^p-1)a]}{2^p}]$$

Propositions

1) The sequence (I_{KDLp}) is increasing.

$\forall p,q \in N, p>q \Rightarrow I_{KDLp}(a,b) \geq I_{KDLq}(a,b)$

$\lim_{p\to 1} I_{KDLp} = I_{KDLp}$ and $\lim_{p\to\infty} I_{KDLp} = I_Y$

2) The sequence (I_{Yp}) is decreasing.

$\forall p,q \in N, p>q \Rightarrow I_{Yp}(a,b) \leq I_{Yq}(a,b)$

$\lim_{p\to 1} I_{Yp} = I_Y$ and $\lim_{p\to\infty} I_{Yp} = I_{KDL}$

proof : the proof is similar to **2.1.1**

2.2.2 Between I_Y and I_L: One can generate at least two families (sequences) of fuzzy implication operators.

$$I_{Lp} = \min\,[1-\frac{a(1-b)}{2^p}, (1-a)+\frac{b\,[(2^p-1)+a]}{2^p}]$$

$$I'_{Lp} = \min[1-\frac{(2^{p-1}-1)a(1-b)}{2^p}, (1-a)\frac{b[(2^{p-1}+1)+(2^{p-1}-1)a]}{2^p}]$$

Propositions

1) The sequence (I_{Lp}) is increasing.

$\forall p,q \in N^*$, $p>q \Rightarrow I_{Lp}(a,b) \geq I_{Lq}(a,b)$

$\lim_{p\to 1} I_{Lp} = I_Y$ and $\lim_{p\to\infty} I_{Lp} = I_L$

2) The sequence (I'_{Lp}) is decreasing.

$\forall p,q \in N^*$, $p>q \Rightarrow I'_{Lp}(a,b) \leq I'_{Lq}(a,b)$

$\lim_{p\to 1} I'_{Lp} = I_L$ and $\lim_{p\to\infty} I'_{Lp} = I_Y$

the proof is similar to **2.1.1**

3. SOME PROPERTIES OF THESE FUZZY IMPLICATION FAMILIES

In this section, we give some properties of the previous families ; in Klir and Yuan[7], some of these properties are seen as axioms. For a deeper comprehension of their meaning, we refer the reader to Dubois and Prade [5] and Kerre [6].

3.1 Properties of I_{KDp}, I'_{KDp}, I_{Xp} and I'_{KDL} Families

For instance, we exclude I_{KD}

1) Boundary conditions
2) $I(0,b) = I(a,1) = 1$ $I(a,0) = 1\text{-}a$ $I(1,b) = b$
3) $I(a,b) = 0 \Leftrightarrow a=1$ AND $b=0$
4) $I(a,b) = 1 \Leftrightarrow a=0$ OR $b=1$
5) $a \leq b \not\Rightarrow I(a,b) = 1$
6) for $b\neq 1$, $I(.,b)$ is strictly decreasing (ie $a_1<a_2 \Rightarrow I(a_1,b) > I(a_2,b)$)
7) for $a\neq 0$, $I(a,.)$ is strictly increasing (ie $b_1<b_2 \Rightarrow I(a,b_1) < I(a,ba_2)$)
8) I is continuous
9) $I(a,a) > 1/2$
10) I is contrapositivly symmetric
11) $\forall x \in [0,1]$, $I(a, I(b,x)) = I(b, I(a,x))$ (Exchange property)
12) $I(a,b) \geq \min(a,b)$

Properties 6) and 7)are very interesting in the sense where:

$\forall\, b \neq 1, \forall a_1,a_2 \in [0,1], a_1\neq a_2 \Rightarrow I(a_1,b) \neq I(a_2,b)$

$\forall\, a \neq 0, \forall b_1,b_2 \in [0,1], b_1\neq b_2 \Rightarrow I(a,b_1) \neq I(a,b_2)$.

