HOTGP-Higher-Order Typed Genetic Programming

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ABSTRACT

Program synthesis is the process of generating a computer program following a set of specifications, which can be a high-level description of the problem and/or a set of input-output examples. The synthesis can be modeled as a search problem in which the search space is the set of all the programs valid under a grammar. As the search space is vast, brute force is usually not viable and search heuristics, such as genetic programming, also have difficulty navigating it without any guidance. In this paper we present HOTGP, a new genetic programming algorithm that synthesizes pure, typed, and functional programs. HOTGP leverages the knowledge provided by the rich data-types associated with the specification and the built-in grammar to constrain the search space and improve the performance of the synthesis. The grammar is based on Haskell's standard base library (the synthesized code can be directly compiled using any standard Haskell compiler) and includes support for higher-order functions, λ -functions, and parametric polymorphism. Experimental results show that, when compared to 6 stateof-the-art algorithms using a standard set of benchmarks, HOTGP is competitive and capable of synthesizing the correct programs more frequently than any other of the evaluated algorithms.

CCS CONCEPTS

• Computing methodologies \rightarrow Genetic programming; • Software and its engineering \rightarrow Functional languages.

KEYWORDS

Inductive Program Synthesis, Genetic Programming, Functional Programming

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1 INTRODUCTION

Program Synthesis (PS) is the task of creating a computer program, in algorithmic form, based on a set of specifications [6]. A program specification is a high-level description of the objective of the program [19]. This specification can have different formats, from natural language to a more formal notation. A common approach to specifying a program to solve a problem is providing a set of input-output examples, with special attention to edge cases, to reduce the set of ambiguous solutions. This is called Inductive Synthesis, or Programming-by-Examples (PBE) [7]. In this case, the task of the synthesizer is to find a program that correctly maps each pair of input-output provided by the examples. Depending on

Algorithm 1: One solution to the example specification.					
1 function solution(x):					
2	$s \leftarrow 0$				
3					
4	if $x[i] < 100$ then				
5	if $x[i] < 100$ then				
6	return s				

the completeness of the provided examples, the specification might be ambiguous leading to many alternative programs that do not behave as intended, even if they correctly map those examples.

The generation of a program can be modeled as a search where the search space contains the set of all possible programs valid under a pre-specified grammar. The objective is to find a program that meets the specification. The search space size makes it impractical to employ a naive approach for selecting the best candidate from the enumeration of all possible programs. For this reason, the PS is often done using a (meta-)heuristic approach, usually Genetic Programming (GP) [18].

Even though this approach has presented some success in standard PS benchmarks [11–14, 17], it is still incapable of finding solutions to some tasks that are trivial for humans. Some of the reasons for this are:

- Traversing the search space is challenging, as sometimes a small change in the program code significantly impacts its output;
- (2) Without additional information about the program, the search relies on the completeness of the examples;
- (3) Some synthesizers can create stateful programs that can have unpredictable behavior depending on how the states are changed.

To illustrate these difficulties, let us take as an example a specification for a program that, given a list x, returns the sum of all the elements which are smaller than 100. A valid imperative style program is presented in algorithm 1. Changes to any of the initial values of s (line 2), i (line 3) or the constant 100 (line 4) can drop the accuracy from 100% to 0%. On the other hand, for instance, if the input-example lists do not contain values between 95 and 100, a constant such as 98 in line 4 will be enough to achieve 100% accuracy on the training set, even though the code will produce wrong results when one considers all possible inputs. Finally, any additional statement inside the loop body that affects the value of either i or s might also decrease the achieved accuracy. One possible solution to alleviate these problems is to employ a typed and purely functional paradigm. In this paradigm, a program is a *pure* function and is defined as the composition of pure functions.

A pure function has, by definition, the fundamental property of *referential transparency* [29]. This means that any expression (including the whole program which is, by itself, an expression) can safely be substituted by the result of its evaluation. This property makes input transformations explicit and predictable, constraining the search space to only functions with no side effects. Finally, a typed language contains information about the input and output types which helps us to constrain the search space further.

Moreover, if we also allow for parametric polymorphism¹, we can effectively constrain the search space to contain only well-formed programs. Take, for example, the type signature $\forall a. a \rightarrow a$. This signature only allows a single implementation, which is the identity function. Any other implementation would either violate referential transparency or the type information. Albeit extreme, this example shows how the combination of these properties can constrain the search space, easing the task of the PS algorithm [22].

