Compressive Sparse Binary Signals Reconstruction Algorithm Using Simulated Annealing

Ahmed Aziz
aziz.bfci@gmail.com
Department of International Business Administration, Tashkent State University of Economics
Tashkent, Uzbekistan

Kongratbay Sharipov
k.sharipov@tsue.uz
Tashkent State University of Economics
Tashkent, Uzbekistan

Mansur Eshov
Tashkent State University of Economics
Tashkent, Uzbekistan

Nodir Jumaev
Oliy Majilis of the Republic of Uzbekistan
Tashkent, Uzbekistan

Dilshodjon Rakhmanov
Tashkent State University of Economics
Tashkent, Uzbekistan

ABSTRACT

Compressive Sensing method has been used in several applications specially for Image processing and wireless sensor network applications where Binary Compressive Sensing(BCS) is widely used. However, BCS reconstruction process is considered one of the most challenge in the most of applications. Therefore, this paper aims to address the mentioned problem by proposing Compressive Sparse Binary Signals Reconstruction Algorithm Using Simulated Annealing (CSBCSA). CSBCSA uses the advantage of Simulated Annealing algorithm in terms of finding the optimal solutions using lightweight computation and the advantage of the easy and fast implementation for the greedy algorithm to solve the reconstruction problem. This integration makes CSBCSA outperform the baseline reconstruction algorithm as shown in the simulation results section.

CCS CONCEPTS

• **Internet of Things** → **WSNs**; *Data Reduction*; Routing; • **Compressive Sensing** → Reconstruction.

KEYWORDS

IoT, WSNs, Binary Data, CS

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1 INTRODUCTION

In the last decade, Compressive sensing (CS) method [1–7]proved itself as a novel and efficient data reduction methods for many applications such as Internet of Things(IoT), Wireless Sensor Networks (WSNs) and Biological applications. CS method has efficient contribution specially in solving IoT network data transmission [19, 20] power consumption process. That's because, based on CS theory the Base Station(BS) needs $M \geq K \log N/K$, measurements from the entire network which is consists of N nodes ,where M is compressed sampled size, K sparsity level and $M << N$, instead of collecting N to recover he original data $x \in R^N$ from only $y \in R^M$ such that the CS framework cab be expressed as follows: that $y = \Phi x$ where Φ is the CS matrix. However, the CS reconstruction process is considered as NP-hard problem [8] because in the reconstruction process the BS tries to reconstruct the original data vector N from only M samples such that $M << N$ where the number of unknown N is bigger than the input M. The CS recovery process can be shown as following:

$$min_x \parallel x \parallel_0 \ s.t. y = \Phi x \tag{1}$$

Eq.1, targets to reconstruct the non zeros value of the signal x (K sparsity level) such that Φ and t y (measurements vector) are given. There are many reconstruction algorithms have been proposed to solve this problem such as convex relaxation and greedy algorithm. during the convex relation reconstruction process , the problem in 1 is solved by replacing L_0 to L_1 [9] as following:

$$min_x \parallel x \parallel_1 \ s.t. y = \Phi x \tag{2}$$

and then the magic toolbox [10] can be used to solve the problem in Eq.2. Convex reconstruction based algorithms can provide stabile reconstruction process where the full signal can be recovered correctly. However, all of them have complex computations process which consume more communication process leads to consume the IoT network energy. That's why convex relation algorithms are not suitable for IoT network. On the other hand, Greedy algorithms present them self as sufficient reconstruction algorithm. During the greedy algorithm reconstruction process, one or more CS matrix Φ's columns are iteratively choose based on their correlation to the current residual. There are different greedy algorithms can be used such as OMP [11] algorithm, in which one column is selected from Φ and then OMP algorithm remove its orthogonality from the current residual and then repeats till obtain the estimated signal x'. Based on OMP algorithm a lot of algorithms have been proposed such as ROMP [12] and StOMP [13]. in addition to, CoSaMP [14], SP [15], IHT [16] and FBP [17]algorithms, which uses backward steps to prune the wrong elements that have been added during the forward step. All of these algorithms are sufficient but cannot obtain the optimal solution.

