Analyzing Location-Based Advertising for Vehicle Service Providers Using Effective Resistances

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ABSTRACT

Vehicle service providers can display commercial ads in their vehicles based on passengers' origins and destinations to create a new revenue stream. We study a vehicle service provider who can generate different ad revenues when displaying ads on different arcs (i.e., origin-destination pairs). The provider needs to ensure the vehicle flow balance at each location, which makes it challenging to analyze the provider's vehicle assignment and pricing decisions for different arcs. To tackle the problem, we show that certain properties of the traffic network can be captured by a corresponding electrical network. When the *effective resistance* between two locations is small, there are many paths between the two locations and the provider can easily route vehicles between them. We derive the provider's optimal vehicle assignment and pricing decisions based on effective resistances.

CCS CONCEPTS

• **Networks** → **Network economics**; *Location based services*; • **Information systems** → *Display advertising*; • **Social and professional topics** → *Pricing and resource allocation.*

KEYWORDS

In-vehicle advertising; spatial pricing; effective resistances

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1 INTRODUCTION

In-vehicle advertising has been emerging as a promising approach for vehicle service providers (e.g., taxi companies and ride-sharing platforms) to monetize their service. Vehicle service providers can display ads to users of their service, and receive payment from the corresponding advertisers. In this work, we study the impact of in-vehicle advertising on a vehicle service provider who displays banner ads or in-stream ads to its users regularly. We use "an arc" to represent an origin-destination pair, and a "unit ad revenue" to represent the revenue that the provider can gain by displaying ads to a user for a time slot. The advertisers have different target arcs to display their ads, and have different willingnesses to pay. Hence, the provider's unit ad revenues on different arcs can be different.

The provider can set different prices for the vehicle service on different arcs [1, 3]. Given the unit ad revenues, we aim to analyze the provider's optimal service prices for different arcs. The analysis is challenging, because the provider's vehicle assignment and pricing decisions for different arcs (i.e., origin-destination pairs) are tightly coupled by the *vehicle flow balance constraints*. The flow balance constraints do not exist in other advertising models (e.g., web advertising [2] and mobile advertising [5]), and constitute a unique challenge to the study of in-vehicle advertising.

We show that the traffic network, which is determined by the network topology and the traffic demand and travel time on different arcs, can be interpreted by an associated electrical network. Each location corresponds to a node in the electrical network. If there is a positive traffic demand between two locations, there is a resistor between the two corresponding nodes, and its resistance is determined by the traffic demand and travel time between the two locations. Given the associated electrical network, we compute the *effective resistance* between any two nodes. Based on the effective resistances, we derive closed-form expressions for the provider's optimal prices.

2 MODEL

We consider a discrete-time model, and denote the set of locations by $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$. In each time slot, a continuum of users of mass $\theta_{ij} \geq 0$ consider taking the provider's vehicle service to *depart* from location i to location j ($i, j \in \mathcal{N}$). In particular, we have $\theta_{ii} = 0$ for all $i \in \mathcal{N}$. Note that $\left\{\theta_{ij}\right\}_{i,j\in\mathcal{N}}$ induces a directed graph $\mathcal{G}_{\rm d} = (\mathcal{N}, \mathcal{A})$, where there is an arc from i to j (i.e., $(i, j) \in \mathcal{A}$) if and only if $\theta_{ij} > 0$. We focus on the case where $\mathcal{G}_{\rm d}$ is weakly connected. We use $\left\{\xi_{ij}\right\}_{i,j\in\mathcal{N}}$ to denote the travel time between different locations. If a user takes the vehicle service, the number of time slots required for the travel from i to j is $\xi_{ij} > 0$ ($j \neq i$).

