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Benchmarking of Two Implementations of CMA-ES with Diagonal Decoding on the bbob Test Suite

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ABSTRACT

In this paper, we present a comparative benchmark of two implementations of CMA-ES, both with and without diagonal decoding. The benchmarked variants of CMA-ES with diagonal decoding adaptively split the update of the covariance matrix into an update with the original CMA-ES method and an update with the separable-CMA-ES method. Thus, the diagonal decoding should allow for improved performance on separable functions with minimal loss on nonseparable ones. To gain insight into how diagonal decoding impacts CMA-ES runs, an assessment of the performance gain or loss due to the use of diagonal decoding relative to the original CMA-ES, was performed on bbob problems using the COCO platform. We were also interested in variances that might emerge from the difference in the implementations. The data presented in this paper shows improved performance of the CMA-ES on separable functions when using diagonal decoding, without any apparent loss on nonseparable ones. In addition, a few performance variances were spotted in the weakly structured functions, which appeared uncorrelated with the use of diagonal decoding. However, they can be traced back to implementation differences, such as the stopping conditions that may result in different runs, as the data suggests.

CCS CONCEPTS

•Computing methodologies → Continuous space search;

KEYWORDS

Benchmarking, Black-box optimization

ACM Reference format:

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1 INTRODUCTION

The covariance matrix adaptation evolution strategy CMA-ES [9], is recognized as a state-of-the-art algorithm for continuous black-box optimization. It is a stochastic comparison-based algorithm that maintains a multivariate normal distribution for sampling new candidate solutions. It does so by updating the sampling distribution parameters such as (but not exclusive to) the covariance matrix and the mean vector.

The benchmarked implementations introduce an update of the covariance matrix, with both the original CMA-ES and separable CMA-ES at the same time. The initial assumption is that those

variants should allow for an improvement of the algorithm's performance on separable functions without deteriorating it on non-separable ones.

Using the COCO platform [8], we are interested in assessing the effect of the diagonal decoding acceleration added to the CMA-ES algorithm, on a diverse portfolio of problems in the bbob suite. Moreover, we want to underline any variance in performance that may originate from differences in the implementations.

In this paper we compare 4 variants of the CMA-ES algorithm: the first two are from a new version of `pycma` module [6] that allows for diagonal decoding, referred to as `pycma1` and `pycma0`, respectively with and without diagonal decoding. The second two are from the implementation by Akimoto and Hansen [2] [1] of CMA-ES also with and without diagonal decoding, referred to as `ddcma1` and `ddcma0` respectively.

2 ALGORITHM PRESENTATION

The original CMA-ES algorithm maintains a multivariate normal distribution $\mathcal{N}(m, \sigma^2 DCD)$ for sampling candidate solutions by repeating the steps shown below until at least one of the stopping criteria is met:

- 1) Sample λ candidate solutions $\{s_1, \dots, s_\lambda\} \sim \mathcal{N}(m, \sigma^2 DCD)$.
- 2) Evaluate the sampled candidate solutions on the objective function and sort them $\{f(s_{1:\lambda}) \leq \dots \leq f(s_{\lambda:\lambda})\}$
- 3) Update the mean vector m , the covariance matrix C and the step size σ [9]. However, the diagonal decoding matrix D is not updated instead $D = I$.

Whereas separable-CMA-ES updates D instead of C , which in this case set to I .

The `dd-CMA-ES` on the other hand, which stands for CMA-ES with diagonal decoding, is a variant of CMA-ES that allows for a speed up of the covariance matrix adaptation by splitting the update into D update and C update with different methods. The C matrix is updated by the original CMA-ES method, and the D matrix is updated by the separable CMA-ES method with a larger learning rate. However, this rate is adapted using a dampening mechanism in order to avoid disturbing the update of C , by keeping the Kullback-Leibler divergence of consecutive sampling distributions as small as possible.

3 IMPLEMENTATION AND EXPERIMENTAL PROCEDURE

The `pycma` and `ddcma` implementations benchmarked in this paper are both written in Python, though they are different codes (*e.g.* different stopping conditions). The diagonal decoding is activated by setting the respective options of each algorithm:

- `CMA_diagonal_decoding` $\leftarrow$ 1 for `pycma1`
- `flg_variance_update` $\leftarrow$ True for `ddcma1`

Algorithm	2-D	3-D	5-D	10-D	20-D	40-D
pycma0	3.4	2.9	2.6	3.3	3.2	3.4
pycma1	4.0	3.6	3.3	4.2	4.3	5.0
ddcma0	2.3	2.0	1.8	2.2	2.2	2.5
ddcma1	2.7	2.4	2.1	2.5	2.5	2.7

Table 1: CPU time in 1×10^{-4} seconds per function evaluation for varying dimensions

The code used to run benchmark experiments can be found in the repository [4]. The details of the CMA-ES-2019 algorithm used as baseline can be found in [5]. For all algorithms, the bboB problem is evaluated for the origin of the search space at the beginning of each run to avoid unfair advantage given to algorithms that exploit this strategy. The initial solution is given by the bboB function *problem.initial_proposal*, the initial sigma value is set to 2.0 and the population size is the default value for each dimension [9]. The optimization runs are done with IPOP restarts (*i.e.* the population size is doubled after each restart), with a maximum number of restarts of 9. The restart's initial proposed solution is given by calling the function *problem.initial_proposal*, the rest of the parameters are kept at the implementation's default values. The experiments' initial conditions presented are the same ones used to generate the performance data of CMA-ES-2019 except the budget, which was set to $5 \times 10^5 \times n$ function evaluations, n being the dimensionality of the problem.

