TriPoll: Computing Surveys of Triangles in Massive-Scale Temporal Graphs with Metadata

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ABSTRACT

Understanding the higher-order interactions within network data is a key objective of network science. Surveys of metadata triangles (or patterned 3-cycles in metadata-enriched graphs) are often of interest in this pursuit. In this work, we develop `TriPoll`, a prototype distributed HPC system capable of surveying triangles in massive graphs containing metadata on their edges and vertices. We contrast our approach with much of the prior effort on triangle analysis, which often focuses on simple triangle counting, usually in simple graphs with no metadata. We assess the scalability of `TriPoll` when surveying triangles involving metadata on real and synthetic graphs with up to hundreds of billions of edges. We utilize communication-reducing optimizations to demonstrate a triangle counting task on a 224 billion edge web graph in approximately half of the time of competing approaches, while additionally supporting metadata-aware capabilities.

KEYWORDS

distributed graph processing, asynchronous communication

1 INTRODUCTION

Network scientists seek meaningful higher-order structure within their relational datasets. Real-world relational datasets tend to have graph topology (the relational data) with structured or unstructured metadata living on vertices and/or edges. The discovery of how topology, timing, and other non-relational data combine to form meaningful higher-order structure facilitates exploratory data analysis of network datasets, network-based machine learning capabilities, and foundational understanding of network function. Such pattern discovery is a computationally challenging task; massive relational datasets often require distributed storage and computing for scalable pattern analysis strategies.

A common case we focus on here is where records contain a relationship between two vertices, a time of observation (or timestamp), and structured metadata on the vertices and edges. This vertex and edge metadata can take the form of discrete labels and/or text fields.

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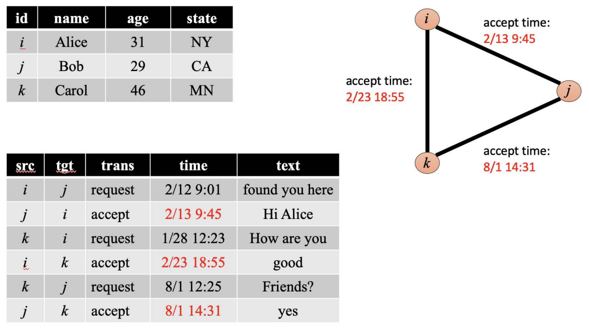


id	name	age	state
i	Alice	31	NY
j	Bob	29	CA
k	Carol	46	MN

src	tgt	trans	time	text
i	*j*	request	2/12 9:01	found you here
j	*i*	accept	2/13 9:45	Hi Alice
k	*i*	request	1/28 12:23	How are you
i	*k*	accept	2/23 18:55	good
k	*j*	request	8/1 12:25	Friends?
j	*k*	accept	8/1 14:31	yes

Figure 1: Scenarios in a social network where analysis seeks to understand higher-order structures. This relational dataset is organized into (Top-left) a vertex table, listing users and their attributes, and (Bottom-left) an edge table, listing the times users request to connect and when connections are accepted. (Right) Studying presence or absence of various higher-order structures (e.g., triangles with metadata) may aide in enhancing several network analysis tasks like link prediction (*Who is likely connected in real life but not in our dataset?*) and anomaly detection (*Which accounts are fake?*).

Consider, for example, a large online marketplace listing interactions (edges) between users (vertices). Vertex fields could include a user rating such as a floating point number, a discrete label such as buyer, seller, or both, and a short string username. Meanwhile, edge fields could include a discrete type label such as message, purchase, or rating, a floating point timestamp, a floating point rating, and a possibly long message string. In this work we will refer to such a graph as a *decorated temporal graph*. See the example in Fig. 1.

Triangles (graph 3-cycles) are fundamental patterns that are commonly difficult to enumerate in massive distributed real-world networks [49, 50]. These difficulties arise because networks associated with data analysis have scale-free topology (heavy-tail degree distribution, small effective diameter, and the expander graph property [29]) complicating many aspects of serial and distributed computation. Often, techniques that approximate triangle counts suffice for an application, and processing every triangle is not necessary [6]. However, when an analysis is performed involving *metadata triangles* (3-cycles involving specific categories involving labels, topics, or timings) and there are a wide variety of metadata triangles, processing every single triangle may be required. Additionally, processing every triangle is of interest when metadata triangle incidence upon vertices, edges, or subgraphs inform feature vectors in downstream machine learning models.

In this work we design `TriPoll`, a distributed, asynchronous graph framework that affords the exploration of the types of triangles that exist within massive datasets. Our prototype system allows users to answer hypotheses regarding the relevance of metadata triangles on their datasets and leverage their discoveries for automated analyses (see scenarios in Fig 1).

Most previous works use *triangle counting* as the final goal, as the number of triangles can be much larger than the edge set of a massive graph. This is often global counting (returning a single integer for the number of triangles in the entire graph) or local counting of triangle participation at vertices and/or edges. Several applications make use of the latter, including performing truss decomposition [15], enhancing community detection [11], bounding the sizes of cliques [60], computing clustering coefficients [7], and using the local triangle counts in downstream machine learning including vertex role analysis [26]. In some cases, the goal is to find triangles of a particular type within labeled or temporal graphs. Work exists leveraging local vertex/edge participation in various metadata triangle types for more customizable community detection in labeled graphs [10], as well as dynamic graphs [40]. Moreover, [45] uses local edge participation in various metadata triangle types for performance gains in interactive labeled pattern matching. As many relational datasets are highly decorated with many fields at every vertex and edge, these works are just scratching the surface of what network scientists could ask of their datasets regarding metadata triangles and how the metadata triangles can be leveraged for various discovery tasks.

This provides us with one unique capability of `TriPoll`: our users provide a *callback function* to perform on the metadata associated to the respective vertices and edges as each triangle is found. This allows the same code to be used for counting of simple or metadata triangles in a decorated temporal graph, full triangle enumeration if desired, or more interesting data collection that is more in line with the queries a network scientist wants answered during data exploration.

It is worth noting `TriPoll` identifies all triangles in a graph and applies the user's callback to the metadata of each one as they are discovered. The end goal is not to search for particular patterns of vertex and edge metadata within a graph, but instead to allow for custom surveys of triangles in a graph. A pattern matching approach would answer the question "Where can I find all triangles involving 1 red vertex and 2 blue vertices?", whereas we seek answers to questions of the form "Of the triangles with 1 red vertex and 1 blue vertex, what is the distribution of colors of the third vertex?"

Additionally, `TriPoll` has no output in the traditional sense. We rely on the user-defined callbacks to have the effects desired for output, whether that is incrementing counters to be read later, writing information on individual triangles out to file, or preparing data for use in a downwind application.

Our work builds on `YGM` [2], our recently released open source asynchronous communication library, to allow for scalable performance. `YGM` leverages buffering techniques and message serialization to allow truly asynchronous computations where messages of heterogeneous types can be exchanged simultaneously. Using `YGM`, we are able to build vertex-centric graph data structures that are able to handle metadata and can easily express the communication necessary for identifying triangles in a graph. `TriPoll` provides an example application for performance and usability considerations as we further develop `YGM`.

The research contributions made in this work are:

- We introduce `TriPoll`, a scalable `C++` framework for triangle processing on graphs with metadata that, through user-defined callback functions for executing on the edge and vertex metadata of each triangle identified, allows the computation of customizable metadata triangle surveys on massive graphs.
- We showcase new capabilities in `YGM`, our asynchronous communication library[1], that builds on message buffering techniques seen in previous libraries [34, 44] and adds a serialization layer to allow messages of heterogeneous and complicated types, e.g., `C++` strings and STL containers, to be exchanged simultaneously.
- We implement a novel *Push-Pull* optimization method allowing choice of direction for sending adjacency information during triangle identification to reduce communication.
- We evaluate performance on real-world and synthetic graphs with up to 224 billion edges. As a subset of the functionality offered by `TriPoll`, we are able to perform simple triangle counting to compare against previous work tailored to triangle counting. We find `TriPoll` gives comparable or better performance than these works, including counting triangles in a 224 billion edge web graph on 256 compute nodes in just over half of the time required by the only openly available code able to solve this problem.
- We use `TriPoll` to survey the distribution of triangle closing times seen in a temporal graph derived from Reddit including 9.4 billion edges with timestamps.

