Problem of Optimum Control Connected with Environmental Problems

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ABSTRACT

The problem of optimization of time of passing through area is considered by an object which behavior is described by a linear differential system with delay. Necessary conditions of optimality in the form of the maximum principle of Pontryagin are obtained. It is based on the reduction of an initial task to the general problem of mathematical programming and use of the rule of multipliers of Lagrange on any linear spaces. Differential properties of the selected criterion of quality are investigated.

CCS CONCEPTS

• **e-government**; • **e-government development index**; • **GDP**; • **human capital index**; • **unemployment rate**; • **online service index**; • **tax revenue**; • **telecommunication infrastructure index**; • **inflation rate**; • **agriculture**; • **industry**;

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1 INTRODUCTION

The problem of optimization of time of passing through area is considered by an object which behavior is described by a linear differential system with delay. Necessary conditions of optimality in the form of the maximum principle of Pontryagin are obtained. It is based on the reduction of an initial task to the general problem of mathematical programming and use of the rule of multipliers of Lagrange on any linear spaces. Differential properties of the selected criterion of quality are investigated.

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1.1 Accessibility

Problem statement and necessary facts

The task has the source important practical problems. It solves such tasks as minimization of time of passing in the infected area, time of passing of the airplane through area and others nonconventional problems of optimum control.

For clearness we will remind that in a task of management in sense of high-speed performance it is required to minimize time of hit of a dynamic system from start state of x_0 in end state of x_1.

Let's assume, now, between areas of start and end states of a dynamic system (is higher as x_0and x_1it was possible to consider such areas) there is some area. It is required to select such management that the time spent of a dynamic system in this area would be minimum (Fig. 1).

Indeed, in reality in the problems of optimum control connected with ecology there is a problem when it is required to find a trajectory of some dynamic system which the minimum time is in the infected area, and the area can move over time. For example, when there is a need to pass through radiation infected cloud moved with wind. To similar setting life can the problem of the fastest passing by airplane of the storm front at its sudden, not predicted emergence and impossibility of a bypass is reduced.

If in a problem of optimum high-speed performance to leave all conditions the same, and in integrated functionality to replace sub integral function with the function having an appearance

$$f^0(x,u) \equiv \delta(x,u) = \begin{cases} 1, if x belongs to set \\ (infected)\ area, \\ 0,\ if x does not belong to set \\ (infected)\ area, \end{cases}$$

that is obvious that the time spent of a trajectory x(•) in the set, perhaps moving area, is expressed by integral

$$I \equiv T(x(\cdot)) = \int_0^1 \delta(x(t), u(t), t)\,dt,$$

where the integral is understood in Lebesgue's sense. The feature of this functionality, consists that in it is mute under integral function discontinuous (i.e. there was a rough criterion of quality of management) that direct use of classical results of the theory of optimum processes does not allow. Such setting a task almost literally matches problem definition of optimum high-speed performance, except for one, we minimize functionality is not normal. Therefore, its research presents to a certain difficulty [3, 17].

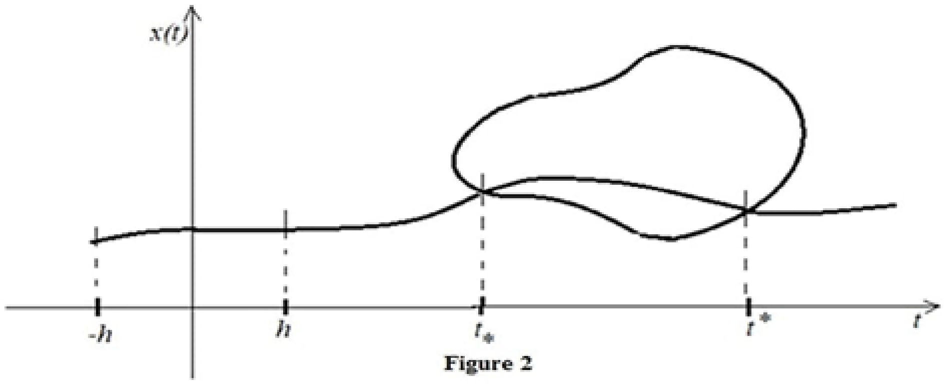

Figure 1: Illustration of management task performance

In the minimized integrated functionality sub-integral function is explosive function that direct use of the maximum principle of Pontryagin does not allow. Assessment of the minimized functionality demands introduction of conditions of coherence of a class of managements and the set area that qualitatively changes the analysis of a problem. It is supposed that the area of start and end states is not crossed from the set area.