4 CPU TIMING

In order to evaluate the duration of each algorithm, we have reported here the timing results when running the experiments with a budget of 1×10^2 times the dimension with a single batch. Both codes were run on a linux machine with 64 cores, Intel®Xeon®E7 to E3 v4 processors. We are interested in CPU time per function evaluation for varying dimensions. The results can be found in Table 1. The numbers in the table should be taken with a grain of salt, as the experiments are handled by a job manager on a shared machine, which makes it difficult to track the conditions and the specs of the processor on which the experiment was run. Nevertheless, the results consistently show that ddcma is faster than pycma with a factor less than 2. This can be attributed to the fact that the latter is a larger module designed for more diverse use of CMA-ES compared to ddcma, which is a lightweight code designed around diagonal decoding.

5 RESULTS

Results from experiments according to [10] and [7] on the benchmark functions given in [3] are presented in Figures 1, 2, 3, 4 and 5 and Tables 2.

The experiments were performed with COCO [8], version 2.6, the plots were produced with version 2.6.

The **expected runtime (ERT)**, used in the tables, depends on a given target precision, $I_{\text{target}} = f_{\text{opt}} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t , summed over all trials and divided by the number of trials that

actually reached f_t [10, 11]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t using, for each trial, either the number of needed function evaluations to reach f_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

The impact of using diagonal decoding. According to the figures 2 and 3, we can clearly confirm that on **the separable functions group**, at lower budgets the algorithms with the diagonal decoding are better than the others. We can also observe that this effect is more expressed with higher dimensions. By looking further into single functions ECDFs we can see clearly in figure 5 that most of the performance gain comes from **f2**. On the other hand, besides **the weakly structured functions group**, the ECDFs shows no significant difference between the algorithms performance on the other groups.

The impact of the implementations differences. In the case of functions (**f20 - f24**) aka the **weakly structured functions**, we see a variance in the ECDFs that is uncorrelated with the usage of the diagonal decoding. If we consider for example the **Lunacek bi-Rastrigin** function **f24** in dimension 5 in figure 4, we see that both ddcma algorithms (*i.e.* with and without diagonal decoding) outperform the rest of the algorithms including best 2009. We can also see that ddcma0 is better than ddcma1, however the same thing cannot be said about cma algorithms, where cma0 is worse than the rest of the benchmarked algorithms.

The ranking of the algorithms changes again for **f24** in dimension 20, however, it is clear that the ddcma on the **the weakly structured functions group** outperforms the cma on the majority of the dimensions tested. The additional runs of cma with modified stopping conditions (not reported here) suggest that this difference is a result of the difference in restarts run length (*i.e.* number of iterations per restart), caused by the stopping conditions difference between the 2 implementations.

6 CONCLUSION / DISCUSSION

Two implementations of the CMA-ES algorithm with diagonal decoding were benchmarked in this paper, using bboB test suite on COCO platform. The goal was to assess both any gain or loss of performance over the suite's problems, and highlight any variance that might come from the different codes of CMA-ES besides the usages of diagonal decoding. The data confirms the expected outcome of applying the diagonal decoding acceleration, which is a gain in performance on separable functions with almost no loss on other function groups. Although for weakly structured functions a variance in the ECDFs was observed, and *a priori* unrelated to the usage of diagonal decoding, but more influenced by the stopping conditions used to trigger restarts. Further investigation of single runs is needed to determine how these stopping conditions and the diagonal decoding interacts.

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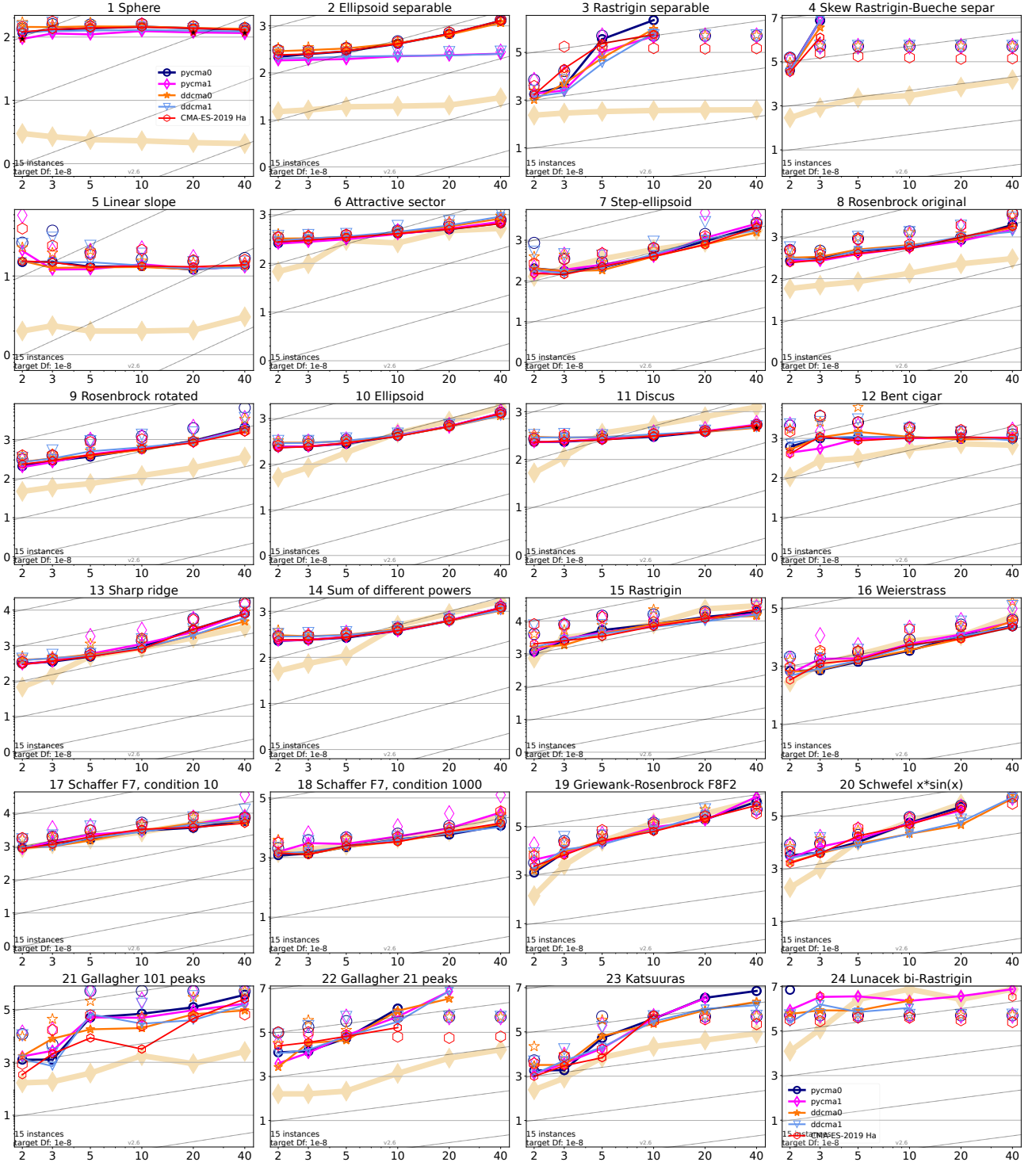


