JiuZhang 2.0: A Unified Chinese Pre-trained Language Model for Multi-task Mathematical Problem Solving

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ABSTRACT

Although pre-trained language models (PLMs) have recently advanced the research progress in mathematical reasoning, they are not specially designed as a capable multi-task solver, suffering from high cost for multi-task deployment (e.g., a model copy for a task) and inferior performance on complex mathematical problems in practical applications. To address these issues, in this paper, we propose JiuZhang 2.0, a unified Chinese PLM specially for multi-task mathematical problem solving. Our idea is to maintain a moderate-sized model and employ the cross-task knowledge sharing to improve the model capacity in a multi-task setting. Specially, we construct a Mixture-of-Experts (MoE) architecture for modeling mathematical text, so as to capture the common mathematical knowledge across tasks. For optimizing the MoE architecture, we design multi-task continual pre-training and multi-task fine-tuning strategies for multi-task adaptation. These training strategies can effectively decompose the knowledge from the task data and establish the cross-task sharing via expert networks. In order to further improve the general capacity of solving different complex tasks, we leverage large language models (LLMs) as complementary models to iteratively refine the generated solution by our PLM, via in-context learning. Extensive experiments have demonstrated the effectiveness of our model.

CCS CONCEPTS

• Information systems \rightarrow Language models.

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KEYWORDS

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1 INTRODUCTION

Recently, the mathematical reasoning capacity of machines has been largely empowered by the progress of pre-trained language models (PLMs) [30, 39, 41, 56]. By pre-training on large-scale mathematical corpus with specially designed tasks, PLMs can understand the mathematical formulas and logic to a certain extent [56], achieving better performance on a variety of math-related tasks.

Despite the progress, existing PLM based approaches still have two major limitations in real-world math-related applications. (1) *Limited task performance*: due to the limit of model capacity and pre-training data, PLMs are less capable of understanding complex mathematical problems, thus suffering from performance degradation on difficult tasks. (2) *Large maintenance cost*: an online application often supports multiple math-related tasks (*e.g.*, similar problem retrieval and knowledge point classification), while PLMs need to be fine-tuned task by task when dealing with different downstream tasks, taking a significant cost of maintaining multitask solvers (*e.g.*, a model copy for a task).

By exploring the scaling laws, large language models $(LLMs)^1$ [4, 5] can overcome the above issues to some extent with stronger mathematical reasoning ability. While, they are very costly to be tuned for task or domain adaptation. Although in-context learning [4] can be applied to solve different tasks in an efficient way (with no need for fine-tuning), it is still difficult to adapt them to

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¹In this paper, PLMs and LLMs refer to mathematical language models with *moderate sizes* (e.g., BERT [10]) and *huge sizes* (e.g., GPT-3 [4]), respectively.

specific tasks that require rich domain knowledge, *e.g.*, Englishfocused LLMs such as GPT-3 [4] and CodeX [5] cannot perform very well on Chinese mathematical problems (as shown in Table 2).

Considering the above issues, we aim to develop a more effective Chinese PLM that can well adapt to multiple complex mathematical tasks, so as to better support math-related applications. To motivate our solution, we observe that mathematical tasks usually rely on common or related background knowledge, e.g., a multichoice problem and a blank-filling problem might target the same knowledge point though with different problem settings. Thus, it is intuitive to transfer and share mathematical knowledge across tasks by learning a unified model, so that the performance of each individual task can be potentially improved. In a multi-task manner, it also naturally reduces the cost of task-specific fine-tuning, since a joint model is trained with the data of all tasks. While, to become multi-task learner, it requires a higher generalization ability for solving different tasks [4, 44]. For this purpose, we further leverage existing LLMs that implicitly encode large amounts of knowledge to enhance the capacity of complex problem solving for PLMs.

To this end, in this paper, we propose JiuZhang 2.0, a unified Chinese PLM specially for multi-task mathematical problem solving. In order to enhance the multi-task capacity, we make three major technical contributions. Firstly, we design a Mixture-of-Experts (MoE) based architecture to transfer and share mathematical knowledge across tasks. We adopt the MoE architecture to encode mathematical text with an elaborately designed routing mechanism. Secondly, we design multi-task continual pre-training and multi-task fine-tuning strategies to optimize the MoE-based architecture for multi-task adaptation. For multi-task continual pre-training, we construct a group of self-supervised pre-training tasks to warm up the MoE architecture for knowledge sharing; for multi-task fine-tuning, we unify the math-related tasks into two general formats of language understanding and generation, and directly enhance the knowledge sharing across these tasks. Thirdly, in order to further improve the general capacity of solving different complex tasks, we leverage LLMs as complementary models to improve the generated solution by our PLM. The PLM (with a smaller tuning cost) is used for task adaptation and generates a preliminary solution, while the LLM (with a stronger model capacity) mainly refines the generated results without directly solving the problem. Concretely, we retrieve similar examples and iteratively concatenate instructions with them to compose the prompt, gradually guiding the LLM to improve the generation results in a coarse-to-fine manner (overall logic, deduction process and language expressions).

To verify the effectiveness of our proposed JiuZhang 2.0, we conduct extensive experiments on eight tasks, covering both the evaluation settings of *seen tasks* and *unseen tasks*. Experimental results have shown that our approach can consistently outperform a number of competitive baseline methods (even LLM based methods). Besides, we deploy our model in a Chinese education app and online A/B test further verifies the effectiveness of our approach.

2 RELATED WORK

This work focuses on solving mathematical problems, which has been extensively discussed in the literature [37, 38, 48]. Various resources or toolkits are released [8, 22, 28], and also empower a variety of math-related applications [3, 53, 58]. In the following, we will review the related study in three major technical approaches.

Traditional NLP Approaches. Since mathematical problems are described in natural language, it is straightforward to cast the understanding of mathematical problems as a natural language processing (NLP) task. A major difficulty lies in the understanding of the formulas and logic that mathematical text contains. Thus, early NLP approaches typically extract the features for understanding the text and formulas, *e.g.*, semantic parser [47] and operator tree [53]. In recent years, a surge of methods introduce the deep neural network into mathematical problem understanding. They generally leverage advanced NLP models, *e.g.*, RNN [7] and Transformer [31], to encode the mathematical text into meaningful representations.

PLM Based Approaches. Inspired by the success of PLMs in NLP tasks, researchers employ PLMs to deal with mathematical problems [41, 42], showing the superiority in understanding and modeling of mathematical texts. Basically, these methods continually pre-train PLMs (*e.g.*, BERT [10]) with a specific math corpus, and design proper pre-training strategies to capture the semantics of the formulas and logics conveyed in the mathematical texts, *e.g.*, text-formula representation alignment [18, 41], basic-to-advanced curriculum pre-training [56] and unified multi-task learning [39]. However, existing PLM approaches cannot well solve complex mathematical problems and also have a high cost in multi-task deployment.

