Comparing different computation methods of Reduced Google Matrix

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ABSTRACT

The Reduced Google Matrix (RGM) method is used to analyze interactions between a selected subset of nodes within a Big Data network. In this work we aim to compare the convergence and the outcomes of different computation methods of RGM: direct, η and projection methods. We have made our study on French, English, Russian and German Wikipedia versions that include respectively 1.3, 4.2, 0.9 and 1.5 million nodes. Those Big Data networks accumulate a great part of global human knowledge. The Reduced Google Matrix takes into account the direct and hidden links between a selection of 40 nodes/countries (articles) appearing due to all paths of a random surfer moving over the whole network. As a result we argue that even η and direct methods were showing significant results on hidden links, however projection method is reflecting better the hidden links without being affected by other factors.

CCS CONCEPTS

• **Big Data Network**; • **Google Matrix**; • **Reduced Google Matrix**;

KEYWORDS

PageRank, Matrix

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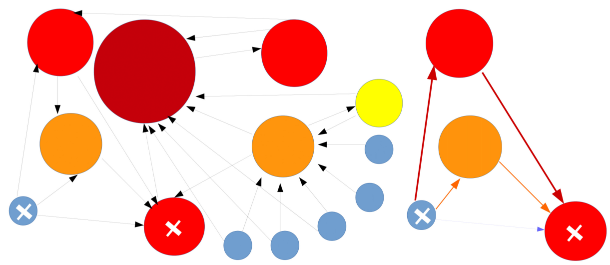

Figure 1: Sketch of a random network (left) with a zoom in capture showing direct and indirect links between two nodes.

1 INTRODUCTION

During recent years, modern societies have developed several networks. Their classification and treatment of information research has become an important and essential task for the company. Due to the rapid growth of the web and communication networks, new mathematical methods have been invented to characterize the properties of these networks in detail. Various search engines widely use these methods in order to classify the information and pages according to their importance. With the growth of these networks, it is very important to develop new tools to classify and categorize this huge amount of information and find the links between them. Given a network of "n" nodes, it is very important to be able to classify these nodes according to their levels of importance as well as being able to assess/evaluate the links between these nodes. Google matrix [1, 2] has been developed to analyze the network and classify the nodes in order of importance as well as to study the direct relationship between them. Although the evaluation and study of the direct relationship between the nodes of a network is very important, it is important to point out that in different areas, studying the indirect links between nodes is of the same importance. We cite here the biological area and the policy area. Let us consider a multi-node network classified by order of importance according to their size as well as the links between them (see figure 1). Although there is a direct link between the two marked nodes, it is possible to have important indirect links between them which will allow us to analyze the relationships between the nodes from a different point of view. Indeed, in the right side of figure 1, we note the existence of strong indirect links between these two nodes. These indirect links can be significant. In the following sections, we will study the impact of indirect links.

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2 REDUCED GOOGLE MATRIX

Reduced Google matrix was proposed by Frahm and Shpeylyansky in [3] to study indirect relationships between nodes of a network. We have implemented three different calculation methods of Reduced Google matrix in order to verify the impact of indirect links

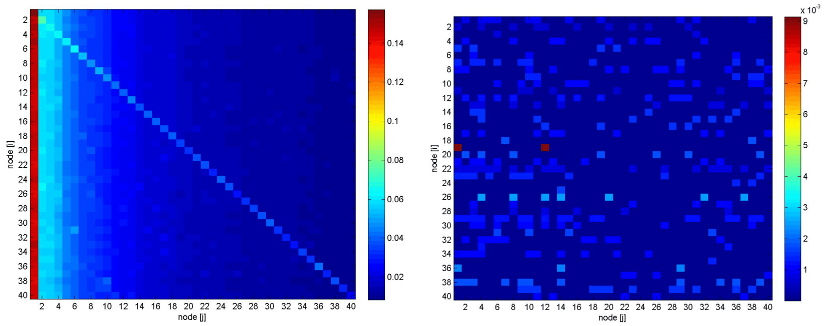

Figure 2: Direct (left) and indirect (right) links between the 40 selected countries of English Wikipedia.

