Abstracts of the Fifth Spanish Meeting on Computer Algebra and Applications EACA-99

Communicated by Isabel Bermejo

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This is the collection of the abstracts of the talks given during the EACA-99, the Fifth Spanish Meeting on Computer Algebra and Applications held in Tenerife (Canary Islands, Spain) on September 8-11, 1999.

The main goal of the EACA–Meetings is to provide a forum for researchers from Computer Algebra and researchers that essentially use these techniques in their investigation. As in previous events of the series (Santander 1995, Sevilla 1996, Granada 1997 and Sigüenza 1998), participation of young researchers was especially encouraged.

The EACA-99 was supported by Cabildo de Tenerife, Cajacanarias, Consejería de Educación, Cultura y Deportes del Gobierno de Canarias (Dirección General de Universidades e Investigación), Ministerio de Educación y Cultura (Dirección General de Enseñanza Superior e Investigación Científica. Referencia CO99–0319), Real Sociedad Matemática Española, SUN Microsystems Ibérica S.A. and Universidad de La Laguna.

The first six abstracts correspond to the invited lectures.

On the ubiquity of Newton polyhedra

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The notion of a Newton polyhedron goes back to Newton (1670). After introducing formal power series in one variable by analogy with decimal expansions of (real) numbers, he outlines an algorithmic procedure yielding fractional power series solutions of the equation $f(x, y) = 0$, the first step of which consists in solving numerical equations given by the edges of a certain convex polygon attached to f.

Although implicit, Newton polyhedra also play an important role in the resolution of singularities of an algebraic variety defined over a field of characteristic 0 (Hironaka 1964).

In connection with toric geometry, another viewpoint shows to be fruitful. Instead of dealing with an individual polynomial, one looks for properties which hold for almost all polynomials (or series) sharing the same Newton polyedron. This leads to the notion of non-degeneracy of a system of functions (Laurent polynomials, polynomials, power series) for its Newton polyhedra, and to the computation of a number of discrete analytic invariants (global or local) of complete intersections defined by such a system, in terms of the convex geometry of the given polyhedra (genus, Hodge-Deligne numbers, Milnor numbers, ...) (Bernstein, Danilov, Khovanskii, Varchenko 1975-1980).

The state polytope of an homogeneous ideal I in a polynomial ring over a field may be viewed as a generalization of the Newton polytope of a polynomial. Its vertices are in 1-1 correspondence with the initial ideals of I for all possible term-orderings on the monomials (Bayer, Morrison, Mora, Robbiano 1988).

J. Mac Donald generalized Newton's algorithm to polynomial equations in n variables (1995). The solutions are fractional power series with exponents lying in strongly convex polyhedral cones given by admissible edges of the Newton polyhedron of the polynomial. The construction is related to the theory of fiber polytopes of Billera and Sturmfels (1992).

Computing Weil's descente variety

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Given a variety V, implicitly defined over an algebraic field extension $k(\alpha)$ of characteristic zero, A. Weil (*Adèles et groupes algébriques,* Séminaire Bourbaki, no. 186. (1959)) developed a restriction technique (called by him a *descente* method), that associates to V a suitable k-variety W, such that many properties of V can be analyzed by merely looking at W, that is, by descending to the base field k. In this talk we present a parametric counterpart, for curves, of Weil's construction. As an application, we state some simple algorithmic criteria over the variety W that translate, for instance, the k-definability of a parametric curve V, or the existence of an infinite number of k-rational points in V. The criteria are based on recognizing a special class of curves, dubbed *hypercircle curves,* associated to the algebraic extension $k(\alpha)$. Further details appear in the full paper "Base field restriction techniques for parametric curves" by Andradas, Recio and Sendra, at the proceedings of the ACM-ISSAC'99 conference.

