REMARK ON ALGORITHM 424

Clenshaw-Curtis Quadrature [01]
[W.M. Gentleman, *Comm. ACM 15,* 5 (May 1972), 353–355]

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This algorithm may be used to compute the Chebyshev series coefficients for a function F which is continuous on the interval $[-1, 1]$, as noted in [1]. For this purpose, function *CCQUAD* would be called with $A = -1$, $B = 1$, and appropriate values of *TOLERR* and *LIMIT*. (For some applications, one would prefer instead to state the number of Chebyshev series coefficients desired.) The comments in function *CCQUAD* indicate that the array *CSXFRM* contains, on return, $N = USED - 1$ times the discrete cosine transform of F. Therefore, the values

$$CSXFRM(K)/N, \; 1 \leq K \leq NUMBER,$$

for some $NUMBER \leq USED$, should be estimates for the first *NUMBER* Chebyshev series coefficients of F.

However, the published code produces an array *CSXFRM* with an incorrect sign on each value *CSXFRM*(K) for K even (i.e. the odd Chebyshev series coefficients will all have incorrect signs). This error does not affect the value of the definite integral computed by the algorithm because only the even terms in the Chebyshev series enter into the computation of the definite integral. The error does, however, affect the stated claim that "because the cosine transform is an explicit representation . . . , an approximation to the indefinite integral . . . can be obtained from the indefinite integral [of the truncated Chebyshev series]." The error can be corrected as follows.

Change the eighth and ninth executable statements

```
CSXFRM(1) = F(A)   to   CSXFRM(1) = F(B)
CSXFRM(7) = F(B)   to   CSXFRM(7) = F(A)
```

Change the statements one and four lines below this

```
SHIFT = WIDTH*RT3*.5EO   to   SHIFT = -WIDTH*RT3*.5EO
SHIFT = WIDTH*.5EO       to   SHIFT = -WIDTH*.5EO
```

Change the second and fifth statements following the eight nested "DO 120" statements

```
SHIFT = WIDTH*COS(ANGLE)   to   SHIFT = -WIDTH*COS(ANGLE)
SHIFT = WIDTH*SIN(ANGLE)   to   SHIFT = -WIDTH*SIN(ANGLE)
```

REFERENCES

1. GEDDES, K.O. Near-minimax polynomial approximation in an elliptical region *SIAM J Numer. Anal. 15* (1978), 1225–1233