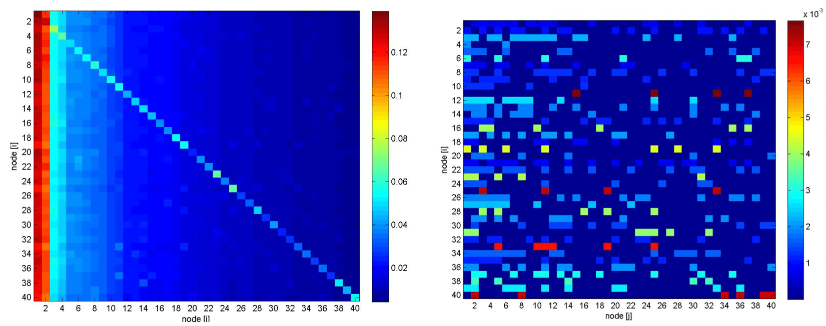

Figure 3: Direct (left) and indirect (right) links between the 40 selected countries of French Wikipedia.

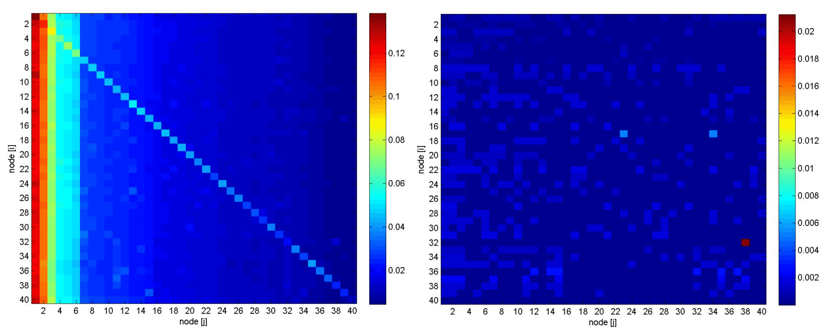

Figure 4: Direct (left) and indirect (right) links between the 40 selected countries of German Wikipedia.

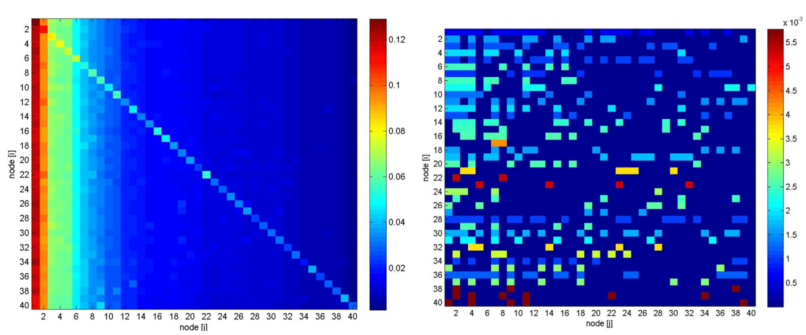

Figure 5: Direct (left) and indirect (right) links between the 40 selected countries of Russian Wikipedia.

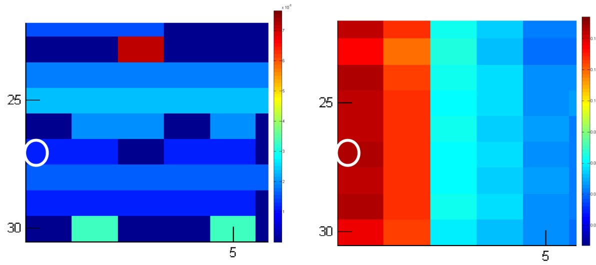

Figure 6: Direct links (left) and indirect links (right) between the countries on Wikipedia French (e.g. the relationship between Iran and France).

Table 1: The 40 selected countries

Country	CC	en	fr	de	ru
Argentina	AR	28	27	30	33
Australia	AU	7	13	14	18
Austria	AT	26	16	4	14
Belgium	BE	25	9	16	29
Brazil	BR	16	17	20	20
Canada	CA	5	7	9	12
China	CN	10	20	32	9
Denmark	DK	32	28	18	31
Egypt	EG	35	23	29	24
Finland	FI	34	33	25	26
France	FR	2	1	3	3
Germany	DE	4	3	2	4
Greece	GR	27	26	23	25
Hungary	HU	37	18	21	23
India	IN	6	14	15	13
Indonesia	ID	36	30	36	34
Iran	IR	15	32	34	30
Israel	IL	33	31	28	28
Italy	IT	8	4	6	6
Japan	JP	9	8	11	11
Mexico	MX	19	22	24	22
Netherlands	NL	14	12	12	15
New Zealand	NZ	18	34	33	36
Norway	NO	21	25	22	27
Pakistan	PK	31	38	39	37
Philippines	PH	29	36	35	39
Poland	PL	13	15	10	10
Portugal	PT	30	21	19	17
Romania	RO	22	35	27	32
Russia	RU	11	11	7	1
South Africa	ZA	24	29	26	35
South Korea	KR	39	39	37	38
Spain	ES	12	6	8	8
Sweden	SE	17	19	13	19
Switzerland	CH	20	10	5	16
Taiwan	TW	38	40	38	40
Turkey	TR	23	24	17	21
Ukraine	UA	40	37	31	5
United Kingdom	UK	3	5	40	7
United States	US	1	2	1	2

