Microeconomics and the Market for Computer Services

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Microeconomics has much to offer the computer services manager. This article reviews some of the traditional topics in microeconomics and shows how they can be applied to the market for computer services The topics covered include supply, demand, costs, and pricing The most significant application of microeconomics is in setting prices—so much so that microeconomics is frequently called "price theory " Accordingly, the thrust of the article is towards providing a sound framework for the pricing of computer services

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INTRODUCTION

The computer services industry provides an excellent environment for the study of microeconomic principles [48]. Even a single computer installation may be viewed as an economic system in miniature wherein all the forces of supply and demand may be observed. In this paper we offer the results of such an observation, both for individual installations and for the industry as a whole. We follow the canonical approach to microeconomics, and consider in turn the topics of supply, demand, costs and pricing. The emphasis is on relating microeconomic theory to the practical management of computer services.

Classical microeconomics focuses on the behavior of producers and consumers in a market setting. The behavior of producers is examined in terms of production functions (the relationship of factor inputs such as labor and capital to product outputs) and positioning relative to the market. Of particular concern is the effect of firm size and organization on the production functions for its various products. There is a continuing change in the scale of production, differentiation of products, and integration of supply in the search for the most profitable overall strategy.

The introspection characteristic of rational producers is uncommon in consumers; the behavior of consumers is more often studied by producers than by consumers themselves. Thus, consumer demand for goods and services is typically examined in terms of its external manifestations, such as response to pricing changes or periodic variations, rather than in terms of the intrinsic nature of the demand.

Supply characteristics and demand characteristics are linked through the functioning of the market. The marketplace for computer services ranges from highly competitive, as in the case of independent time-sharing companies, to completely monopolistic, as in the case of a corporate data center with a

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captive clientele. Given these different market situations, the prices for similar commodities may be quite different.

Pricing is the key factor, and developing a rationale for setting prices is a prime objective. Indeed, microeconomics is often referred to simply as "price theory." Too few computer center directors realize that they are operating in a marketplace. An understanding of supply and demand relationships as they exist in the computer services marketplace, and an understanding of the economic effects of a pricing policy should lead to a more reasoned approach to setting prices. This will work to the advantage of both the computer center and its users [42].

SUPPLY

The computer services industry is a subset of the entire computer industry. Under a definition suggested by Selwyn [46], computer services includes all of the computer industry except hardware manufacture and maintenance. It includes service bureaus, time-sharing firms, consultants, software producers, and data-bank organizations. It also includes in-house computer facilities, encompassing their operation, programming, systems analysis and systems management functions. In an economic sense, firms that operate their own computing systems are, in effect, suppliers of computing services, although they may limit the sale or provision of these services to themselves. At the present time, in-house computer facilities produce the overwhelming majority of computing services in this country. Service bureaus and time-sharing suppliers represent a very small fraction of all such services produced.

Types of Services

Basic computer services are produced by the execution of a program or predefined sequence of instructions on a hardware complex of a CPU, main memory, and peripherals, referred to as a computer system. The basic service is the action of this program on a set of data which is provided for the particular execution. A basic service supplier will offer the use of the computer system for the time necessary for the particular program to be executed.[1] Service provided may be measured in terms of resources used for periods of time, e.g., core-seconds, or in terms of work processed, e.g., cards read or lines printed.

[1] Since the execution of the program does not consume or in any way harm the computer system (except for the infinitesimally small amount of aging of active electronic components), and since for any given configuration capacity is strictly limited, payment for the use of the computer system may be considered as a true rent in the economic sense. (Strictly speaking, if we consider that system capacity may grow over time through the addition of new equipment in response to high demand, then we would have to speak of a quasirent).

Such "raw computation" is not, however, the only service generally offered. Organizations engaged in the provision of computer services tend to be vertically integrated in that they supply computer time, application software, systems analysis, consulting, training, and other services to users. As Selwyn [46] points out, these services are characterized by significantly different production functions, thereby providing opportunities for specialized suppliers, operating on a scale different from that of the integrated supplier, to produce certain services more efficiently.

Economies of Scale

The production of raw computation has been shown to exhibit increasing economies of scale over the range of currently available machines. In the 1940s, Herbert Grosch asserted that the power of a computer system increased as the square of its cost. Although unpublished by Grosch at the time, this part of the computing profession's early oral tradition has become firmly entrenched in the literature as "Grosch's Law" [53].

Grosch's Law has been empirically tested by a number of investigators, including Knight [28] and Solomon [53]. Both of these studies found that the law generally held, although results were more in conformity for processor-bound tasks than for I/O-bound tasks, reflecting the more rapid drop with increased size in the average costs of main memories and logic elements than in mass storage devices and communications. Littrell's study [30] found economies of scale for scientific calculations, but not for commercial data processing, thus suggesting that such economies might be a function of the application as well. These results are also in conformity with those of Knight and Solomon, since commercial applications are generally characterized as being I/O rather than processor-bound.

Grosch's Law is not without its critics. Adams was one of the first to question it, back in 1962, with a chart of memory access time versus monthly rental for 48 then current computers [1]. More recently, Hobbs has claimed that the law was more a reflection of the pricing policy of a major manufacturer than an inherent law of computer systems design [23].[2] Basing his argument on a perceived change in the relative costs of different parts of a computer and communications system. Hobbs states:

> To the extent that Grosch's Law could be considered a law, it has been limited by the Software Amendment of 1964 and the Integrated Circuit Amendment of 1967 and has been repealed by the LSI Act of 1970.