While the mentioned features already allow for an expressive language, typical functional programming languages also provide constructs for the implementation of higher-order functions [15]. In this context, a higher-order function is a function that receives a function as one of its arguments. Commonly used higher-order functions are map, filter, fold, which generalize many common patterns required by a program.

This work proposes a new GP algorithm, named HOTGP (Higher-Order Typed Genetic Programming), that searches for pure, typed, and functional programs. The grammar supports higher-order functions, parametric polymorphism in functions, and parametric types (such as lists and tuples). HOTGP was evaluated against 29 benchmark problems and its results compared to 6 other algorithms from the literature. Results show that a pure functional approach can significantly improve the results of the standard GP algorithm in terms of the frequency that it finds correct programs.

The remainder of this paper is organized as follows. Section 2 presents related work. Section 3 describes HOTGP. The experimental evaluation is outlined in Section 4, and we conclude in Section 5.

2 RELATED WORK

To the best of our knowledge, Automatic Design of Algorithms Through Evolution (ADATE) [23] is the earliest example of PS targeting functional code. This work aimed at synthesizing recursive ML language programs using incremental transformations. The algorithm starts with an initial program described by the token "?" that always returns a *don't* $know^2$ value. After that, ADATE systematically expands the expression into a pattern matching of the input type, synthesizing a program for each branch of the pattern match, and replacing the general case with a recursive call.

Montana [22] proposes the Strongly Typed Genetic Programming (STGP) algorithm, an adaption of GP that considers the types of each function and terminal during the PS. The purpose of taking types into consideration is to further constrain the search space by allowing only correctly-typed programs to exist (*i.e.*, programs in which all functions operate on values with the appropriate data types). In contrast to standard GP, where a given nonterminal must be capable of handling any data type, STGP imposes extra constraints to enforce type-correctness. Another important contribution of the STGP is that the types of the nonterminals can employ parametric polymorphism.

The main benefit of having parametric polymorphism is that there is no need for multiple similar functions whose difference is only in their types. Experiments on four different problems (regarding matrix and list manipulations) have shown that STGP generally outperforms untyped GP.

STGP and a standard untyped GP were compared by Haynes et al. [8] using the "Pursuit Problem". This problem models a game where four predators pursue a prey. The goal is to create an algorithm for the predators to capture the prey as fast as possible. The prey always runs away from the nearest predator, and the predators only have information about themselves and the prey, but not about the other predators. Results show that a good STGP program can be generated faster than a good GP program. Moreover, the best STGP program has a higher capture rate than the best GP program.

PolyGP [32, 33] extends STGP with support to higher-order functions and λ -functions. It also differs from STGP by using a type unification algorithm instead of a lookup table to determine the concrete types when using polymorphic functions. The λ -functions use the same initialization procedure of the main PS, but the available terminals are limited to the input parameters. Because these λ -functions do not have any type restriction, they can be invalid in which case it must be discarded and regenerated. The overall algorithm is a simple search for a composition of λ -functions with a user-defined set of terminals and nonterminals as in STGP.

Katayama [16] proposes MagicHaskell, a breadth-search approach that searches for a correctly-typed functional program using SKIBC [31] combinators. This simplifies the PS by reducing the search space. MagicHaskell also introduces the use of the de Bruijn lambda to find equivalent expressions and memoization to improve performance [2]. Additionally, it implements fusion rules to simplify the synthesized program further. This particular approach was reported not to work well with larger problems [24].

Strongly Formed Genetic Programming (SFGP) [1] is an extension to STGP. SFGP not only assigns known data-types to terminals but also node-types to functions. A node-type identifies if a given node is a variable, an expression, or an assignment. Each subtree of a function can also be required to be of a certain node-type. The authors argue that this extra information is helpful to build correctly typed *imperative* programs (*e.g.*, the first child of an assignment must have the "Variable" node-type and match the data-type of the second child). They conducted experiments on 3 datasets, with a reduced grammar that deals mainly with integers, and reported high success rates with a lower computational effort than competing methods.

Santos et al. [28] discuss desiderata for PS approaches by further constraining the search space, similar to what is done by STGP. They propose the use of Refinement Types to this aim. As this is an ongoing project, to the best of our knowledge, there are still no experimental evaluations or comparative results.

¹Also known as generics in some programming languages.