LLM Based Approaches. In contrast to PLMs with moderate sizes, large language models (LLMs) [4, 5, 57] are introduced to solve mathematical problems [8, 21, 30, 39]. Further, external modules or tools are used to assist LLMs in complex math problem solving, *e.g.*, program interpreter [6, 13, 16]. Since it is very costly to tune LLMs, in-context learning [4] has been widely used to solve different tasks, *e.g.*, chain-of-thought (CoT) method that uses multi-step reasoning [51]. Based on CoT, several improvements have been proposed for mathematical reasoning, including selecting more appropriate samples [15, 54], designing better instructions [25], generating multiple results for ranking [32, 50, 60] and decomposing problem into sub-problems [59]. However, it is hard for LLMs to adapt to the domains or tasks with large differences from the pre-training setting [21], *e.g.*, Chinese mathematical problem solving.

Besides, our model is built on MoE architecture [23], which aims to scale up the model capacity with controllable computational cost. For MoE architectures, it is important to design suitable expert network [43], routing mechanism [19, 26, 52] and training strategies [46, 52, 61]. While, our work has presented a novel application of MoE for dealing with mathematical tasks, with specific improvements. Our work is also related to multi-task learning based on language models [1, 35], while our focus is to share mathematical knowledge across. We design specific architecture and corresponding training strategies for mathematical problem solving, which distinguishes it from prior work on multi-task learning.

3 APPROACH

In this section, we present our **JiuZhang 2.0**, which is developed based on the former version of JiuZhang by introducing specific improvements for multi-task mathematical problem solving.

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Figure 1: The overview of our model JiuZhang 2.0, consisting of two major parts: MoE extension with multi-task training based on the PLM (the primary role) and iterative refinement via LLM (the complementary role). The red bold tokens are errors generated by JiuZhang, which are corrected by LLM in the later iterative refinement process.

3.1 Backbone Model: JiuZhang

We first introduce the backbone model JiuZhang [56] for mathematical problem understanding. Unlike general-purpose PLMs (*e.g.*, BERT [10]), JiuZhang considers the pre-training corpus of *mathematical text*, in which each text consists of a sequence of *n* tokens (either a text word or a math symbol) corresponding to a mathematical problem (including both problem statement and possible solution), denoted as $d = \{t_1, t_2, \dots, t_n\}$. Next, we introduce the original architecture and pre-training tasks for JiuZhang [56].

Architecture. Since both understanding and generation capacities are needed for mathematical problem solving, JiuZhang adopts an architecture consisting of *one shared encoder* and *two task-specific decoders*: one decoder for understanding tasks (*U*-decoder) and the other decoder for generation tasks (*G*-decoder). It employs bidirectional Transformers to implement the shared encoder and the *U*-decoder, and an auto-regressive Transformer to implement the *G*decoder. In order to enhance the representation ability, the shared encoder is built with more layers than the two decoders (*i.e.*, 10 layers *v.s.* 2 layers). Given a mathematical text $d = \{t_1, \dots, t_n\}$, the shared encoder can produce contextualized token representations $\{\mathbf{h}_1^{(L)}, \mathbf{h}_2^{(L)}, \dots, \mathbf{h}_n^{(L)}\}$ (*L*-layer architecture) by capturing mathematical semantics from the input text. Then, the *U*-decoder and *G*-decoder will solve the understanding and generation tasks based on the contextualized representations, respectively.

Pre-training Tasks. In the former version, JiuZhang sets up three types of pre-training tasks and schedules them in a curriculum learning approach. The basic course is constructed based on masked token prediction following general-purpose PLMs, with two pre-training tasks of masked language modeling (L_{MLM}) and denoised auto-encoder (L_{DAE}). The advanced course is constructed based

on specific considerations of mathematical text, including mathematical logic recovering and solution checking. For mathematical logic recovering, we introduce the pre-training tasks of shuffled sentences recovering (L_{SSR}) and shuffled formulas recovering (L_{SFR}), in order to enhance the understanding of mathematical logic; for solution checking, we introduce the pre-training tasks of dual-decoder solution checking (L_{GSC} and L_{USC}), which improve the model's ability to detect and correct errors in its own generated outputs. These pre-training tasks can gradually adapt JiuZhang to mathematical problem solving. Due to space limit, please refer to original paper [56] for more details.

Although JiuZhang can better model mathematical text compared with general-purpose PLMs, it is not specially designed for multi-task mathematical problem solving. In order to enhance the multi-task capacity, we next introduce two important improvements, namely MoE extension with multi-task training (Section 3.2) and iterative refinement with LLM (Section 3.3). In the following, we introduce the two parts in detail.

3.2 MoE Extension with Multi-task Training

By leveraging a corpus of mathematical text, JiuZhang implicitly captures mathematical knowledge with specially designed pretraining tasks. While, such information is encoded via a whole model (*i.e.*, the shared encoder), and it is difficult to transfer mathematical knowledge across different tasks. To better decompose and share the mathematical knowledge, we propose to enhance the backbone model with Mixture-of-Experts (MoE) [46] extension, and introduce multi-task continual pre-training and multi-task finetuning strategies based on MoE-enhanced architecture. 3.2.1 MoE Extension for Knowledge Sharing. MoE [23] is a widely used technique to increase model capacity by incorporating multiple expert networks (the same architecture yet different parameters). While, we employ MoE to decouple and share mathematical knowledge across tasks: common knowledge for related tasks can be captured in one specific expert and less irrelevant knowledge across different tasks is distributed among multiple experts.

MoE Layer for Mathematical Text. In our approach, we only extend the deep shared encoder (capturing the essential mathematical knowledge) with MoE, but not the shallow decoders (supporting different types of tasks). As the encoder is composed of multiple bidirectional Transformer layers, we incorporate the MoE layer to substitute for the original feed-forward layer. Each MoE layer consists of a routing network $R(\cdot)$ and multiple expert networks $\{E_i(\cdot)\}_{i=1}^K$, where *K* denotes the number of expert candidates. To reuse the encoded knowledge from JiuZhang, we utilize the parameters of its feed-forward layer to initialize the parameters of the expert networks, which can also improve the training stability. Since a mathematical problem is usually related to diverse knowledge points, we adopt a token-wise routing mechanism [46] to decouple its associated mathematical knowledge, by assigning experts individually for each token. Given an input mathematical text $d = \{t_1, \dots, t_n\}$, in each Transformer layer, the multi-head self-attention layer first produces the aggregated representations of all these tokens $\{h_1, \dots, h_n\}$. Then, for each token, the routing network estimates the probability distribution over the K experts:

$$R(\boldsymbol{h}) = \operatorname{softmax}(\boldsymbol{W} \cdot \boldsymbol{h}), \tag{1}$$

where W is the trainable matrix for deriving the routing distribution. Further, we employ a weighted combination to integrate the outputs from the K experts:

$$MoE(\boldsymbol{h}) = \sum_{i=1}^{K} R(\boldsymbol{h})_i \times E_i(\boldsymbol{h}).$$
(2)