While Hobbs does not present any empirical evidence to support his assertations, the implication is clear. Grosch's Law is essentially a statement about central processing units, and it may lose its validity as it is extended to the other components of a computer system. Still, Selwyn found users of computing equipment "behaving as if there were significant economies of scale" [43].

When we move from raw computation to other types of computer services, the question of economy of scale becomes more complex. In the operation of a computer service center, for example, there are opposing factors introducing both economies and diseconomies of scale. As Selwyn noted [43], hardware costs represent only one part of the total cost of running a computer installation. Other cost categories include peripheral devices, keypunching and other data collection activities, programming support personnel, system management personnel, physical site facilities, air conditioning, maintenance, magnetic tapes and disk packs, and expendable supplies such as punched cards, continuous forms, and the like.

In general, these costs will rise as hardware costs rise, since a larger operation is needed to support a larger machine. Using cost data on more than one thousand federal computer

[2] As Selwyn [47] has noted, we should distinguish between real economies of scale and pecuniary economies of scale. A real economy of scale is evident in the production function for the good or service in question, as the scale of production increases, unit costs decrease A pecuniary economy of scale results only from a pricing decision, without regard to the cost of production Unit costs may stay the same (or even increase), but the supplier chooses to charge less for larger orders in order to make them more attractive.

installations, Selwyn found that the rate of increase in overall operating expense was slower than the rate of increase in hardware system rent. Solomon's study of personnel costs for commercial (nongovernmental) installations [54] also revealed lower average costs for larger installations. However, both Selwyn and Solomon noted factors contributing to diseconomies of scale. For example, there are costs of sharing larger systems, such as additional hardware and software required for multiprogramming a large computer and for providing privacy protection for each user, costs of access, such as communications facilities, and the loss of individual control over operations which inevitably accompanies the move to a large central machine.

While economies of scale have been noted for support personnel, in software development projects the reverse appears to be true. The effective span of control for programming managers is limited, resulting in larger managerial overhead for larger projects, and the complexity of programming projects has been observed to grow exponentially with size [6, 37].

Communications facilities are rapidly becoming important components of data-processing systems. Economies of scale have been noted for "raw" communications and for data networks [33], though in this area it is particularly important to distinguish between real and pecuniary economies. For example, the current Bell System TELPACK high-speed offerings are nothing more than bundles of lower speed lines; their lower unit prices, therefore, do not reflect any intrinsic economies in providing service (except for the small fixed-cost component associated with issuing bills to customers). Higher speed services implemented in different ways do, however, exhibit significant economies. On the other hand, Cerf has shown how a growing, ground-based packet network suffers from diseconomies if delay is to be kept bounded [8].

The final issue to be considered relating to economies of scale concerns the point at which such economies are exhausted. In 1970, Selwyn [43] concluded that "none of the evidence, in fact, suggested that even the largest size system available is the most efficient possible size of 'plant.' " On this basis, he recommended public policies that encouraged, to the greatest possible extent, the shared use of large systems. Three years later, however, Bower [5] found rapidly increasing costs associated with expansion to serve different types of clientele. He concluded that "once a computer services firm expands beyond customers for a particular type of information in a particular package, its size offers no advantage." [3] These findings are all consistent with the basic premise that economies of scale apply primarily to computer system hardware, not to overall operations.

Product Differentiation

Selwyn [46] has argued that computing services, taken as a whole, are relatively undifferentiated from one another—providing that the relative scale of hardware is selected properly, and assuming intelligent system designs—since the development of most types of computer applications can be accomplished with almost equal success on any general-purpose computer. Thus, prior to the actual commitment of resources to software development, the application developer should be relatively indifferent in choosing among the alternative hardware configurations that may be available to him. However, as Selwyn recognizes, the services of a general-purpose computer become highly differentiated when they are provided in conjunction with access to a specific application program. Users with a heavy investment in not-easily-converted software are often locked-in to a specific system. Given this observation, it would seem desirable to restate the original argument to be that only raw computation is a relatively undifferentiated product. Furthermore, even raw computation may not be so undifferentiated as Selwyn believes. Computers are not, in general, compatible with one another, and brand

[3] As Bower noted, this conclusion implies that the long-run average total unit cost curve in computer services is L-shaped, a common finding.

loyalty does exist, based on real or perceived differences, or simply on which system the buyer was trained.

The other types of services which have been considered along with raw computation —software development, systems analysis, consulting, training and user services—are already highly differentiated. Most buyers demand more than just raw computation, so that even if raw computation is completely undifferentiated, through the other services the supplier may establish for himself an oligopolistic (for example, offering the services of a particular operating system or compiler) or monopolistic (for example, offering proprietary applications software) position.[4]

The benefits of product differentiation accrue principally to suppliers who are able to establish themselves as monopolists for their particular product or service. If customers come to depend on the product to the point where they could not easily find a substitute, the supplier can frequently increase profits by raising prices. On the other hand, product differentiation (and the expectation of monopoly profits) does work to benefit consumers by inducing suppliers to develop specialized products which uniquely satisfy the needs of smaller groups of customers.

Economies of Integration

A production function is a statement about the relationship between the inputs used in production and the resulting output(s). It describes a technological relationship: with a given technology, certain combinations of inputs will make possible a given level of output. Production functions usually apply to one activity or to a closely related group of activities. A firm that produces several different types of products is said to be an integrated supplier, and is subject to all of the production functions that apply to the individual products. If these are parallel and not directly related to one another, the firm is said to be horizontally integrated. If, on the other hand, all the products are related in that they represent intermediate stages of the production of some final good or service, then the firm is said to be vertically integrated.