Figure 1: Expected running time (ERT in number of f -evaluations as $\log_{10}$ value), divided by dimension for target function value 10^{-8} versus dimension. Slanted grid lines indicate quadratic scaling with the dimension. Different symbols correspond to different algorithms given in the legend of f_1 and f_{24} . Light symbols give the maximum number of evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with $p < 0.01$ and Bonferroni correction number of dimensions (six). Legend: $\circ$: CMA-ES-2019 Hansen, $\diamond$: ddcma0, $\star$: ddcma1, $\triangle$: pycma0, $\circ$: pycma1.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	43	43	43	43	43	43	43	15/15	f13	652	2021	2751	3507	18749	24455	30201	15/15
pycma0	6.7(1)	13(1)	19(1)	26(2)	32(2)	45(2)	58(3)	15/15	pycma0	2.9(1)	3.8(2)	4.0(2)	3.3(2)	0.77(0.4)	1.1(0.6)	1.4(0.5)	15/15
pycma1	5.6(1)	11(1)	16(2)	22(2)*	27(2)*	38(2)*	49(2)*2	15/15	pycma1	3.5(4)	2.9(2)	4.3(2)	4.3(3)	1.1(0.5)	1.0(0.5)	1.0(0.5)	15/15
ddcma0	7.3(1)	13(2)	19(2)	26(2)	33(2)	45(4)	58(3)	15/15	ddcma0	2.8(0.9)	2.7(2)	2.6(1)	3.1(2)	0.70(0.3)	1.0(0.8)	1.0(0.6)	15/15
ddcma1	6.5(1)	12(2)	18(2)	24(2)	30(2)	41(3)	53(3)	15/15	ddcma1	3.0(3)	2.8(2)	3.5(2)	3.7(2)	0.77(0.4)	1.0(0.4)	1.1(0.6)	15/15
CMA-ES	6.2(0.9)	13(1)	19(2)	25(2)	32(3)	44(2)	58(3)	15/15	CMA-ES	2.9(2)	2.8(2)	4.4(3)	4.6(2)	1.0(0.3)	1.2(0.6)	1.1(0.5)	15/15
f2	385	386	387	388	390	391	393	15/15	f14	75	239	304	451	932	1648	15661	15/15
pycma0	20(4)	25(4)	27(2)	29(2)	31(2)	32(2)	33(2)	15/15	pycma0	2.5(1)	2.4(0.4)	3.3(0.4)	3.8(0.3)	3.2(0.3)	3.8(0.2)	0.67(0.1)	15/15
pycma1	6.1(1)	7.2(1)	7.9(1)	8.5(1.0)	9.1(1.0)	10(0.9)	12(0.9)	15/15	pycma1	2.2(0.9)	2.0(0.3)	2.7(0.2)*2	3.4(0.5)	3.0(0.4)	3.7(0.3)	0.66(0.0)	15/15
ddcma0	20(2)	23(3)	27(4)	28(3)	29(2)	31(2)	32(2)	15/15	ddcma0	2.9(0.8)	2.5(0.4)	3.3(0.4)	3.8(0.4)	3.1(0.2)	3.8(0.3)	0.65(0.0)	15/15
ddcma1	5.5(0.7)	6.4(0.9)	7.2(0.8)	7.9(0.9)	10(1)	11(1.0)	11(1.0)	15/15	ddcma1	3.1(0.9)	2.4(0.3)	3.1(0.2)	3.6(0.5)	3.0(0.3)	3.7(0.3)	0.67(0.0)	15/15
CMA-ES	22(3)	26(4)	28(4)	30(3)	31(2)	33(2)	34(2)	15/15	CMA-ES	2.9(1)	2.5(0.5)	3.4(0.6)	3.9(0.5)	3.2(0.4)	3.9(0.3)	0.66(0.0)	15/15
f3	5066	7626	7635	7637	7643	7646	7651	15/15	f15	30378	1.5e5	3.1e5	3.2e5	3.2e5	4.5e5	4.6e5	15/15
pycma0	14(6)	1.8e4(3e4)	∞	∞	∞	∞	∞	0/15	pycma0	0.87(0.6)	1.0(0.5)	0.73(0.2)	0.74(0.2)	0.75(0.2)	0.55(0.2)	0.56(0.2)	15/15
pycma1	5.3(5)	1.9e4(2e4)	∞	∞	∞	∞	∞	0/15	pycma1	0.93(0.4)	1.1(0.5)	0.66(0.2)	0.66(0.2)	0.66(0.2)	0.49(0.1)	0.49(0.1)	15/15
ddcma0	6.4(2)	∞	∞	∞	∞	∞	∞	0/15	ddcma0	0.83(0.3)	1.0(0.5)	0.71(0.2)	0.71(0.2)	0.71(0.2)	0.52(0.1)	0.53(0.1)	15/15
ddcma1	7.8(6)	∞	∞	∞	∞	∞	∞	0/15	ddcma1	0.97(0.4)	0.84(0.2)	0.55(0.2)	0.55(0.2)	0.55(0.2)	0.41(0.1)	0.41(0.1)	15/15