Later, one can see that both I_{KD} and I_L lack this property

3.1.1 Proof of 6). Let $a_1<a_2$, and let us consider the I'_{KDp} class :

$$I'_{KDp} = \mathrm{Max}\left[\frac{1+a(b-1)+(2^p-1)b}{2^p},\ (1\text{-}a)+\frac{ab}{2^p}\right]$$

$I(a_1,b)\text{-}I(a_2,b) =$

$$\mathrm{Max}\left[\frac{1+a_1(b-1)+(2^p-1)b}{2^p},\ (1-a_1)+\frac{a_1 b}{2^p}\right] -$$

$$\mathrm{Max}\left[\frac{1+a_2(b-1)+(2^p-1)b}{2^p},\ (1-a_2)+\frac{a_2 b}{2^p}\right]$$

Three different cases emerge for computing this difference.

a) <u>case1</u> : $a_1>1\text{-}b$ and $a_2>1\text{-}b$

$$I(a_1,b)\text{-}I(a_2,b) = [(1+a_1(b\text{-}1)+(2^p\text{-}1)b) - (1+a_2(b\text{-}1)+(2^p\text{-}1)b)] / 2^p$$
$$= (b\text{-}1)(a_1 - a_2)/2^p > 0 \qquad (1)$$

b) <u>case2</u> : $a_1<1\text{-}b$ and $a_2<1\text{-}b$

$$I(a_1,b)\text{-}I(a_2,b) = ((1\text{-}a_1) + a_1 b/2^p) - ((1\text{-}a_2)+a_2 b/2^p)$$
$$= (a_2 - a_1)\ (1 - b/2^p) > 0$$

c) <u>case3</u> : $a_1<1\text{-}b$ and $a_2>1\text{-}b$

Let $\alpha = (a_2 - a_1) > 0$

$$I(a_1,b)\text{-}I(a_2,b) = [2^p(1\text{-}a_1\text{-}b) + (a_2 + b\text{-}1) \text{-} b(a_2 - a_1)]/2^p$$
$$= [2^p(1\text{-}b) - (1\text{-}b) - 2^p a_1 + (a_1 - a_1) + a_2 - b(a_2 - a_1)]/2^p$$
$$= [(2^p\text{-}1)(1\text{-}b) - a_1(2^p\text{-}1) + (a_2 - a_1) - b(a_2 - a_1)]/2^p$$
$$= [(2^p\text{-}1)(1\text{-}b\text{-} a_1) + (a_2 - a_1)(1\text{-} b)]/2^p$$

now, since $(a_2 - a_1) > 0$ and $(1\text{-}b\text{-}a_1) > 0$, it follows that $I(a_1,b)\text{-}I(a_2,b) > 0$

3.1.2 Proof of 7). Let $b_1<b_2$, and let us consider once more the I'_{KDp} class :

$I(a,b_1)\text{-}I(a,b_2)=$

$$\mathrm{Max}\left[\frac{1+a(b_1-1)+(2^p-1)b_1}{2^p},\ (1-a)+\frac{ab_1}{2^p}\right] -$$

$$\mathrm{Max}\left[\frac{1+a(b_2-1)+(2^p-1)b_2}{2^p},\ (1-a)+\frac{ab_2}{2^p}\right]$$

Similarly to the precedent proof, three different cases emerge to compute this difference :

a) <u>case1</u> : $b_1>1\text{-}a$ and $b_2>1\text{-}a$

$$I(a,b_1)\text{-}I(a,b_2) = [(1+a(b_1\text{-}1)+(2^p\text{-}1)b_1) - (1+a(b_2\text{-}1)+(2^p\text{-}1)b_2)]/2^p$$
$$= (b_1 - b_2)(a+2^p\text{-}1)/2^p < 0$$

b) <u>case2</u> : $b_1<1\text{-}a$ and $b_2<1\text{-}a$

$$I(a,b_1)\text{-}I(a,b_2) = ((1\text{-}a) + ab_1/2^p) - ((1\text{-}a)+ab_2/2^p)$$
$$= (b_1 - b_2)\ (a/2^p) < 0 \qquad (2)$$

c) <u>case3</u> : $b_1<1\text{-}a$ and $b_2>1\text{-}a$

$$I(a,b_1)\text{-}I(a,b_2) = [(1\text{-}a) + ab_1/2^p] - [(1+a(b_2\text{-}1)+(2^p\text{-}1)b_2)/2^p]$$
$$= [2^p(1\text{-}a\text{-}b_2) - (a+b_2\text{-}1) + a(b_1\text{-}b_2)]/2^p$$
$$= [(1\text{-}a\text{-}b_2)(2^p\text{-}1) + a(b_1\text{-}b_2)]/2^p < 0$$