Simulated annealing (SA) [8] is an efficient heuristic algorithm to find the optimal solution for many problems such as optimization problems [8]. Thus, in this paper we aim to utilize the advantage of greedy algorithm in term of simple implementation to integrate it with SA algorithm. To achieve this aim, this paper propose a Compressive Sparse Binary Signals Reconstruction Algorithm Using Simulated Annealing (CSBCSA) which combine between greedy algorithm and SA algorithm. During this paper, we are interested on reconstruct binary data $x \in [0, 1]$. Our contributions can be summarized as follows:

- Convert the CS reconstruction problem into optimization problem
- Utilizes the advantage of SA algorithm in term of finding the optimal solution to solve this optimization problem.
- Proposes an efficient fitness function that aims to improve the performance of the proposed algorithm
- The simulation results explains that the reconstruction performance of the proposed algorithm outperforms existing baseline algorithms.

The rest of the paper is organized as follows: Section 1 presents Simulated Annealing algorithm background . In Section 2, the proposed reconstruction algorithm is described. In Section 3, we present the performance results of our approach and the comparison with existing algorithms. In Section 4, conclusion is presented.

section SA Algorithm Background

Metals annealing process is stimulated by iterative random search algorithm which is called Simulated annealing (SA) [18]. The aim of SA is to find a new solution by searching into current solution's neighbors. Due to the comparison of the new solution with the current one in each iteration, SA can avoid being stuck in local optimum. In case new solution is better (based on its fitness value), it is selected and saved by SA as the base for its next iteration. Moreover, in contrast to all other algorithms, SA has an ability to move to this solution (depending on acceptance probability) without ignoring it. In Algorithm SA algorithm's main procedure is summarized 1.

Algorithm 1 SA Algorithm

initialize Temperature T and minimum temperature T_{min}
Maximum number of iterations i_{max}
SA generates a random solution *sol*
Calculate the fitness value for the *sol* by using the predefined fitness function $cost(sol)$
old_{cost}= cost(sol)
while ($T > T_{min}$) **do**
    i=1
    while ($i < i_{max}$) **do**
        new_{sol} = neighbor(sol)
        new_{cost} = cost(new_{sol})
        ap = $acceptance_{probability}(old_{cost}, new_{cost}, T)$
        if (ap < random()) **then**
            sol = new_{sol}
            $old_{cost} = new_{cost}$
        end if
        $i = i + 1$
    end while
    Update T
end while
Return sol, cost

2 THE PROPOSED CSBCSA ALGORITHM

In this section, the proposed reconstruction algorithm which called Compressive Sparse Binary Signals Reconstruction Algorithm Using Simulated Annealing (CSBCSA) is going to be explained. CSBCSA consists of three phases: initialization phase, selection phase and stop criteria phase. CSBCSA initializes as any greedy algorithms by selecting the largest K amplitude components from the Matched Filter Detection process $\Phi^t y$ i.e $H = max_K(\Phi y^t)$. In addition to, CSBCSA initializes the SA algorithm parameters such as T_{min} and T. Then, CSBCSA starts the selection phase in which: firstly, CSBCSA randomly selects indices of q columns from the matrix Φ such that $q = M/2 - K$. Secondly, CSBCSA creates the set C which equal to the union of set H and q.

Then CSBCSA solves the least square problems $\Phi_C^{\dagger} y$ and selects the largest K amplitude components from $I = max_K(\Phi_C^{\dagger} y)$, where I is called support set, as a solution to this iteration, where Φ_C are the columns of Φ with indies equal to C and † means pseudo inverse. These solution I is then evaluated by using the fitness function $F(I)$. Finally, CSBCSA checks the the stopping criteria to decides either to stop if the number of iteration exceeds the maximum number of iteration O_{max} or $F(I) = 0$, or Update the temperature T and increase the number of iterations to repeat the selection phase.