Table 2: Ratio between the PageRank calculated based on G and G_R

Country	Ratio between PageRank calculated based on G and G_R			
	en	fr	de	ru
Argentina	51	55	57	37
Australia	51	55	57	37
Austria	51	55	57	37
Belgium	51	55	57	37
Brazil	51	55	57	37
Canada	51	55	57	37
China	51	55	57	37
Denmark	51	55	57	37
Egypt	51	55	57	37
Finland	51	55	57	37
France	51	55	57	37
Germany	51	55	57	37
Greece	51	55	57	37
Hungary	51	55	57	37
India	51	55	57	37
Indonesia	51	55	57	37
Iran	51	55	57	37
Israel	51	55	57	37
Italy	51	55	57	37
Japan	51	55	57	37
Mexico	51	55	57	37
Netherlands	51	55	57	37
New Zealand	51	55	57	37
Norway	51	55	57	37
Pakistan	51	55	57	37
Philippines	51	55	57	37
Poland	51	55	57	37
Portugal	51	55	57	37
Romania	51	55	57	37
Russia	51	55	57	37
South Africa	51	55	57	37
South Korea	51	55	57	37
Spain	51	55	57	37
Sweden	51	55	57	37
Switzerland	51	55	57	37
Taiwan	51	55	57	37
Turkey	51	55	57	37
Ukraine	51	55	57	37
United Kingdom	51	55	57	37
United States	51	55	57	37

on real data and to compare the efficiency of those methods. Matrices that represent direct and indirect links between the different nodes of the network have been calculated. The direct method is based on equation 1 to calculate the indirect links.

Table 3: Comparison between G_{rr} and G_I on our 4 networks.(Node i is pointing on Node j)

Links			Wikipedia			
direct	indirect	node	ru	de	fr	en
strong	low	i	MX (22)	TR (17)	RU (11)	MX (19)
		j	IR (30)	IR (34)	NO (25)	ES (12)
low	strong	i	PK (37)	PL (10)	CA (7)	TR (23)
		j	RU (1)	US (1)	DE (3)	UK (3)
low	acceptable	i	MX (22)	AT (4)	PK (38)	ZA (24)
		j	ES (8)	CH (5)	IT (4)	IN (6)
acceptable	low	i	ZA (35)	AU (14)	BE (9)	CH (20)
		j	NZ (36)	IN (15)	PK (38)	PT (30)
strong	strong	i	MX (22)	BE (16)	SE (19)	MX (19)
		j	US (2)	DE (2)	FR (1)	US (1)

$$G_R = G_{rr} + G_{rs}\left(1 - G_{ss}\right)^{-1} G_{sr} \tag{1}$$

G_R is the sum of two matrices of the direct and indirect links G_{rr} and $G_I \left(= G_{rs}\left(1 - G_{ss}\right)^{-1} G_{sr}\right)$. Equation (1) is calculated based on equation (2) :

$$GP = P \tag{2}$$

taking into account that:

- $G = \begin{pmatrix} G_{rr} G_{rs} \\ G_{sr} G_{ss} \end{pmatrix}$
- $P = \begin{pmatrix} P_r \\ P_s \end{pmatrix}$
- G represents Google Matrix
- G_{rr} represents the links between the selected nodes
- G_{rs} represents the links between the selected nodes and the remains nodes of the network
- G_{sr} represents the links between the remains nodes of the network and the selected ones
- G_{ss} represents the links between the remains nodes of the network
- P_r and P_s are the values of PageRank of the selected nodes and the remains nodes respectively.