2 RELATION TO EXTERNAL WORK

Triangle counting is a widely studied graph algorithm. With the rapid growth of the scale of real-world datasets, the demand for scalable, high-performance solutions has been growing as well. From the earlier parallel node-iterator [51] family of algorithms, in recent years, a number of solutions have been developed targeting multi-core CPUs [63], GPUs [59], (heterogeneous) distributed platforms [42], as well as FPGAs [31]. Here, we focus on related work that targets distributed platforms.

Vertex-Centric Frameworks. Popular vertex-centric graph frameworks, often optimized for low-complexity traversal algorithms, have been shown to support triangle counting: GraphLab [22], Galois-Gluon [17], HavoqGT [43], GraphMat [54], Giraph [21], GraphX [23], Oracle PGX.D [28] and Chaos [48] (the core processing engine is edge-centric) are some of the most well-known systems. Since these frameworks are designed to support a wide variety of graph problems, while they can scale to large datasets, typically they are unable to offer algorithm-specific optimizations.

General Pattern Matching Tools. Since triangle counting is also a primitive subgraph matching problem, distributed subgraph graph mining solutions such as Arabesque [56], QFrag [52], Fractal [19], PruneJuice [46], TriAD [25], G-Miner [14] and GraphFrames [18], and graph databases such as Neo4j [36], OrientDB [37]

[1]available at https://github.com/LLNL/ygm

and TigerGraph [57] seamlessly support triangle matching queries; however, they are aimed at more general pattern matching and are unable to match the scale and performance offered by tailored triangle counting systems.

Tailored Triangle Counting. Depending on the target architecture (i.e., CPU, GPU or heterogeneous), distributed triangle counting solutions embrace different graph partitioning [58] and load balancing [39] techniques (distributed [20] vs fully/partially replicated [39]), preprocessing and/or pruning strategy (e.g., degree-based sorting, wedge identification [42]), computation model (i.e., vertex/edge-centric [27] or linear algebra [5]). In the most expensive operation in a triangle counting kernel, i.e., computing the intersection of adjacency lists, three fundamental approaches are commonly used, namely, binary search, merge-path, and hashing (of a vertex's neighborhood) [39].

Early examples of distributed triangle counting were MapReduce implementations of node-iterator algorithms [16, 55]. These use a degree-ordered, 2D graph partitioning technique. The works [41, 42] embrace asynchronous MPI communication; a preprocessing step iteratively prunes degree-one vertices, orders vertices by degree, followed by querying wedges for closure to detect presence of triangles. This work showed scalability using a 224 billion edge real-world webgraph on ~9K cores, one of the largest scale to date. The work [58] utilizes a 2D cyclic decomposition of the data graph; its processing resembles Cannon's parallel matrix-matrix multiplication algorithm and uses hashing for adjacency list intersection. Both [7] and [32] use 1D graph partitioning, and [7] demonstrated that partially replicating partitions offers communication efficiency and subsequently showed application of their technique to clustering coefficient computation. [32] combined shared memory and distributed memory optimization techniques to offer improved distributed memory performance; similar to [42], this solution operates on precomputed wedges; and employs a message aggregation technique to avoid latency-prone short messages. TriC [20] is a distributed memory solution that exploits the knowledge about the graph structure to improve inter-node communication; it creates edge-balanced partitions, performs parallel edge enumeration to identify triangles; accompanied by a batch-oriented scalable communication substrate. The authors demonstrated scalability using up to 8K cores on the Cori Supercomputer on up to 3.6B edge real-world graphs. [5] presents a linear algebra-based solution which utilizes 2D partitioning at the cluster level, and 1D partitioning at the node level, and achieves high throughput using MPI and Cilk for parallelism. Scalability was demonstrated using up to 1K Cores on a 3.6B edge graph. Another linear algebra-based solution is presented in [61], which offers heterogeneous (CPU-GPU) processing on multi-GPU platforms like the Nvidia DGX.

TriCore [30] targets GPU clusters and exploits streaming partitions to accommodate large graphs. TriCore follows an edge-centric computation model and uses binary search for adjacency list intersection (which can improve memory coalescing on GPUs). DistTC's [27] design is comparable to that of TriCore, however, it uses static graph partitions consisting of vertex replicas, which drastically reduces the volume of inter-node communication. Using up to 32 GPUs, authors showed that DistTC comfortably outperforms TriCore. H-Index [38] offers innovations for hashing-based adjacency list intersection on GPUs; the technique enables search space pruning leading to improved throughput. TRUST [39], which targets distributed GPUs, also uses a hashing-based solution for computing adjacency list intersection. Unlike TriCore and DistTC, TRUST embraces vertex-centric processing on GPUs. It follows a hashing-based 2D partitioning (also performs full/partial graph replication) and offers load balancing through a collaborative workload partitioning technique. TRUST [39] has demonstrated the largest scale among distributed GPU-based solutions: up to 1,000 GPUs on the Summit Supercomputer (using up to 42B edge real-world graphs).

3 PRELIMINARIES

Throughout this work, we will use $\mathcal{G}(\mathcal{V}, \mathcal{E})$ to denote a graph with vertex set $\mathcal{V}$ and edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. Associated with any vertex $v \in V$ is metadata $meta(v)$ (containing vertex labels, strings, and/or floating point values), and similarly we have edge metadata $meta(u, v) = meta(v, u)$ for any $(u, v) \in \mathcal{E}$ (containing edge labels, timestamps, strings, and/or floating point values). We assume input graphs $\mathcal{G}$ are *undirected*, that is $(u, v) \in \mathcal{E} \Leftrightarrow (v, u) \in \mathcal{E}$. For a vertex $v \in \mathcal{V}$, we define its *adjacency list* $Adj(v) = \{(v, u) \in \mathcal{E}\}$. We then let $d(v) = |Adj(v)|$ be its *degree*.

A *triangle* (or 3-cycle) is a 3-way interaction between vertices $p, q, r \in \mathcal{V}$, defined by the existence of edges $(p, q), (p, r), (q, r) \in \mathcal{E}$ (and all reciprocal edges). This triangle will be represented by Δ_{pqr}. We let $\mathcal{T}(\mathcal{G})$ be the set of all triangles present in $\mathcal{G}$.

Vertex degree is a natural ordering for computational savings in triangle counting, as it tends to turn most or all of a high degree vertex's undirected edges into directed *in-edges* [16]. We will use $<_+$ as our comparison operator between vertices according to degree. That is, $u <_+ v$ if and only if

- $d(u) < d(v)$, or
- $d(u) = d(v)$ and $hash(u) < hash(v)$

where we have chosen a deterministic function *hash* to use in breaking ties.

We define an augmented graph known as the *degree-ordered directed graph* [16] *(DODGr)*, denoted as $\mathcal{G}_+$, to be the directed graph obtained by changing the pair of edges $(u, v), (v, u) \in \mathcal{E}$ into the single directed edge (u, v) such that $u <_+ v$.

With a fixed ordering $p <_+ q <_+ r$ on $\mathcal{G}_+$, Δ_{pqr} is uniquely identifiable, as none of the alternative vertex orderings occur. See the left part of Fig. 2. Throughout this work, we will canonically use p, q, and r as the vertices on individual triangles with $p <_+ q <_+ r$, and p is referred to as the *pivot* (or anchor) vertex.