2.1 Fitness Function

in this section, we propose the following fitness function to be used during the selection phase:

LEMMA 2.1. *Assume that we have the original signal x is binary vector such that $x \in [0, 1]$ and y is the compressed samples of x such that $y = \Phi x$ then the estimated solution x' with support set $I = \{I_1, I_2, \ldots .I_K\}$ is the correct solution i.e $x' = x$ if and only if* $F(I) = y - (\sum_{i=1}^{i=K} \Phi_{I_i}) = 0$. proof: *according to CS theory, the compressed samples y is generated from the multiplication result of the non zeros values of x with the corresponding columns from the matrix Φ. Let $I = I_1, I_2, \ldots .I_K$ is the indices of non-zero values of x then the CS can be expressed as:*

$$y = \Phi_I x_I, \textit{where}\ \Phi_I = \begin{pmatrix} \Phi_{I_1} & \Phi_{I_2} & . & . & \Phi_{I_K} \end{pmatrix} \textit{and}\ x_I = \begin{pmatrix} x_{I_1} \\ x_{I_2} \\ . \\ . \\ x_{I_K} \end{pmatrix} \tag{3}$$

Eq.3 can be written as:

$$y = \Phi_{I_1} x_{I_1} + \Phi_{I_2} x_{I_2} + \ldots.. + \Phi_{I_K} x_{I_K} \tag{4}$$

Since x is a binary vector, so all the non-zero values equal to 1. Then Eq.4 can be expressed as:

$$y = \Phi_{I_1} + \Phi_{I_2} + \ldots. + \Phi_{I_K} = \sum_{i=1}^{K} \Phi_{I_i} \tag{5}$$

from Eq.5, it is clear that if the support set I is correct then $y - (\sum_{i=1}^{i=K} \Phi_{I_i}) = 0$ ■.

So, our algorithm aims to find the estimated signal x' with the support set I that achieves $F(I) = 0$. To clarify Lemma2.1 we provide the following example:

Let $x = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\Phi = \begin{pmatrix} 0.1818 & 0.1361 & 0.5499 & 0.6221 \\ 0.2638 & 0.8693 & 0.1450 & 0.3510 \\ 0.1455 & 0.5797 & 0.8530 & 0.5132 \end{pmatrix}$.
From vector x we can say that the non-zero values of x are located at indices $I = \{1, 3\}$. Then the compressed samples vector y can be computed as following:

$$y = \Phi_I x(I) = \begin{pmatrix} 0.1818 \\ 0.2638 \\ 0.1455 \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix} + \begin{pmatrix} 0.5499 \\ 0.1450 \\ 0.8530 \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix} = \begin{pmatrix} 0.7317 \\ 0.4088 \\ 0.9986 \end{pmatrix}$$

Assume that the support set I is equal to $I = \{1, 4\}$, then $\sum_{i=1}^{K} \Phi_{I_i} = \begin{pmatrix} 0.1818 \\ 0.2638 \\ 0.1455 \end{pmatrix} + \begin{pmatrix} 0.6221 \\ 0.3510 \\ 0.5132 \end{pmatrix} = \begin{pmatrix} 0.8039 \\ 0.6148 \\ 0.6588 \end{pmatrix}$ According to the proposed fitness function $F(I) \neq 0$, then I isn't the correct solution.

2.2 CSBCSA Description

This section provides the detailed descriptions of the proposed algorithm. During the initialization phase, CSBCSA initializes all SA parameters such as T_{min} and T. In addition to, CSBCSA uses Matched Filter Detection process Φy^t and selects the largest K amplitude components from it to initialize the support set H and then calculates the fitness value to H i.e. $F(H)$ using the proposed lemma. The selection Phase is considered the main search steps, in which the CSBCSA starts by selects random q columns from the matrix Φ ,where $q = M/2 - K$ depends on the fact that the CS reconstruction problem can be resolved if the sparsity level $K \leq M/2$ [15].