Let the behavior of an object be defined by the system of differential equations as follows

$$\dot{x}(t) = Ax(t) + A_1 x(t-h) + Bu(t)$$

where $x(t)$ - a n-dimensional vector function of a statusA, A_1, B – a matrix of the corresponding dimensions, $h > 0$- continuous delay, $u(t)$ - m - a measured vector function of management from the set class of piecewise continuous vector functions of U at $t \in [0,1]$.

The following set is given.

$$M_i = \left\{x \in R^n : \varphi_j(x) \le 0, \varphi_j(x) \in C^{(1)}, j = 1, ..s\right\},$$

multiple-valued mapping $M(\cdot)$of the segment $[0,1]$on the set of all possible subsets of space of $[0,1]$

$$M(t) = \left\{x \in R^n : \varphi_0(x,t) \le 0, t \in [0,1]\right\},$$

where $\varphi_0(x,t)$- continuously differentiable function on both variables and initial condition

$$x(t) = x_0(t), t \in [-h, 0],$$

satisfy conditions

$$M_1 \cap M(1) = \emptyset, \qquad x(0) \notin M(0).$$

It is required to select a control of $u \in U$ so, that $x(1) \in M_1$, and the time during which the inclusion of $x(t) \in M(t)$is executed would be minimum.

Introducing the characteristic function

$$\delta(x,t) = \begin{cases} 1, & x \in M(t) \\ 0, & x \notin M(t), \end{cases}$$

the initial problem can be formulated as follows

$$T(x(\cdot)) = \int_0^1 \delta(x(t), t)\, dt \to min, \tag{1}$$

under conditions

$$\dot{x}(t) = Ax(t) + A_1 x(t-h) + Bu(t), \tag{2}$$

$$x(t) = x_0(t), \quad t \in [-h, 0], \tag{3}$$

$$\mathrm{M}_1 \cap \mathrm{M}(1) = \emptyset, \qquad \mathrm{x}(0) \notin \mathrm{M}(0) \tag{4}$$

$$x(1) \in M_1, \tag{5}$$

$$u(t) \in U, \quad t \in [0,1]. \tag{6}$$

1.2 Definition 1.

Let's say that the coherence condition is satisfied, if solutions of a system (2) with initial condition (3) at $u(t) \in U, \quad t \in [0,1]$belong to a subspace of L, spaces of absolutely continuous functions and derivative

$$\frac{d}{dt}\varphi_0(x(t), t) = \varphi'_{0x}(x(t), t)\dot{x}(t) + \dot{\varphi}_{0t}(x(t), t),$$

exists for all $t \in [0,1]$ and it is continuous at t

For a coherence condition illustration, we will consider $L_B \in R^n$subspace which pulled to column-vectors of B matrix, its orthogonal complement of $L_B^L$and the operator of orthogonal projection$P$ in space of $L_B$of a type of $= B$. Let's take a function $\varphi_0$in the following form

$$\varphi_0(x,t) = \varphi((I-P)x, t),$$

where I - unity operator. Then

$$\frac{d}{dt}\varphi_0(x(t), t) = \frac{d}{dt}\varphi((I-P)x, t)$$
$$= \varphi'_x((I-P)x(t), t)(I-P)(Ax(t) + A_1 x(t-h)) + \varphi'_t((I-P)x(t), t)^{\cdot}$$

From this follows that $\frac{d}{dt}\varphi_0(x(t), t)$- is continuous, and the subspace of $L_B$consists of absolutely continuous functions for which the vector $(I-P)x(t)$ has continuous derivatives.

Let's assume that the trajectory $x(t)$, $t \in [0,1]$of the system (2) belonging to space of L, satisfying conditions (3) – (6) and crosses a set M (t) at two points of $t_*$and t^* (see fig. 1). At the same time $t^*$is an input moment of trajectory to $M(t)$and t^* an output moment of trajectory from $M(t)$ (Fig. 2)

The following relations ate satisfied at points $t_*$and t^*.

$$\varphi_0(x(t_*), t_*) = 0, \varphi_0\left(x\left(t^*\right), t^*\right) = 0.$$