In order to achieve a faster convergence, in their second method, Frahm and Shepelyansky have changed slightly equation 1 by adding a dumping factor ' η ' as shown in the following equation :

$$G_{mod} = \begin{pmatrix} 1 & (1-\eta)U_{rs} \\ 0 & \eta 1 \end{pmatrix} \times \begin{pmatrix} G_{rr} & G_{rs} \\ G_{sr} & G_{ss} \end{pmatrix} \tag{3}$$

with :

$$- U_{rs} = (1/N_r)\, E_r E_s^T$$

$$- E^T = (1, \ldots\ldots, 1) = \left(E_r^T, E_s^T\right)$$

$$- 0.5 \leq \eta < 1$$

By combining equations 2 and 3 we get the following equation :

$$\begin{aligned} G_{Rmod} &= G_{rr} + (1-\eta)U_{rs}G_{sr} + \\ &\eta\left[G_{rs} + (1-\eta)U_{rs}G_{ss}\right]\left(1 - \eta G_{ss}\right)^{-1} G_{sr} \end{aligned} \tag{4}$$

So the modification will affect the indirect links matrix G_I and it will be $(1-\eta)U_{rs}G_{sr} + \eta\left[G_{rs} + (1-\eta)U_{rs}G_{ss}\right](1- \eta G_{ss})^{-1} G_{sr}$.

The third method also aims to solve the problem of slow convergence. This method will be based on using the second largest eigenvalue λ_c of matrix G_{ss} instead of unity eigenvalue as follows: We denote by ψ_R and ψ_L^T the corresponding right and left eigenvectors such that $G_{ss}\psi_R = \lambda_c\psi_R$ (and $\psi_L^T G_{ss} = \lambda_c \psi_L^T$). A projector of λ_c onto the eigenspace $\mathcal{P}_c \left(= \psi_R \psi_L^T\right)$ can verifies $\mathcal{P}_c G_{ss} = G_{ss}\mathcal{P}_c = \lambda_c \mathcal{P}_c$. Therefore we can write:

$$(1 - G_{ss})^{-1} = (\mathcal{P}_c + \mathcal{Q}_c)\,(1 - G_{ss})^{-1}\,(\mathcal{P}_c + \mathcal{Q}_c) \tag{5}$$

$$= \mathcal{P}_c \frac{1}{1-\lambda_c} + \mathcal{Q}_c (1 - G_{ss})^{-1} \mathcal{Q}_c \tag{6}$$

$$= \mathcal{P}_c \frac{1}{1-\lambda_c} + (1 - \bar{G}_{ss})^{-1} \mathcal{Q}_c \tag{7}$$

$$= \mathcal{P}_c \frac{1}{1-\lambda_c} + \mathcal{Q}_c \sum_{l=0}^{\infty} \bar{G}_{ss}^l \tag{8}$$

with $\mathcal{Q}_c = 1 - \mathcal{P}_c$ and $\bar{G}_{ss} = \mathcal{Q}_c G_{ss} \mathcal{Q}_c$.

As a result we get a G_R divided into three matrix. The first represents the direct links G_{rr}. The second $G_{pr} = G_{rs}\left(\mathcal{P}_c \frac{1}{1-\lambda_c}\right) G_{sr}$ represents a part of the indirect links but it is highly affected by the classification and the score of PageRank. The third $G_{qr} = G_{rs}\left(\mathcal{Q}_c \sum_{l=0}^{\infty} \bar{G}_{ss}^l\right) G_{sr}$ will show us another clear part of indirect links without the affectation of importance score. This method will be referred as projection method [4].

$$G_R = G_{rr} + G_{pr} + G_{qr} \tag{9}$$

3 DIRECT AND INDIRECT RELATIONSHIP BETWEEN THE MOST IMPORTANT COUNTRIES OF WIKIPEDIA NETWORK

We have applied our code on different Wikipedia networks namely, English, French, Russian and German to verify the direct and indirect links between the most important countries [5]. We started by computing the PageRank vector of English Wikipedia network, then with the selection of our reduced network by choosing the most important 40 countries from that network. The PageRank vector was also calculated for French (fr), German (de) and Russian (ru) versions of Wikipedia. The study of direct and indirect links between these countries was established in a second stage on the mentioned reduced networks. In this paragraph we introduce the different results between the selected countries (see table 1). We cite here the French, English, Russian and German Wikipedia that include respectively 1.3, 4.2, 0.9 and 1.5 million nodes [6].

3.1 Direct method

Here we mention that the relationship between the PageRank calculated based on G and GR according to equation 1 shows that the order of importance of the countries remains stable (see table 2).

Figures 2, 3, 4 and 5 represents the direct and indirect links between the 40 countries according to the mentioned networks. The importance of indirect links between the nodes of a network is clear. The figures show that even there is a very weak link between Iran and France, there is a strong indirect link between them(see figure 6).