Then CSBCSA creates the set $C = q \cup H$ to solve the least square problem $\Phi_C^{\dagger} y$ and creates the support set I which contains the largest K amplitude components in $\Phi_C^{\dagger} y$. This support set is then evaluated by using the proposed fitness function Lemma2.1. CSBCSA algorithm checks the value of $F(I)$, If $F(I) < F(H)$ then set $H = I$ and $F(H) = F(I)$. The inner loop is repeated till the maximum inner iterations In_{max} is reached. Finally, CSBCSA algorithm checks the the stoping criterion i.e. the number of outer loop iterations exceed the maximum numbers Out_{max} or $F(I) = 0$. If the stoping criterion is met then the estimated signal will calculated as $x'_I = \Phi_I y$ and $x'_{N-I} = 0$. Otherwise, CSBCSA updates the T value and repeats the selection phase. CSBCSA is summarized in Algorithm 2.

Algorithm 2 CSBCSA Algorithm

Input: The compressed sample= y, selection size= q and CS matrix Φ
Initialization Phase
Maximum number of inner iterations In_{max} and Maximum number of outer iterations Out_{max}
initialize Temperature T, minimum temperature T_{min}, outer loop counter $O_i = 1$ and inner loop counter $E_i = 1$
H={the largest K amplitude components in $\Phi^t y$}
Calculates the fitness value of H i.e $F(H)$using Lemma .2.1
while Stopping criterion is not met || $O_i < Out_{max}$ **do**
  Selection Phase:
  while $E_i \leq In_{max}$ **do**
    Choose randomly q columns from Φ
    $C = H \cup q$
    I = {the largest K amplitude components in $\Phi_C^{\dagger} y$ }
    Calculates $F(I)$ using Lemma 2.1
    if $F(H) > F(I)$ **then**
      H=I and $F(H) = F(I)$
    end if
    $E_i = E_i + 1$
  end while
  Stopping Criteria Phase
  if $O_i \geq Out_{max} || F(I) == 0$ **then**
    Stop, and Return $x'_I = \Phi_I y$ and $x'_{N-I} = 0$
  end if
  Updates T and $O_i = O_i + 1$
end while
Output: x'

3 SIMULATION RESULTS

In this section, MATLAB environment is used for performing all simulations and we use Gaussian and Bernoulli matrices Φ with size $M \times N$, where $N = 256$ and $M = 128$ as Φ CS matrix. CSBCSA algorithm is applied to reconstruct computer-generated sparse binary signals. We evaluate the performance of LSARA reconstruction algorithm in comparison to OMP[11], COSAMP [14] and SP [15] in term of Average Normalized Mean Squared Error (ANMSE) which can be defined as average $\|L + \|_2$ difference between the original reading and the reconstructed one, divided by $\|x\|_2$ which can be expressed as: $\frac{\|x - x'\|_2}{x}$ where x is the original signal and x' is the estimated one, and the average runtime.

In Fig. 1 where the Gaussian matrix is used to compress the sparse binary signal, CSBCSA algorithm clearly provides lower ANMSE comparing to COSAMP, OMP and SP. In addition, ANMSE for RSMP algorithm is started to increase only when $K > 58$ while it increases when $K > 45$, $K \geq 41$ and $K \geq 48$ for COSAMP, OMP and SP algorithms respectively as shown in Fig 1.

Fig.2 shows ANMSE results where the Bernoulli matrix is used to compress the sparse binary signal. In Fig.2, CSBCSA algorithm still provides lowest ANMSE result comparing to COSAMP, OMP and SP, as $K > 56$,$K \geq 45$, $K > 38$ and $K > 49$, respectively.