Important Remarks :

- In **3.1.1 case1**, result **(1)** shows that when p tends to infinite (I becomes I_{KD}) and,

$I_{KD}(a_1,b) = I_{KD}(a_2,b)$

example : $a_1=0.4$, $a_2=0.6$, $b=0.7$; we have $1-b < a_1 < a_2$

$0.4 \xrightarrow{KD} 0.7 = 0.6 \xrightarrow{KD} 0.7 = 0.7$

- In **3.1.2 case2**, result **(2)** shows that when p tends to infinite (I becomes I_{KD}) and,

$I_{KD}(a,b_1) = I_{KD}(a,b_2)$

example : $b_1 = 0.5$, $b_2 = 0.7$, $a = 0.3$; we have $b_1<b_2<1-a$

$0.3 \xrightarrow{KD} 0.5 = 0.3 \xrightarrow{KD} 0.7 = 0.7$

so it follows that the Kleene-Dienes fuzzy implication is not pseudo-strictly monotonic.

3.2 Properties of I_{KDLp}, I_{Yp}, I_{Lp} and I'_{Lp} Families.

For instance, we exclude I_L

1) Boundary conditions
2) $I(0,b) = I(a,1) = 1$ $I(a,0) = 1-a$ $I(1,b) = b$
3) $I(a,b) = 0 \Leftrightarrow a=1$ AND $b=0$
4) $I(a,b) = 1 \Leftrightarrow a=0$ OR $b=1$
5) $a \le b \not\Leftrightarrow I(a,b) = 1$
6) for $b\neq 1$, $I(.,b)$ is strictly decreasing (ie $a_1<a_2 \Rightarrow I(a_1,b) > I(a_2,b)$)
7) for $a\neq 0$, $I(a,.)$ is strictly increasing (ie $b_1<b_2 \Rightarrow I(a,b_1) < I(a,ba_2)$)
8) I is continuous
9) $I(a,a) \ge 3/4$
10) I is contrapositivly symmetric
11) $\forall x \in [0,1]$, $I(a,I(b,x))=I(b, I(a,x))$ (Exchange property)
12) $I(a,b) \ge \min(a,b)$

As stated in section **3.1**, properties 6) and 7) assure the surjectivity, that is

$\forall\, b \neq 1, \forall a_1,a_2 \in [0,1], a_1\neq a_2 \Rightarrow I(a_1,b) \neq I(a_2,b)$

$\forall\, a \neq 0, \forall b_1,b_2 \in [0,1], b_1\neq b_2 \Rightarrow I(a,b_1) \neq I(a,b_2)$.

The proofs of properties 6) and 7) are almost similar to the previous ones.

The same remarks as those in section **3.1.2** can be stated for these families. Indeed, all the fuzzy implications of these families are pseudo strictly monotonic except the supremum one, namely I_L.

$I_L(a_1,b) = I_L(a_2,b)$

example : $a_1=0.4$, $a_2=0.6$, $b=0.7$; we have $a_1<a_2<b$

$0.4 \xrightarrow{L} 0.7 = 0.6 \xrightarrow{L} 0.7 = 1$

$I_L(a,b_1) = I_L(a,b_2)$

example : $b_1 = 0.5$, $b_2 = 0.7$ and $a = 0.3$; one has $a<b_1<b_2$

$0.3 \xrightarrow{L} 0.5 = 0.3 \xrightarrow{L} 0.7 = 1$

4. THE WEAKNESS OF AN INSTANCE OF THE K SUB-TRIANGLE INFERENCE STRUCTURE

Fuzzy relational products, tools for analysing the behaviour of complex artificial and natural systems were first introduced by Bandler and Kohout ([1],[2]) and later revised and improved by De Baets and Kerre [4].