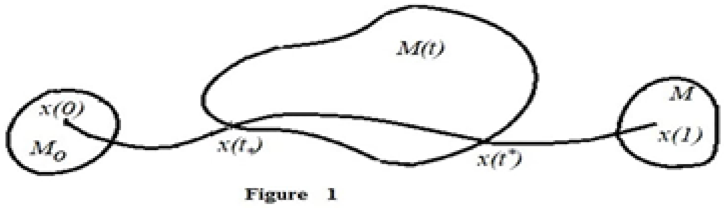

Figure 2: A coherence condition illustration

1.3 Definition 2.

We will call the points $t_*$and t^* regularity points of a trajectory $x(t)$, if

$$\frac{d}{dt}\varphi_0(x(t),t)|_{t=t_*} = \varphi'_{0x}(x(t_*),t_*)x'(t_*) + \varphi'_{0t}(x(t_*),t_*) = \gamma(x(\cdot),t_*) \neq 0, \quad (7)$$

$$\frac{d}{dt}\varphi_0(x(t),t)|_{t=t^*} = \varphi'_{0x}(x(t^*),t^*)x'(t^*) + \varphi'_{0t}(x(t^*),t^*) = \gamma(x(\cdot),t^*) \neq 0.$$

Because of the above assumptions, (1) can be written as follows

$$T(x(\cdot)) = t^* - t_* \quad (8)$$

The problems (1) – (6), taking into account the above assumptions and a type of sets $M(t)$and M_1 are reduced to the general problem of mathematical programming

$$T(x(\cdot)) = t^* - t_* \to min,$$

$$\varphi_j(x(1)) \leq 0, j = 1,2,...,s \quad (9)$$

$$x(\cdot) \in E,$$

in which, t_* and t^* - the moments of an input and output to the set $M(t)$, E subspace of space of L, for which in the assumptions of [1] that
there is a convex cone K_E„ such that if $e \in K_E$, then

$$x(\lambda) = x^0 + \lambda e + o(\lambda) \in E, \quad (10)$$

where x^0- a solution of a task (9), λ - rather small positive number.
for $j = 0,1,2,...s$

$$\overline{\lim_{\lambda\to 0}}\frac{\varphi_j(x(\lambda)) - \varphi_j(x^0)}{\lambda} \leq h_j(e),$$

where $x(\lambda)$, $h_j(e)$- functions, convex on K_E, and $domh_j = L$. is defined (10), and. It is fair.

1.4 Theorem 1

[1]. If x^0a solution of a task (9) conditions a), b) also are satisfied, there will be such numbers λ_j, not all equal to zero that

$$\sum_{j=0}^{s}\lambda_j h_j(e) \geq 0 \quad for\ all \quad e \in K_E,$$

$$\lambda_j \geq 0 \quad for j = 0,1,2,...,s, \quad (11)$$

$$\lambda_j\varphi_j\left(x^0\right) = 0, \quad j = 1,2,...,s.$$

2 NECESSARY CONDITIONS OF OPTIMALITY

Let's consider variations of a trajectory $x(\cdot)$problems (9) of a look

$$x(\cdot) + \varepsilon\delta x(\cdot), \quad \delta x(\cdot) \in K$$

and equation

$$g(\varepsilon,t) = \varphi_0(x(t) + \varepsilon\delta x(t)) = 0 \quad (12)$$

The equation 12) defines t implicitly as function ε, i.e$t = t(\varepsilon)$. Really, since$(0,t_*) = 0$ also satisfied a coherence condition

$$\left.\frac{\partial g(t,\varepsilon)}{\partial t}\right|_{\substack{\varepsilon=0\\ t=t_*}} = \gamma(x(\cdot),t_*) \neq 0,$$

$$\left.\frac{\partial g(t,\varepsilon)}{\partial\varepsilon}\right|_{\substack{\varepsilon=0\\ t=t_*}} = \varphi'_{0x}(x(t_*),t_*)\delta x(t_*),$$

taking into account continuous differentiability of function $g(\varepsilon,t)$ in the neighborhood of a point $(0,t_*)$on variables, the approvals of the theorem of implicit functions are fair [2]:

The equation 12) is solvable concerning t, i.e. there is a display $t = t(\varepsilon)$differentiated in a point $\varepsilon = 0$ and

$$\frac{dt(\varepsilon)}{d\varepsilon} = -\frac{\varphi'_{0x}(x(t_*),t_*)\delta x(t_*)}{\gamma(x(\cdot),t_*)}. \quad (13)$$

Believing $(x(\cdot) + \varepsilon\delta x(\cdot),t_*) = t(\varepsilon)$, (13) it is possible to consider as a derivative in the direction $\delta x(\cdot)$functions $t(x(\cdot),t_*)$, i.e.

$$t'(x(\cdot),\varepsilon\delta x(\cdot),t_*) = -n(x(\cdot),t_*)\delta x(t_*) \quad (14)$$

$$n(x(\cdot),t_*) = \frac{\varphi'_{ox}(x(t_*),t_*)\delta x(t_*)}{\gamma(x(\cdot),t_*)} \quad (15)$$

external normal to a set of $M(t)$in a point $x(t_*)$.