The parametrization problem for algebraic surfaces

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The parametrization problem asks for a parametrization of an implicitly given surface, in terms of rational functions in two parameters. Relevant classical results are the criterion of Castelnuovo and the classification of complex-parametrizable surfaces by Enriques/Manin. These results have recently been made effective by an algorithm given by the author (JSC 98). Using the method of adjoints, one can reduce the parametrization problem to the problem of solving a quadratic form over the function field of a curve, or over the parametrization of a del Pezzo surface. The hardest part of this method is the analysis of the singularities.

Constructive Algebraic Topology

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The classical "computation" methods in Algebraic Topology most often work by means of highly infinite objects and in fact *are not* constructive. Typical examples are shown to describe the nature of the problem. The Rubio-Sergeraert solution for Constructive Algebraic Topology is recalled. This is not only a theoretical solution: the concrete computer program *Kenzo* (joint work with Xavier Dousson and Yvon Siret) has been written down which precisely follows this method. This program has been used in various cases, opening new research subjects and producing in several cases significant results unreachable by hand. In particular the Kenzo program can compute the first homotopy groups of a simply connected *arbitrary* simplicial set.

Bounds in the computation of the integral closure

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There are two general problems in the calculation of integral closures: of ideals and of affine domains. The latter has been the focus of several modern algorithms but the former only indirectly has received that kind of attention. We will treat here one related problem: the number of generators (its *embedding dimension*) the integral closure of an affine domain may require. This number and the degrees of the generators in the graded case are major measures of costs of the computation. We report on progress in joint work with Bernd Ulrich on this question for various kinds of algebras, particularly homogeneous algebras with a small singular locus.

For an affine equidimensional graded reduced algebra of dimension d and multiplicity e, satisfying the Serre's condition R_ℓ, two of our results are the following:

- If A satisfies R_{d-3}, the embedding dimension of its integral closure B is bounded by

$$(e(e-1))^{2^{d-3}} - (2e(e-1))^{2^{d-4}} + 5.$$

- If A is defined over an algebraically closed field of characteristic zero, satisfies R_1 and the Eisenbud-Goto's conjecture holds in dimension $\leq d-1$, the integral closure of A is generated by elements of degree at most $(e-1)^2$.

Using OMEGA for the effective construction, coding and decoding of block error-correcting codes

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In this work we show how to implement effective constructions, and effective coding and decoding, of algebraic codes by means of OMEGA, a system specifically designed and programmed for general mathematical computations. The main class we consider is the class of alternant codes (which includes the classes of BCH codes, RS codes, and classical Goppa codes), and for them we give an implementation of the Euclidean division Berlekamp–Massey decoding algorithm. For cyclic codes we implement the Meggitt decoder, and we illustrate its working implementing the Meggitt syndrome tables for the two Golay codes. Finally we present several other groups of functions and the computations and problems (still almost in the area of error-correcting codes) they solve.

On some real problems in computer vision which yield to algebraic system of equations

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In this work, we present some real problems which appear in computer vision which yields to nonlinear system of algebraic equations. We study the problem of camera calibration. Roughly speaking, camera calibration consists in looking at the camera position in the $3-D$ world using as information the projection of a $3-D$ scene in a $2-D$ plane (the photogram). The problem is quite different when we use a single view or several views (stereo vision) of the $3-D$ scene. We will show in this work how these problems yields to nonlinear algebraic system of equations.

Convex polytopes over ordered fields

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A well known result from classical Convex Geometry (that is, over the real numbers $\mathbb{R}$) is that any closed convex set C is the intersection of the half spaces determined by the hyperplanes tangent to its boundary points. This result is false over arbitrary ordered fields (consider for instance $C = \{y - \sqrt{2}x \geq 0\}$ in $\mathbb{Q}^2$).

In this work we extend this result to the class of *semilinear* sets over an arbitrary ordered field K, that is, sets describable by a boolean combination of linear equalities and inequalities. We show that for closed semilinear sets to be convex is equivalent to be a polyhedra, that is, the intersection of a finite number of halfspaces. This result allows us to give an algorithm to decide whether a closed semilinear set is convex. Moreover, in the affirmative it gives a description of the set as intersection of hyperplanes.