CMA-ES	15(8)	∞	∞	∞	∞	∞	∞	0/15	CMA-ES	1.0(0.6)	1.1(0.3)	0.65(0.3)	0.66(0.3)	0.66(0.3)	0.49(0.2)	0.50(0.2)	15/15
f4	4722	7628	7666	7686	7700	7758	1.4e5	9/15	f16	1384	27265	77015	1.4e5	1.9e5	2.0e5	2.2e5	15/15
pycma0	∞	∞	∞	∞	∞	∞	∞	0/15	pycma0	1.7(1.0)	0.80(0.7)	0.76(0.8)	0.88(0.9)	1.0(0.7)	1.0(0.7)	1.0(0.6)	15/15
pycma1	1.4e4(2e4)	∞	∞	∞	∞	∞	∞	0/15	pycma1	1.3(0.4)	0.88(0.4)	0.93(0.6)	0.84(0.6)	1.2(1.0)	1.2(1.0)	1.1(0.9)	15/15
ddcma0	∞	∞	∞	∞	∞	∞	∞	0/15	ddcma0	1.7(0.7)	0.89(0.6)	0.72(0.2)	0.74(0.6)	0.74(0.5)	0.84(0.5)	0.78(0.4)	15/15
ddcma1	∞	∞	∞	∞	∞	∞	∞	0/15	ddcma1	1.4(0.5)	0.68(0.6)	0.91(0.6)	0.77(0.7)	0.83(0.8)	1.0(0.8)	0.96(0.7)	15/15
CMA-ES	∞	∞	∞	∞	∞	∞	∞	0/15	CMA-ES	2.1(1)	0.94(0.4)	0.95(0.6)	0.69(0.3)	0.72(0.5)	0.84(0.6)	0.78(0.5)	15/15
f5	41	41	41	41	41	41	41	15/15	f17	63	1030	4005	12242	30677	56288	80472	15/15
pycma0	4.9(0.9)	5.7(1)	5.9(1)	5.9(1)	5.9(1)	5.9(1)	5.9(1)	15/15	pycma0	0.95(1.0)	0.81(0.3)	0.82(1)	0.86(0.6)	0.76(0.4)	0.87(0.3)	0.85(0.3)	15/15
pycma1	5.0(1)	5.8(0.9)	6.0(0.9)	6.1(0.9)	6.1(0.9)	6.1(0.9)	6.1(0.9)	15/15	pycma1	1.0(1)	0.93(0.2)	2.0(2)	1.7(0.9)	1.2(0.8)	1.2(0.8)	0.99(0.6)	15/15
ddcma0	4.9(1)	5.7(1)	5.9(1)	5.9(1)	5.9(1)	5.9(1)	5.9(1)	15/15	ddcma0	1.2(2)	1.0(0.4)	1.3(2)	1.5(1)	1.3(0.8)	1.2(0.4)	1.0(0.3)	15/15
ddcma1	5.0(1.0)	5.9(1)	6.0(1)	6.0(1)	6.0(1)	6.0(1)	6.0(1)	15/15	ddcma1	1.3(2)	0.87(0.2)	2.3(2)	2.1(0.9)	1.1(0.4)	0.93(0.3)	1.1(0.5)	15/15
CMA-ES	5.1(0.8)	6.1(1)	6.4(1)	6.4(1)	6.4(1)	6.4(1)	6.4(1)	15/15	CMA-ES	1.1(0.9)	0.81(0.2)	0.94(2)	1.2(1)	0.77(0.3)	0.79(0.4)	0.87(0.3)	15/15
f6	1296	2343	3413	4255	5220	6728	8409	15/15	f18	621	3972	19561	28555	67569	1.3e5	1.5e5	15/15
pycma0	1.4(0.2)	1.2(0.1)	1.0(0.1)	1.1(0.1)	1.1(0.1)	1.1(0.1)	1.1(0.2)	15/15	pycma0	0.83(0.1)	0.71(0.3)	0.86(0.5)	1.1(0.4)	0.94(0.3)	0.83(0.5)	0.80(0.4)	15/15
pycma1	1.4(0.3)	1.2(0.2)	1.1(0.2)	1.1(0.1)	1.1(0.2)	1.1(0.2)	1.1(0.2)	15/15	pycma1	0.83(0.4)	2.2(4)	1.4(0.5)	1.6(1.0)	1.1(0.7)	1.3(0.5)	1.3(0.4)	15/15
ddcma0	1.5(0.3)	1.2(0.2)	1.1(0.2)	1.2(0.2)	1.2(0.2)	1.2(0.1)	1.2(0.1)	15/15	ddcma0	0.87(0.2)	0.47(0.2)	1.1(0.9)	1.7(0.9)	1.3(0.5)	0.87(0.3)	0.83(0.3)	15/15
ddcma1	1.5(0.3)	1.3(0.4)	1.2(0.3)	1.2(0.2)	1.2(0.2)	1.3(0.2)	1.3(0.2)	15/15	ddcma1	0.74(0.2)	0.92(2)	1.1(0.9)	1.4(0.2)	0.96(0.7)	0.86(0.4)	0.85(0.3)	15/15
CMA-ES	1.5(0.3)	1.2(0.2)	1.1(0.1)	1.1(0.1)	1.0(0.1)	1.1(0.1)	1.1(0.1)	15/15	CMA-ES	0.94(0.3)	0.75(0.2)	1.1(0.6)	1.2(1)	0.93(0.9)	1.0(0.4)	1.0(0.4)	15/15
f7	1351	4274	9503	16523	16524	16524	16969	15/15	f19	1	3.4e5	4.7e6	6.2e6	6.2e6	6.7e6	6.7e6	15/15
pycma0	1.3(1)	2.2(2)	1.6(1.0)	1.1(0.4)	1.1(0.4)	1.1(0.4)	1.1(0.4)	15/15	pycma0	1(0)	1(0)	1.2(0.5)	0.57(0.2)	0.53(0.2)	0.56(0.2)	0.57(0.2)	15/15
pycma1	1.2(1)	4.0(2)	2.2(0.6)	1.3(0.4)	1.3(0.4)	1.3(0.4)	1.3(0.4)	15/15	pycma1	1(0)	1(0)	1.3(0.5)	0.52(0.2)	0.51(0.2)	0.53(0.2)	0.53(0.2)	15/15