Replacing a point $(0,t_*)$with a point $(0,t^*)$ and repeating the previous reasoning, in (14), (15) instead of the moment of t_*having used t^* moment, we will determine a derivative by the direction $\delta x(\cdot)$ for $T(x(\cdot))$ (see (8)):

$$T'(x(\cdot),\delta x(\cdot)) = -n\left(x(\cdot),t^*\right)\delta x\left(t^*\right) + n(x(\cdot),t_*)\delta x(t_*) \quad (16)$$

2.1 Theorem 2.

(necessary conditions of optimality). Let $x^0(\cdot)$ and $u^0(\cdot)$an optimum trajectory and the optimum control corresponding to it in a task (9). If conditions of coherence and in points of t_q, $q = 1,3,...,2m-1$ an input in a set of $M(t)$, and exit of t from it $t_{q+1}, q = 1,3,$ are satisfied $2m-1$ the optimum trajectory is regular, then there are numbers such not all equal to zero $\lambda_j > 0, j = 0,1,2,...s$ and a vector function $\psi(\tau)$, $\tau \in (t_q, t_{q+1})$, $q = 1, 3, ..., 2m-1$ that ratios take place

$$\dot{\psi}(\tau) = -A'\psi(\tau), \quad \tau \in (t_q, t_{q+1}), \quad q = 1, 3, ..., 2m-1 \tag{17}$$

$$\psi(t_q + 0) - \psi(t_q - 0) = (-1)^q n\left(x^0(\cdot), t_q\right), \quad q = 1, 2, ..., 2m \tag{18}$$

$$\psi(1) = \sum_{j=1}^{s} \lambda_j \varphi'_{jx}\left(x^0(1)\right) \tag{19}$$

$$\psi'(1)x^0\left(1, u^0(\cdot)\right) = \min_{u(\cdot)\in U} \psi'(1)x(1, u(\cdot)) \tag{20}$$

2.2 Proof.

Let $x^0(\cdot)$- the optimum trajectory having only one point of entry to area $M(t)$and an exit from it, $u^0(\cdot)$- the optimum control corresponding to it. Then variations of an optimum trajectory will be $x^0(\cdot)+\varepsilon\delta x^0(\cdot)$, $\quad \delta x^0(\cdot) \in E$, and a derivative in the direction $\delta x^0(\cdot)$ for$T(x^0(\cdot)) = t_{q+1} - t_q$

$$\begin{aligned} T\left(x^0(\cdot), \delta x^0(\cdot)\right) = -n\left(x^0(\cdot), t_{q+1}\right)\delta x^0\left(t_{q+1}\right) \\ + n\left(x^0(\cdot), t_q\right)\delta x^0\left(t_q\right) \end{aligned} \tag{21}$$

Taking into account the assumption b) in item 1

$$\overleftarrow{\min_{\varepsilon\to 0}} \frac{T\left(x^0(\cdot) + \varepsilon\delta x^0(\cdot), t\right) - T\left(x^0(\cdot)\right)}{\varepsilon} = T'\left(x^0(\cdot), \delta x^0(\cdot)\right) \tag{22}$$

and

$$\begin{aligned} \frac{d\varphi_j(x^0(1)+\varepsilon\delta x^0(1))}{d\varepsilon} &= \overline{\lim_{\varepsilon\to 0}} \frac{\varphi_j(x^0(1)+\varepsilon\delta x^0(1)) - \varphi_j(x^0(1))}{\varepsilon} \\ &= \varphi'_{jx}x^0(1))\delta x^0(1), \quad j = 1, 2, \ldots, s \end{aligned} \tag{23}$$

By the form (21), (22), their (23) right parts linear functions on $\delta x^0(\cdot)$.