On equations defining monomial varieties

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Our results are the following:

1. In characteristic $p > 0$ every simplicial toric affine or projective variety with full parametrization is a set-theoretic complete intersection. This extends previous results by R. Hartshorne and T.T. Moh.
2. In any characteristic, every simplicial toric affine or projective variety with full parametrization is an almost set-theoretic complete intersection. This extends previous known results by M. Barile-M. Morales and A. Thoma.
3. In any characteristic, every simplicial toric affine or projective variety of codimension two is an almost set-theoretic complete intersection.
4. Let V be a simplicial toric variety of codimension r over a field of characteristic zero. Then $bar(I(V)) = r$ if and only if V is a complete intersection.

Moreover the proofs are constructive and the equations we find are binomial ones.

On Castelnuovo-Mumford regularity of codimension two monomial varieties

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Let I be an ideal of the polynomial ring $K[z, y, x_1, \ldots, x_n]$ defining a projective monomial variety $\mathcal{V} \subset \mathbb{P}_K^{n+1}$ of codimension two. We give a formula for the Castelnuovo-Mumford regularity of $\mathcal{V}$, $\operatorname{reg}\mathcal{V}$, in terms of the degrees of the minimal generators of I. As a consequence, we obtain that $\operatorname{reg}\mathcal{V} \leq \deg\mathcal{V} - 1$, where $\deg\mathcal{V}$ is the degree of $\mathcal{V}$, and characterize when equality holds.

When is a finitely generated algebra of Poincaré–Birkhoff–Witt type?

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A PBW algebra R over a field $\mathbb{k}$ can be understood as the associative algebra generated by finitely many elements $x_1, \ldots, x_n$ subjected to the relations

$$Q = \{x_j x_i = q_{ji} x_i x_j + p_{ji} \ (1 \leq i < j \leq n)\}, \qquad (1)$$

where each p_{ji} is a finite $\Bbbk$–linear combination of standard monomials $\vec{x}^{\vec{\alpha}} = x_1^{\alpha_1} \cdots x_n^{\alpha_n}$, $\vec{\alpha} = (\alpha_1, \ldots, \alpha_n) \in \mathbb{N}^n$, and each q_{ji} is a non-zero scalar in the field $\Bbbk$. The algebra is required to satisfy the following two conditions:

1. There is an admissible order $\preceq$ on $\mathbb{N}^n$ such that $\exp(p_{ji}) \prec \vec{\epsilon}_i + \vec{\epsilon}_j$ for every $1 \leq i < j \leq n$.

2. The standard monomials $\vec{x}^{\vec{\alpha}}$, $\vec{\alpha} \in \mathbb{N}^n$ form a basis of R as a $\Bbbk$-vector space.

The aim of this work is to provide algorithms to check in a effective way both conditions and, hence, to decide whether a given algebra satisfying relations like (1) is a PBW algebra.

The search for rational $\mathcal{A}$-hypergeometric functions

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We develop algebraic and combinatorial tools with the goal of describing all rational solutions of the $\mathcal{A}$-hypergeometric systems of linear partial differential equations introduced by Gelfand, Kapranov and Zelevinsky. All such systems have Laurent polynomial solutions, but we find severe combinatorial restrictions for the existence of rational solutions with non monomial denominator. A related preprint is available at: http://xxx.lanl.gov/abs/math/9911030.

Open problems in Computer Algebra

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Recently, Erich Kaltofen has given talks on some of his favourite open problems in computer algebra. He has allowed David Jeffrey and I to add an appendix to his associated paper, on a favourite open problem of our own, namely to design an extension of the Risch algorithm capable of producing antiderivatives continuous on domains of maximum extent.

In brief, the difficulty is that the Risch algorithm is fundamentally an *algebraic* algorithm, whilst the notion of (Riemann) integral is an *analytic* one; the algebraic approach ignores difficulties with piecewise constants arising from branch choices. The canonical example is $\int_a^x 1/(2+\sin t)\,dt$, for which most systems (except Derive) give an answer which is not continuous in x (but obviously should be).