ddcma0	0.90(0.1)	1.8(1.0)	1.3(0.7)	0.92(0.3)	0.92(0.3)	0.92(0.3)	0.91(0.3)	15/15	ddcma0	1(0)	1(0)	1.2(0.8)	0.48(0.1)	0.49(0.3)	0.58(0.3)	0.58(0.3)	14/15
ddcma1	1.2(1)	2.9(2)	1.8(0.5)	1.2(0.3)	1.2(0.3)	1.2(0.3)	1.2(0.3)	15/15	ddcma1	1(0)	1(0)	1.1(0.6)	0.63(0.4)	0.74(0.3)	0.92(0.8)	0.92(0.8)	13/15
CMA-ES	1.4(1)	2.3(0.8)	1.5(0.6)	0.92(0.3)	0.92(0.3)	0.92(0.3)	0.90(0.3)	15/15	CMA-ES	1(0)	1(0)	1.3(0.5)	0.49(0.1)	0.56(0.2)	0.61(0.4)	0.62(0.4)	13/15
f8	2039	3871	4040	4148	4219	4371	4484	15/15	f20	82	46150	3.1e6	5.5e6	5.5e6	5.6e6	5.6e6	14/15
pycma0	3.2(0.6)	3.2(0.3)	3.5(0.3)	3.6(0.3)	3.6(0.3)	3.7(0.3)	3.7(0.3)	15/15	pycma0	3.6(0.7)	3.6(2)	0.92(0.4)	0.75(0.2)	0.76(0.2)	0.76(0.2)	0.76(0.2)	15/15
pycma1	3.0(0.5)	3.2(0.2)	3.3(0.2)	3.4(0.2)	3.4(0.2)	3.5(0.2)	3.5(0.2)	15/15	pycma1	2.9(0.8)	2.2(0.9)	0.67(0.4)	0.56(0.2)	0.56(0.2)	0.56(0.2)	0.56(0.2)	15/15
ddcma0	3.6(0.8)	3.5(0.4)	3.8(0.4)	3.8(0.4)	3.9(0.4)	3.9(0.4)	3.9(0.4)	15/15	ddcma0	4.0(0.7)	1.2(0.5)	0.22(0.2)	0.16(0.1)	0.16(0.1)	0.16(0.1)	0.16(0.1)	15/15
ddcma1	3.2(1)	4.1(3)	4.3(3)	4.4(3)	4.4(3)	4.4(3)	4.4(3)	15/15	ddcma1	2.9(0.9)	1.4(0.5)	0.23(0.2)	0.20(0.1)	0.20(0.1)	0.21(0.1)	0.21(0.1)	15/15
CMA-ES	3.6(0.9)	3.9(3)	4.2(3)	4.2(3)	4.3(3)	4.3(3)	4.3(2)	15/15	CMA-ES	3.7(0.8)	3.9(1)	0.69(0.4)	0.62(0.2)	0.62(0.2)	0.63(0.2)	0.63(0.2)	15/15
f9	1716	3102	3277	3379	3455	3594	3727	15/15	f21	561	6541	14103	14318	14643	15567	17589	15/15
pycma0	3.8(0.6)	4.4(0.4)	4.7(0.4)	4.7(0.4)	4.8(0.3)	4.8(0.3)	4.8(0.3)	15/15	pycma0	3.1(5)	102(171)	170(238)	168(234)	164(229)	155(216)	137(191)	14/15
pycma1	3.4(0.9)	4.2(0.4)	4.5(0.4)	4.6(0.4)	4.6(0.4)	4.6(0.4)	4.6(0.4)	15/15	pycma1	3.8(4)	173(126)	132(258)	130(254)	127(248)	120(234)	106(207)	14/15
ddcma0	4.0(1)	4.2(0.7)	4.5(0.6)	4.5(0.6)	4.6(0.5)	4.6(0.5)	4.6(0.5)	15/15	ddcma0	7.3(5)	93(230)	92(138)	91(136)	89(133)	84(126)	75(111)	15/15
ddcma1	3.4(0.6)	4.2(0.4)	4.4(0.4)	4.5(0.3)	4.5(0.3)	4.5(0.3)	4.5(0.3)	15/15	ddcma1	1.6(4)	91(70)	55(65)	54(64)	53(63)	50(59)	45(52)	15/15
CMA-ES	3.6(0.5)	3.9(0.3)	4.2(0.3)	4.3(0.3)	4.3(0.3)	4.4(0.3)	4.4(0.3)	15/15	CMA-ES	4.7(11)	121(253)	62(65)	62(93)	61(42)	57(69)	51(43)	9/15
f10	7413	8661	10735	13641	14920	17073	17476	15/15	f22	467	5580	23491	24163	24948	26847	1.3e5	12/15
pycma0	1.2(0.2)	1.2(0.2)	1.0(0.1)	0.86(0.0)	0.81(0.0)	0.74(0.0)	0.76(0.0)	15/15	pycma0	5.1(5)	541(1792)	∞	∞	∞	∞	∞	0/15
pycma1	1.2(0.2)	1.2(0.1)	1.1(0.1)	0.86(0.0)	0.81(0.0)	0.75(0.0)	0.76(0.0)	15/15	pycma1	122(31)	335(901)	6190(5961)	6017(7244)	5828(6215)	5416(5216)	1079(965)	1/15
ddcma0	1.1(0.2)	1.1(0.1)	0.98(0.1)	0.82(0.0)	0.78(0.0)	0.72(0.0)	0.74(0.0)	15/15	ddcma0	23(38)	198(255)	2876(3393)	2796(2306)	2708(1814)	2516(2411)	501(928)	2/15
ddcma1	1.2(0.2)	1.2(0.2)	1.0(0.1)	0.82(0.1)	0.79(0.1)	0.73(0.0)	0.75(0.0)	15/15	ddcma1								