For a vertex $v \in \mathcal{V}$, we use $Adj_+(v)$ as the adjacency list containing v's out-edges in $\mathcal{G}_+$, and we use $d_+(v) = |Adj_+(v)|$ as the out-degree of v.

A triangle Δ_{pqr} exists if and only if we are able to identify the directed edges $(p, q), (p, r)$, and (q, r) in the $\mathcal{G}_+$. As a first step, we can identify the pair of edges (p, q) and (p, r), and we must then check for the closing edge (q, r). This *wedge check* is the basic unit of work in triangle enumeration, and we let $|\mathcal{W}_+|$ be the number of such wedges in $\mathcal{G}_+$.

For a triangle Δ_{pqr}, we will use the shorthand $meta(\Delta_{pqr})$ to mean the six pieces of metadata associated to Δ_{pqr}'s vertices and edges.

Lastly, we use $Rank(u)$ to denote the MPI rank responsible for storing the associated adjacency lists $Adj(u)$ or $Adj_{+}(u)$ (depending on context) and metadata $meta(u)$ and $meta(u, v)$ for all $v \in Adj_{+}(u)$, and all computations corresponding to u.

4 DISTRIBUTED TRIANGLE PROCESSING

In this section, we will describe our algorithm for identifying triangles. During triangle identification, `TriPoll` must ensure the correct vertex and edge metadata is gathered on the correct process for use in callbacks on discovered triangles; this data cannot be simply read after a triangle is found as it is likely to be scattered across multiple processes.

`TriPoll` uses our recently-developed open source, asynchronous communication mechanism built on MPI [2] to handle sending messages between ranks. We use custom distributed data structures as building blocks for our graph storage. Our communication library is designed to handle irregular communication patterns. Triangle processing in graphs provides a communication-intensive problem for us to assess performance and usability when integrated into a full application.

In this section we will describe our communication schema, discuss our graph storage, and explain our distributed vertex-centric, merge-path based algorithm in detail, and give examples of analyses that can be performed through defining triangle callbacks.

As mentioned in Sec. 3, we consider all graphs to be undirected. Since our algorithms operate on $\mathcal{G}_{+}$, a directed version of the original graph, instead of $\mathcal{G}$ itself, this approach could be used for directed graphs as well. In the directed case, our augmented graph would be the original graph with many edges having their directionality reversed and any bidirectional edges having one direction removed. Additionally, each directed edge in the augmented graph may need an additional two bits of storage to give the original directionality (as-seen, reversed, or bidirectional) for use in the user callback if the directionality is important for an application.

4.1 Asynchronous Communication using `YGM`

In designing software to survey metadata triangles in massive graphs, two major challenges arise. The first is the presence of highly non-uniform communication patterns, which confounds the scalability of traditional HPC codes and also occurs in simple triangle counting workflows. The second is the handling of vertex and edge metadata while affording the user the flexibility to execute arbitrary callback functions when surveying triangles, which requires careful data management to maintain efficiency.

We meet these challenges with `YGM`, our asynchronous communication library built on top of MPI [2]. To achieve scalability, we leverage an asynchronous communication scheme and message buffering. To grant the expressiveness necessary for our triangle surveys, we use remote procedure call (RPC) semantics, a collection of pre-built storage containers to perform many of the basic tasks we frequently face, and serialize messages and function arguments before sending them through MPI to allow any serializable type to be easily sent to remote MPI ranks.

4.1.1 Message Buffering. Like many graph algorithms, naïve triangle enumeration in distributed memory generates a large number of small messages as compute nodes transmit small amounts of adjacency information. As each MPI data message creates additional overhead in the form of bits of header information and handshake protocol messages, such graph analysis workflows result in poor bandwidth utilization and slow wall time compared to more traditional MPI workflows.

Rather than relying upon application-specific message engineering in applications, `YGM` opaquely buffers small messages until the buffer reaches a size threshold or is directed to flush. By aggregating all of the small buffered messages destined for a next destination into a single large message, we can dramatically reduce the overhead of a naïve workflow without adding design complexity to the application. Although some individual messages may take slightly longer to reach their destinations, buffering strategies of this sort have been shown to improve overall latency in benchmark experiments [34, 44].

4.1.2 Serialization. MPI was designed for numerically-typed data in traditional HPC applications rooted in physical simulations. As the community continues to use MPI to solve problems in the data science realm, the limitations imposed by these design decisions become more obvious. Most notably, sending non-fixed-width data structures such as *hash tables*, user-defined *structs* or *objects*, or even *strings* is a challenging task.

We overcome this inconvenience by serializing structured message contents into variable-length byte arrays. `YGM`'s message buffering strategy concatenates and transmits these byte arrays as conventional MPI messages and destination nodes deserialize the arrays back into their original data structure forms. This pattern affords the transparent communication of structured data with no additional complexity imposed on the application, at the cost of a small amount of computing overhead in the serialization and deserialization steps.

Serializing messages in this manner provides two main benefits. First, we are able to send variable length objects, such as strings, without padding. We take advantage of this capability in the experiment of Sec. 5.8. Second, we are able to easily send messages with payloads of different types in arbitrary orders. This feature relies upon the `cereal` C++ serialization library in its implementation [1].

4.1.3 RPC Semantics. Messages in `YGM` have three basic components: a function to execute, arguments to pass to the function, and an MPI rank at which to evaluate the function. By wrapping the user-provided function in a `C++` *lambda* function and serializing the user-provided function arguments along with a pointer to the user function (offsets to `C++` lambdas are handled by senders and receivers to account for address space randomization), `YGM` can send a collection of serialized bytes through MPI, with the target rank unpacking and evaluating the user-provided function with its arguments appropriately upon receipt.

This method of serializing function arguments and wrapping functions mimics some functionality of a *remote procedure call* (RPC) system, allowing users to interleave several different functions to be executed asynchronously. One major deviation from traditional RPC programming is that responses are not sent upon completion of requests, giving `YGM` a *fire-and-forget* model. This paradigm lends itself naturally to many graph problems, where a vertex's MPI rank performs a small computation and sends a message to activate some number of its neighbors' ranks so they can perform a further calculation.

4.1.4 Containers. The interleaving of messages afforded by *fire-and-forget* RPC semantics makes it possible to build composable containers on top of YGM's communication framework to handle many simple tasks that arise frequently in distributed computing. One example that gets used frequently in this work is a distributed map. This structure stores *key-value* pairs at deterministic MPI ranks based on a hash of the keys.

One particular use for this distributed map is in building a distributed *counting set* that keeps individual counts of different items seen across ranks. This structure stores a small cache on each rank to keep values seen recently, which must be flushed and have its contents sent across the network occasionally.

This *counting set* is not necessary if merely doing simple triangle counting. However, for complicated surveys with multiple types of metadata triangles, it is highly useful and we utilize it for the experiments in Sec. 5. Through the interleaving of messages calling different functions, the counting sets are free to increment counters on remote MPI ranks when their caches are flushed without ever interfering with the messages used to identify triangles. This composability with pre-built distributed containers allows our system to answer nontrivial analysis questions involving topology and metadata jointly.

4.2 Graph Storage

For triangle identification, TriPoll stores the degree-ordered directed graph $\mathcal{G}_+$, as defined in Sec. 3, in a custom structure built on YGM's distributed map container, as described in Sec. 4.1.4.

For a vertex u, TriPoll stores a unique identifier associated to u as the key. Values are a pair containing $meta(u)$ and an adjacency list augmented to contain the necessary metadata, $Adj_+^m(u)$, where

$$Adj_+^m(u) = \{(v, meta(u,v), meta(v)) | v \in Adj_+(u)\}.$$

In $Adj_+^m(u)$, we consider the elements to be ordered by degree.