In this talk I describe some progress, and some limitations on any progress (in some contexts, the problem is undecidable).

Bezoutian formulas à la Macaulay for the multivariate resultant

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We present Macaulay style formulas for the multivariate resultant in terms of Bezoutians. They are of smaller size than the classical Macaulay formulas and correspond to explicit computations for the resultant as the determinant of a complex.

Both authors are supported by Universidad de Buenos Aires, grant TX094. The second author is also supported by CONICET (Argentina).

Algorithms to compute the eigenvalues of a p-adic matrix

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In this work we present some algorithms to compute eigenvalues of a p-adic matrix. These algorithms are more flexible than those that compute first the characteristic polynomial and then the roots of this polynomial.

New non recursive formulas for irreducible representations of $\mathrm{GL}(\mathbb{C}^{m+1})$ and systems of equations for the symmetric powers $\mathrm{Sym}^n\mathbb{P}_m$

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We apply the "Boerner's footnote lemma" for a drastic simplification of the theory and formulas involving such representations, the geometry of the attached flag manifolds, and new kind of homogeneous polynomial equations defining the n^{th} symmetric power $\mathbb{S}_m^n := \mathrm{Sym}^n\mathbb{P}_m = \mathbb{P}_m^n/\mathcal{S}_n$, where $\mathcal{S}_n$ stands for the symmetric group, and $\mathbb{P}_m = \mathbb{P}(\mathbb{C}^{n+1})$, different from the badly known identical vanishing of the classical Brill-Gordan covariant.

Polar invariants and topology

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We attach to the decomposition of the generic polar curve of C a matrix depending only of the topological type of C, which is moreover completely determined by this matrix. This work generalises the results of M. Merle (*Invariants polaires des courbes planes*, Inventiones Math. 41, 103-111, 1977) and others.

Commutative ideal extensions of abelian groups

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We give characterizations of the property of being an ideal extension of an abelian group. As a consequence of these characterizations we obtain algorithms for deciding whether a finitely generated commutative monoid is an ideal extension of an abelian group and for computing the set of idempotents of a finitely generated commutative monoid.

This work was supported by the project PB96-1424.

A Newton-Puiseux algorithm for the ring $\mathbb{C}\{X_1, \ldots, X_d\}[Y]$

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We prove a generalization of the Newton-Puiseux Theorem. We construct all solutions $Y(X_1, \ldots, X_d)$, of a polynomial equation $F(X_1, \ldots, X_d, Y) = 0$, defined by a reduced polynomial in Y with coefficients in the ring of convergent complex power series in d variables, $\mathbb{C}\{X_1, \ldots, X_d\}$. These solutions belong to some rings of fractional power series determined by the Newton polyhedron of the discriminant of the polynomial F. The terms appearing on the solutions are computed by an algorithm which we provide.

This result continues the work of J. McDonald (see: "Fiber Polytopes and fractional power series", in *J. Pure and App. Algebra*, 104, (1995), 213-233). Further details will appear in the full paper "Singularités quasi-ordinaires toriques et polyèdre de Newton du discriminant" in the *Canadian J. of Mathematics*. The author is grateful for the hospitality offered by *DMI of ENS-Paris*, and the financial support of the *DGUI del Gobierno de Canarias*.

Subfields in pure transcendental extensions

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In this work we study the following computational problem: "Given a field $\mathbb{K}$ and rational functions $f_1, \ldots, f_m \in \mathbb{K}(x_1, \ldots, x_n)$, we want to know if there exist $h_1, \ldots, h_t \in \mathbb{K}(x_1, \ldots, x_n)$ such that $\mathbb{K}(f_1, \ldots, f_m) \subset \mathbb{K}(h_1, \ldots, h_t)$ is a proper extension. And in the affirmative case, compute $h_1, \ldots, h_t \in \mathbb{K}(x_1, \ldots, x_n)$ and rational functions $g_1, \ldots, g_m \in \mathbb{K}(y_1, \ldots, y_t)$ such that

$$f_i = g_i(h_1, \ldots, h_t) \text{ for all } i \in \{1, \ldots, m\}."$$

We present an exponential time algorithm to solve the above problem for zero characteristic fields. The method requires Gröbner basis computation and polynomial factorization over algebraic extension. The basic idea underlying our approach is the translation of decomposition issues to factorization over algebraic extensions. The algorithm compares favorably to the previous methods.