²This is equivalent to a function that always returns null.

Pantridge et al. [25] proposes an adaptation of the Code Building Genetic Programming (CBGP) [26] as a means to incorporate elements of functional programming such as higher-order functions and λ -functions. CBGP uses the same representation of PushGP with three primary constructs: APP, to apply a function; ABS, to define a function of 0 or more arguments and; LET, to introduce local variables in the current scope. It also uses concepts from type theory to ensure the correctness of the polymorphic types. CBGP achieved higher generalization rates for a subset of benchmark problems. However, for other problems, the generalization rate was close to 0. The authors noted that the evolutionary search avoided using λ -functions and preferred to employ pre-defined functions in higher-order functions such as map. These results show some indirect evidence of the benefits provided by type-safety to PS, in particular, with regard to the stability of the solutions over different executions of the search algorithm.

In this same line, Garrow et al. [5] compared the generation of Python and Haskell programs using a grammar-guided system [21]. Similar to our work, they employ a different grammar for each set of types instead of a different grammar per benchmark problem. Their approach supports higher-order functions, but limits the function arguments to pre-defined commonly used functions. Experimental results showed that the Haskell version consistently outperforms Python in most selected benchmarks. Implementing general λ -functions was left as future work by the authors since that would add complexity to the search space and must be carefully handled as a different construct from the main program.

He et al. [9] investigate the reuse of already synthesized programs as subprograms to be incorporated in the nonterminal set. The main idea is that, if the algorithm has already synthesized solutions to simpler tasks, these solutions can be used to build more complex solutions, in an incremental process. Their results show a significant benefit could be obtained by adding handcrafted modules in 4 selected benchmarks.

Forstenlechner et al. [3] criticize a common technique in GP, which is to provide a different grammar for each problem. They argue that this leads to difficulties in grammar reuse, as they are specifically tailored to each problem. They propose a general grammar to the G3P algorithm and perform experiments on the benchmark introduced by Helmuth and Spector [12]. Since the proposed grammar had difficulty with the benchmark problems involving characters and strings, the authors proposed an improved and expanded grammar leading to G3P+ [4].

3 HIGHER-ORDER TYPED GENETIC PROGRAMMING

This section introduces Higher-Order Typed Genetic Programming (HOTGP). To the best of our knowledge, STGP was among the first to propose and employ types for GP. As such, it naturally has influenced following works, such as Castle and Johnson [1], Santos et al. [28], and HOTGP. We now present the main concepts needed for typed GP which are shared by all these synthesizers.

In Strongly-Typed Genetic Programming (SGTP), every terminal has an associated data type, and nonterminals have associated input types and one output type. To enforce correctness, the algorithm imposes two constraints: i) the root of the tree must have the same output type as the intended program output type; ii) every non-root node must have the output type expected by its parent.

Due to these restrictions, the main components of the evolutionary search must be adapted. At every step of the initialization process, a node will be considered only if it matches the type expected by its parent node. STGP also builds type-possibility tables to keep track of which data-types can be generated by a tree of each depth, one for each initialization method (*grow* and *full*). Those tables are dictionaries whose keys represent the depths and the values the types representable by trees of that depth, for *full* or *grow*. At depth 0, they contain only terminals. At depth *i*, they contain the output types of all the functions that take the types at i - 1 as an argument; and for *grow*, it also contains the types at i - 1.

The mutation operator replaces a random subtree with a new subtree generated with the same algorithm of the initialization procedure, using the *grow* method. The crossover, as expected, also takes into account the types. The crossover point of the first parent is chosen entirely at random, while the point of the second parent is limited to those whose type is the same as the first parent. If no such candidate exists, it returns one of the parents. There are also some additional changes to the original GP algorithm regarding the evolutionary process such as the use of *steady-state replacement* [30] and *exponential fitness normalization*, which select parents for reproduction based on their ranks. The probability of picking the *n*th best individual is given by $p(n) = P_{scalar} \times p(n-1)$, where $0 < P_{scalar} < 1$ is a hyperparameter.

Montana [22] also argues in favor of handling runtime errors as part of the evolutionary process, penalizing individuals that present them. This is the opposite of the original GP approach [18], which enforces that a value must always be returned.

STGP also introduces the Void type for functions that do not return anything (*i.e.*, procedures) and local variables which can be statefully changed during computation.