We use a *reservation price* to refer to the highest price that a user is willing to pay. If the vehicle service's price is no greater than the reservation price, the user is willing to take the provider's vehicle service; otherwise, the user will travel to its destination by other approaches. We assume that the reservation prices of all the θ_{ij} users on arc $(i, j) \in \mathcal{A}$ are uniformly distributed in $[0, 1]$. Therefore, if the vehicle service's price is p, the mass of users that want to take the service from i to j is $\theta_{ij} \max\{1 - p, 0\}$. We call $\theta_{ij} \max\{1 - p, 0\}$ the *actual demand* on arc (i, j).

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The provider's decisions include the vehicle assignment and pricing. We use q_{ij} to denote the mass of vehicles that depart from i to j in each time slot. We use p_{ij} to denote the vehicle service's price for arc (i, j), measured in dollars per time slot. If a user travels from i to j via the vehicle service, since the travel time is ξ_{ij}, the user's total payment is $\xi_{ij} p_{ij}$.

Besides the users' payment, the provider gets revenue by displaying ads to the users. Different origin-destination pairs can correspond to different values of ad revenue. Let $a_{ij} \geq 0$ denote the provider's *unit ad revenue* (i.e., the ad revenue per user per time slot) on arc (i, j). If a user travels from i to j via the vehicle service, the provider can get $\xi_{ij} a_{ij}$ by displaying ads to the user for ξ_{ij} time slots. We use c to denote the provider's *unit cost*, and the provider's total cost for transporting a user from i to j is $\xi_{ij} c$. In this work, we focus on the case where $c < 1$, i.e., the unit cost is smaller than the users' maximum reservation price.

The provider decides the vehicle assignment and pricing to maximize its time-average payoff in the system's steady state. We assume that the provider's supply of vehicles on each arc is no greater than the actual demand, i.e., $q_{ij} \leq \theta_{ij} \max\left\{1 - p_{ij}, 0\right\}$ for all $(i, j) \in \mathcal{A}$. Then, we can formulate the provider's problem as follows:

$$\max \sum_{(i,j)\in\mathcal{A}} q_{ij}\xi_{ij}\left(p_{ij} + a_{ij} - c\right) \tag{1a}$$

$$\text{s.t.} \quad q_{ij} \leq \theta_{ij}\max\left\{1 - p_{ij}, 0\right\}, \forall (i, j) \in \mathcal{A}, \tag{1b}$$

$$\sum_{j:(i,j)\in\mathcal{A}} q_{ij} = \sum_{j:(j,i)\in\mathcal{A}} q_{ji}, \forall i \in \mathcal{N}, \tag{1c}$$

$$\text{var.} \quad q_{ij} \geq 0, p_{ij} \leq 1, \forall (i, j) \in \mathcal{A}. \tag{1d}$$

The objective function in (1a) captures the provider's time-average payoff. In each time slot, a continuum of vehicles of mass q_{ij} *depart* from i to j, i.e., q_{ij} is the vehicle departure rate for the travel from i to j. In any time slot, the mass of vehicles traveling on arc (i, j) is $q_{ij}\xi_{ij}$. Each vehicle carries a user, and the provider gets $p_{ij} + a_{ij} - c$ by serving the user *in this time slot*. Therefore, the provider's time-average payoff from arc (i, j) is $q_{ij}\xi_{ij}\left(p_{ij} + a_{ij} - c\right)$.

Constraint (1b) states that the supply of vehicles is no greater than the actual demand, which is the assumption made above. Constraint (1c) captures the vehicle flow balance at each location i. For arc (i, j), both the rate that the vehicles depart i and the rate that the vehicles arrive at j are q_{ij}. Considering all arcs, the vehicles' departure rate at i is $\sum_{j:(i,j)\in\mathcal{A}} q_{ij}$, and the arrival rate at i is $\sum_{j:(j,i)\in\mathcal{A}} q_{ji}$. The two rates are equal under constraint (1c).

3 RESULTS

We first construct an electrical network corresponding to the directed graph $\mathcal{G}_{\mathrm{d}}$, and then analyze the provider's optimal decisions based on properties of the electrical network.