Using *Schubert* to enumerate conics in $\mathbb{P}^3$

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We obtain, by means of *Schubert* MAPLEV package, the intersection numbers of the variety of conics in $\mathbb{P}^3$ that satisfy eight conditions of the following types: ρ, that the plane determined by the conic go through a point; μ, that the conic intersect a line; ν, that the conic be tangent to a plane. To do so, we construct several compactifications of the variety of non-degenerate conics in $\mathbb{P}^3$, we study their boundaries and, finally, we compute all the aforementioned intersection numbers using a well-known degeneration formula, which we do justify.

Computing epistasis through Walsh transforms

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Many optimization problems do not consider directly the objects of the search space, but rather a binary encoded version of the latter. In this case the efficiency of the optimization process is highly influenced by the (in)dependency of the separate bits in the encoding. The notion of *epistasis* has been shown to be an efficient tool to describe the degree of dependency of separate bits, but may be very difficult to calculate, however. In this work, we show how the computation of epistasis may be greatly simplified through the use of Walsh transforms.

A. Verschoren was partially supported by the GOA "Generic Optimization" at the University of Antwerp and C. Vidal by a research grant of the University of La Coruña.

Arithmetic Nullstellensätze

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We present sharp estimates for the degree and the height of the polynomials in the Nullstellensatz over $\mathbb{Z}$. This result improves previous work of Berenstein-Yger and Krick-Pardo.

We also present degree and height estimates of intrinsic type, which depend mainly on the degree and the height of the input polynomial system. As an application, we derive an effective arithmetic Nullstellensatz for sparse polynomial systems.

Simplicial sets in the EAT system

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In a recent paper (*Specifying implementations*, Proceedings ISSAC'99, ACM Press, 1999, pp. 245–251), we have characterized in the Category Theory setting a class of canonical implementations of Abstract Data Types. This class has been suggested by the way of working in a system for Symbolic Computation in Algebraic Topology called EAT (Effective Algebraic Topology, `ftp://fourier.ujf-grenoble.fr/pub/EAT`).

In this work, we focus that study on the case of simplicial sets and we prove that the aforementioned canonical implementation is isomorphic to the implementation actually used in EAT.

This work has been partially supported by DGES, project PB97-1025-C02-01 and by Universidad de La Rioja, project API-98/B22.

Real Solving of ill-conditioned sine-polynomials equations

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In this work we present a method for localizing all the real roots of a sine-polynomial in a bounded interval even if this function is ill-conditioned. This method arises from the exclusion method of J.P.Dedieu and J.C.Yakoubsohn. We give a theoretical analyze of its efficiency and some numerical experiences.

Radicals of primary submodules.

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Because of the generalization of the notion of prime ideal from the category of rings to the category of modules, the last ten years have witnessed an increasing interest in the study on radicals of submodules in modules. In this work we first use a characterization of the radical of a submodule which allows us to calculate some radicals of submodules of free modules. Next we tackle the natural question whether the radical of a primary submodule is a prime submodule. We give a condition for detecting if a primary submodule of a finitely generated module has prime radical and it is then applied to determine, by using computational methods, if some primary submodules of a free module have prime radical.

Basic algorithms for specialization in Gröbner bases

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Weispfenning algorithm for computing a comprehensive Gröbner basis of an ideal in $K[\overline{a}, \overline{x}]$ provides a complete discussion of the solutions of a polynomial system of equations with parameters. Nevertheless a large set of cases is often obtained, many of them repeated and/or inconsistent. Our aim is to improve the output, reducing the number of cases.