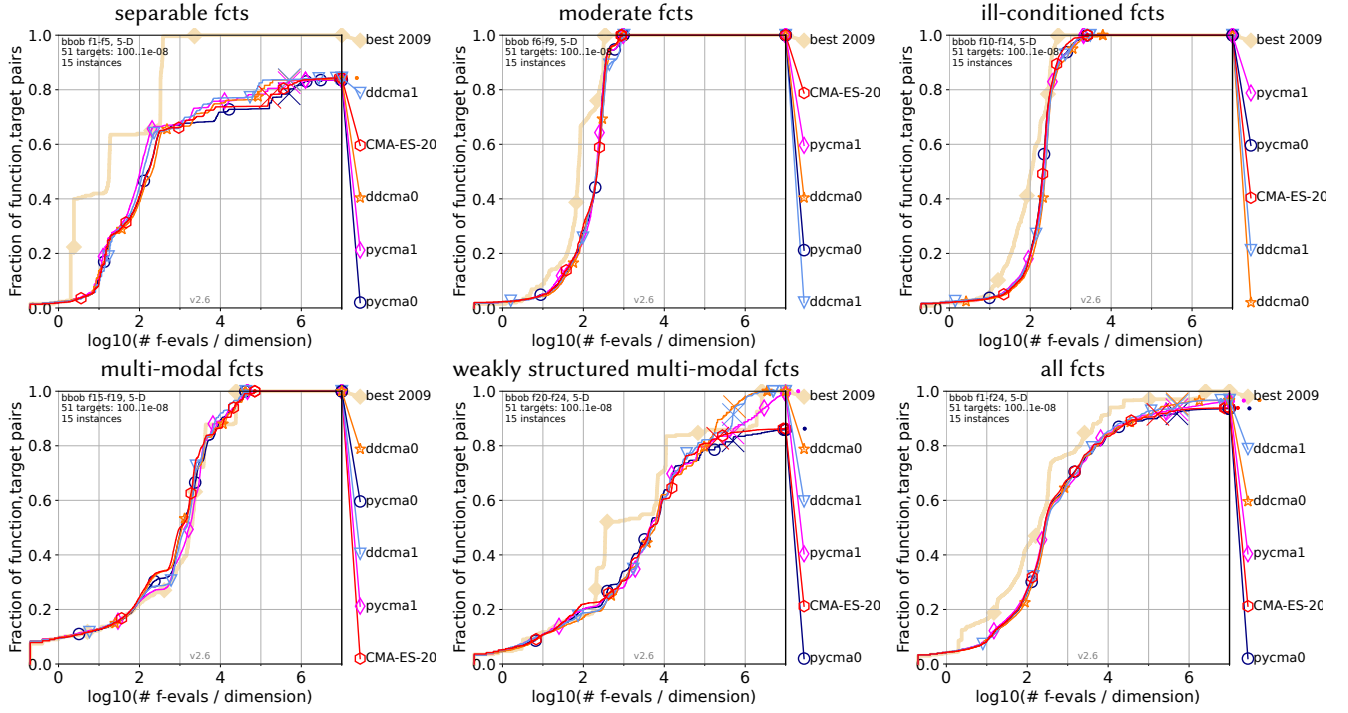


Figure 2: Bootstrapped empirical cumulative distribution of the number of f -evaluations divided by dimension (FEvals/DIM) for 51 targets with target precision in $10^{[-8..2]}$ for all functions and subgroups in 5-D. As reference algorithm, the best algorithm from BBOB 2009 is shown as light thick line with diamond markers.