Sparsely Routing with Jitter Noise. To save the computational cost in MoE layers, we introduce the sparse activation mechanism [46] to selectively utilize expert networks for each token. Specifically, according to the estimated probability distribution $R(\mathbf{h})$, we first rank all the expert networks and then select the top-k ones $(k \leq K)$ in Eq. (2) to derive the token representation. Here, we set k = 1, *i.e.*, only the most related expert will be routed for each token. In this way, for each token, the computational cost of the expert network is roughly the same as the original feed-forward layer of JiuZhang. More detailed analysis about inference latency can be found in Appendix B. However, prior studies [14] have found that such a sparse expert assignment approach would deterministically choose the best-ranking expert, causing the expert network easy to overfit. Therefore, we introduce randomness into the expert selection process by using the *jitter noise* [14] in the routing network. We multiply the estimated probability distribution in Eq. (1) by a jitter noise ϵ (a randomly scaling distribution vector) as:

$$R(\boldsymbol{h}) = \operatorname{softmax}((\boldsymbol{W} \cdot \boldsymbol{h}) \odot \boldsymbol{\epsilon}), \tag{3}$$

where $\boldsymbol{\epsilon} \in \mathbb{R}^{K}$ is a randomly sampled vector and each entry is from a uniform distribution $[1-\eta, 1+\eta]$ (with the noise degree controlling hyper-parameter η), and " \odot " is the element-wise product. In this

way, the probability scores of different experts would be increased or decreased randomly, making the expert networks more robust to perturbations on the routing results.

3.2.2 Multi-task Pre-training for MoE Adaptation. In order to support the MoE architecture, we design multi-task continual pretraining strategies for adapting to the multi-task setting.

Multi-task Continual Pre-training. The goal of multi-task pretraining is to decouple and transfer mathematical knowledge via expert sharing, according to task supervision. Since there is no task data during the pre-training stage, we consider reusing the original pre-training tasks of JiuZhang discussed in Section 3.1, including masked token prediction (L_{MLM} and L_{DAE}), mathematical logic recovering (L_{SSR} and L_{SFR}) and solution checking (L_{GSC} and L_{USC}). Instead of using a curriculum learning way as in [56], we treat the six pre-training losses as equal optimization goals, and set a multitask pre-training objective:

$$L_{MT} = L_{MLM} + L_{DAE} + L_{SSR} + L_{SFR} + L_{USC} + L_{GSC}.$$
 (4)

Note that our model has been initialized with the parameters of the former JiuZhang, so that it also implicitly benefits from the curriculum learning strategy proposed in the previous paper [56]. While, based on the MoE-based architecture, we employ these pre-training tasks to decouple and share mathematical knowledge across tasks.

Auxiliary Losses for Improved Optimization. For MoE methods, there are two major training problems that affect the performance, *i.e.*, the unbalanced load among experts [46] and the training instability [61]. To alleviate these problems, we adopt two auxiliary losses [46, 61] as the regularizers in our approach. Specially, the unbalanced load problem refers that certain experts are extremely frequently routed, which may cause the overfitting problem on these experts and the underfitting problem on other experts. Therefore, we aim to improve the unbalanced routing among all *K* experts. Formally, we encourage the accumulated estimated probabilities for each expert to be uniform, denoted as:

$$L_U = \alpha \cdot K \cdot \sum_{i=1}^{K} f_i \cdot s_i, \tag{5}$$

where f_i is the number of tokens dispatched to the *i*-th expert, and s_i is the accumulated routing score estimated by the routing network for the *i*-th expert, and α is the coefficient to control the influence. According to [46], this loss encourages uniform routing since it would be minimized under a uniform distribution. Further, the training instability problem is often caused by the large volatility of the probability scores in the routing network. In order to control the volatility, we adopt the *Z*-loss [61] that encourages the routing logits of all tokens (size *n*) to remain small as:

$$L_Z = \beta \cdot \frac{1}{n} \log \sum_{j=1}^{n} \exp\left(R(\boldsymbol{h}_j)\right)^2,\tag{6}$$

where β is the coefficient for this loss.

3.2.3 *Multi-task Fine-tuning for MoE Adaptation.* To apply the pretrained model, a typical way is to fine-tune it on some downstream tasks. While, it cannot sufficiently leverage the merits of MoE-based architectures (*i.e.*, decoupling and sharing), without considering inter-task relationships. Thus, we design a multi-task fine-tuning strategy, which boosts the capacity of our MoE architecture by leveraging the data of all (available) downstream tasks.

Unifying the Fine-tuning Tasks. For multi-task fine-tuning, we combine the available training data from multiple downstream tasks for jointly optimizing our model. Since these tasks that we consider are math related, they tend to rely on common mathematical knowledge for task solving, which can be captured via the MoE-based architecture. However, the formats of the input and output data for downstream tasks are generally different, making it hard to be jointly fine-tuned. Recall that our backbone model has included two specific decoders that can handle both understanding and generation tasks for mathematical text. Thus, we unify the math-related tasks into two general formats, either understanding or generation. Specially, for all text classification tasks, we merge the annotation labels and consider an extended multi-label setting, where the label dictionary covers the labels from all classification tasks. In this way, we can equip our U-decoder with a multi-label classifier head to simultaneously accomplish all these classification tasks. Further, for all text generation tasks, we adopt a standard sequence-to-sequence format and utilize the *G*-decoder to solve them. To better distinguish the different tasks for our model, given the training data from *m* tasks, we also devise *m* task prompt embeddings, denoted as $\{p_1, \dots, p_m\}$. For each instance, we insert its task prompt embedding after the [CLS] token embedding.

Routing with Task Prompt. During multi-task fine-tuning, as the task type may be useful to determine the selection of different experts with specific mathematical knowledge, we further revise the routing mechanism by incorporating task-level instruction. Specially, in each MoE layer, we add the input token representation h with the representation of the task prompt p, to compose the input of the routing layer for estimating the probability distribution over the experts as:

$$R(\boldsymbol{h}) = \operatorname{softmax}((\boldsymbol{W} \cdot (\boldsymbol{h} + \boldsymbol{p})) \odot \boldsymbol{\epsilon}), \tag{7}$$

where we also use jitter noise to improve the robustness.

3.3 Iterative Refinement via LLM

Although MoE extension is employed to enhance the backbone model, we keep a moderate-sized model (*i.e.*, 276*M* for K = 4) with an affordable cost for downstream applications. Due to the limit in model size and pre-training data, it still has difficulty in generating solution text for some complex mathematical problems. Our solution is to leverage large language model (LLM) [4, 5] with stronger general modeling capacities for refining the generation results of our PLM. To achieve this, we first design a retrieval strategy to select the most relevant exemplars for constructing the prompts, and then devise an iterative prompting method that utilizes in-context learning to gradually correct the generated results.