Thus, all conditions of the theorem 1 and, therefore, are satisfied if $x^0(\cdot)$ a solution of a task (9), then exist such numbers $\lambda_j \geq 0, j = 0, 1, 2, ..., s$ not all equal to zero that for all variations of a trajectory $\delta x^0(\cdot)$

$$\lambda_0 T'\left(x^0(\cdot), \delta x^0(\cdot)\right) + \sum_{j=1}^{s} \lambda_j \varphi'_{jx}\left(x^0(1)\right)\delta x^0(1) \geq 0. \tag{24}$$

Let's integrate on segment $[t-\bar{h}, t]$a system (2) with initial condition (3) where $\bar{h}(\bar{h} < h)$ - it is commensurable with any point of $k\bar{h}$ (k – the integer positive number) an interval $[0, t)$, we have

$$x(t) = e^{A\bar{h}}\left[x\left(t-\bar{h}\right) + \int_{t-\bar{h}}^{t} e^{-A\tau}\left(A_1 x(\tau - h) + Bu(\tau)\right)d\tau\right]. \tag{25}$$

Let's replace in (25)$x(t-\bar{h})$the expression received by integration of a system (2) on segment$[t-2\bar{h}, t-\bar{h}]$

$$\begin{aligned} x(t) &= e^{A\bar{h}}\left[e^{A\bar{h}}\left[x\left(t-\bar{h}\right) + \int_{t-2\bar{h}}^{t-\bar{h}} e^{-A\tau}\left(A_1 x(\tau-h) + Bu(\tau)\right)d\tau\right]\right. \\ &\quad \left. + \int_{t-\bar{h}}^{t} e^{-A\tau}\left(A_1 x(\tau-h) + Bu(\tau)\right)d\tau\right] = \\ &= e^{2A\bar{h}}x\left(t-2\bar{h}\right) + e^{2A\bar{h}}\int_{t-2\bar{h}}^{t-\bar{h}} e^{-A\tau}\left(A_1 x(\tau-h) + Bu(\tau)\right)d\tau \\ &\quad + e^{A\bar{h}}\int_{t-\bar{h}}^{t} e^{-A\tau}\left(A_1 x(\tau-h) + Bu(\tau)\right)d\tau. \end{aligned}$$

Having used the expressions received by integration of a system (2) on segments

$$\left[t-3\bar{h}, t-2\bar{h}\right], \left[t-4\bar{h}, t-3\bar{h}\right], ..., \left[t-k\bar{h}, t-(k-1)\bar{h}\right],$$

$$x(t) = e^{kA\bar{h}}x\left(t-k\bar{h}\right) + \sum_{i=1}^{k} e^{kA\bar{h}}\int_{t-i\bar{h}}^{t-(i-1)\bar{h}} \alpha(\tau)\,d\tau \tag{26}$$

where

$$\alpha(\tau) = e^{-A\tau}\left(A_1 x(\tau-h) + Bu(\tau)\right).$$

Letss_1, s_2, s_3 - integer positive numbers, such that$t_q = s_1\bar{h}, t_{q+1} = s_2\bar{h}, 1 = s_3\bar{h}, \quad h = s\bar{h}$. Then, using representation (26), we have

$$x\left(t_q\right) = e^{s_1 A\bar{h}}x(0) + \sum_{i=1}^{s_1} e^{iA\bar{h}}\int_{t_q-i\bar{h}}^{t_q-(i-1)\bar{h}} \alpha(\tau)\,d\tau \tag{27}$$

$$x\left(t_{q+1}\right) = e^{s_2 A\bar{h}}x(0) + \sum_{i=1}^{s_2} e^{iA\bar{h}}\int_{t_{q+1}-i\bar{h}}^{t_{q+1}-(i-1)\bar{h}} \alpha(\tau)\,d\tau \tag{28}$$

$$x(1) = e^{s_3 A\bar{h}}x(0) + \sum_{i=1}^{s_3} e^{iA\bar{h}}\int_{1-i\bar{h}}^{1-(i-1)\bar{h}} \alpha(\tau)\,d\tau \tag{29}$$

We use representations (21), (27) – (29) in the ratio (24) we have

$$\begin{aligned} &\lambda_0\left[-n\left(x^0(\cdot), t_{q+1}\right)\delta x^0\left(t_{q+1}\right) + n\left(x^0(\cdot), t_q\right)\delta x^0\left(t_q\right)\right] + \\ &\quad + \sum_{j=1}^{s} \lambda_j \varphi'_{jx}\left(x^0(1)\right)\delta x^0(1) \geq 0 \end{aligned} \tag{30}$$

where

$$\delta x^0(t) = x(t) - x^0(t).$$

Let's define$\delta x^0(t_q), \quad \delta x^0(t_{q+1}), \quad \delta x^0(1)$it agrees (27) – (29).