Most firms in the computer services area are vertically integrated, offering services comprised of raw computation, specialized application programs, contract programming, consulting, and user services. The larger firms, such as IBM, Sperry Rand, and Xerox, are also horizontally integrated, offering a wide range of business products in addition to computer services.

The advantages of vertical integration to a supplier of computer services are the protection to the supply of factor inputs it affords, the internal demand for intermediate outputs it creates, and the economies which often result from control over the entire production process. So attractive are the advantages of vertical integration, that in the computer service field it has occurred by growth in both directions—hardware manufacturers have integrated upwards by the creation of service bureau subsidiaries, and service firms have integrated backwards into hardware manufacture (as University Computing Company did for its line of COPE remote batch terminals).

Against the advantages of integration must be weighed certain disadvantages. As Selwyn [46] explained, each activity of an integrated firm is characterized by its own production function. Software production, for example, is more efficiently done by smaller firms, in contrast to the economies of scale associated with raw computation. Each function is likewise characterized by a most efficient scale of production. Thus, a single integrated firm of any given size is not likely to be the most efficient size for the production of all its products. When this occurs, other firms, competing with the integrated supplier, may be able to produce similar goods

[4] Oligopoly is the market situation in which a few producers control the demand from many buyers It is midway between a free market situation, with many producers and many buyers, and monopoly, with a single producer and many buyers The suppliers of system software services are in an oligopolistic situation, since that software can only be run on the limited number of computers of a given type, while suppliers of proprietary application packages may be in a monopolistic situation, since they may be the only supplier for that software

or services more efficiently, and hence capture a large share of the particular market. Of course, even an inefficient component of an integrated firm may be protected from competition by the more efficient components.[5]

Market Structure

As we have noted, the market supply pattern for computer services has been toward integration. Dis-integration, where it occurred, was generally limited to the labor intensive portions of computer services—principally software development (where a type of economy of scale results from the negligible marginal cost for additional copies of a program), but also consulting, facilities management, and training. Dis-integration serves to benefit consumers by widening competition, and thereby encouraging production by firms of the most efficient size for each commodity or service. In the past, however, the dis-integration of basic computer services was hampered by technological difficulties associated with delivery of the services.

Recently this has changed. The development of computer networks has provided a marketplace for the widespread sale and distribution of basic computer services [7, 14, 21, 25]. Users with nothing more than a terminal and access to the telephone may select from a large number of potential suppliers. Since geography is no longer of major concern, the economies of scale which were observed to exist for basic services makes the dis-integration of such services practical. Thus, a sufficiently large complex, wherever located, may offer basic services to users anywhere in the country at prices lower than they can obtain locally.

Several models have been proposed to describe the functioning of this marketplace. Grobstein, Uhlig and Stefferud have been the principal proponents of the "wholesale-retail" model of the network marketplace [20, 56, 57]. This model views raw computation as essentially a wholesale commodity, best sold at retail by local suppliers who add specialized software and supporting personnel services. The concept is perhaps best exemplified by the operation of the TUCC network [59]. Basic computer services are supplied at wholesale to the three participating universities and the North Carolina Educational Computing Service (NCECS), who act as retailers for their customers. All supporting services are provided by the local computer centers or, in the case of NCECS, by "circuit riders" who make regular visits to the smaller institutions receiving service remotely.

R. Moore has suggested an alternate model of the network marketplace based on the "international trade analogy" [35]. He views competing resource suppliers in a network as similar to nations engaged in international trade, each concerned about its balance of payments. While this model is not fully developed, it does explain such financial squabbles as occurred in the early days of the MERIT network [21]. Thus, Moore's model addresses somewhat different aspects of the marketplace than the wholesale-retail model. As the widespread distribution and exchange of computer services becomes more common through networks, an understanding and appreciation of both of these marketplace theories will become more important [36]. Cotton has surveyed current managerial practices in existing networks [9].

DEMAND

Demand, in the microeconomic sense, is a function or schedule relating the willingness of consumers to buy differing quantities of a product at differing prices. It is not a description of the buying habits of any single consumer, but of all consumers of that product, in aggregate. Each consumer may have a limit to the quantity he would accept at any price and a price above which he would not buy at all, but it is presumed that not all consumers have the same limits. As price goes up or down, fewer or more consumers will be drawn into the market (perhaps each also buying less or more). The demand func-

[5] This may be accomplished by either an active or passive strategy The price may be kept artificially low in the marketplace (relative to actual production costs), or it may simply be hidden as a component of some other end-product.

tion maps the aggregate quantity of product this group of consumers is willing to buy at each given price.

On a Cartesian coordinate system with price on the ordinate and quantity on the abscissa, demand is generally shown as downward sloping, reflecting the lesser quantities of product consumers are willing to buy at increasingly higher prices. Economists are quite particular about distinguishing between changes in the quantity demanded (in response to price changes), which is expressed by moving along the curve to a new point, as opposed to a change in the demand function itself (as a result of a change in consumer preferences).