3.3.1 Constructing Prompts Using Retrieved Samples. Since existing LLMs are mainly English-focused, they cannot sufficiently capture the necessary mathematical knowledge to effectively accomplish math-related tasks in Chinese (see experiments in Section 4.2). Thus, instead of directly solving the tasks, LLM plays a complementary role in our approach for refining the generated results of our PLM.

Specifically, given a mathematical problem q, we first utilize the PLM (Section 3.2) to generate the solution text \hat{a} , and then employ the LLM via in-context learning [4] to refine \hat{a} into \tilde{a} with improved quality. To provide effective guidance on the LLM, we construct the prompts with retrieved relevant exemplars and specially designed natural language instructions.

Retrieving Exemplars. As empirical studies [34] have revealed that the exemplars in the prompts of LLMs are important to the task performance, we retrieve relevant instances from the training data as the exemplars. Since exemplar finding is essentially an unsupervised text retrieval task, we further employ SimCSE [17] to enhance the representation capacity of our backbone model for semantic matching. Following SimCSE, we incorporate the dropout mechanism to augment positive representations and utilize the contrastive learning objective for training. In the retrieval stage, given the target problem q and the training data set as the retrieval candidate pool, we first encode all the mathematical problems into dense vectors by our backbone model, and then select the top-ranking problems as relevant exemplars, denoted as $C = \{\langle q_j, a_j \rangle\}_{j=1}^B$, where a_j is the associated solution text for problem q_i . Note that we do not use the solution text for the target problem, while only utilizing the solution texts of the problems from training data.

Building Prompts. In order to guide the LLM to refer to the retrieved exemplars for revising the generated result \hat{a} from our PLM, we utilize the in-context learning method with specially designed prompts. Specifically, the input of the LLM consists of four parts, *i.e.*, the given question q, the generated result \hat{a} , the retrieved exemplars $C = \{\langle q_j, a_j \rangle\}_{j=1}^B$, and a natural language instruction I. We concatenate the above four parts into a long sentence, to compose the prompt template as:

$$[q; \hat{a}; C; I] \to \text{prompt(LLM)}, \tag{8}$$

where the instruction I can be flexibly set according to different tasks. We will discuss how to set it in the following part.

3.3.2 Iterative Prompting for Result Refinement. Generally, the generated results from the PLM may contain a variety of mistakes (e.g., inconsistent logic and language typos), and it is hard for the LLM to completely check and correct all these mistakes at once. Therefore, we devise a three-stage iterative refining strategy that gradually improves the generated results following a coarse-to-fine manner. Concretely, based on the prompt template in Eq. (8), we design three specific instructions for the three stages, which guide the LLM to refine the generation results from the three perspectives of overall logic, deduction process and language expressions, respectively. We present the above instructions in the Appendix (Table 9).

Further, to better cooperate with the above instructions, we also revise the way of retrieving exemplars in the three stages:

- at the first stage, we only rely on the problem statement q for finding similar problems, referring to their overall logic;
- at the second stage, we leverage both *q* and the generated solution text *â* for retrieving relevant problems with similar solution text, checking the deduction process;
- at the third stage, we only utilize the generated solution text *â* for retrieval to find other similar solution texts, correcting improper language expressions.

Table 1: Statistics of the datasets for eight evaluation tasks. "Seen" and "Unseen" refer that the task data is *used* or *not used* during multi-task fine-tuning, respectively.

Setting	Туре	Task	Train	Dev	Test
	OA tooks	MCQ	22,000	3,982	7,466
	QA LASKS	BFQ	14,795	1,786	1,778
Soon	Concretion	CAG	16,000	1,976	1,977
Seen	Generation	BAG	14,795	1,786	1,778
	Classification	KPC	8,721	991	1,985
	Classification	QRC	10,000	2,000	4,000
Uncoon	Constian	JCAG	8,000	1,000	1,000
Unseen	Generation	JBAG	8,000	1,000	1,000

To accomplish the goal for each individual stage, we find that it needs multiple iterations for LLM to produce ideal outputs. Thus, we perform *T*-step (*T* = 3) iterations for each stage. At each step, the refined output $\tilde{a}^{(t)}$ will be used as the input of the next step $\hat{a}^{(t+1)}$ to compose the prompt and the retrieved exemplars can also be updated according to new query $\hat{a}^{(t+1)}$. In this way, we can iteratively refine the generated results until the expected goal is fulfilled at each stage, and finally generate high-quality results.

4 EXPERIMENTS

4.1 Experimental Settings

We utilize the same pre-training corpus of JiuZhang [56], consisting of 1,276,952 high-school math problems collected from Zhixuewang, and each problem is associated with the problem type, problem statement and solution text. We preprocess these collected texts in the same way as JiuZhang.

Evaluation Tasks. We consider two different settings for evaluation, namely *seen tasks* and *unseen tasks*, referring to the task data that are *used* and *not used*, respectively, during multi-task fine-tuning. We split each task dataset into training/development/test sets. The statistics of these tasks are shown in Table 1.

• Seen tasks consist of six tasks based on high-school math problems, including (1) two question answering tasks, *i.e.*, Multiple-Choice Question Answering (MCQ) and Blank-Filling Question Answering (BFQ); (2) two analysis generation tasks, *i.e.*, Multiple-Choice Analysis Generation (CAG) and Blank-Filling Analysis Generation (BAG); and (3) two classification tasks, *i.e.*, Knowledge Point Classification (KPC) and Question Relation Classification (QRC). For these tasks, we perform multi-task fine-tuning with all training sets, select the model checkpoint with the best average performance on development sets, and then evaluate the results on test sets.

• Unseen tasks consist of two analysis generation tasks based on junior high school math problems, *i.e.*, Junior-high-school Multiple-Choice Analysis Generation (JCAG) and Junior-high-school Blank-Filling Analysis Generation (JBAG), which are *not used* in multitask fine-tuning for our model. For the two tasks, we perform task-specific fine-tuning, *i.e.*, the multi-task fine-tuned model is separately optimized, tuned and evaluated for each task.

We use the evaluation metrics following JiuZhang [56]. For classification tasks (KPC and QRC), we adopt Accuracy and F1-macro as the evaluation metrics. For question answering tasks (MCQ and BFQ), we adopt Accuracy for evaluation. For generation tasks (CAG, BAG, JCAG and JBAG), we use BLEU-4 [40], ROUGE-2 and ROUGE-L [33] to evaluate the quality of the generated analysis, and also adopt Accuracy to evaluate the generated answers.

Baseline Methods. We select the following four types of baselines: • *Non-pretraining methods* consist of classic neural network methods for text classification or generation, *i.e.*, TextCNN [24], TextR-CNN [27], Seq2Seq [2] and Transformer [49].