In contrast to STGP, HOTGP introduces higher-order functions and λ -functions, drops the support for impure functions, and uses a general-use grammar extracted from Haskell's base library. The main differences, detailed in the next subsections, are:

- HOTGP builds programs using a pure functional program paradigm (a subset of the Haskell programming language) while STGP is modeled after a combination of typed-LISP and ADA allowing impure functions (see Section 3.1);
- Since HOTGP is designed to only support pure functions, all side effects, including local variables (mutable state) and IO, are disallowed by design (see Sec. 3.1);
- As we shifted to a different language, appropriate changes to the grammar were performed (see Section 3.1);
- Instead of specifying a strict set of terminals and nonterminals which are specific to each problem, we specify generic sets based on the input and output types³ (see Section 3.1);
- Moreover, we use a more generic set of non-terminals (all available in the standard Haskell base library) instead of very specific functions that often need to be implemented by the user. This characteristic, combined with the use of a subset of the Haskell language, allows for all the synthesized code

³This is a design choice that was not explored by STGP nor PolyGP.

to be immediately consumed by a Haskell compiler without modification (see Section 3.1);

 Finally, HOTGP has support for higher-order functions (functions that accept λ-functions as input) to handle advanced constructs in the synthesized programs (see Section 3.2);

3.1 Functional Grammar

Even though both HOTGP and STGP share the use of strong types, in both experimental evaluations of STGP [8, 22], the authors employed a limited grammar specifically crafted for each one of the benchmark problems. For example, to solve the Multidimensional Least Squares Regression problem, they used a minimal set of functions with matrix and vector operators such as *matrix_transpose*, *matrix_inverse*, *mat_vec_mult*, *mat_mat_mult*. Instead, this paper uses a more general set of functions, common to all problems, all of which were extracted from the standard Haskell base library.

We argue that, in a practical scenario, providing only the functions needed for each problem is undesirable since it involves giving too much information to the algorithm. This is, in our opinion, not ideal since this piece of information might not be readily available beforehand. A much more reasonable demand on the user is to ask them for the acceptable result type for each problem. This kind of information usually only requires as much intuition on the problem as providing examples.

HOTGP primitive types currently includes 32-bit integers, singleprecision floating-point numbers, booleans, and UTF8 characters. The following parametric types are also supported: pairs (2-tuples); linked lists; and λ -functions. Types can be combined to create more complex types, *e.g.*, a list of pairs of λ -functions or, something simpler such as a string (represented as a list of characters).

As a consequence of using a subset of the Haskell language, HOTGP precludes the use of impure functions. The use of pure functions is often associated to a reduction of the number of possible bugs [27]. An essential property of pure functions is that, being without side effects, they are easier to compose. Thus, whenever the return type of one function is the same as the input type of another function, they can be composed to form a new, more complex pure function.

The full list of the functions allowed by HOTGP's grammar is shown in Table 1⁴. Most functions are common operations for their specific types. Since we employ a strongly-typed language, we also require conversion functions. Additional functions of common use include sum and product for lists of numbers (integers and floating points), Range, which generates a list of numbers (equivalent to Haskell's [x, y..z]; Zip, that pairs the elements of two lists given as input; Take, that returns the first *n* elements of a list; and Unlines, that transforms a list of strings into a single string, joining them with a newline character. In particular, Unlines is needed for the benchmarks requiring the program to print text to the standard output (in our case, since we are working on a pure language, we simply return the output string).

It is worth noting that we included three *constructor* functions in the grammar: ToPair, Cons, and Singleton. This is a deliberate

Table 1: Functions supported by HOTGP.