Elasticity

Elasticity is a fundamental concept in microeconomics because it characterizes relationships between variables in a way that permits "what if?" questions to be answered. Elasticity is basically a measure of the sensitivity of the dependent component of a functional relationship to changes in the independent component. It may be defined as the ratio of the relative changes of the two components when the independent variable is changed by a small amount. If the ratio has an absolute value greater than 1, sensitivity is high and the relationship is said to be elastic. In this case a change in one variable (the "what if?") elicits a larger change in the other. Elasticity is zero when the variables are not functionally related. If the ratio has an absolute value less than 1, the relationship is said to be inelastic, since a change in one variable elicits a smaller change in the other.[6]

The concept of elasticity is most frequently used to describe the relationship between price and quantity in the demand function. Demand is said to be "price-elastic," or simply elastic, when a relatively small change in price induces a relatively large change in quantity demanded. The price elasticity of demand is almost always negative, so that the sign is commonly ignored. (As was just indicated, a ratio greater than one is considered elastic, a ratio less than one, inelastic). Only in the most perverse cases does raising the price of a commodity increase the quantity demanded.

The demand for computer services is a derived demand. Computer services are not required for their own sake, but are used for accounting, inventory control, market forecasting—in short, for all the myriad buisness problems to which the computer has been applied. As a derived demand, the demand for computer services on the whole could be expected to be somewhat inelastic. While this may be true for existing applications (the automation of a particular function is rarely reversible), it does not appear to hold for new applications [10]. For new applications, the reduction in the unit cost of computing (over time) which has been characteristic of the industry has been a major factor in promoting the continued development of these applications.

The cross elasticity[7] of demand for services from different vendors varies according to the homogeneity of the particular service. In the case of cross elasticity, the sign of the ratio is significant and cannot be ignored. As a rule, homogeneous or undifferentiated services such as raw computation have high positive cross elasticities, reflecting the easy substitutability of products from different vendors. (Perhaps it would be simpler to say that the market for undifferentiated computer services is highly competitive. When the price for one service is raised, customers shift to another, raising the quantity demanded.) The cross elasticity of highly differentiated services such as specific ap-

[6] The elasticity of a functional relationship between two variables is defined as the ratio of the relative changes of the two variables when the independent variable is changed by a small amount Expressed this way we have $e = (dQ/Q)/(dP/P)$. (It is often easier to rearrange terms to obtain $e = (dQ/dP)(P/Q)$ In this form, the elasticity is determined by taking the first derivative of the function—with respect to the independent variable—times the ratio of the independent to the dependent variable)

[7] Cross elasticity expresses the sensitivity of demand for one product to changes in price of a different product. Cross elasticity can be positive or negative, depending on whether the products are substitutes or complements. For two products, A and B, the cross elasticity of demand (for the product B to the price of A) is $e = (dQ[B]/Q[B])/(dP[A]/P[A])$

plications programs will be lower, perhaps even zero. (Thus, as has been explained, a firm can establish itself as a monopolist through the development of proprietary software.) Negative cross elasticities of demand between complementary services could also be expected; e.g., a decrease in the cost of raw computation could stimulate increased consumption of programming or consultation services to accompany the increased consumption of raw computation.

Cyclical Variations

In contrast to many other industries where industry-wide demand characteristics are well known but the demand facing an individual firm is not, in the computer services industry the demand facing individual suppliers has been most thoroughly investigated. Many computer service suppliers face captive markets,[8] so that the aspect of the demand facing the firm which has been most intensively studied is the regular variation in demand which often occurs on daily, weekly, and/or annual cycles.

Computer installations are frequently faced with wide cyclical variations in the quantities of service demanded by users [26]. Typically, demand for service is greater during prime shifts than at night. Demand may be greater one day a week when a payroll program must be run, or at the end of a semester, when student projects must be finished. (Variations in quantity demanded may be estimated by the length of service queues at different times in the cycle.) The demand function itself may fluctuate, since identical users may have greater or lesser intrinsic need for services at different times, or the users may be drawn from different populations at different times in the cycle. A major objective of computer center managers should be to level out these fluctuations so as to make more efficient use of the system and reduce the disutility to users who cannot obtain service at times of peak loading. As will be demonstrated later, the price mechanism provides a means to accomplish this.

COSTS

The provision of computer services is characterized by a high ratio of fixed to variable costs. This is most true for the supplying of raw computation, since, for the most part, machine rental accrues whether or not the system is running production jobs. For this reason, considerable attention has been devoted in the literature to the equitable allocation of these fixed costs among the various users [3, 4, 11, 17, 19, 24, 29, 44, 55]. We shall refer to this cost allocation as "billing," rather than "pricing," since pricing has other objectives which will be discussed later. For a modern computer system, the design of an equitable billing algorithm is not simple. (Other types of computer services, such as contract programming and consulting, present less of a problem since variable costs are a more significant portion of total costs, and fixed costs may be allocated as overhead in proportion to variable costs.)

The earliest computers were operated in a sequential batch processing mode, wherein each job occupied the computer fully for the length of time necessary to run to completion. Accounting was simple, as each user could simply be charged according to elapsed, or so-called "wall clock" time. Time-sharing and multiprogramming changed this, since multiple jobs could occupy the computer simultaneously. The elasped time for any given job was no longer a function only of that job, but was also a function of the job mix. Timing was not a problem, since most advanced operating systems could determine actual running time for each program. More serious was the fact that each job used a different set of machine facilities. Depending on the job mix, conflicts could occur, resulting in less than optimal use of the total computer system. Thus, a given system could take different times to run a set of jobs, depending on the order in which they were loaded.