After applying our code on the different mentioned networks, we noticed that the links between the nodes can be classified according to 5 main relations shown in table 3.

Based on the results, we can see the importance of the indirect links on the rank of the country. For example, Canada is the 5^{th} in the order of the countries on the English network despite the fact that she has just 3 direct low (incoming) links with 3 countries. In addition, the France that has the 1^{st} rank in the PageRank on the French network, has occupied this importance based on its indirect links.

3.2 η method

In order to reduce the convergence time, we applied our code base on η method. Indeed, the results showed that η plays a very important role on the level of influence between the nodes. In their paper [3], Fraham et al. set the value of η between 0.5 and 1. The results showed that the value of the gap between the order of PageRank G and G_{Rmod} decrease when η is high. We found that the value of the PageRank plays a very important role on the convergence time. Whereas the Wikipedia networks mentioned before, we applied our code on different networks in order to find the direct and indirect links between the countries. Assuming different values of η, we found that a high value of η decrease the difference between the order of the countries using the Google PageRank matrix and η method.

Comparing the figures of G_I with and without η, we found that with the use of η, the relationships between the nodes becomes more readable and clear. Thus the level of influence becomes more significant. Figures 7, 8, 9 and 10 represent the matrices G_I from our selection of 40 countries on different networks for two different η 0.8 and 0.97.

Tables 4 and 5 represent the difference between the PageRank order of the countries using the Google matrix G and G_{Rmod} methods.

Table 4: Comparison of the values of PageRank between G and G_{Rmod} for English and French Wikipedia. (O :Order based on matrix G, NO :new order based on G_{Rmod}, Gap :gap between O and NO)

CC	English Wikipedia					French Wikipedia				
	O	η=0.97		η=0.8		O	η=0.97		η=0.8	
		NO	Gap	NO	Gap		NO	Gap	NO	Gap
AR	28	30	2	35	7	27	28	1	32	5
AU	7	7	0	9	2	13	13	0	13	0
AT	26	26	0	30	4	16	17	1	21	5
BE	25	25	0	27	2	9	9	0	11	2
BR	16	16	0	15	1	17	16	1	19	2
CA	5	5	0	7	2	7	7	0	9	2
CN	10	10	0	6	4	20	21	1	23	3
DK	32	32	0	32	0	28	26	2	25	3
EG	35	35	0	34	1	23	22	1	20	3
FI	34	34	0	29	5	33	32	1	26	7
FR	2	2	0	2	0	1	1	0	1	0
DE	4	4	0	4	0	3	3	0	3	0
GR	27	29	2	36	9	26	27	1	36	10
HU	37	38	1	39	2	18	18	0	18	0
IN	6	6	0	5	1	14	14	0	12	2
ID	36	33	3	22	14	30	29	1	24	6
IR	15	15	0	17	2	32	33	1	35	3
IL	33	36	3	37	4	31	31	0	37	6
IT	8	8	0	8	0	4	4	0	4	0
JP	9	9	0	10	1	8	8	0	8	0
MX	19	19	0	23	4	22	24	2	28	6
NL	14	14	0	13	1	12	12	0	15	3
NZ	18	18	0	18	0	34	35	1	34	0
NO	21	21	0	20	1	25	23	2	14	11
PK	31	31	0	31	0	38	38	0	30	8
PH	29	28	1	26	3	36	34	2	29	7
PL	13	13	0	14	1	15	15	0	17	2
PT	30	27	3	25	5	21	20	1	22	1
RO	22	22	0	24	2	35	36	1	39	4
RU	11	11	0	11	0	11	10	1	6	5
ZA	24	24	0	33	9	29	30	1	40	11
KR	39	39	0	38	1	39	39	0	38	1
ES	12	12	0	12	0	6	6	0	7	1
SE	17	17	0	16	1	19	19	0	16	3
CH	20	20	0	19	1	10	11	1	10	0
TW	38	37	1	28	10	40	40	0	31	9
TR	23	23	0	21	2	24	25	1	27	3
UA	40	40	0	40	0	37	37	0	33	4
UK	3	3	0	3	0	5	5	0	5	0
US	1	1	0	1	0	2	2	0	2	0
Average			0.4		2.5			0.6		3.4