Let us use the definition of the K Sub-Triangle inference structure (one kind of fuzzy relational products) defined by De Baets and Kerre as an improvement of Bandler and Kohout's definition.

$$(R \lhd_k S)_{ik} = \min\left(\inf_j (R_{ij}\to S_{jk}) , \sup_j \Im(R_{ij},S_{jk})\right)$$

where $\Im$ is a t-norm. One of the abstract inference template can have the following form (Yew [8]).

$(R \lhd_k S)_{ik}=\min(\text{AndTop}(R_{ij}\to S_{jk}),\text{OrBot}(\text{AndBot}(R_{ij},S_{jk}))$

where, $\text{AndBot}(a,b) = \max(a+b-1,0)$,
$\text{AndTop}(a,b) = \min(a,b)$ and $\text{OrBot}(a,b) = \max(a,b)$

The two extreme bounds of the interval result are obtained by instanciating the above inference with the two fuzzy implications I_L and I_{KL}. The I_L implication derives the upper bound, whereas I_{KL} derives the lower one.

let us examine the case where $R_{ij}\le S_{jk}$ and $R_{ij}+S_{jk} \le 1$.

From a formal standpoint, it can be easily seen that

$\text{AndBot}(R_{ij},S_{jk}) = 0$, $I_L(R_{ij},S_{jk}) = 1$, and

$I_{KD}(R_{ij},S_{jk}) = \max(S_{jk}, 1-R_{ij}) = 1-R_{ij}$

$\Rightarrow (R \lhd_k S)_{ik}=\min[\min(R_{ij}\to S_{jk})],\max(\text{AndBot}(R_{ij},S_{jk}))]= 0$

$\Rightarrow$ Lower bound = Upper bound = 0

4.1 Illustrating the abnormality

Practically speaking, the above can be elucidated by the following example (Medical Diagnosis):

Let P_S the fuzzy relation between *Patients* and *Signs/Symptoms*, denoting to which degree a patient P_i is showing a sign/symptom S_j ; and,

S_I the fuzzy relation between *Signs/Symptoms* and *Illness*, denoting to which degree a sign/symptom S_j is characterising the illness I.

	S1	S2	S3	S4
P1	.4	.3	.5	.4
P2	.4	.2	.4	.4
P3	.3	.3	.4	.3

P_S: P to S relation

	I
S1	.6
S2	.7
S3	.5
S4	.6

S_I: S to I relation

Provided that $R_{ij}+S_{jk} \leq 1$, any k sub-relational product namely $(R \triangleright_k S)_{ik} = \min(\inf_j (R_{ij} \to S_{jk}), \sup_j \Im(R_{ij}, S_{jk}))$ taking the t-norm $\Im$ as AndBot will lead to zero (0) as a result.

Comment: It means that whenever the sum of the sign/symptom shown by the patient with the same sign/symptom in the knowledge base (denoting its degree of membership in the Illness) is less or equal to unity, the diagnosis result is crisply zero!

	P_1I	P_2I	P_3I
Upper Bound	**0**	**0**	**0**
Lower Bound	**0**	**0**	**0**

The inferred result: P to I Relation

The non pseudo-strict monotonocity of AndBot connective , is the reason of such result.

Even more, in the case where $\min[(R_{ij} \to S_{jk})]$ is less than $\max(\text{AndBot}(R_{ij}, S_{jk}))$, this inference template can infer identical results (intervals) for totally different (see ordered) n-uplet of data.

5. CONCLUDING REMARKS AND FURTHER WORK

In this paper, we presented some families of fuzzy implications that are laid between the Kleene-Dienes implication operator "I_{KD}", and the Lukasiewicz implication operator "I_L" which are respectively the attainable lower and upper bounds of measure m1 introduced by Bandler and Kohout ([1],[2],[3]). We showed also that they (I_{KD} and I_L) are particular cases of some of the generated families.

Excluding I(0,.) and I(.,1), one can prove that the strict monotonicity hold for all the operators of the previous families except for I_{KD} and I_L. The lack of this property may lead to utterly wrong inference in very special cases.(as shown in section 4)

Our present work is directed towards defining other instances of inference structures that may behave normally in such special (critical) cases.

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