Despite this inherent variability, billing algorithms were sought which conformed to the principles of reproducibility (result in

[8] In this case the cross-elasticity of demand for service from different suppliers is zero, and the focus is on the price elasticity of demand (which is not zero).

the same charges for the same job, no matter when run) and equitability (be a function of only the resources actually used by the job) [29]. Additional suggested attributes of a billing algorithm were auditability, understandability, and demurrage (charging for resources which, though they may not be in active use, cannot be used by others—for example, dedicated peripherals or memory space) [24]. There is no general solution which satisfies all these requirements. Most approaches have been to bill at average costs, which are determined from analysis of a past "typical" time period. This results in repeatability by using constant billing factors for all identifibale resources used (e.g., CPU time, memory space used, lines printed), and approximates equitability, since users are charged in proportion to resources actually used [22, 29, 60]. However, such an approach ignores fluctuations in true cost resulting from job-mix idiosyncrasies, inevitably results in inequities as average factors for resources change over time, and may fail to encourage efficient use of the hardware by driving some users away with high prices.

PRICING

Any economic system must solve the problem of how to use scarce resources. The price system is the vehicle by which economic units express their preferences in a market context. When these preferences are uniformly expressed in terms of price, the strategy of allocating resources to those willing to pay the highest price insures the maximization of total utility realized by the use of these resources.

It has been observed that computer services are among today's scarcer resources [39]. However, prices are not presently the dominant allocative mechanism for these services. Pricing has been used for a number of other objectives [41], and other mechanisms have been used to allocate resources [50]. In this section we review some of the uses to which pricing has been put, and some of the alternative mechanisms for the allocation of services. We conclude with an exposition of our view of the proper role of price in the market for computer services.

Pricing Objectives

It is widely recognized that organizations operate according to many different objectives, be they stated explicitly or not. Naturally, the pricing policy of an organization should bear some relationship to the organization's objectives. Selwyn [45] has identified some of the different objectives, and indicated how they may be expressed in pricing policy.

The desire to fully utilize existing resources is a particularly strong objective of pricing policies for most in-house computer centers [19]. When users are essentially captive, the installation manager is free to manipulate prices so as to encourage more efficient usage patterns, without fear of driving away customers and losing business.[9]

Profit Maximization

Long-term profit maximization is the ultimate goal toward which all good business school students are taught to strive. One approach to maximizing long-term profits is to continuously work toward maximizing short-term profits. Total short-term profits are maximized by increasing production (and accepting a continually lower price, in accordance with normal supply-demand considerations) up to the point where marginal costs just equal marginal revenue (see Figure 1). For firms in the computer service industry, the bulk of the costs in the short run are fixed, so that virtually all marginal revenue represents a contribution to profit. There is, therefore, a strong motivation to establish prices so that all machine time is sold (this is the point in Figure 1 where the marginal cost curve becomes vertical, indicating that any increase in capacity in the short run is impossible). However, the approach most frequently taken is to set prices according to the average demand.

For firms in the computer service business, adherence to this policy may be far from

[9] Even the in-house computer services manager is not completely free to manipulate prices, since captive customers will likely still be price-elastic as regards their demand for services.

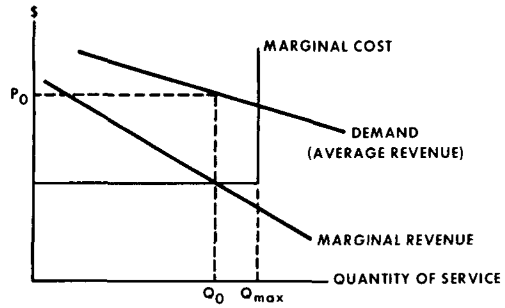

FIGURE 1 Short run demand and cost functions for a computer service firm, output at less than capacity.

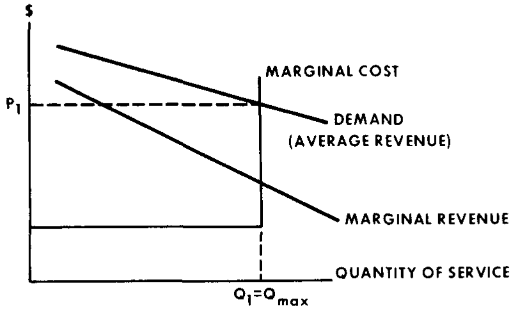

FIGURE 2 Short run peak-hour demand; output is at capacity.

optimal in terms of the long-term profit maximization objective. First, it ignores temporal variations in demand. These variations may result in all prime time being sold, but no sales for night time use. Second, the policy ignores the monopoly potential of specialized computer services. We have already discussed how a firm may establish itself as a monopolist by differentiating its products. Third, by selling to capacity during the peak hours, the quality of service may become substantially degraded (particularly important for time-shared systems), which may result in a loss of customers if they grow dissatisfied with the service they are receiving (see Figure 2).

Market Penetration

A policy of increasing market penetration may accomplish more than mere short-term profit maximization in achieving long term objectives.[10] By foregoing current profits, the firm may be able to capture a much larger share of the market than would be possible without this policy. With an increased customer base, the firm can move to expand capacity, possibly resulting in lower average costs. Since demand is less elastic for established users than potential users, the firm may then be able to alter its policies with respect to short-term profit and still retain a major portion of its customer base. Over the long term, such a policy may result in greater total profits.[11]

Tie-in With Other Services

As has been discussed, integration is presently typical of most firms in the computer services industry. The appropriate pricing policy must consider the impact on all of the firm's products, not just the one for which a price is being established. For example, a time-sharing firm may establish a very low charge for initially connecting to its system in the hope of stimulating usage for which it can charge. Here too, the hope is to accustom clients to a differentiated service to which they will become price-insensitive.