3.3 Projection method

In the previous methods, even though we see the importance of reduced google matrix in showing the indirect links, but we still have two problems: one is the time of convergence which still large and the second problem is the dominance of importance/PageRank score on the resultant matrix showing the indirect links G_I. For that reasons we implement the projector method [4] on our selection of 40 countries for the selected Wikipedia networks. Our point of interest is to compare the matrix of direct links G_{rr} and the matrix G_{qr} which is not affected by PageRank values (with all diagonal values rendered to zero). In figures 12, 13, 14 and 15 we can see side by side direct links G_{rr} and indirect links G_{qr}. Some of our observations are cited in table 6 to figure out the weights of links

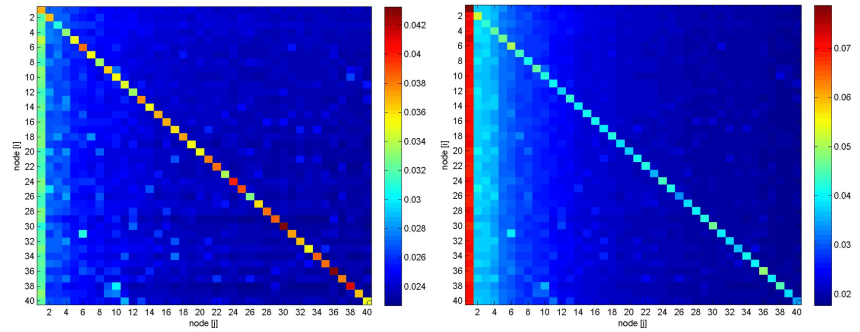

Figure 7: G_I with η = 0.8 (left) and η = 0.97 (right) of English Wikipedia.

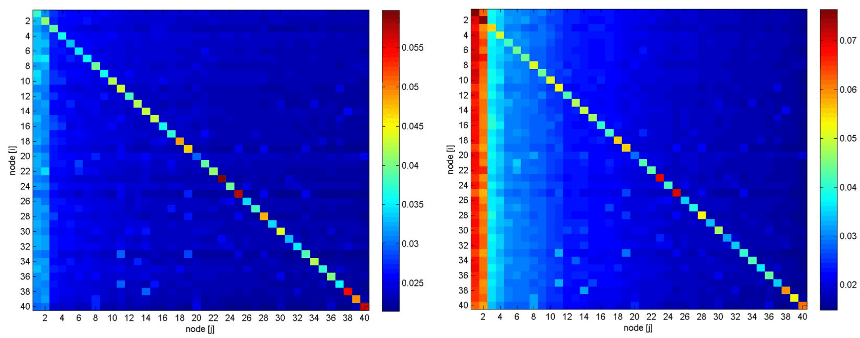

Figure 8: G_I with η = 0.8 (left) and η = 0.97 (right) of French Wikipedia.

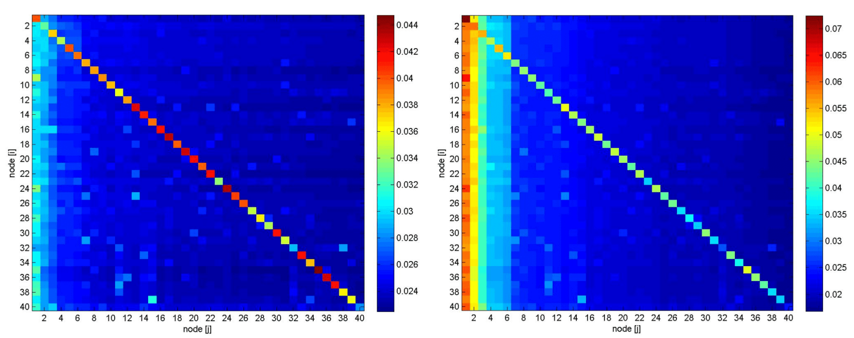

Figure 9: G_I with η = 0.8 (left) and η = 0.97 (right) of German Wikipedia.

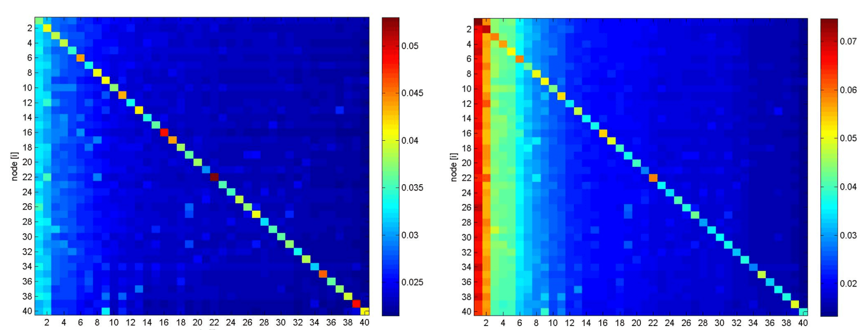

Figure 10: G_I with η = 0.8 (left) and η = 0.97 (right) of Russian Wikipedia.