We use random or cyclic partitionings of vertices across MPI ranks and do not attempt to do more sophisticated partitionings in this work. In large scale-free graphs, high-degree vertices tend to cause the computation and storage imbalances that are one reason to partition a graph differently. In this case, we construct $\mathcal{G}_+$ to reduce the degree of these hub vertices. This alleviates the imbalances in triangle identification to a point that cyclic partitioning becomes palatable. Previous work has shown partitioning to be helpful, specifically if data replication is allowed [39, 64].

This DODGr structure is designed to be used in a vertex-centric manner. For the high-level algorithms we describe, the only operations we will need are a way to iterate over all vertices, and a $DODGr.visit(v, func, args)$ operation that sends an RPC call to $Rank(v)$ to execute the function $func(v, args)$, where passing v to $func$ is understood to give it access to v's metadata and adjacency information.

The decision to store target vertex metadata along edges changes the storage requirement of vertex metadata from $O(|\mathcal{V}|)$ to $O(|\mathcal{E}|)$. This additional memory overhead is required because a user-defined callback must have access to all metadata associated with p, q, r and

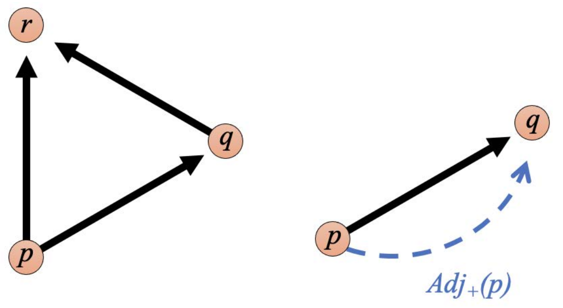


Figure 2: Roles of vertices $p <_+ q <_+ r$ in each triangle within the degree-ordered directed graph (DODGr) $\mathcal{G}_+$ (left) and a push message sent from i to j containing the neighborhood of i (right).

the 3 edges between them once Δ_{pqr} is identified. The ordering of $\mathcal{G}_+$ allows us to enumerate Δ_{pqr} without visiting r, the highest-degree vertex involved. Including $meta(r)$ in $Adj_+(q)$ maintains this communication advantage at the cost of some additional memory. It is also possible to aggregate all target metadata within an MPI rank to avoid storing redundant copies of $meta(r)$. This, however, does not avoid the worst case storage requirement of $O(|\mathcal{E}|)$ for vertex metadata.

4.3 *Push-Only* Algorithm

For each vertex p, the construction of $\mathcal{G}_+$ guarantees that $p <_+ q$ for all $q \in Adj_+(p)$. Vertex p can be a part of $O(d_+(p)^2)$ triangles, one for each pair of vertices in $Adj_+(p)$. For the purposes of triangle identification, we can imagine the process of spawning these $O(d_+(p)^2)$ wedge checks as iterating through $Adj_+(p)$ in increasing order, popping the vertex q currently at the front, and sending the remaining adjacency list $(Adj_+(p) \setminus \{v : v <_+ q\})$ to $Rank(q)$ as potential vertices (the 'r' vertices) to search for in the adjacency list $Adj_+(q)$. This is depicted on the right of Fig. 2.

Additionally, $Rank(p)$ must send $meta(p)$, $meta(p,q)$, and $meta(p,r)$ for every r to $Rank(q)$ to be used in a callback after a triangle Δ_{pqr} is found. Ultimately, the communication for triangle identification just described occurs using $Adj_+^m(p)$, rather than $Adj_+(p)$. It is worth noting our definition of $Adj_+^m(p)$ includes storing $meta(r)$ for all $r \in Adj_+(p)$, which is not needed by $Rank(q)$, as $Rank(q)$ will already have a copy of $meta(r)$ if Δ_{pqr} exists. In reality, this extra metadata is never actually transmitted, but for ease of discussion, we ignore the fact that $Rank(q)$ receives a subset of the metadata found in $Adj_+^m(p)$.

When $Rank(q)$ receives a collection of vertices to search for, we can consider these as a batch of wedge checks, reducing the number of checks that must be performed. We can identify the common elements between the sorted lists $Adj_+^m(q)$ and the subset of $Adj_+^m(p)$ sent to q by traversing each list simultaneously, a process known as a *merge-path* intersection [24].

As triangles are identified using this intersection technique, the callback provided by the user is executed using the metadata of all vertices and edges in each triangle. As a check, $Rank(p)$ sent $meta(p)$, $meta(p,q)$, and $meta(p,r)$, to $Rank(q)$, and $Rank(q)$ already had $meta(q)$ and $meta(q,r)$, as well as $meta(r)$ by our construction

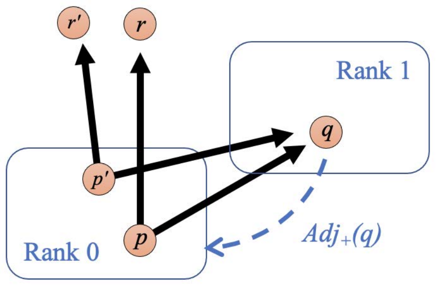


Figure 3: A situation depicting when the pull approach is useful. Assume p, p' are stored on $Rank(p)$ and $|Adj_+(p) \setminus \{v : v <_+ q\}| + |Adj_+(p') \setminus \{v : v <_+ q\}| >> |Adj_+(q)|$. Then it is potentially advantageous to send $Adj_+(q)$ to $Rank(p)$ so that communication is minimized.

of $Adj_+^m(q)$. Therefore, all of the necessary metadata is colocated at $Rank(q)$ at this moment in time, allowing the callback to execute correctly.

Pseudocode for this algorithm is given in Alg. 1. The triangle survey takes in arguments that are a graph, a callback operation defined by the user to be executed on the metadata of every triangle, and optional user-provided arguments that will be passed to the callback. These extra arguments often have their state modified by the user's callback to capture the results of the desired survey.

We refer to this simplistic vertex-centric merge-path based triangle identification algorithm as the *Push-Only* implementation of `TriPoll`, for reasons we will discuss in the next section.

Algorithm 1 TriPoll (Push-only)

```
1:  procedure TRIANGLE_SURVEY($\mathcal{G}$, $user\_callback$, $user\_args$)
2:      $\mathcal{G}_+ \leftarrow DODGr(\mathcal{G})$                ▷ described in Sec. 3
3:      for all $p \in \mathcal{G}_+$ do
4:          for all $q \in Adj_+(p)$ do
5:              $\mathcal{G}_+.visit(q, \lambda\_intersection, Adj_+^m(p))$
6:      barrier
7:
8:  $\lambda\_intersection(q \in \mathcal{G}_+, Adj_+^m(p)))$   ▷ invk. on $q$, recv. $Adj_+^m(p)$
9:      for all $r \in Adj_+(p) \cap Adj_+(p)$ do      ▷ merge-path intersection
10:         $user\_callback(meta(\Delta_{pqr}), user\_args)$
```

4.4 *Push-Pull* Optimization

Given the $O(d_+(p)^2)$ wedge checks generated by every vertex, each of which involves checking for the existence of an edge potentially stored on another compute node, distributed triangle identification spawns massive amounts of network traffic. Any reduction in this volume of communication has the potential to greatly speed up computations.

The process we described in Section 4.3, which we refer to as *pushing*, involves $Rank(p)$ identifying a vertex q and a list of candidate r vertices, and then "*pushing*" these candidates to $Rank(q)$ to identify vertices that complete triangles. Alternatively, we can envision $Rank(p)$ identifying vertex q and asking $Rank(q)$ to send the adjacency list $Adj_+^m(q)$ for $Rank(p)$ to enumerate triangles, instead "*pulling*" vertex q's adjacency list.