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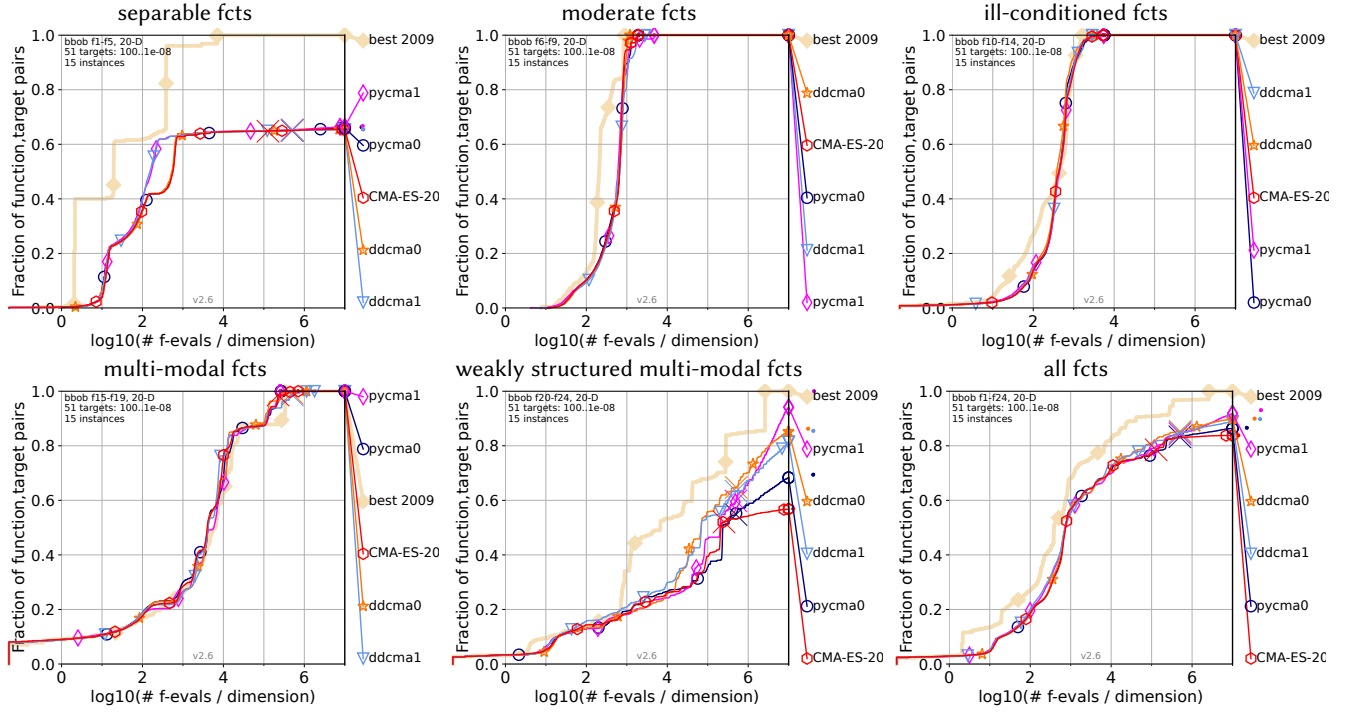


Figure 3: Bootstrapped empirical cumulative distribution of the number of f -evaluations divided by dimension (FEvals/DIM) for 51 targets with target precision in $10^{[-8..2]}$ for all functions and subgroups in 20-D. As reference algorithm, the best algorithm from BBOB 2009 is shown as light thick line with diamond markers.