Optimal Use of Computer Resources

For any given computer installation there is an absolute limit to its capacity to offer service. However, this limit is rarely approached, due to imperfect matching of demand for use of the individual resources to the availability of those resources. A gross example might be the idle time occurring at off-peak hours. A more subtle example is provided by a system whose printer is saturated. A pricing policy which, for the first example, encourages off-peak utilization, and, for the second example, discourages excessive use of the printer, may dramatically increase the total throughput of the system. Agnew

[10] In economic terms, the "short term" is defined as that period of time for which productive capacity is fixed. By contrast, in the "long term" productive capacity may be altered, either positively or negatively.

[11] Of course, expected future profits must be discounted at an appropriate rate in order to compare alternative short- and long-term policies Such discounting adjusts for the time value of money.

has shown how a pricing system may be used to control congestion in systems prone to saturation [2].

Pricing Alternatives

Having determined the (set of) goal(s) of its pricing policy, management must then examine the tools available for the establishment of rate plans and policies.

Pricing for Cost Recovery

One alternative pricing policy is to estimate utilization over a given period and set prices so that they cover all costs of operation, including profit in the case of a commercial installation [52]. However, such a policy assumes demand to be perfectly inelastic and, as Smidt [51] has shown, can often be self-defeating. The best example of this is provided by the case of the newly installed computer system with considerable excess capacity available that is expected to be gradually used up as demand increases. The cost per unit time of owning and operating the computer is fairly constant over its life and depends only slightly on the amount of work done. From a common sense point of view, it is clearly advisable to encourage users to make full use of the available capacity early in the life of the computer system, when excess capacity exists, and to discourage usage (or encourage more efficient utilization) later, when usage approaches the capacity of the system.[12]

However, if charges for the computer are determined by allocating its total cost over the total usage for a given time interval (usually a year) which is considerably less than its economic life (say, 4 to 10 years), the charges provide incentives that are exactly the opposite of what is desired. When the computer is new, the fixed costs are allocated over a small volume of work, leading to a high cost per unit of work. When the computer is old and nearing capacity, approximately the same fixed costs are spread over a much larger volume of work, leading to a low cost per unit of work. Insofar as users respond to the costs charged, they tend to economize on the use of the computer in the early days when excess capacity is available, and to be liberal in their use of it later on when capacity is being approached.

The only way out of this dilemma is to recognize that the price at any point in time need not bear any relation to the cost of production at that time. If demand for a good is low, its price may well fall below current average cost, but thereby elicit greater utilization. So long as marginal costs are covered, such operations will make a contribution to profit. Unless price is permitted to fall below cost, the proper information about demand may never be obtained, and the allocation of resources can never adjust to the unprofitability of that good. Smidt [51] and Neilsen [38] have recognized the shortcomings of average cost pricing, and advocate the use of "flexible" pricing schemes where the price is allowed to vary to adjust to demand at any given time so that the quantity sold will be close to the quantity available.

Pricing According to Value

The characteristic negative slope of an aggregate demand curve arises, in part, from the fact that the value of a product or service—perceived or actual—may vary substantially from one buyer to another, and, in part, from the decreasing marginal utility of additional quantities of the product or service to a single user. In order to sell a larger quantity, it is normally necessary to lower the price to all buyers, even those who would be willing to pay more than is being asked, and to charge the same price for all quantities sold to the same buyer.

Price discrimination is a technique by which groups of users are isolated and charged prices that are closer to the maximum price which they would be willing to pay. Price discrimination may be accomplished by segregating users into groups defined according to their demand schedules, and charging the groups different prices, or by charging individual users different prices for successive quantities of the same com-

[12] This may also be viewed as a penetration strategy, as previously discussed.

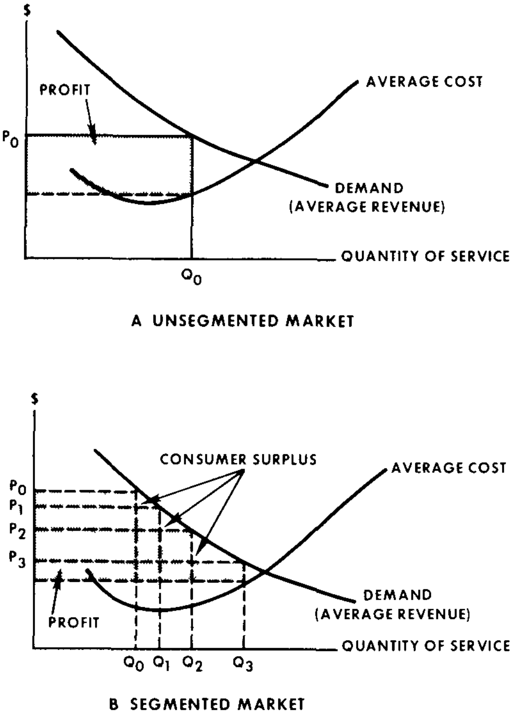

FIGURE 3 Effect of market segmentation By decreasing the price charged for quantities above Q_0, additional service is sold and total profit (shaded area) is increased

modity.[13] As shown in Figure 3, this has the effect of increasing total profit to the vendor. The larger the number of individual segments that can be isolated, the more profitable the technique will be. Profit is maximized when each user is charged the maximum that he is willing to pay for each unit of service.[14]

The requirements for price discrimination are that it be possible (practical and legal) to segment the market, and that users in low-cost segments should not be able to resell services to users in higher-cost segments. The ideal may be achieved by selling each unit of service at auction, so that the maximum price possible is always obtained. Sutherland [58] described a bidding technique for computer time, though he intended it as an efficient allocation mechanism rather than as a means to maximize profit.