The addition of a pull capability is beneficial if $Rank(p)$ knows $d_+(q)$ in advance, which requires only a small constant amount of additional memory per edge. However, its true advantage occurs when pulls are coalesced within a computational process, resulting in each MPI rank pulling $Adj_+^m(q)$ at most once, rather than potentially once per vertex stored locally. For each process $Proc$, if

$$|Adj_+(q)| < \sum_{p' \in Proc} \left|Adj_+(p') \setminus \{v : v <_+ q\}\right|,$$

the pulling approach likely saves on total communication. See Fig. 3 for a simple illustration. Note that sending metadata for use when triangles are detected makes the savings difficult to estimate precisely a-priori.

In our novel *Push-Pull* `TriPoll` algorithm, we choose whether to push from $Rank(p)$ or pull from $Rank(q)$ for each target vertex by taking an initial conditional pass over the data. This pass computes the total number of edges that will be sent to each target vertex summed across all local vertices, but does not actually transmit any adjacency information. The pass also stores pointers to efficiently iterate over source vertices stored locally, preparing for the eventuality that the target's adjacency list is pulled.

After this communication-free counting is done, each MPI rank sends a single message to each processor responsible for one of its target vertices q with the total number of edges it intends to send. Then $Rank(q)$ determines whether pulling would result in less communication than pushing. Correspondingly, $Rank(q)$ either adds the source rank to a list of ranks to which to send $Adj_+^m(q)$, or sends a reply informing the source rank that the proposed pull is ill-advised. Due to this determination of whether remote vertices should be pulled or not mimicking the *Push-Only* algorithm's pass over adjacency lists, but without sending adjacency information, we call this the *Push vs Pull Dry-Run*.

After these determinations are made, each rank iterates over its local vertices in a *Push Phase*: for each p, it sends a subset of $Adj_+^m(p)$ to vertex q for all (p, q) such that $Adj_+^m(q)$ is not set to be pulled. Finally, each rank iterates over the list of ranks each of its local vertices is being pulled from to complete the sensible pulls, coalesced where possible, in what we call the *Pull Phase*.

4.5 Callback Examples

In Alg. 1, we see there is nothing returned by `TriPoll`. Instead, we rely on the user-defined callback that gets executed on Line 11 to produce the side-effects desired to function as an output.

The simplest example of a callback is incrementing a counter to perform triangle counting. This is shown in Alg. 2. In this case, the callback is given all of the metadata associated to a triangle, which it completely ignores. At the end of this survey, each MPI rank has a local count of triangles seen that must be combined in an *All_Reduce*-type operation.

As a slightly more involved example, suppose we wish to know the distribution of maximum edge labels seen among all triangles in which all vertex labels are distinct. This data can be gathered using Alg. 3, where we have made use of the *distributed counting set* described in Sec. 4.1.4.

At the conclusion of Alg. 3, the returned *counters* object contains the result of this particular triangle survey. At that point, the user

Algorithm 2 Triangle Counting

```
1: procedure TRIANGLE_COUNT($\mathcal{G}$)
2:     $tc \leftarrow 0$
3:     $Triangle\_Survey(G, \lambda\_triangle\_count, tc)$
4:     $global\_tc \leftarrow all\_reduce(tc, SUM)$        ▷ triangle count
5:     return $global\_tc$
6:
7: $\lambda\_triangle\_count(meta(\Delta_{pqr}), tc)$
8:     $tc \leftarrow tc + 1$
```

Algorithm 3 Max Edge Label Distribution

```
1: procedure MAX_EDGE_LABEL_DISTRIBUTION($\mathcal{G}$)
2:     $counters \leftarrow distributed\_counting\_set$
3:     $Triangle\_Survey(G, \lambda\_max\_edge\_counts, counters)$
4:     return $counters$
5:
6: $\lambda\_max\_edge\_count(meta(\Delta_{pqr}), counters)$
7:     if $meta(p) \neq meta(q) \neq meta(r)$ then
8:         $max\_edge \leftarrow max(meta(pq), meta(pr), meta(qr))$
9:         $counters.increment(max\_edge)$
```

may use this data for any desired visualization purposes or as part of some further processing.

5 EVALUATION

Here we present an evaluation of the performance of `TriPoll`, as well as the results of the surveys. We begin with strong and weak scaling experiments in Sec 5.4 and 5.5, followed by comparisons to related work in Sec 5.6. These studies are all done in the case of simple triangle counting, a subset of the functionality `TriPoll` provides. We then show the results of experiments on a large Reddit dataset with timestamps to gather distributions of the time required for triangles to fully form (which we refer to as *closure rate*) in Sec. 5.7 and provide the strong scaling behavior. Following this, we give results of another experiment designed to study the domain name metadata present in the 224 billion edge Web Data Commons 2012 graph. We finish by looking at the performance impact of including nontrivial metadata (Sec. 5.9) and the effects of the *Push-Pull* optimization (Sec. 5.10) on synthetic and real datasets.

5.1 Test System

Experiments presented here are performed on the *Catalyst* cluster at LLNL with nodes featuring dual Intel Xeon E5-2695v2 processors totaling 24 cores along with 128GB of DRAM per node. *Catalyst* uses an Infiniband QDR interconnect.

5.2 Datasets

For the strong scaling experiments and comparison to related work, we use several openly available datasets [3, 12, 33, 35, 53], most of which are available from [47]. For weak scaling experiments, we use R-MAT graphs [13] up to scale 32.

We downloaded the Reddit dataset from [9], and the portion we use contains all comments and posts scraped from Reddit in the time period of December 2005 to April 2020. This amounts to 835 million comment authors and 7.2 billion unique comments. The graph we construct uses authors as vertices and comments between authors as undirected edges. This data is naturally given in the form of a multigraph with authors possibly replying to each others' comments multiple times. The graph we use keeps the chronologically-first comment between two authors and discards the remaining comments. This reduces the edge count to 9.4 billion edges.

Graph	$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{E}\rvert$	$\lvert\mathcal{T}\rvert$	d^{max}	d_{+}^{max}
LiveJournal [8]	4.85M	69.0M	286M	20333	686
Friendster [53]	66M	3.6B	4.2B	5214	868
Twitter [33]	42M	2.4B	34.8B	3.0M	4102
uk-2007-05 [12]	106M	6.6B	286.7B	975K	5704
web-cc12-hostgraph [35]	101M	3.8B	415B	3.0M	10654
Web Data Commons 2012 [3]	3.56B	224.5B	9.65T	95M	10683
Reddit	835M	9.4B	88.1B	1.70M	3301

Table 1: Datasets used for experiments

Tab. 1 gives an overview of the non-synthetic graphs used. All graphs were treated as undirected. For consistency, we report edge counts as the number of directed edges in a graph after symmetrizing or, equivalently, the number of nonzeros in a symmetrized graph's adjacency matrix.

5.3 Triangle Counting with `TriPoll`

Triangle counting is the simplest example using `TriPoll`, in which the callback for when a triangle is identified only increments a counter. This, however, is the extent of functionality present in most distributed triangle processing solutions. Exceptions are distributed versions of computing truss decompositions [15], where counts of triangles are desired at edges, and computing clustering coefficient where local counts of triangles are desired at vertices. Callbacks designed for these local participation counts would merely increment local counters. On the other hand, [45] represents an example where counts of metadata triangles (involving vertex labels) are needed at vertices or edges for use in accelerating general pattern matching code.

In order to count simple triangles globally or locally in our highly generalizable system, we must affix dummy metadata to all vertices and edges of the graph in question. We use booleans for this purpose. This addition adds a slight overhead to our storage requirements for a graph as well as the size of every adjacency list sent across the network.

The pseudocode to perform triangle counting with `TriPoll` that ignores metadata is given in Alg. 2.

5.4 Triangle Counting Strong Scaling Results

We provide a collection of strong scaling studies using `TriPoll`. For this endeavor, we use the openly available topology-only Friendster, Twitter, uk-2007-05, and web-cc12-hostgraph graphs[12, 33, 35, 53]. The results of these triangle counting studies for each of these datasets are given in Fig. 4.