In this work two basic facts are presented. First, we prove that the ordinary Buchberger algorithm, acting on $K(\overline{a})[\overline{x}]$ with some minor modifications, specializes for all values of the parameters outside a $m-1$ dimensional *singular variety* $V = \bigcup_i \mathbb{V}(f_i(\overline{a}))$. We show that it also allows to compute a sufficient singular variety V' containing V. Second, we present a new *generalized Gaussian elimination algorithm* that when used as first step, produces a reduced V', where unnecessary irreducible components are dropped.

These facts allows us to give a new complete algorithm for discussing polynomial systems of equations with parameters, whose output contains a reduced set of cases. It will be presented in a forthcoming paper.

The author was partially supported by Proyecto DGES-MEC PB96-0005-C02-02

Index of nilpotency of binomial ideals

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In this work, we prove that, in zero characteristic, it is possible to compute the index of nilpotency of cellular binomial ideals. Furthermore, in positive characteristic, we find a linear bound of it. This results can be generalized to binomial ideals. Thus, given a binomial ideal I we can also get bounds for the index of nilpotency of I, from the indices of its cellular components.

Both authors were partially supported by Universidad de Sevilla, ayuda a grupos precompetitivos. The second author *was also partially supported by Junta de Andalucía, ayuda* a grupos 1144.

Implicitization of a general union of parametric varieties

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We reconduct the computation of the equations defining a union V of parametric varieties, up to a given degree d, to the computation of the equations, of degree less or equal to d, vanishing on a finite set of points of V. If V is general this gives a new algorithm for implicitizing V.

Graded Codes by G-sets

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Since S.D.Berman showed in 1967 that cyclic codes and Reed Muller codes can be studied as ideals in a group algebra KG (K being a finite field and G finite cyclic group and 2-group respectively), several authors have considered these codes, since if you have more algebraic structure then their study is more effective. Following this way, we introduce the concept of graded code by G-sets. We show that some important properties about graded codes can be got from its homogeneous components and we generalize some results about codes as ideals in group algebras. In particular, we show some examples of cocyclic codes and how can be studied, in a easy way, as codes graded by G-sets.

The two authors have been supported by grant PB91-1068 from DGES, grant CRG 971543 from NATO and PAI 1136.

Solving problems symbolically by using Poisson Series Processors

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Different aspects must be taking into account when the symbolic approach to non-linear dynamical problems is considered. All these aspects are analyzed in this work in the context of the artificial satellite theory.

Usually, high order theories in Celestial Mechanics include series with a huge number of terms. To handle the algebraic complexity of such problems we require analytical methods specifically designed to the computer implementation of the problem. The Lie–Deprit method based on the Lie canonical transformations is a good example of adaptation among algebraic, symbolic and computer requirements in non-linear problems.

The existence of a well defined mathematical object, with a close algebraic structure, guarantees the use of efficient symbolic tools specially adapted to the problem. In our case, the properties of the Poisson Series permit the use of special software named Poisson Series Processors that shows to be dramatically more efficient than general algebraic and symbolic processors in this kind of problems.

Finally, we show that different symbolic and computational structures may be used to handle Poisson Series, however, the efficiency of Poisson Series Processor depends on this structure. We see, for instance, the advantages of the use of bidimensional lists to store series when we try to evaluate them.

Optimal reparametrization of polynomial algebraic curves

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In this work, we present an algorithm for optimally parametrizing polynomial algebraic curves. Let $\mathcal{C}$ be a polynomial plane algebraic curve given by a polynomial parametrization $\mathcal{P}(t) \in \mathbb{L}[t]^2$, where $\mathbb{L}$ is a finite field extension of a field $\mathbb{K}$ of characteristic zero. Then, we prove that if $\mathcal{C}$ is polynomial over $\mathbb{K}$ then Weil's *descente* variety (see Andradas C., Recio T., Sendra J.R. (1999), *Base field restriction techniques for parametric curves.* Proc. ISSAC 99, ACM Press.) associated with $\mathcal{P}(t)$ is surprisingly simple; it is in fact a line. Applying this result we are able to derive an effective algorithm to algebraically optimal reparametrize polynomial algebraic curves.