In Fig.3, we aims to test CSBCSA reconstruction performance when different measurement vector lengths- M are used with Gaussian CS matrices distribution matrix. To achieve this aim, the length

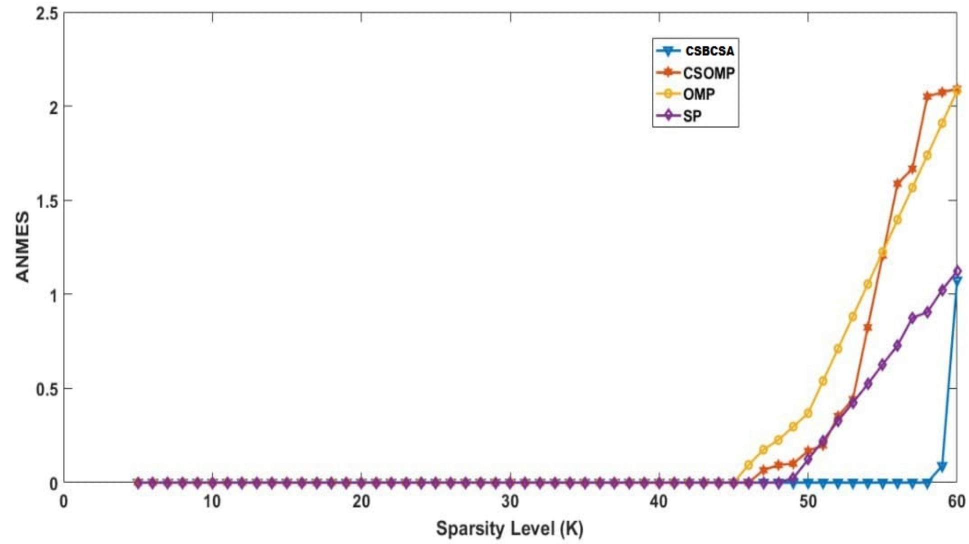

Figure 1: ANMSE vs Sparsity Level for sparse binary signals using Gaussian matrix.

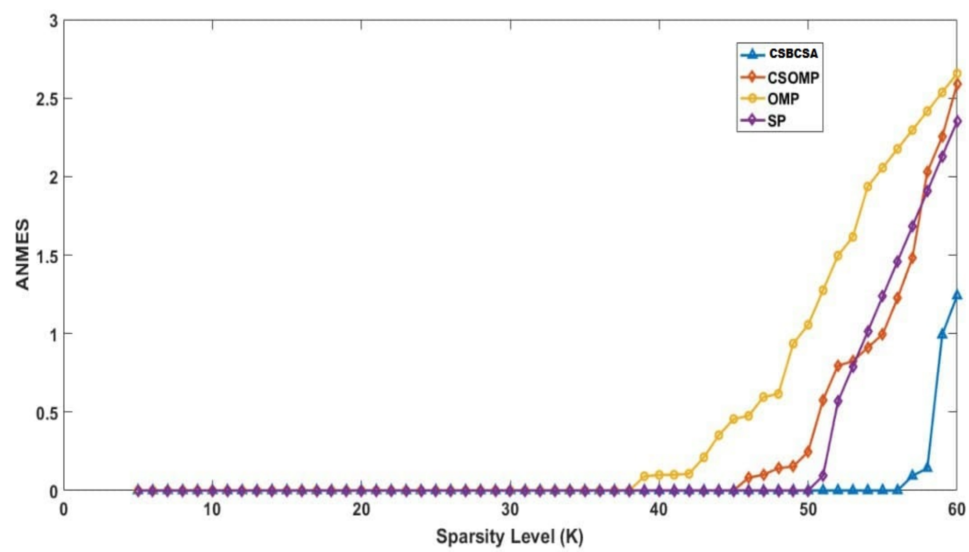

Figure 2: ANMSE vs Sparsity Level for sparse binary signals using Bernoulli matrix.

of the sparse binary signals drawn from uniform distribution is $N = 120$ is used and M values ranges from 10 to 60 with step size 1. From those figures, we observe that CSBCSA algorithm still provides the lowest ANMSE values comparing to the others.

From Fig.4,it is clear that CSBCSA average run time is higher than the greedy algorithm but still slightly fast.