On these datasets, we find the *Push-Pull* implementation of `TriPoll` scales well to between 64 and 256 compute nodes, depending on the dataset. By the nature of our *Push-Pull* algorithm, we do not expect to see ideal speedup in a strong scaling experiment. As the number of MPI ranks increases with a fixed graph size, the number of edges allocated to each rank decreases. With

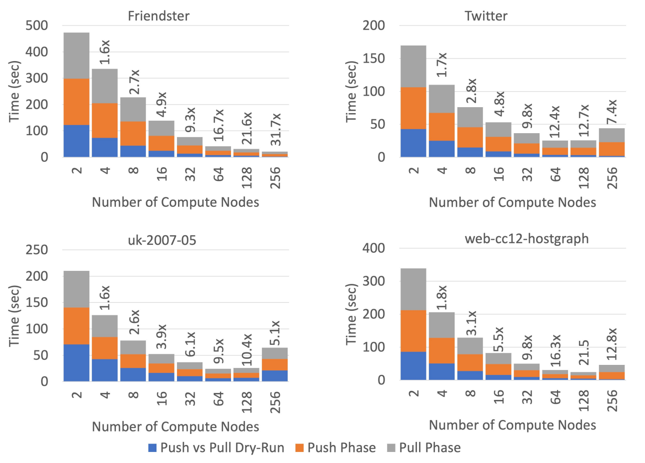


Figure 4: Strong scaling of each phase of *Push-Pull* algorithm on 2 to 256 compute nodes. Numbers above bars indicate the overall speedup on each dataset relative to times using 2 nodes.

this decrease in edges per rank comes fewer opportunities for the aggregation of edges we exploit in our *Push-Pull* optimization.

For all datasets except Friendster, running times for `TriPoll` increase between the 128 and 256 compute node configurations. This is likely due to limitations in the current implementation of our communication library. At 256 compute nodes, we have 6144 MPI ranks that may be communicating with each other, giving $\binom{6144}{2} \approx 18.8$ million pairs of ranks potentially communicating. This likely leads to a significant number of small messages being sent because of a lack of opportunities for message aggregation. This can likely be remedied by adding extra aggregation of messages at the level of compute nodes (instead of just individual MPI ranks), similar to other work [34, 44].

5.5 Weak Scaling in R-MAT Graphs

We performed weak scaling experiments using R-MAT graphs. These graphs provide a standard test for scalability despite their relatively low triangle count for their size. Here we again use triangle counting as our application to test scalability.

To assess the weak scalability of our *Push-Pull* algorithm with `TriPoll`, we want to make sure the rate at which each compute node is performing work remains roughly constant. Because we have an algorithm whose computational complexity is $O(|\mathcal{W}_+|)$ (the number of wedges in the DODGr graph $\mathcal{G}_+$) it makes sense to gauge the weak scalability of a distributed version of the algorithm versus $|\mathcal{W}_+|/(N * t)$ where N is the number of compute nodes and t is the time required to count triangles. We therefore use this as the vertical axis of our weak scaling results in Fig. 5.

We can see that the rate of work per compute node steadily decreases as the number of compute nodes increases. As in the case of strong scaling, we expect to see this effect, although for a different reason. As the graph grows while keeping roughly the same number of edges on each compute node, each edge stored has more potential target vertices to connect to. Likewise, any pair of edges stored on an MPI rank has a decreasing probability of connecting to the

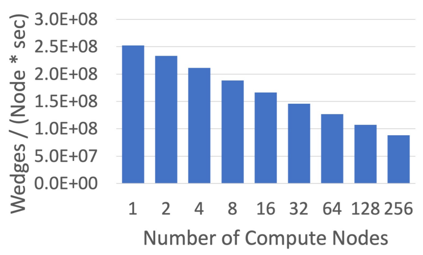


Figure 5: Weak scaling of triangle counting using `TriPoll`. Experiments are configured to use a scale 24 R-MAT per compute node, starting with a scale 24 on 1 node and reaching up to a scale 32 on 256 nodes.

same target vertex as the size of the graph increases. This leaves us with fewer opportunities for the aggregated communication our algorithm exploits. We will investigate the implications of this effect in Sec. 5.10.

5.6 Comparison with the Related Work

Since we are not aware of any system that directly solves the problem addressed by `TriPoll`, we focus on evaluating the performance of the core operation of our system, i.e., triangle identification/counting. Using four real-world graphs, we present empirical comparisons with three tailor-made, MPI-based, distributed triangle counting solutions that target processing on CPU clusters. Tab. 2 lists end-to-end runtimes of our work, and solutions by Pearce et al. [42], Tom et al. [58] and, the 2020 GraphChallenge [4] winner, TriC [20]. We wanted to compare with [5] which also targets CPU clusters, and demonstrated respectable performance and scalability; unfortunately the implementation was not publicly available.

The graphs chosen for these comparisons were the LiveJournal [8], Friendster [53], Twitter [33], and Web Data Commons 2012 [3] graphs. These graphs form a widely-used subset of the openly available datasets used in benchmarking graph algorithms and provide a wide range of sizes. Details of these graphs can be found in Tab. 1

We ran most of these experiments on 1,024 cores (64 compute nodes), except for the Web Data Commons graph. This configuration was chosen because the work of Tom et al. requires a number of MPI ranks that is a perfect square. Due to its high memory demand, we had to run TriC with 256 compute nodes and 4 processes per node to finish triangle counting on Friendster; for Twitter, experiments with TriC crashed due to insufficient memory, even when using 256 compute nodes. For the medium-sized graphs, `TriPoll` outperforms Pearce et al.: for Friendster, ~1.8× faster; for Twitter, ~6.8× faster.

Although, for the Friendster and Twitter graphs, the work by Tom et al. achieves the fastest time-to-solution, we were unable to get their code to run with more than 1024 MPI ranks, which confirms its optimizations are geared towards throughput rather than scalability. In comparison, we demonstrate scalability of `TriPoll` using up to 224 billion edge graphs and 6144 cores, to the best of our knowledge, the largest scale for real-world graphs to date. The only previous work that could count triangles in this graph is that of Pearce et al.; however, `TriPoll` is ~1.8× faster when using 256 compute nodes with 24 processes per node.

To the best of our knowledge, [62] demonstrated the second largest scale for real-world graphs: using the 2014 version of the Web Data Commons graph which has 124B edges (note: the 2014 graph is approximately half the size of the 2012 graph), on an Nvidia DGX machine with eight GPUs (each with 32GB memory), it counted over four trillion triangles in under two minutes. Although, they offer superior throughput, other distributed GPU-based solutions (discussed in Sec. 2) did not demonstrate scalability using real-world graphs as large as we do in this paper; hence, we limit comparison to solutions that target CPU clusters.

Graph	TriPoll	Pearce et al. [42]	Tom et al. [58]	TriC [20]
LiveJournal	1.01s	1.08s	1.45s	1.24min
Friendster	38.62s	69.79s	23.78s	5.55min*
Twitter	28.96s	196.10s	16.43s	N/A
Web Data Commons	456.7s	808.7s	N/A	N/A

Table 2: End-to-end runtime comparison using four real-world graphs. All experiments were run on 1024 cores on a 64 node deployment, except for Web Data Commons experiments which use 6144 cores on 256 nodes, and the TriC Friendster experiment. *Experiments were run on 256 compute nodes, 4 processes per node.