Function Type	Function names
$Int\toInt\toInt$	AddInt, SubInt, MultInt,
	DivInt, ModInt, MaxInt,
	MinInt
$Bool \rightarrow Bool$	Not
$\texttt{Bool} \to \texttt{Bool} \to \texttt{Bool}$	And, Or
$\texttt{Bool} \to \texttt{a} \to \texttt{a} \to \texttt{a}$	If
$Float \rightarrow Float$	Sqrt
$\texttt{Float} \rightarrow \texttt{Float} \rightarrow \texttt{Float}$	AddFloat, SubFloat,
	MultFloat,DivFloat
$a \rightarrow [a]$	Singleton
$a \rightarrow [a] \rightarrow [a]$	Cons
[a] → a	Head
[a] → [a]	Reverse
[[a]] → [a]	Concat
$a \rightarrow b \rightarrow (a,b)$	ToPair
$(a,b) \rightarrow a$	Fst
$(a,b) \rightarrow b$	Snd
$Char \rightarrow Char \rightarrow Bool$	EqChar
$Char \rightarrow Bool$	IsLetter, IsDigit
$Int \rightarrow Float$	IntToFloat
$Float \rightarrow Int$	Floor
$\texttt{Int} \rightarrow \texttt{Int} \rightarrow \texttt{Bool}$	GtInt, LtInt, EqInt
$[a] \rightarrow Int$	Len
Int \rightarrow [a] \rightarrow [a]	Take
$\texttt{Int} \rightarrow \texttt{Int} \rightarrow \texttt{Int} \rightarrow \texttt{[Int]}$	Range
$[Int] \rightarrow Int$	SumInts, ProductInts
$[Float] \rightarrow Float$	SumFloats, ProductFloats
$[[Char]] \rightarrow [Char]$	Unlines
Int \rightarrow [Char]	ShowInt
$[a] \rightarrow [b] \rightarrow [(a,b)]$	Zip
$(a \rightarrow b) \rightarrow [a] \rightarrow [b]$	Мар
$(a \rightarrow Bool) \rightarrow [a] \rightarrow [a]$	Filter

choice to simplify the grammar. Let us take 2-tuples (pairs) as an example. Our grammar must be able to cope with constructions such as (1, 2) or (1 + 2, 3 * 4) (pairs of literals and pairs of expressions). However these same pairs can be easily represented as applications of ToPair. The first example can be represented as ToPair 1 2, which means applying the ToPair function to the arguments 1 and 2. Following the same representation, the second example becomes just ToPair (AddInt 1 2) (MultInt 3 4).

In other words, the construction of a pair is a simple function application with no special treatment by our grammar. This has the added benefit of being directly compatible with the mutation and crossover operators already defined for regular nodes. Under the same reasoning, the evolution process can generate linked lists using a combination of the Cons and Singleton functions. For example, the list of the literals 1, 2, 3 can be represented as Cons 1 (Cons 2 (Singleton 3)); and the list of the expressions 1 + 2, 3 + 4, 5 - 6 can be represented as Cons (AddInt 1 2) (Cons (MultInt 3 4) (Singleton (SubInt 5 6))). As was also the case with pairs, this has the added benefit of enabling crossover

 $^{^{4}}$ For the sake of space and legibility, in this text we represent pairs and lists using ML-inspired conventions: (7, 42) is a pair containing 7 and 42, and [42, 7, 6] is a list with elements 42, 7 and 6. Similarly, we write [a] in lieu of the type List a and (a, b) in lieu of the type Pair a b.

and mutation to happen on just the head or just the tails of such lists.

HOTGP also allows the user to select which types the program synthesis algorithm can use, to constrain the search space further. Whenever the user selects a subset of the available types, the non-terminal set is inferred from Table 1 by selecting only those functions that support the selected types. For example, if we select only the types Int and Bool we would allow functions such as AddInt, And, GtInt, but would not allow functions such as Head, Floor, ShowInt.

For a future implementation of this algorithm, we plan to add support to ad-hoc polymorphism, employing Haskell's type-classes, so we can simply have Add, Mult, Sub that determine their types by the context instead of having specific symbols for each type.

Another important distinction from STGP to HOTGP is the absence of the Void type, and constructs for creating local variables. Therefore, impure functions and mutable state are not representable by HOTGP's grammar. By construction, HOTGP does not allow side effects and can only represent pure programs. On the other hand, similarly to STGP, runtime errors (such as divisions by zero) can still happen. When they do, the fitness function assigns an infinitely bad fitness value to that solution.

3.2 Higher-order Functions and λ -functions

The main novelty of HOTGP is the use of higher-order functions. To that end, adding support to λ -functions is essential. A λ -function, or anonymous function, or simply lambda, is a function definition not bound to a name. As first-class values, they can be used as arguments to higher-order functions.

The introduction of lambdas requires additional care when creating or modifying a program. When evaluated, HOTGP's lambdas only have access to their own inputs, and not to the main program's. In other words, they do not capture the environment in which they were created or in which they are executed. This means that lambda terminals can be essentially considered "sub-programs" inside our program, and are generated as such. We use the same initialization process from the main programs, using the function type required by the current node and employing the *grow* method. However, two additional constraints must be respected.