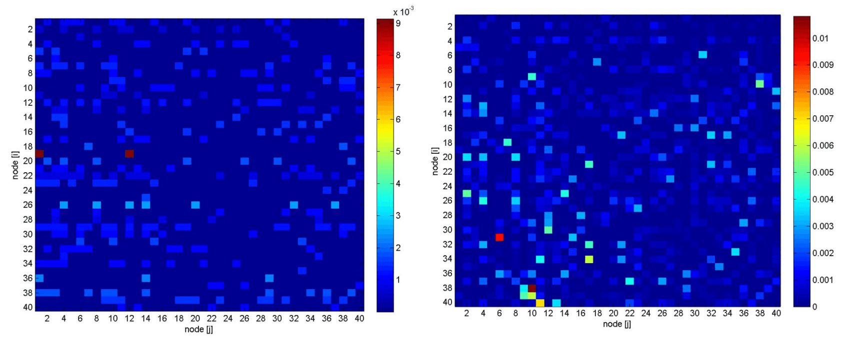

Figure 11: Comparison between G_{rr} and G_{qr} for English Wikipedia

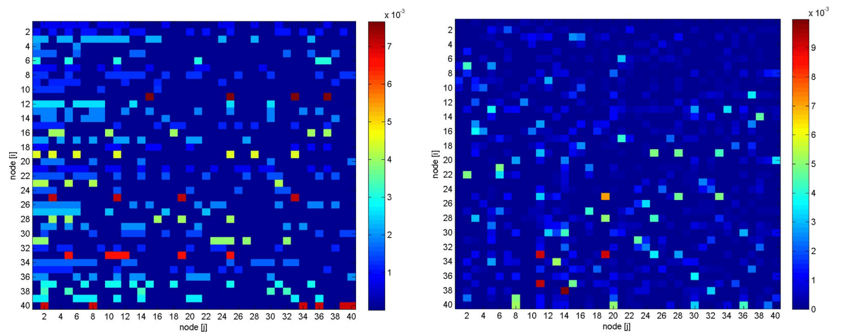

Figure 12: Comparison between G_{rr} and G_{qr} for French Wikipedia

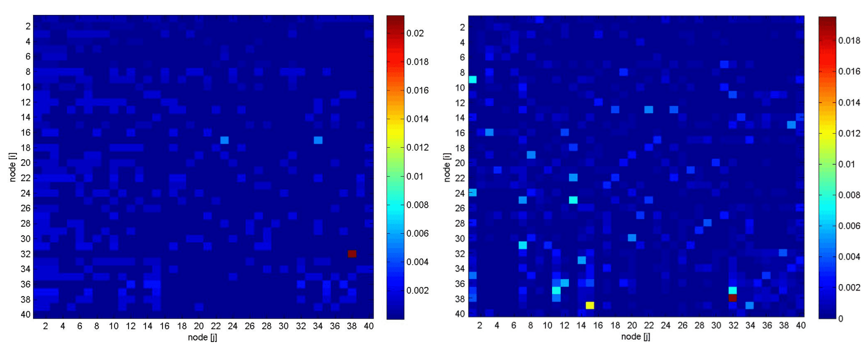

Figure 13: Comparison between G_{rr} and G_{qr} for German Wikipedia

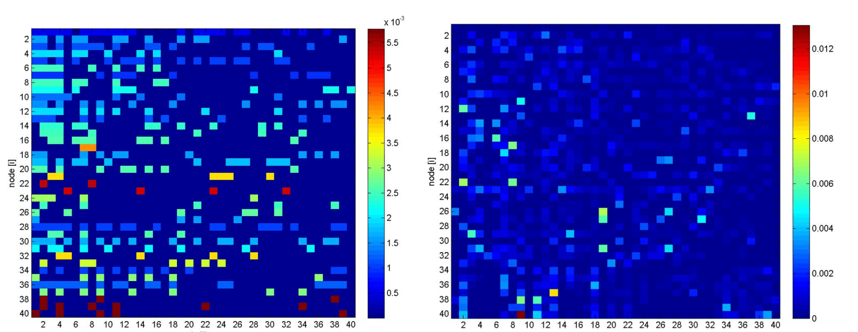

Figure 14: Comparison between G_{rr} and G_{qr} for Russian Wikipedia

Table 5: Comparison of the values of PageRank between G and G_{Rmod} for German and Russian Wikipedia. (O :Order based on matrix G, NO :new order based on G_{Rmod}, Gap :gap between O and NO)