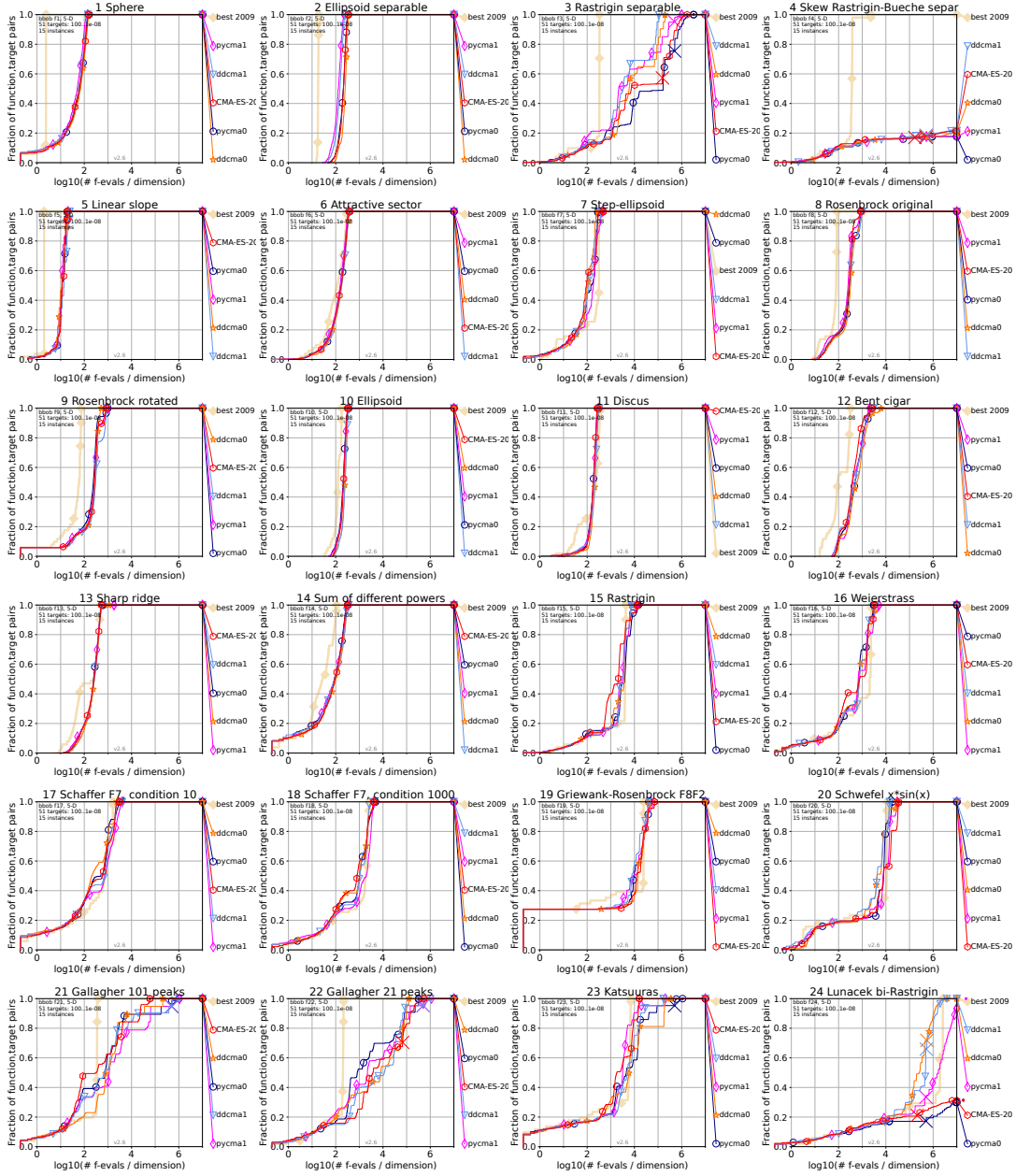


Figure 4: Empirical cumulative distribution of simulated (bootstrapped) runtimes, measured in number of f -evaluations, divided by dimension (FEvals/DIM) for the 51 targets $10^{[-8..2]}$ in dimension 5.

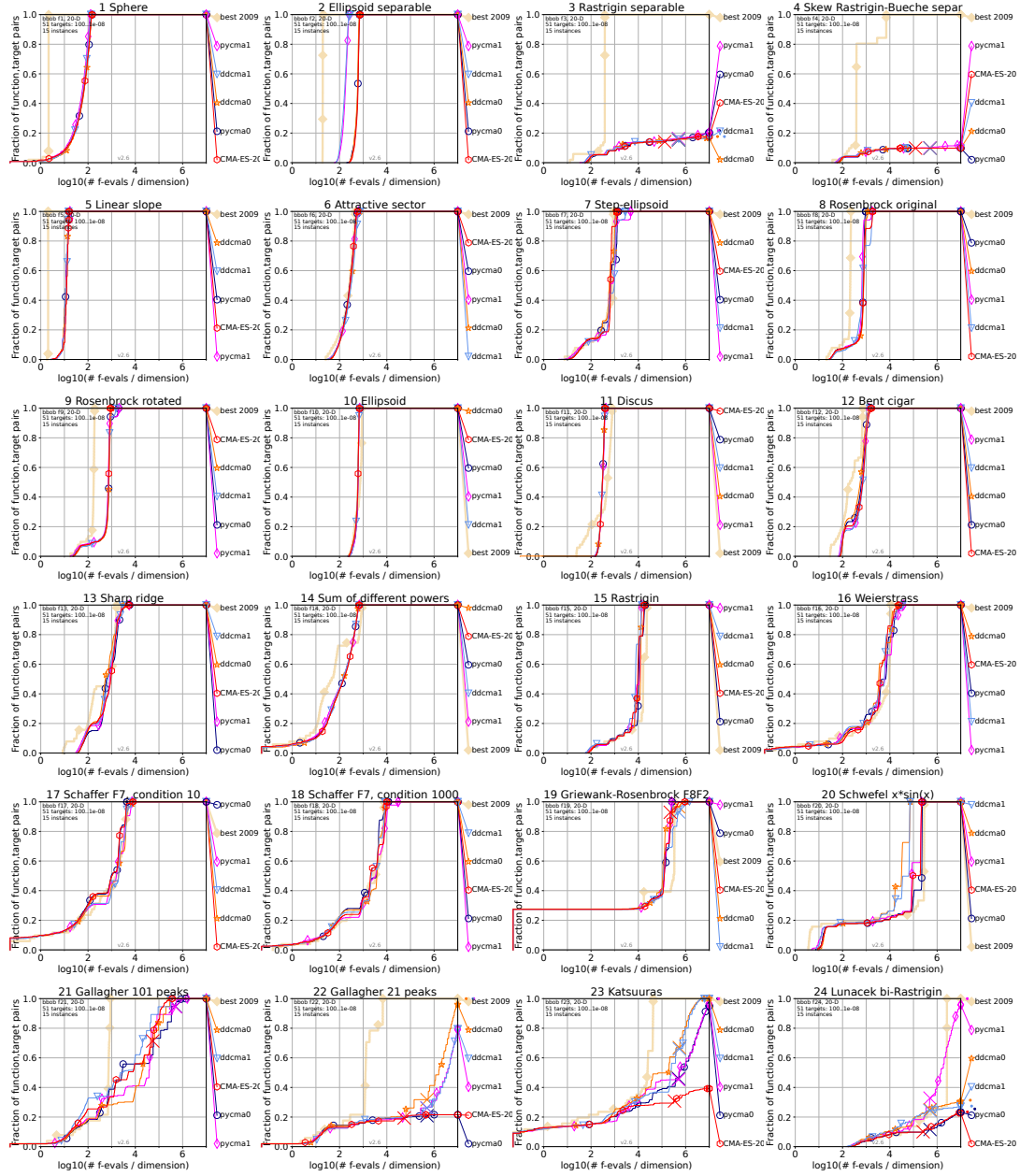


Figure 5: Empirical cumulative distribution of simulated (bootstrapped) runtimes, measured in number of f -evaluations, divided by dimension (FEvals/DIM) for the 51 targets $10^{[-8..2]}$ in dimension 20.