5.7 Triangle Closure Times in Reddit

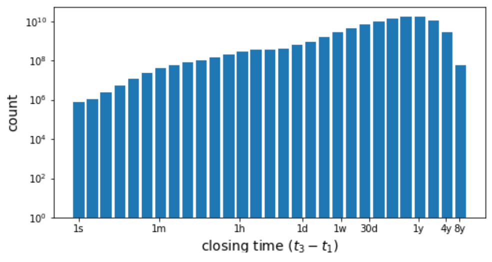
In our Reddit experiments, we want to determine how quickly the closing edge for each triangle appears relative to the speed that the initial wedge forms. This may be viewed by a network scientist as an interesting dynamic property of triangles, and different networks will likely have very different distributions of closure time. Moreover, within a single network, closure times are likely to indicate the nature of certain activity. For example, a triangle formed by human peer-2-peer connectivity should happen in reasonable time scales (e.g., the time it takes for a human to notice, read, and reply to another's Reddit comment) whereas much faster completion time might indicate a coordinated machine activity.

The graph we construct for these experiments uses authors as vertices and comments between authors as undirected edges. Details of this 9.4 billion edge graph are given in Sec. 5.2.

The question we are answering concerns the time of comments. We store these timestamps as edge metadata and do not make use of vertex metadata.

Once a triangle is found, the *wedge opening time*, $\Delta t_{open} = t_2 - t_1$, and *triangle closing time*, $\Delta t_{close} = t_3 - t_1$, are computed, where $t_1 \leq t_2 \leq t_3$ are the timestamps of the three edges involved in a triangle in sorted order. A counter for the pair $(\lceil \log_2(\Delta t_{open}) \rceil, \lceil \log_2(\Delta t_{close}) \rceil)$ is then incremented using a distributed counting data structure. Pseudocode for this application is given in Alg. 4. In this case, our counting set is counting pairs of objects because we are looking for the joint distribution of Δt_{open} and Δt_{close}.

Fig. 6 shows the distributions of closing and opening times generated by this experiment. In the joint distribution, we see that wedges are often formed quickly, but triangles are not (on average) systematically closed rapidly after wedges are formed.



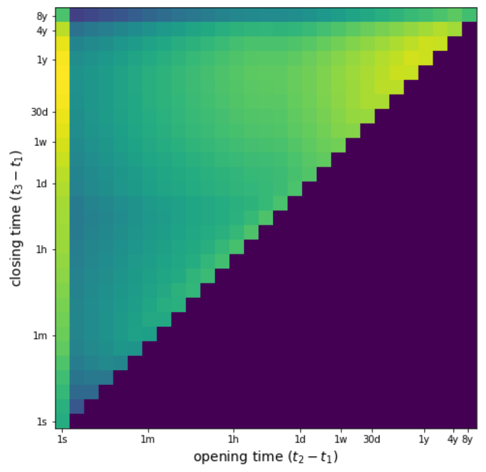


Figure 6: Distribution of closing time and joint distribution of closing time versus opening time for the Reddit graph. Axes and counts are in log-scales.

Algorithm 4 Reddit Triangle Closure Times

```
1:  procedure REDDIT_TRIANGLE_TIME_JOINT_DISTRIBUTION($\mathcal{G}$)
2:      $counters \leftarrow distributed\_counting\_set$
3:      $Triangle\_Survey(G, \lambda\_triangle\_times, counters)$
4:      return $counters$
5:
6:  $\lambda\_triangle\_times(meta(\Delta_{pqr}), counters)$
7:      if $meta(p) \neq meta(q) \neq meta(r)$ then
8:          $t\_1 \leftarrow min(meta(pq), meta(pr), meta(qr))$
9:          $t\_2 \leftarrow median(meta(pq), meta(pr), meta(qr))$
10:         $t\_3 \leftarrow max(meta(pq), meta(pr), meta(qr))$
11:         $open\_time \leftarrow ceil(log_2(t\_2 - t\_1))$
12:         $close\_time \leftarrow ceil(log_2(t\_3 - t\_1))$
13:         $counters.increment(pair(open\_time, close\_time))$
```

Fig. 7 shows the strong scaling performance of the *Push-Pull* implementation of `TriPoll` out to 256 compute nodes. Performance scales well out to 256 compute nodes for this problem. We do not appear to witness the same issues with strong scaling discussed in Sec. 5.4. This is likely due to the topology of this graph, as we

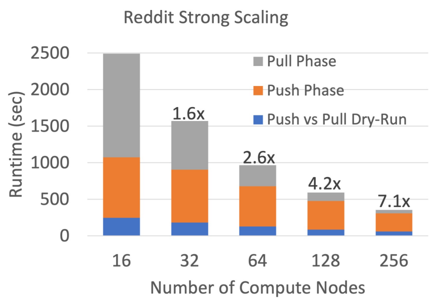


Figure 7: Strong scaling of triangle closure time collection on up to 256 nodes using *Push-Pull* algorithm. Times are divided into 3 pieces: the time to determine which vertices to pull, the push phase, and the pull phase. Numbers shown above bars are the overall speedup relative to 16 nodes.

Nodes	Avg. Pulls Per Rank
16	861K
32	466K
64	228K
128	101K
256	42.2K

Table 3: Average number of vertices pulled per rank as number of compute nodes increases.

see the same scaling behavior from Friendster, another large social network graph.

Within Fig. 7, it is important to note that the breakdowns of times into the different phases of computation does not necessarily show our pull phase scaling well and the push phase showing limited scalability. These results are actually indicative of a shift in the *Push-Pull* algorithm from pulling many vertices when a small number of nodes is used, to an almost entirely push-based algorithm on larger allocations. This is due to the decreased opportunities for the aggregation necessary in our *Push-Pull* optimization when each MPI rank has fewer edges. This can be seen by the decrease in average vertices pulled per rank shown in Tab. 3.

5.8 FQDN Analysis on Web Data Commons 2012

We have seen in Sec. 5.6 that the work of [42] provides the only previous solution which is capable of counting triangles in the Web Data Commons 2012 graph, although TriPoll outperforms the former handily. Next, we look to perform an analysis of the triangles found in this webgraph, a capability not possible within the confines of [42].

The vertices of the Web Data Commons graph represent webpages, each with a URL. For this experiment, we extract the *fully qualified domain name* (FQDN) from each vertex's URL and attach it as vertex metadata. We store this vertex metadata as C++ strings. We are able to do so without padding the strings to a fixed size or making use of lookup tables to retrieve the FQDNs by leveraging the serialization features described in Sec. 4.1. This deliberate

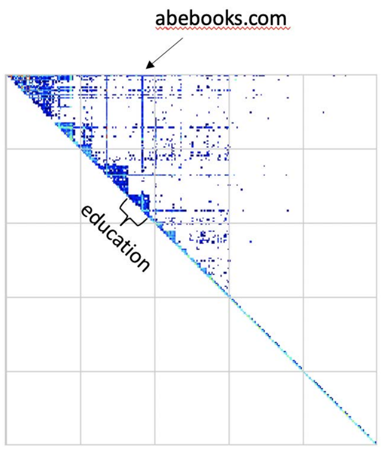


Figure 8: Distribution of FQDNs involved in triangles with "amazon.com".

choice of using strings prepares us for situations where metadata can truly be arbitrary in length.

While identifying triangles in this graph, we use a callback that counts 3-tuples of FQDNs involved in triangles, only counting triangles with 3 distinct FQDNs. This triangle analysis completes in 1694.6, compared to the 456.7 seconds required to count triangles without vertex metadata included.

During this distributed computation, TriPoll identifies 248.7 billion triangles where all FQDNs are distinct with a total of 39.2 billion unique 3-tuples of FQDNs. After this, we post-process the results on a single machine to investigate these FQDN triangles.

As an example, we searched the 39.2 billion 3-tuples of FQDNs in triangles to find all triangles involving "amazon.com". This generates a 2D distribution of pairs of FQDNs. This distribution is shown in Figure 8, where the FQDNs are ordered based on communities identified by the Louvain method.