CC	German Wikipedia					Russian Wikipedia				
	O	η=0.97		η=0.8		O	η=0.97		η=0.8	
		NO	Gap	NO	Gap		NO	Gap	NO	Gap
AR	30	29	1	32	2	33	33	0	34	1
AU	14	14	0	13	1	18	17	1	17	1
AT	4	4	0	6	2	14	14	0	16	2
BE	16	17	1	18	2	29	31	2	31	2
BR	20	20	0	17	3	20	20	0	21	1
CA	9	10	1	14	5	12	12	0	13	1
CN	32	32	0	36	4	9	9	0	8	1
DK	18	18	0	20	2	31	30	1	25	6
EG	29	31	2	38	9	24	24	0	26	2
FI	25	25	0	28	3	26	26	0	29	3
FR	3	3	0	3	0	3	3	0	4	1
DE	2	2	0	2	0	4	4	0	3	1
GR	23	23	0	26	3	25	25	0	27	2
HU	21	22	1	25	4	23	23	0	22	1
IN	15	15	0	11	4	13	13	0	11	2
ID	36	36	0	30	6	34	34	0	35	1
IR	34	33	1	23	11	30	29	1	30	0
IL	28	27	1	29	1	28	28	0	28	0
IT	6	6	0	4	2	6	6	0	7	1
JP	11	9	2	8	3	11	10	1	10	1
MX	24	24	0	24	0	22	21	1	19	3
NL	12	11	1	10	2	15	15	0	14	1
NZ	33	34	1	35	2	36	38	2	39	3
NO	22	21	1	22	0	27	27	0	24	3
PK	39	39	0	39	0	37	37	0	38	1
PH	35	35	0	27	8	39	39	0	32	7
PL	10	12	2	12	2	10	11	1	12	2
PT	19	19	0	19	0	17	19	2	20	3
RO	27	28	1	34	7	32	32	0	36	4
RU	7	7	0	7	0	1	1	0	1	0
ZA	26	26	0	31	5	35	35	0	37	2
KR	37	38	1	37	0	38	36	2	33	5
ES	8	8	0	9	1	8	8	0	9	1
SE	13	13	0	15	2	19	18	1	18	1
CH	5	5	0	5	0	16	16	0	15	1
TW	38	37	1	21	17	40	40	0	40	0
TR	17	16	1	16	1	21	22	1	23	2
UA	31	30	1	33	2	5	5	0	5	0
UK	40	40	0	40	0	7	7	0	6	1
US	1	1	0	1	0	2	2	0	2	0
Average			0.5		2.9			0.4		1.7

Table 6: Comparison between G_{rr} and G_{qr} on our 4 networks. Node i is pointing on Node j

Links			Wikipedia			
direct	indirect	node	ru	de	fr	en
strong	low	i	PH (39)	CN (32)	RU (11)	MX (19)
		j	DE (4)	TW (38)	AT (16)	US (1)
low	strong	i	-	TW (38)	PK (38)	TW (38)
		j	-	CN (32)	IN (14)	CN (10)
low	acceptable	i	-	PT(39)	PT (21)	-6
		j	-	IN (15)	ES (6)	-31
acceptable	low	i	PT (17)	TR (17)	TW (40)	-26
		j	UK (7)	GR (23)	NZ (34)	-14
strong	strong	i	TW (40)	-	FI (33)	-
		j	CN (9)	-	RU (11)	-

between nodes in a comparison view between G_{rr} and G_{qr} on our 4 networks. The projection method have been used in [4, 7–9].

4 CONCLUSION

In this paper, we represented a detailed comparison of implementing the Reduced Google Matrix algorithm. The difference between Google matrix and reduced google matrix has been detailed. In addition to demonstrate the importance of indirect links between a selected subset of nodes within a network, the objective of this paper was to show and compare the implementation of η method to the projection and the direct method. The results showed the importance of indirect links on the ranking of nodes of a network. The results showed that a small value of eta leads to a small time of convergence, which means more homogeneity between the selected nodes. However the gap between the results of the two methods, namely, with and without eta, becomes more important, and the outcome becomes less significant. This allows us to analyze the influence of a node on a given network.

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