• *Pre-trained language models* have been pre-trained on largescale general corpus. We select BERT-Base [11], BART-Base [29], RoBERTa-wwm [9], CPT [45] and Mengzi [55]. For generation tasks, we fine-tune RoBERTa-wwm in a UniLM way [12], and utilize bi-directional attention for input and unidirectional attention for output to implement the Seq2Seq based training and inference.

• Continual pre-training methods further pre-train PLMs on domain-specific corpus (our collected math corpus), and also adopt specially designed pre-training tasks. We select MathBERT [41], DAPT-BERT [20], DAPT-CPT, COMUS [18], JiuZhang [56]. Since our approach is also related to multi-task learning [1, 35], we also add a variant that extends JiuZhang [56] in a multi-task training strategy, MTDNN [35] for fine-tuning.

• *Chain-of-thought (CoT) methods* add explanations to the exemplars in the input prompt of LLMs, to better guide them to generate correct answer [51]. We employ CoT on GPT-3 [4] and CodeX [5], *i.e.*, GPT3-CoT and CodeX-CoT.

Note that CoT methods rely on intermediate reasoning steps of the sampled exemplars in input to guide the solving of math problems, which are not available in the two classification tasks of KPC and QRC. While, in MCQ, BFQ, CAG and BAG tasks, we can utilize the analysis text to derive the intermediate reasoning steps, hence we only report the results of CoT methods on the four tasks.

Implementation Details. For GPT3-CoT and CodeX-CoT, we follow the standard chain-of-thought way to construct the input prompts [51], and the numbers of sampled exemplars are set to 5 and 8, respectively, since GPT-3 has a smaller maximum input length than CodeX. During training, we use AdamW [36] as the optimizer with the learning rate of 3e-5, and warm up the learning rate for the first 5% steps then decay the weight with a ratio of 0.01. The coefficients of the auxiliary loss (Eq. (5)) and the Z-loss (Eq. (6)) are 1e-3 and 1e-4, respectively. For the MoE structure, we set the number of experts K = 4 and the number of activated experts k = 1. For continual multi-task pre-training, we pre-train our model with a batch size of 256 for 700000 steps. For multi-task fine-tuning, we fine-tune our model with a batch size of 32 for 80 epochs and adopt the routing mechanism with task prompt. For iterative refinement, we use CodeX [5] as the LLM and retrieve top-8 similar problems from the training set as exemplars for each input problem. More details are reported in Appendix A.

4.2 Main Results

4.2.1 *Evaluation on Seen Tasks.* For seen tasks, we evaluate the performance of our approach after multi-task fine-tuning. The results of the seen QA/generation and classification tasks are shown in Table 2 and Table 3, respectively, and we can observe that:

Tasks	MCQ	BFQ		CAG				BA	G	
Metrics	Acc.	Acc.	BLEU-4	ROUGE-2	ROUGE-L	Acc.	BLEU-4	ROUGE-2	ROUGE-L	Acc.
Seq2Seq	37.61	44.32	39.91	47.79	67.88	42.63	39.86	48.15	68.06	39.91
Transformer	35.33	46.57	41.39	48.50	67.09	41.02	41.91	48.80	67.76	45.95
RoBERTa-wwm	37.29	47.24	47.29	53.81	70.61	47.70	44.62	51.5	69.54	42.35
BART	36.15	46.82	48.20	55.04	71.66	48.92	45.46	52.16	69.62	43.92
CPT	37.90	46.31	47.98	54.97	71.67	47.03	44.82	52.29	70.01	40.68
DAPT-CPT	46.26	53.41	49.54	55.97	72.52	50.46	46.33	53.69	70.91	48.98
JiuZhang	47.73	54.60	50.05	56.51	72.99	54.51	47.73	54.36	71.17	51.82
JiuZhang-MTDNN	48.81	54.95	49.15	56.28	72.77	56.80	47.58	54.16	71.22	53.09
GPT3-CoT	36.15	50.39	46.93	53.59	70.65	55.18	45.82	52.35	69.43	50.39
CodeX-CoT	40.36	53.82	43.65	54.28	70.43	56.30	42.96	53.45	69.89	53.82
JiuZhang 2.0 w/o IRL	49.75	55.85	50.17	56.72	73.02	58.83	48.33	54.79	71.48	54.78
JiuZhang 2.0	50.37	58.77	50.72	56.97	73.14	60.19	49.39	55.61	71.69	58.77

Table 2: Main results on two question answering tasks and two analysis generation tasks in the setting of seen tasks. Here, "Acc." denotes the metric Accuracy, and "w/o IRL" denotes removing the iterative refinement strategy using LLMs. The best and the second-best methods are denoted in bold and underlined fonts respectively.

Table 3: Main results on two basic classification tasks in the seen setting. Iterative refinement via LLM is not applicable to the two tasks.

Tasks		КРС	(QRC
Metrics	Acc.	F1-macro	Accu.	F1-macro
TextCNN	47.4	26.8	73.3	52.9
TextRCNN	55.3	38.8	79.6	59.0
BERT	59.6	34.9	82.7	63.4
RoBERTa-wwm	61.0	37.0	84.2	65.2
Mengzi	56.6	29.5	81.7	62.8
BART	62.7	41.9	82.0	63.0
CPT	66.2	48.4	82.8	63.4
DAPT-BERT	68.7	46.5	86.5	68.5
MathBert	68.9	47.1	85.3	69.8
COMUS	71.0	63.3	88.0	73.3
DAPT-CPT	72.0	58.0	88.8	76.7
JiuZhang	73.3	59.4	89.4	79.2
JiuZhang-MTDNN	71.5	58.4	89.2	77.1
JiuZhang 2.0 (w/o IRL)	73.5	61.2	89.9	79.8

First, continual pre-training methods (*i.e.*, COMUS, DAPT-CPT, JiuZhang, JiuZhang-MTDNN) achieve better performance than general-purpose PLMs such as BART and CPT. The reason is that these methods have been continually pre-trained on the math corpus, which can learn useful mathematical knowledge from such texts. Among these continual pre-training methods, the two methods based on JiuZhang (*i.e.*, JiuZhang and JiuZhang-MTDNN) mostly outperform all other methods. It is mainly because that JiuZhang incorporates three types of pre-training tasks, which is further pretrained in a curriculum learning way. While, JiuZhang-MTDNN revises the fine-tuning process of JiuZhang by adopting multi-task learning, which can improve the performance on MCQ and BFQ, but has worse performance on KPC and QRC tasks. A possible reason is that there exists negative interference among these tasks during multi-task learning. Besides, COMUS also performs well on the KPC task. Since the KPC task requires a deep understanding of the formulas in mathematical problems for predicting the knowledge points, COMUS specially designs graph neural networks and memory networks for modeling the formulas.