Selwyn [45] has discussed a number of bases for market segmentation applicable to the sale of computer services:

> Segmentation by type of customer—for example, by offering discounts to educational customers, who would not purchase services were they priced according to their value to commercial firms
>
> Segmentation by type of application—for example, a software supplier can price individual program products according to their value to users, rather than their cost of production. User isolation is obtained by definition, since they are using different products.
>
> Segmentation by time of day—this has been suggested by a number of authors as a means of more evenly spreading the overall load on a computer system over the total time available.

Finally, Nunamaker and Whinston [40] show how price discrimination (setting charges according to perceived value) can even be used within a single company (with several different groups of internal users) to influence users to adjust their demands to that level most beneficial to the overall organization. This procedure allocates a larger portion of the costs to those users who would have to pay a proportionately higher amount to obtain equivalent service from an alternate source. Thus, the pricing procedure not only helps allocate the resources of a new or existing system, but also provides a guideline for any additional purchases for computer services.

Priority Mechanisms

Priority mechanisms have received wide attention in the literature on managerial and operations research problems. In contrast, they have been virtually ignored by economists. One group of authors [16, 49] suggests that the reason for this is that priorities are

[13] The latter is a technique commonly employed by public utilities in the pricing of such commodities as water, gas, and electricity It should be noted that a portion of the price differential for successive quantities represents a passing on to the consumer of economies of scale in supplying the commodity; the remainder represents the "discriminatory" price decrease offered in order to sell additional quantities An example in the computer world would be the reduced incremental rentals charged by equipment lessors for adding second and third shift operation.

[14] In this case, profit is the total area under the demand curve The consumer surplus is the total quantity users "save" by being charged less than they would be willing to pay. As the segments get smaller and smaller, the consumer surplus is gradually, but completely eliminated

simply a surrogate set of prices that may in some instances work as well as a true price mechanism, but will almost never be superior. For their part, operations analysts seem unaware that priorities are a form of pricing; thus Kleinrock [27] discusses "bribes" which are merely prices, and Greenberger [18] tries to minimize the cost of delay, a cost which can never be known except in terms of the price users would pay to avoid the delay.

Two types of priority rules are recognized [49]: one type that governs the access pattern for a given set of users, and another that offers incentives to potential users in determining their demands for computer time. The problem with the first class of rules is that an implicit assumption must be made about the value placed on computer time by each user. In general, users will not value time equally, nor consider waiting equally costly; consequently, such rules will not allocate time so as to maximize total utility to users. The second set of rules often suffers from inflexibility in the face of changing user requirements, and may discourage efficient substitution of other resources for computer use.

In defense of priority mechanisms, it is recognized that they may serve to reduce the level of disutility that users cause each other through their presence in service queues—a function attributed by Marchand [32] to "advisable" pricing mechanisms. Priority mechanisms are also inexpensive to administer and are fequently automated [15, 31]. Finally, a pay-for-priority scheme can permit users to control the quality of service they will receive according to the price they are willing to pay [34].

The Dual Role of Price

The controversy regarding the proper function of price has centered around whether it is a mechanism for the recovery of costs (including profit), or for allocating resources. Singer, Kanter and Moore [49] are quite emphatic: "This point should be stressed: prices are a rationing device, not a mechanism for recovering cost."

On the other hand, as Oliver [41] recognizes: "It is a sad fact of life that pricing is generally the only way a center has to recover costs. Someone has to pay for the center."

How are these opposing views to be reconciled?

A possible reconciliation may be achieved by recognizing that price has a dual nature and satisfies dual objectives. Any pricing policy will serve as an allocation mechanism (but with varying efficiency). As Nielsen [39] observes, "if resource allocation is not done explicitly, it will be done implicitly; there is no such thing as 'no allocation.'" The concern of those who insist that pricing be viewed purely as an allocation mechanism is that this allocation be optimized for some set of criteria such as total user utility or system throughput [12, 13].

The main factors in the criticism of the cost-recovery objective are that it often focuses on the short term to the detriment of the long term, is frequently inflexible in its implementation, and thus may lead to inefficient utilization. Such objections are well taken, but can be met by aiming to cover costs for a more appropriate period of time, and by adjusting prices in response to both secular changes and cyclical fluctuations in demand.

It is necessary to establish a pricing policy which considers both objectives of price. The overall result of the policy must be to achieve some cost-recovery objective (maximize profit for a commercial installation, recover actual costs for an internal corporate installation, limit losses to a budgeted amount for a university center) as well as to allocate resources on an equitable basis. Such a flexible pricing scheme can serve to promote more efficient use of the hardware, and may even result in greater total revenues.

Break-even Analysis

Microeconomic theory offers no prescriptions guaranteeing that costs can be recovered for a particular product or service. What it does offer are tools with which to analyze the level of production necessary for all costs to be recovered. One such tool is the so-called "break-even" chart (Figure 4).