Constraint 1 requires all lambdas to use their argument in at least one of its subtrees, which significantly reduces the possibility of the creation of a lambda that just returns a constant value. We argue that, for higher-order-function purposes, a lambda is required to use its argument in order to produce interesting results; otherwise the program could be simplified eliminating the use of this higher-order function and returning a constant⁵.

Constraint 2 takes the form of a configurable maximum depth of the lambdas, which is imposed to prevent our programs from growing too large. However, as these lambdas can be nested, this hyperparameter alone is not enough to properly constrain the size of a program. For instance, take a lambda as simple as $\lambda x \rightarrow map$ otherLambda x. Depending on the allowed types, otherLambda = $\lambda x \rightarrow map$ yetAnotherLambda x would be a valid function and so on, which could lead to lambdas of arbitrarily large size. Therefore, to prevent excessively large lambda nesting, we constrain nested functions to always be $\lambda x \rightarrow x$ (the identity function).

To enforce these constraints, similarly to STGP, HOTGP employs type-possibility tables to generate lambdas. For the main program tree, as the argument and output types are known beforehand, both STGP and HOTGP only need to create two tables: one for the *grow* and one for the *full* method. However, HOTGP needs to generate lambdas involving every possible type allowed by the current program. Due to the recursive nature of the table, different argument types can lead up to vastly different type-possibility tables, so we need to keep one separate table for each possible argument type. As a corollary of Constraint 1, those tables are also guaranteed never to grow too large, as they never need to calculate possibilities for depths larger than the maximum lambda depth. These tables also differ from the main tables in the sense that they only consider a node valid if at least one of its subtrees can have an argument leaf as a descendent, enforcing Constraint 2.

In terms of mutation and crossover, lambdas are treated as regular terminals. They are always discarded and regenerated (using the process described above) or moved in their entirety, being treated essentially as a single unit.

3.3 Code Refinements

A well-known difficulty faced by GP algorithms is the occurrence of *bloat* [20], an unnecessary and uncontrollable growth of a program without any benefit to the fitness function. This happens naturally as some building blocks that apparently do not affect the program's output survive during successive applications of crossover and mutation. Not only do these bloats make the generated program longer and unreadable, but they can also affect the performance on the test set. For example, consider the task of doubling a number and the candidate solution $x0 * (\min x0 900)$. If the training set does not contain input cases such that x0 > 900, then this will be a correct solution from the algorithm's point-of-view.

Helmuth et al. [10] empirically show that simpler programs often have a higher generalization capability, in addition to being easier to understand and reason about. Pantridge et al. [25], for example, applies a refinement step at the end of the search, repeatedly trying to remove random sections of the program and checking for improvements.

To alleviate the effect of bloats, we also apply a refinement procedure on the best tree found, considering the training data. Refinement starts by applying simplification rules, which remove redundancies from the code:

- Constant evaluations: if there are no argument terminals involved in a certain tree-branch, it can always safely be evaluated to a constant value, *e.g.* head [4*5, 1+2] → 20;
- General law-application: the simplifier has access to a table of hand-written simplification procedures, which are known to be true (laws) (*e.g.* if True then a else b ≡ a, a > a ≡ False, length (singleton b) ≡ 1, etc).

After this step, HOTGP applies a Local Search procedure aiming at the removal of parts of the tree that do not contribute to, or even reduce, the correctness of the program considering the training set. The local search replaces each node with each one of its children and keeps the modified version if it improves or returns the same

 $^{^5\}mathrm{This}$ is only true because HOTGP's grammar precludes the generation of expressions with side effects.

Algorithm 2: The local search procedure							
1 f	unction localSearch(tree):						
2	<pre>if not hasNext(tree.node) then return tree</pre>						
3	$bestTree \leftarrow tree$						
4	foreach $child \in tree.node.children$ do						
5	<pre>if child.outputType = tree.node.outputType then</pre>						
6	newTree \leftarrow replace(<i>tree.node</i> , <i>child</i>)						
7	if accuracy(<i>newTree</i>) \geq accuracy(<i>bestTree</i>)						
8	&& nNodes(<i>newTree</i>) < nNodes(<i>bestTree</i>)						
9	then						
10	bestTree ← newTree						
11	if bestTree ≠ tree then						
12 return localSearch(<i>bestTree</i>)							
13	<pre>13 return localSearch(nextPreOrder(tree))</pre>						