Second, the chain-of-thought methods based on powerful LLMs (*i.e.*, GPT3-CoT and CodeX-CoT) overall perform worse than continual pre-training methods on generation metrics (*i.e.*, BLEU-4, ROUGE-2 and ROUGE-L). The reason might be that these LLMs mainly focus on English tasks, and cannot well adapt to Chinese math-related tasks. In contrast, these continual pre-training methods have been trained over the math corpus, thus having an adaptation capacity in downstream tasks. While, for the Accuracy metric, chain-of-thought methods perform relatively better than other baselines. It shows that LLMs are more skilled in accurately predicting the answer, since they have a stronger mathematical reasoning capacity due to the huge model size and large-scale pre-training corpus (also including large amounts of mathematical texts).

Finally, our proposed JiuZhang 2.0 outperforms all the baselines in most cases. By integrating the MoE architecture with multi-task training, our model can better capture the mathematical knowledge across various math-related tasks. Even without iterative refinement via the LLM, our model (*i.e.*, *JiuZhang 2.0 w/o IRL*) can still outperform all the baselines. After incorporating the iterative refinement via the LLM, the performance of our approach can be further improved, especially on the Accuracy metric. It demonstrates that our approach can further benefit from the mathematical reasoning capacity of the LLM. In this way, JiuZhang 2.0 can combine both the advantages of the PLM and LLM: PLM can be tuned for domain adaptation to Chinese math-related tasks, while LLM has stronger reasoning and generation capacities.

4.2.2 Evaluation on Unseen Tasks. Since multi-task fine-tuning cannot cover all math-related tasks, we continue to examine the performance of our model on new tasks that are not seen before. In order to enlarge the domain gap between existing and new tasks, we select the two tasks of multiple-choice analysis generation

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Methods	JCAG			JBAG				
	BLEU-4	ROUGE-2	ROUGE-L	Accuracy	BLEU-4	ROUGE-2	ROUGE-L	Accuracy
BART	50.50	59.67	73.15	50.40	54.54	60.75	74.51	30.60
CPT	49.38	59.27	72.91	48.20	53.50	60.32	74.23	27.60
DAPT-CPT	52.06	60.84	73.53	54.50	54.66	61.36	74.78	32.30
JiuZhang	52.13	61.43	73.87	55.30	55.69	61.73	75.00	34.50
JiuZhang 2.0 w/o IRL	53.37	61.74	74.00	55.60	56.19	62.13	75.36	38.10
JiuZhang 2.0	55.73	63.76	75.37	63.20	<u>54.45</u>	64.81	77.14	53.80
CAG BLEU-4	58.5	CAG A	ж.	48.5	BAG BLE	EU-4	54.0	BAG Acc

47 5

46 5

45 4

MIPT

Table 4: Main results on two analysis generation tasks for junior high school in the unseen setting.

Figure 2: Ablation study of our approach on CAG and BAG tasks. "¬" indicates that the corresponding technique is removed from our model, while the rest are kept. We abbreviate the terms Multi-task Continual Pre-Training, Multi-Task Fine-Tuning, Mixture-of-Experts, and Task embedding in Routing network as MTPT, MTFT, MoE and TR respectively.

(JCAG) and blank-filling analysis generation (JBAG) from *junior high schools*, which has a different distribution with those from *high schools* (in multi-task fine-tuning). For these two unseen tasks, we fine-tune our model (task by task) on them after multi-task fine-tuning, as the same way in the baselines.

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From Table 4, we can see that the overall experimental findings are similar to those discussed in Section 4.2.1, where we have the overall performance order: PLMs < continual pre-training methods < JiuZhang < JiuZhang 2.0 w/o IRL < JiuZhang 2.0. In particular, the variant of *JiuZhang 2.0 w/o* IRL also performs better than all these baselines, since it employs MoE extension with multi-task training, thus having an improved ability for capturing common mathematical knowledge across tasks. Further, by adopting the iterative refinement via LLMs (IRL), our JiuZhang 2.0 achieves a significant improvement on the Accuracy metric (*i.e.*, 55.60 \rightarrow 63.20 on JCAG, 38.10 \rightarrow 53.80 on JBAG). The results show that the proposed IRL strategy can effectively leverage the strong generation and reasoning capacities of LLMs via in-context learning, which can gradually improve the generation quality of our PLM.

4.3 Detailed Analysis

4.3.1 Ablation Study. In JiuZhang 2.0, we have proposed a series of improvement techniques for enhancing the capacity for mathematical problem solving. Next, we study how each technique contributes to the model performance. We keep the complete model with all improvement techniques as a reference, then remove one specific technique each time, and compare the performance *with* and *without* it. We consider the following variants: (1) ¬ *MoE* removes the MoE extension, (2) ¬ *MTPT* removes multi-task continual pre-training, (3) ¬ *MTFT* removes multi-task fine-tuning, and (4) ¬ *TR* removes the task embedding from the routing network. Note that ¬ *MoE*



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Figure 3: Varying the number of experts (K) in our approach.

can be considered as an implementation of the multi-task learning method [35] with JiuZhang as the backbone model. We report BLEU-4 and Accuracy of these variants on the CAG and BAG tasks.

From Figure 2, we observe that removing any of these improvements would lead to performance degradation, which indicates the effectiveness of these proposed techniques in mathematical problem solving. In particular, the removal of multi-task pre-training or fine-tuning leads to a larger performance drop, which shows the two training strategies are more important to improve the model performance. These two tasks are well suited to the MoE architecture, and they can help capture the mathematical knowledge via the expert networks.

4.3.2 Hyper-parameters Analysis. In our MoE architecture, there are two major hyper-parameters to tune, *i.e.*, the number of experts K and the number of activated experts k in the MoE layers. Next, we investigate the effect of each hyper-parameter on our approach. We conduct the analysis experiments on CAG and BAG tasks and report the results on BLEU-4 and Accuracy metrics for the two hyper-parameters in Figure 3 and Figure 4, respectively.



Figure 4: Varying the number of activated experts (k).

Table 5: Online A/B test of JiuZhang 2.0 and JiuZhang via the automatic math problem solving function on Zhixuewang.

	JiuZhang 2.0 Wins	JiuZhang Wins
Ratio	53.5 %	46.5%

First, the increase in the number of experts does not necessarily improve the performance of our approach (Figure 3), especially in the Accuracy metric. A possible reason is that the MoE architecture introduces additional parameters, which is more likely to overfit on the training set. Besides, using more experts also leads to larger computational costs. In our experiments, to balance the effectiveness and efficiency, we set K = 4, *i.e.*, using four expert networks, which generally gives a good performance. Second, more activated experts are not useful to improve the model performance, even leading to performance degradation (Figure 4). A possible reason is that activating more experts would cause interference among them, resulting in the conflict utilization of experts. In contrast, by setting k = 1, we can not only achieve a relatively better performance, but also save the computation cost of activated expert networks.

4.3.3 Analysis on the MoE Architecture. A major contribution of our model lies in the architecture extension with MoE. By setting multiple expert networks, we can effectively share the mathematical knowledge learned from the math corpus across tasks, so as to improve multi-task mathematical problem solving. These experts are expected to capture and decompose specific mathematical knowledge for different math tasks. Next, we present an analysis experiment about the encoded knowledge at each expert network.