4 CONCLUSION

In this paper, we proposed CSBCSA algorithm to reconstruct sparse binary signals. CSBCSA integrates between the advantages of the greedy algorithms in easy and fast implementation with the advantage of SA algorithm in finding the optimal solution to improve the reconstruction performance. In addition to, we proposed an efficient fitness function which helped the proposed algorithm to reconstruct the binary signals in perfects way. The simulation results

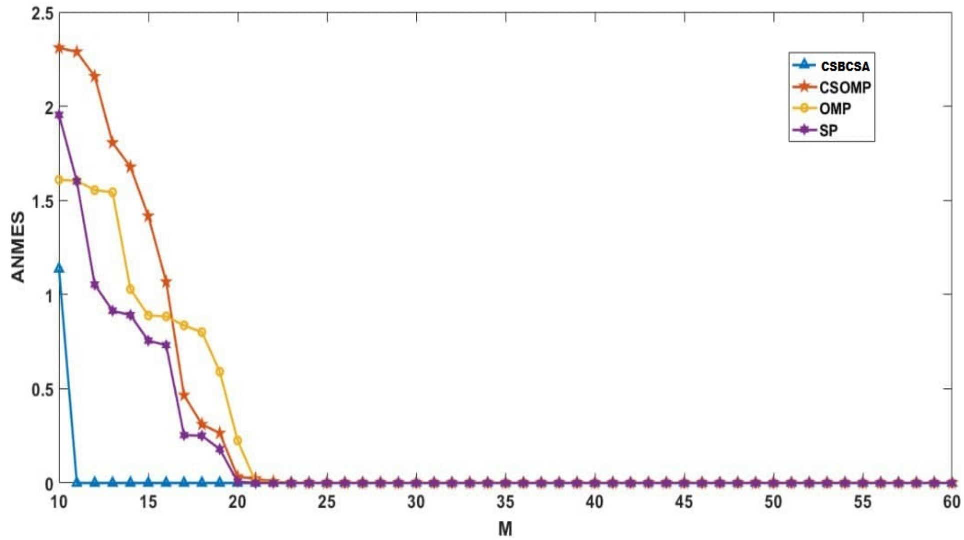

Figure 3: Reconstruction results over Gaussian matrix with different length of M.

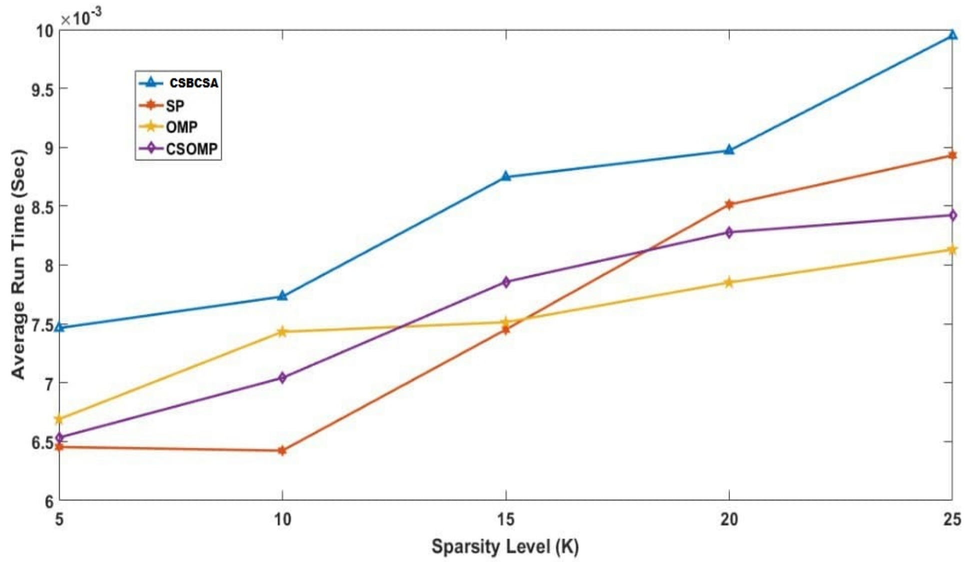

Figure 4: Average run times as a function of sparsity level.

show that CSBCSA outperformed the reconstruction performance of the baselines algorithms with acceptable run time.

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