$$\delta x^0\left(t_q\right) = e^{s_1 A\bar{h}}\left[x(0) - x^0(0)\right] + \sum_{i=1}^{s_1} e^{iA\bar{h}}\int_{t_q-i\bar{h}}^{t_q-(i-1)\bar{h}} \delta\alpha^0(\tau)\,d\tau \tag{31}$$

$$\delta x^0\left(t_{q+1}\right) = e^{s_2 A\bar{h}}\left[x(0) - x^0(0)\right] + \sum_{i=1}^{s_2} e^{iA\bar{h}}\int_{t_{q+1}-i\bar{h}}^{t_{q+1}-(i-1)\bar{h}} \delta\alpha^0(\tau)\,d\tau \tag{32}$$

$$\delta x^0(1) = e^{s_3 A\bar{h}}\left[x(0) - x^0(0)\right] + \sum_{i=1}^{s_3} e^{iA\bar{h}}\int_{1-i\bar{h}}^{1-(i-\bar{1})h} \delta\alpha^0(\tau)\,d\tau. \tag{33}$$

here
$\delta\alpha^0(\tau) = \alpha(\tau) - \alpha^0(\tau) = e^{-A\tau}[A_1(x(\tau-h) - x^0(\tau-h)) + B(u(\tau) - u^0(\tau))]$.

Let's enter notation

$$\sigma(i, \tau, t) = \begin{cases} e^{iA\bar{h}}, & \tau \in [t-i\bar{h}, t-(i-1)\bar{h}], \\ 0, & \tau \notin [t-i\bar{h}, t-(i-1)\bar{h}]. \end{cases}$$

Then (31) – (33) it is possible to write in a look

$$\delta x^0\left(t_q\right) = e^{s_1 A\bar{h}}\left[x(0) - x^0(0)\right] + \int_0^1 \sum_{i=1}^{s_1} \sigma\left(i, \tau, t_q\right)\delta\alpha^0(\tau)\,d\tau \tag{34}$$

$$\delta x^0\left(t_{q+1}\right) = e^{s_2 A\bar{h}}\left[x(0) - x^0(0)\right] + \int_0^1 \sum_{i=1}^{s_2} \sigma\left(i, \tau, t_{q+1}\right)\delta\alpha^0(\tau)\,d\tau \tag{35}$$

$$\delta x^0(1) = e^{s_3 A\bar{h}}\left[x(0) - x^0(0)\right] + \int_0^1 \sum_{i=1}^{s_3} \sigma(i, \tau, 1)\,\delta\alpha^0(\tau)\,d\tau. \tag{36}$$

Taking into account (34) – (36), the ratio (30) will change a form

$$\begin{aligned}&\lambda_0\left[-n\left(x^0(\cdot),t_{q+1}\right)\left[\sigma\left(s_2,0,t_{q+1}\right)\left(x(0)-x^0(0)\right)\right.\right.\\&\left.\left.+\int_0^1\sum_{i=1}^{s_2}\sigma\left(i,\tau,t_{q+1}\right)\delta\alpha^0(\tau)\,d\tau\right]\right]+\\&\lambda_0\left[-n\left(x^0(\cdot),t_q\right)\left[\sigma\left(s_1,0,t_q\right)\left(x(0)-x^0(0)\right)\right.\right.\\&\left.\left.+\int_0^1\sum_{i=1}^{s_1}\sigma\left(i,\tau,t_q\right)\delta\alpha^0(\tau)\,d\tau\right]\right]+\\&\sum_{j=1}^{s}\lambda_j\varphi'_{jx}\left(x^0(1)\right)\left[\sigma\left(s_3,0,1\right)\left(x(0)-x^0(0)\right)\right.\\&\left.+\int_0^1\sum_{i=1}^{s_3}\sigma(i,\tau,1)\,\delta\alpha^0(\tau)\,d\tau\right]\geq 0\end{aligned}\tag{37}$$