As shown in Table 6, we select three mathematical texts from two tasks, and show the routed expert for each token (toke-level routing) in different background colors. It can be observed that our routing network can effectively decompose the mathematical knowledge and route them to the corresponding experts. For example, the trigonometric functions (*e.g.*, *sin* and π) are routed to *expert #3*, while the (background or formal) words and numbers are mainly assigned to *expert #1* and *expert #2*, respectively.

4.4 **Online** *A*/*B* **Test**

Besides offline evaluation, we further conduct the online A/B test on Zhixuewang² for examining the practical performance of our approach. Zhixuewang is designed as a teacher assistant app that provides personalized education services to students, accumulating about 51 million users in China mainland. Specially, we employ Table 6: Case study on the token-level expert routing. We use the background color to indicate different experts: expert #1, expert #2, expert #3.

Task	Problem Content
BAG	$\frac{5}{5}\sin 90^\circ + \frac{2}{5}\sin \frac{0^\circ - 3}{5}\sin \frac{270^\circ}{5} + \frac{10}{5}\cos \frac{180}{5}\circ = _$
	Known the domain of definition of function
KPC	$y = 2a\cos(2x - \frac{\pi}{3}) + b$ is $[0, \frac{\pi}{2}]$ and the domain of func-
	tion is $[-5, 1]$. Find the value of a, b .
VDC	A seagoing ship starts from A, sails in a straight line at a speed
KPC	of 40 nautical miles per hour in the direction of $\frac{40}{20}^{\circ}$

the function of *automatic math problem solving* on Zhixuewang for conducting online A/B test. Given a math problem (*e.g.*, blankinfilling problem), this function aims to automatically generate the answer with a detailed analysis of the solving process. Here, we compare our JiuZhang 2.0 with the original JiuZhang [56], and both models are fine-tuned by the training data provided by this app. For comparison, we sample a small population of requests of this function, and a user will be asked to select her/his preferred answer and analysis provided by the two models in each request.

Table 5 reports the winning ratio of the two methods. As we can see, our proposed JiuZhang 2.0 performs better than the baseline JiuZhang. The major reason is that our model adopts the multi-task training with MoE layers to better capture the shared knowledge across multiple math-related tasks, and also leverages LLMs to iteratively refine the generated results. In this way, our model can generate more accurate answers and high-quality analysis.

5 CONCLUSION

In this paper, we proposed JiuZhang 2.0, a unified Chinese PLM for multi-task mathematical problem solving. Different from previous PLM approaches for math domain, we focus on improving the multi-task capacity of PLMs, especially on complex tasks. For this purpose, we designed a MoE-based encoder for modeling the mathematical text, aiming to share the mathematical knowledge across different tasks. To support the MoE architecture, we specially designed multi-task continual pre-training and multi-task fine-tuning strategies for learning the shared knowledge via expert networks. Further, we leveraged the powerful LLMs as a complementary role to iteratively refine the generation results by our PLM, with the elaborately designed prompts. Experimental results (both offline evaluation and online A/B test) have demonstrated that our approach is superior to competitive baselines on a variety of math-related tasks.

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²https://www.zhixue.com/

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Algorithm 2: T	he iterativel [.]	v refining a	lgorithm.
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Inp	ut : A mathematical problem q and its analysis generated by						
	JiuZhang with MoE $\tilde{a}^{(0)}$						
Par	ameter : Iteration steps of each stage T , number of exemplars B						
1 for	$s \leftarrow 1 \text{ to } 3 \text{ do}$						
2	for $t \leftarrow 1$ to T do						
3	$\text{iter_step} \leftarrow (s-1) \times T + t;$						
4	$\hat{a}^{(\text{iter_step})} \leftarrow \tilde{a}^{(\text{iter_step-1})};$						
5	switch s do						
6	case 1 do						
7	Use q as the query for retrieval ;						
8	end						
9	case 2 do						
10	Use q and $\hat{a}^{(ext{iter_step})}$ as the query for retrieval ;						
11	end						
12	case 3 do						
13	Use $\hat{a}^{(\text{iter_step})}$ as the query for retrieval ;						
14	end						
15	end						
16	Retrieve exemplars $C = \{\langle q_j, a_j \rangle\}_{j=1}^B$ from the training set						
	using the query;						
17	Construct input prompt using Eq. 8;						
18	Feed the prompt to CodeX to obtain the refined result						
	$\tilde{a}^{(ext{iter_step})};$						
19	end						
o end							

Table 8: The inference latency per batch of different methods on BAG.

M - 41 1-		BAG	
Methods	Latency	BLEU-4	Accuracy
BART	830 ms	45.46	43.92
CPT	330 ms	44.82	40.68
JiuZhang 2.0 w/o IRL	370 ms	48.33	54.78

Table 7: Parameter settings of our models.

Task	Settings
	AdamW, learning_rate=3e-5
	warmup_ratio=0.01
Continual Dra training	batch_size=256
Continuai Fie-training	max_steps=70k
	num_experts=4
	top_k=1
	AdamW, learning_rate=3e-5
Multi-task Fine-tuning	warmup_ratio=0.1
	batch_size=64
	num_experts=4
	top_k=1
	router=task_router
	AdamW, learning_rate=5e-5
	warmup_ratio=0.1
OOD Fine tuning	batch_size=64
OOD Mile-tuiling	num_experts=4
	top_k=1
	router=task_router
In contact Learning	num_examplars=8
In-context Learning	T=1

Supplementary material

Input :Pre-training corpus, Multiple math-related datasets for fine-tuning Parameter: The parameters of the encoder Θ _E , U-decoder Θ _U , G-decoder Θ _G // Multi-task Pre-training for MoE Adaptation while not converged do 2 Sample a batch from the pre-training corpus;					
fine-tuning Parameter: The parameters of the encoder Θ_E , <i>U</i> -decoder Θ_U , <i>G</i> -decoder Θ_G // Multi-task Pre-training for MoE Adaptation 1 while not converged do 2 Sample a batch from the pre-training corpus;					
Parameter: The parameters of the encoder Θ _E , U-decoder Θ _U , G-decoder Θ _G // Multi-task Pre-training for MoE Adaptation 1 while not converged do 2 Sample a batch from the pre-training corpus;					
<pre>G-decoder Θ_G // Multi-task Pre-training for MoE Adaptation 1 while not converged do 2 Sample a batch from the pre-training corpus;</pre>					
<pre>// Multi-task Pre-training for MoE Adaptation 1 while not converged do 2 Sample a batch from the pre-training corpus;</pre>					
 while not converged do Sample a batch from the pre-training corpus; 					
2 Sample a batch from the pre-training corpus;					
3 Compute the six pre-training losses using Eq. 4;					
4 Compute the two auxiliary losses using Eq. 5 and Eq. 6;					
Performing gradient descent to optimize Θ_E , Θ_U and Θ_G ;					
6 end					
// Multi-task Fine-tuning for MoE Adaptation					
7 while not converged do					
8 Sample a batch from the multiple fine-tuning datasets;					
9 Unify the input and output data formats of the batch of					
instances;					
10 Compute the fine-tuning loss using the <i>U</i> -decoder and					
G-decoder;					
Performing gradient descent to optimize Θ_E , Θ_U and Θ_G ;					
12 end					

A IMPLEMENTATION DETAILS.