Within this distribution, we can identify several points of interest. First, there are several relatively dense rows near the top of Figure 8 corresponding to other Amazon domains and products such as "amazon.co.uk", "amazon.ca", and "audible.com". Additionally, we have labeled another relatively dense row/column identified by "abebooks.com", an online retailer competing with Amazon. Unsurprisingly, sites linking to an Amazon product page are likely to link to the corresponding product at this competing retailer.

We have also identified a community among the FQDNs comprising a large number of domains for educational institutions and libraries. This community also features a number of booksellers, including abebooks.com.

This example demonstrates the data we are able to collect and study concerning metadata on triangles in graphs large enough that few previous works could even perform global triangle counts.

5.9 Impact of Metadata on Performance

Next, we investigate the effect including nontrivial metadata in TriPoll has on performance. To see this effect, we repeat the weak scaling experiments of Sec. 5.5, but we add each vertex's degree

Dataset	Measurement		8 Nodes	16 Nodes	32 Nodes	64 Nodes	128 Nodes	256 Nodes
Friendster	Communication Volume (GB)	Push-Only	727.7	729.6	730.6	731.1	731.3	731.4
		Push-Pull	572.1	659.0	713.4	744.1	760.4	768.8
	Runtime (sec)	Push-Only	164.9	80.4	38.1	18.6	13.8	11.8
		Push-Pull	179.6	99.9	52.8	29.5	22.8	15.5
Twitter	Communication Volume (GB)	Push-Only	1283	1286	1288	1289	1289	1289
		Push-Pull	372.9	525.8	684.6	829.8	945.2	1025.8
	Runtime (sec)	Push-Only	201.0	99.2	49.5	25.7	14.5	25.1
		Push-Pull	100.9	60.5	36.8	23.0	22.4	38.4
uk-2007-05	Communication Volume (GB)	Push-Only	3323	3331	3335	3337	3338	3338
		Push-Pull	625.9	943.2	1343	1737	2068	2281
	Runtime (sec)	Push-Only	388.8	190.7	97.1	49.1	28.0	37.0
		Push-Pull	137.9	89.8	57.9	37.1	33.8	69.7
web-cc12-hostgraph	Communication Volume (GB)	Push-Only	18497	18545	18569	18582	18588	18591
		Push-Pull	437.1	580.8	739.6	931.3	1197	1602
	Runtime (sec)	Push-Only	1618.5	858.4	452.8	269.9	159.7	163.1
		Push-Pull	198.2	112.0	63.5	38.1	28.7	48.3

Table 4: Strong scaling results for *Push-Only* and *Push-Pull* implementations including communication costs associated to each.

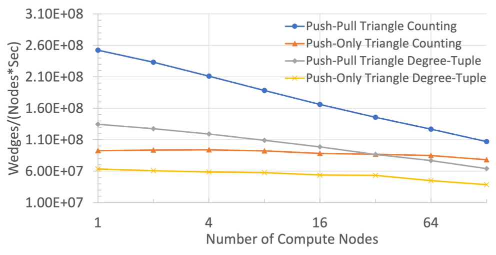


Figure 9: Effects of metadata inclusion on weak scaling of *Push-Pull* and *Push-Only* algorithms in `TriPoll`

as a replacement for the dummy metadata used previously. In this configuration, we add a callback that counts the occurrences of triples ($\lceil\log_2(d(p))\rceil$, $\lceil\log_2(d(q))\rceil$, $\lceil\log_2(d(r))\rceil$) across all triangles discovered. The callback to do this counting involves a simple hash and logarithm of the degrees of vertices p, q, and r. This scenario gives us a simple example with a small amount of vertex metadata and a nontrivial callback operation to compare with the triangle counting results.

With this experiment we track the performance when using the *Push-Only* as well as the *Push-Pull* implementations of `TriPoll`. The results are shown in Fig. 9.

Each algorithm's throughput is cut by a factor of just under 2 across all problem sizes as a direct result of the extra metadata and callback. The scalability of algorithms does not appear to be affected. Although the performance of the *Push-Pull* algorithm degrades more in absolute terms when metadata is excluded, the relative change in performance remains unchanged whether or not metadata is included.

5.10 Push-Pull Optimization Performance

To study the effectiveness of our *Push-Pull* algorithm over the simpler *Push-Only* algorithm, we repeated the triangle counting strong scaling experiments of Sec. 5.4 with the *Push-Only* algorithm. These full strong-scaling experiments can be found in Tab. 4. These results include the total volume of data communicated during the course of each experiment.

These graphs showcase the advantages and disadvantages of the *Push-Pull* implementation. Datasets such as Friendster provide very little opportunity to reduce communication through pulling adjacency lists. In this scenario, the overhead of an additional pass over all of the edges of the graph to determine the pulling and pushing behavior is apparent from the times reported being slower than the *Push-Only* implementation.

This is to be contrasted with the somewhat extreme example provided by the web-cc12-hostgraph, where the communication volume is reduced by more than a factor of 10, even when using 256 compute nodes (6144 MPI ranks). This massive reduction in network traffic leads to the *Push-Pull* implementation beating *Push-Only* by a factor of 6 throughout the regime that both are scaling properly.

Tab. 4 shows a dramatic increase in communication volume for the *Push-Pull* algorithm when strong scaling on the uk-2007-05 and web-cc12-hostgraph datasets, approaching a factor of 4 increase when progressing from 8 to 256 nodes. This is a good indication of the lessened opportunities for the aggregation necessary within an MPI rank to warrant pulling a vertex's adjacency list. On Friendster, we even see the communication volume of the *Push-Pull* algorithm overtake that of the *Push-Only* algorithm. In this case, the communication required to check if vertices should be pulled becomes greater than the communication reduction of pulling the beneficial vertices.

Both algorithms show performance stagnation or regression at 256 compute nodes, however, the negative effect on the *Push-Only* algorithm is less pronounced than for the *Push-Pull* algorithm. This effect can also be seen in the weak scaling of Fig. 9, where the *Push-Only* algorithm shows very little degradation in work rate as the number of compute nodes increases. Despite the lower scalability, the actual performance of *Push-Pull* tends to be comparable to or significantly better than that of the *Push-Only* algorithm even when using thousands of MPI ranks.

6 CONCLUSION

This work demonstrates a novel capability of performing triangle surveys on massive distributed graphs with metadata on the edges and vertices using `TriPoll`. Networks scientists can use such capabilities to better understand the higher-order interactions within their relational datasets, potentially facilitating better models for many graph-related machine learning tasks, and better detection of anomalous activities. We have demonstrated scalability of `TriPoll` out to thousands of cores on hundreds of compute nodes using graphs with up to 224 billion edges. We showed the capabilities of our system by performing a triangle timing survey on a temporal graph built from Reddit. Additionally, we compared the performance of `TriPoll` on the task of triangle counting, a very simple example of our capabilities, to several works tailored to this problem and were able to outperform them on several datasets. This includes counting triangles on a 224 billion edge web graph ~1.8× faster than the only openly available software able to handle a problem of this size that we are aware of.

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Appendix: Artifact Description/Artifact Evaluation

SUMMARY OF THE EXPERIMENTS REPORTED

We ran the proprietary code developed in this paper to survey triangles on LLNL's Catalyst supercomputer.

Author-Created or Modified Artifacts:

```
Persistent ID: https://github.com/LLNL/ygm
Artifact name: YGM
```

BASELINE EXPERIMENTAL SETUP, AND MODIFICATIONS MADE FOR THE PAPER

Relevant hardware details: Catalyst supercomputer, dual Intel Xeon 2695v2

Operating systems and versions: TOSS 3 running Linux kernel 3.10

Compilers and versions: gcc 9.2.0

Libraries and versions: mvapich2 v2.3

Key algorithms: vertex-centric merge-path triangle enumeration

Input datasets and versions: Friendster, Twitter, LiveJournal, uk-2007-05, web-cc12-hostgraph, and Web Data Commons 2012 graphs; R-MAT graph generator