Given the directed graph $\mathcal{G}_{\mathrm{d}} = (\mathcal{N}, \mathcal{A})$, we define a weighted undirected graph $\mathcal{G}_{\mathrm{u}} = (\mathcal{N}, \mathcal{E})$. Specifically, $\mathcal{E}$ contains edge (i, j) if and only if there exists at least one arc between i and j in the directed graph $\mathcal{G}_{\mathrm{d}}$. Moreover, edge (i, j) is associated with a weight, which is $\frac{\theta_{ij}}{\xi_{ij}} + \frac{\theta_{ji}}{\xi_{ji}}$. Then, we can construct an electrical network via replacing the vertices in $\mathcal{N}$ by nodes and the edges in $\mathcal{E}$ by resistors. If $(i, j) \in \mathcal{E}$, we use r_{ij} to denote the resistance of the resistor between nodes i and j, and let $r_{ij} = \frac{1}{\frac{\theta_{ij}}{\xi_{ij}} + \frac{\theta_{ji}}{\xi_{ji}}}$.

In the electrical network, the *effective resistance* between any two nodes i and j $(i, j \in \mathcal{N})$ is defined as the voltage between i and j when a unit current is injected at i and withdrawn at j. Let R_{ij} denote the effective resistance between i and j. We can see that $R_{ii} = 0$ and $R_{ij} = R_{ji}$ for all $i, j \in \mathcal{N}$.

Next, we analyze the provider's decisions based on the effective resistances. Let μ_{ij} be the dual variable corresponding to $p_{ij} \leq 1$ in (1d). We use μ^*_{ij} to denote the optimal dual variable, and can derive the following result.

THEOREM 3.1. *In the case where $\mu^*_{ij} = 0$ for all $(i, j) \in \mathcal{A}$, the provider's optimal price p^*_{ij} is given by*

$$p^*_{ij} = \frac{1 - a_{ij} + c}{2} + \frac{1}{4\xi_{ij}} \sum_{k\in\mathcal{N}} \left(R_{jk} - R_{ik}\right) v_k, \forall (i, j) \in \mathcal{A}, \tag{2}$$

where v_k is defined by

$$v_k \triangleq \sum_{m:(k,m)\in\mathcal{A}} \theta_{km}\left(1 + a_{km} - c\right) - \sum_{m:(m,k)\in\mathcal{A}} \theta_{mk}\left(1 + a_{mk} - c\right).$$

Moreover, the optimal vehicle assignment q^*_{ij} equals $\theta_{ij}\left(1 - p^*_{ij}\right)$ for all $(i, j) \in \mathcal{A}$.

Here, v_k measures the value of having available vehicles at location k. When v_k is large, the traffic demand and unit ad revenues on the arcs originating from k are large, and the provider needs to route more vehicles to k to achieve a high payoff.

Theorem 3.1 characterizes the optimal prices based on the effective resistances, which measure the "distances" between nodes in the electrical network. Suppose that, compared with location i, location j is "farther" from most of the locations with large v_k in the electrical network. In this case, $\sum_{k\in\mathcal{N}}\left(R_{jk} - R_{ik}\right) v_k$ is large. Eq. (2) implies that the provider should charge a high price to reduce the actual demand on arc (i, j). This reduces the mass of vehicles traveling from i to j, which is farther from most "valuable" locations.

In the full version of our work, we generalize Theorem 3.1 to the case where $\mu^*_{ij} \geq 0$ for all $(i, j) \in \mathcal{A}$ [4]. We also include the following results in [4]. First, we characterize the dependence of an arc's optimal price on any other arc's unit ad revenue. Second, we analyze the impact of ad revenues on the provider's optimal payoff and consumer surplus. Third, we study the provider's optimal selection of advertisers when it can only display ads for a limited number of advertisers. Fourth, we relax some assumptions and evaluate our solution's performance based on a real-world dataset.

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