Simple break-even analysis assumes that

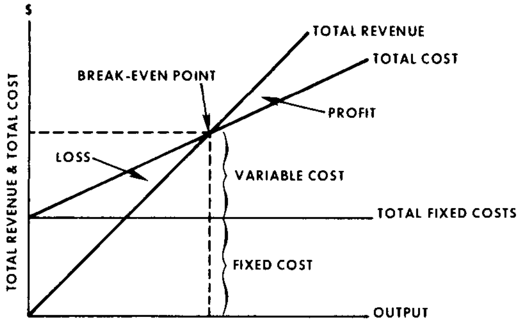

FIGURE 4. Break-even chart.

all costs can be represented as either fixed or variable costs (or some combination of these two types), and that all units are sold at the same price so that marginal revenue is the same from each.[15] The break-even chart graphically illustrates the level of production required—at a given price—for the excess of revenue over variable costs to equal fixed costs. If the simplicity of a straight line intersection can be sacrificed, the requirement for fixed marginal revenue may be relaxed. The lines become curves, or step functions in the chart, but a graphical solution is still possible.

This analysis has two shortcomings. First, the break-even level of production may exceed capacity. If this is the case, cost recovery is impossible at the given price. The immediate temptation is to raise the price—but this gives rise to the second, and more serious shortcoming: the analysis ignores supply-demand considerations.

Elasticity Analysis

If prices are to be raised or lowered, attention must be paid to the resultant changes in the quantity of service demanded. As J. Moore cautions, "one should not establish a pricing mechanism without first determining the implications that such a policy would have on the performance and utilization of the computing system" [34]. The impact of price changes on the quantity of service demanded can be determined through consideration of the price elasticity of demand.

Raising the price will indeed steepen the total revenue line—but there is no guarantee that the quantity sold (at the new given price) will reach the break-even point. Indeed, even the quantity that could have been sold at the old price (had capacity permitted) might have fallen short of the break-even point. The new quantity can be determined from the formula in footnote 5 if the old price and quantity, the change in price, and the coefficient of elasticity are known. If the equilibrium between supply and demand yields a quantity less than that required to break even, cost recovery is truly hopeless.

The effect of raising or lowering price depends, of course, on the price elasticity of the particular product or service under consideration. For products with high elasticities, a small change in price results in a large change in quantity demanded. In this case, increasing the price will not aid in recovering costs. (If, however, the break-even point is below capacity, lowering price may aid in cost recovery by substantially increasing the quantity sold.) For products with low elasticities, raising prices may, indeed, aid in recovering costs. It is for this reason that firms seek to differentiate their products, or establish monopoly positions for themselves. Differentiated products tend to have lower price elasticities, enabling the firm to manipulate price more freely without wide variations in sales.

Reconciliation

By now the discussion should have provided enough information to indicate how a firm ought to undertake the establishment of a pricing policy to satisfy the dual objectives of resource allocation and cost recovery. The firm must have some knowledge of its own cost functions and of the nature of the market in which it is dealing. Any requirements for "normal" profits can be treated as an additional cost. Possible "excess" profits cannot be determined in advance. The firm can then examine its break-even point for several different levels of price. (See Appendix on combining elasticity and break-even analysis, page 109.) This analysis, in con-

[15] For a break-even analysis to be meaningful, all costs must be discounted at an appropriate rate over the life of the project.

junction with the realities of capacity limitations, will permit the firm to rationally manipulate price to recover costs (including profit) and control the allocation of resources. Supply and demand must be in equilibrium at a profitable volume if the firm is to remain in business, but this should be viewed over a reasonable period of time. The firm should also remember that demand is a function of time as well, and is likely to be growing (with possible cyclic variations).

A rational pricing policy will not treat all commodities and all situations alike. Cost recovery and the possibility of earning excess profits will normally be accomplished through commodities with low price elasticity. Where elasticity is high, pricing will be directed more at controling allocation and restricting usage. It is hoped a greater understanding of the underlying economic principles by computer center managers will lead to policies which better satisfy both goals.

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APPENDIX

COMBINING ELASTICITY AND BREAK-EVEN ANALYSIS

If the firm's cost function and the demand function for services can both be expressed mathematically then an analytic solution is possible. For example, consider the case of a linear, downward-sloping demand curve and a linear cost function:

Demand functions of the type shown in Figure 1 (page 104) which appear to express price as a function of quantity demanded may also be interpreted as expressing quantity demanded as a function of price. In this form the function may be expressed as

$$D = Q - kp \qquad (1)$$

where

D is the quantity demanded
Q is the y-intercept (quantity demanded at zero price),
k is the slope of the line, and
p is the unit price.

The cost function we consider is of the type shown in Figure 4 (page 108) with both fixed and variable components. Assuming constant returns to scale (a linear variable cost component), this function may be expressed as

$$C = f + vD \qquad (2)$$

where

C is the total cost,
f is the fixed cost,
v is the variable cost per unit, and
D is the quantity demanded

Profit, which we wish to maximize, is the difference between revenue and cost. Revenue is the product of the quantity supplied and the price. Profit, therefore, is

$$P = pD - C \qquad (3)$$

where

P is the profit,
p is the unit price,
D is the quantity demanded, and
C is the total cost.

Combining (1) and (2) into (3) we obtain

$$\begin{aligned} P &= p(Q - kp) - (f + v\,(Q - kp)) \\ &= kp^2 + p(Q + kv) - vQ - f \end{aligned} \qquad (4)$$

In order to maximize this expression, we set the first derivative equal to zero and solve for the price:

$$dP/dp = 2kp + Q + kv = 0 \qquad (5)$$

$$p = v/2 + Q/2k \qquad (6)$$

This simple analysis does not contain a capacity constraint. Such a constraint can be easily handled, however. The quantity demanded at the optimum price (p in Equation 6) can be found from Equation 1. If this quantity is in excess of the available capacity, then p is increased until the quantity demanded is reduced to exactly the quantity available.

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