Summarizing (21) for m of points of entry and an exit of an optimum trajectory of $x^0(\cdot)$ in numerous of $M(t)$, we will bring a ratio (37) related to

$$\begin{aligned}&-\lambda_0\sum_{q=1}^{m}n\left(x^0(\cdot),t_{2q}\right)\int_0^1\sum_{i=1}^{s_3}\sigma\left(i,\tau,t_{2q}\right)\delta\alpha^0(\tau)\,d\tau+\\&+\lambda_0\sum_{q=1}^{m}n\left(x^0(\cdot),t_{2q-1}\right)\int_0^1\sum_{i=1}^{s_3}\sigma\left(i,\tau,t_{2q-1}\right)\delta\alpha^0(\tau)\,d\tau+\\&+\sum_{j=1}^{s}\lambda_j\varphi'_{jx}\left(x^0(1)\right)\int_0^1\sum_{i=1}^{s_3}\sigma(i,\tau,1)\,\delta\alpha^0(\tau)\,d\tau-\\&-\lambda_0\sum_{q=1}^{m}n\left(x^0(\cdot),t_{2q}\right)\int_0^1\sum_{i=1}^{s_3}\sigma\left(i,\tau,t_{2q}\right)\delta\alpha^0(\tau)\,d\tau+\\&+\lambda_0\sum_{q=1}^{m}n\left(x^0(\cdot),t_{2q-1}\right)\int_0^1\sum_{i=1}^{s_3}\sigma\left(i,\tau,t_{2q-1}\right)\delta\alpha^0(\tau)\,d\tau+\\&+\lambda_0\sum_{q=1}^{s}\lambda_j\varphi'_{jx}\left(x^0(1)\right)\int_0^1\sum_{i=1}^{s_3}\sigma(i,\tau,1)\,\delta\alpha^0(\tau)\,d\tau\geq 0,\ q=\overline{1.m},\end{aligned}$$

Where$s_{1q}$and s_{2q}entire positive integers meeting conditions

$$s_{1q}\bar{h}=t_q,\quad s_{2q}\bar{h}=t_{q+1},\quad q=1,3,5,...,2m-1$$

also we will indicate notation

$$\begin{aligned}\psi(\tau)=&-\lambda_0\sum_{q=1}^{m}n\left(x^0(\cdot),t_{2q}\right)\sum_{i=1}^{s_2}\sigma\left(i,\tau,t_{2q}\right)e^{-A\tau}+\\&+\lambda_0\sum_{q=1}^{m}n\left(x^0(\cdot),t_{2q-1}\right)\sum_{i=1}^{s_1}\sigma\left(i,\tau,t_{2q-1}\right)e^{-A\tau}+\\&\sum_{j=1}^{m}\lambda_j\varphi'_{jx}\left(x^0(1)\right)\sum_{i=1}^{s_3}\sigma(i,\tau,1)\,e^{-A\tau}.\end{aligned}\tag{38}$$

Then the ratio (37) will look like

$$\begin{aligned}&\psi'(0)\left(x(0)-x^0(0)\right)+\int_0^1\psi'(\tau)\left[A_1x(\tau-h)-x^0(\tau-h)\right]d\tau\\&+\int_0^1\psi'(\tau)B\left[u(\tau)-u^0(\tau)\right]d\tau\geq 0.\end{aligned}\tag{39}$$

Multiplying the equation 2) scalar by a vector function $\psi(\tau)$ and the integrative received equality on segment [0.1], we have

$$\begin{aligned}\psi'(1)\,x(1)-\psi'(0)\,x(0)=&\int_0^1\left(\psi'(\tau)+A'\psi(\tau)\right)'x(\tau)\,d\tau\\&+\int_0^1\psi'A_1x(\tau-h)\,d\tau++\int_0^1\psi'(\tau)\,Bu(\tau)\,d\tau.\end{aligned}$$

From the last equality and a ratio (39), equity of a ratio follows (20).

Due to the fact that function $\psi(\tau)$) is continuously differentiated on [0, 1], on intervals $(t_q,t_{q+1}), q=1,3,5,...,2m-1$ implemented ratio (17).

Currently t_q function $\psi(\tau)$changes suddenly, such suddenly changes functions $\sigma(i,\tau,t)$. Follows from expression (38) that upon transition τ through $t_q$moments in expression (38) changes one composed, corresponding to function $\sigma(i,\tau,t_q)$. Considering change of the signs composed for even and odd$q$ in (38) we obtain a ratio (18). Equity a ratio (19), follows from representation (38), at the moment $\tau=1$. The theorem is proved.

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