We report the detailed parameter settings of our approach throughout the experiments in Table 7. In addition to the above settings, we conduct all the experiments on 8 RTX 3090 24G GPUs, where the multi-task continual pre-training and multi-task fine-tuning took about 72 and 12 hours, respectively. During multi-task finetuning, we construct the model inputs of all downstream tasks as follows, and the task embedding will be inserted after the [CLS] token embedding.

KPC, MCQ, BFQ, CAG, BAG, JCAG and JBAG: [CLS] q [SEP]. **QRC:** [CLS] q_1 [SEP] q_2 [SEP].

For the iterative refinement via the LLM, we also design three types of instructions and adopt three ways to construct queries for retrieval, for the three iterative stages, respectively. We show the details in Table 9.

Besides, we also present the Algorithm 1 and Algorithm 2, to better show the multi-task training and iterative refinement processes of our approach, respectively.

B INFERENCE LATENCY ANALYSIS

In our approach, although we scale up the number of parameters in the PLM by incorporating the MoE layers, the sparsely routing mechanism can ensure that only the top-1 most related expert will be activated, leading to relatively less increased computational cost. To investigate it, we conduct the analysis experiments to compare the inference latency per batch of our model with two baselines JiuZhang 2.0: A Unified Chinese Pre-trained Language Model for Multi-task Mathematical Problem Solving

Table 9: The detailed retrieved queries and instructions for constructing the input in each iterative refinement stage.

Iteration Stage	Query for Retrieval	Instruction
First Stage	Problem statement q	根据上文中相似的题目,选取正确的解题思路,修改本题参考解 答中的错误。
		According to similar problems above, choose the correct idea to solve this problem and fix mistakes in the reference solution.
Second Stage	Problem statement q + Generated analysis \hat{a}	根据上文中相似的题目,修改本题的参考解答中的推理错误和逻辑错误,确定本题的答题逻辑,得到正确解答。
		According to similar problems above, correct the reasoning errors and
		logical errors in the reference solution, determine the reasoning logic of this question and get the correct solution.
Third Stage	Generated analysis â	根据上文中相似的题目,修改本题的参考解答中的计算错误和抄 写错误,确定本题的正确答案,得到正确解答。
		According to similar problems above, correct the calculation errors and transcription errors in the reference solution, determine the final answer and get the correct solution.

Table 10: Case study on the analysis generation tasks.

Math Problem	设 $\{a_n\}$ 为等差数列, $a_1 = 22$, S_n 为其前 n 项和, 若 $S_{10} = S_{13}$, 则公差 $d = ?$
	Let $\{a_n\}$ be an arithmetic sequence, $a_1 = 22$, S_n be the sum of the first <i>n</i> items, if $S_{10} = S_{13}$, the common difference $d = ?$
	A2; B1; C. 1; D. 2;
Ground-Truth	因为 $S_{10} = S_{13}$, $a_1 = 22$, 所以 $10 \times 22 + \frac{10(10-1)}{2}d = 13 \times 22 + \frac{13(13-1)}{2}d$, 解得 $d = -2$. 故选A.
	Because $S_{10} = S_{13}$, $a_1 = 22$, so $10 \times 22 + \frac{10(10-1)}{2}d = 13 \times 22 + \frac{13(13-1)}{2}d$, the solution is $d = -2$. So choose A.
Codex-CoT	由 $S_{10} = S_{13}$ 得到 $a_1 + \dots + a_{10} = a_1 + \dots + a_{13}$,即 $a_{13} = 0$.因此 $a_{13} = a_1 + (13 - 1)d = a_1 - 12d = 0$,解得 $d = 1$.故选C.
	From $S_{10} = S_{13}$, get $a_1 + \cdots + a_{10} = a_1 + \cdots + a_{13}$, namely $a_{13} = 0$. Therefore $a_{13} = a_1 + (13 - 1)d = a_1 - 12d = 0$, the solution is $d = 1$. So choose C.
JiuZhang	由 $S_{10} = S_{13}$,得 $\frac{10(a_1+a_{13})}{2} = \frac{13(a_1+a_{13})}{2}$,即22+11 $d = 0$,解得 $d = 2$.故选D.
	From $S_{10} = S_{13}$, get $\frac{10(a_1+a_{13})}{2} = \frac{13(a_1+a_{13})}{2}$, That is, $22 + 11d = 0$, the solution is $d = 2$. So choose D.
Ours	因为 $S_{10} = S_{13}$,所以 $a_{11} + a_{12} + a_{13} = 0$,所以 $a_{12} = 0$,又 $a_1 = 22$,所以 $d = -2$.故选A.
	Because $S_{10} = S_{13}$, so $a_{11} + a_{12} + a_{13} = 0$, so $a_{12} = 0$, and $a_1 = 22$, So $d = -2$. So choose A.

using different model structures, *i.e.*, BART and CPT, in the BAG task. During inference, we adopt greedy search to decode and set the batch size to 16. As shown in Table 8, compared to CPT, the inference latency of our model is slightly increased. It indicates the effectiveness of the sparse routing mechanism to guarantee the efficiency of our approach. Besides, we can see that BART requires double the inference time of CPT and our approach. The reason is that CPT and JiuZhang 2.0 adopt an unbalanced model architecture with a shallower decoder than BART (2 layers VS. 6 layers), which can save the computation cost on the cross-attention layers of the decoder.

C CASE STUDY

To give a qualitative analysis of our proposed approach, we perform a case study that shows the generated analysis from our approach. We select two examples from the CAG and BAG tasks, respectively, and also show the generated analysis by two best performed methods, *i.e.*, JiuZhang and CodeX-CoT.

As shown in Table 10, although CodeX-CoT and JiuZhang have generated a detailed multi-step reasoning process consisting about the two problems, they both make mistakes in the intermediate steps. For the first example, we can see that CodeX-CoT obtains a wrong intermediate conclusion $a_{13} = 0$ by mistakenly simplifying the summation of two arithmetic progressions, which may be caused by the unfamiliarity of the knowledge about arithmetic progressions. JiuZhang makes a small mistake in calculation, *i.e.*, $22 + 11d = 0 \longrightarrow d = 2$, leading to the wrong answer. It also reflects the lack of mathematical computation common sense about JiuZhang. As a comparison, we can see that our approach can generate more proper analysis and successfully produce the true answers. It indicates the effectiveness of our approach in solving complex mathematical problems.