result. Algorithm 2 describes this process. It takes as input *tree*, which has an internal representation of the current position being checked, that can be accessed via *tree.node*. The procedure starts by calling localSearch with the tree we obtained from the simplification rules, and the current position set to the tree root. Next, the algorithm scans the children of the current node that have the same output type as their parent, and creates a new tree by replacing the parent node (Line 6). The best tree is stored, and the process continues recursively, advancing the current position to the next node in pre-order traversal if the tree is not changed, otherwise it will continue using the current position. The process stops when there are no more positions to be checked (Line 2).

4 EXPERIMENTAL RESULTS

In this section, we compare HOTGP to state-of-the-art GP-based program synthesis algorithms found in the literature. For this comparison, we employ the "General Program Synthesis Benchmark Suite" [12], which contains a total of 29 benchmark problems for inductive program synthesis⁶.

Following the recommended instructions provided by Helmuth and Spector [12], we executed the algorithm using 100 different seeds for each benchmark problem. We used the recommended number of training and test instances and included the fixed edge cases in the training data. We also used the same fitness functions described in their paper.

For the evolutionary search, we used a steady-state replacement of 2 individuals per step, with an initial population of 1 000, and using a Parent-Scalar of 99.93%. The maximum tree depth was set to 15 for the main program and 3 for λ -functions. The crossover and mutation rates were both empirically set to 50%. We allowed a maximum of 300 000 evaluations with an early stop whenever the algorithm finds a perfectly accurate solution according to the training data.

We report the percentage of correct solutions found within the 100 executions taking into consideration the training and test data Matheus Campos Fernandes, Fabrício Olivetti de França, and Emilio Francesquini

sets, before and after the refinement process. To position such results with the current literature, we compare the obtained results against those obtained by PushGP [12], Grammar-Guided Genetic Programming (G3P) [3], and the extended grammar version of G3P (here called G3P+) [4], and some recently proposed methods such as Code Building Genetic Programming (CBGP) [25], and G3P with Haskell and Python grammars (G3Phs and G3Ppy) [5]. We have not compared with STGP and PolyGP since their original papers [22, 32] predate this benchmark suite. All the obtained results are reported in Table 2. In this table, all the benchmarks that could not be solved with our current function set are marked with "–" in HOTGP columns. For the other approaches, the dash mark means the authors did not test their algorithm for that specific benchmark.

4.1 Analysis of the results

Compared to the other algorithms, HOTGP has the highest success rate for the test set in 9 of the benchmark problems, followed by PushGP and CBGP, which got the highest rate for 7 and 5 of the benchmarks, respectively. An important point to highlight is that HOTGP obtained a 100% success rate in 4 problems, and $a \ge 75\%$ in 7, a result only matched by CBGP. Moreover, HOTGP obtained at least a 50% success rate in 10 out of the 29 problems, which is not matched by any of the compared methods. This brings evidence to our initial hypothesis that including type information in the program synthesis can, indeed, reduce the search space to improve the efficiency of the evolution process.

For example, in the *compare-string-lengths* problem, the input arguments are of the type String, and the output is a Bool but allowing intermediate Int type. Looking at Table 1, we can see that there are a few ways to convert a string to a boolean, as we only support functions in the character level. The best we can do is to extract the first character with Head and then convert the character into a boolean with IsLetter or IsDigit. We could, for instance, generate a program that does that for both inputs and compares the results with different boolean operators. We could also apply a Map function before applying Head. Also, to convert a string into an integer, the only solution is to use the Length function and the few combinations on how to convert two integers into a boolean. One example of obtained solution is ((length x1) > (length x0)) && ((length x1) < (length x2)).

On the other hand, for the *last-index-of-zero* problem, a possible correct solution using our grammar is fst (head (reverse (filter $(\lambda y \rightarrow 0 == (\text{snd } y))$ (zip (range 0 (length x0) 1) x0)))). So the synthesizer must first enumerate the input, apply a filter to keep only the elements that contain 0, reverse the list, take the first element, and return its index. One of the best obtained solutions was ((length x0) + (if ((head (reverse x0)) == 0) then 1 else 0)) - 2 with 32% of accuracy. It simply checks if the last element is 0, if it is, it returns the length of the list minus one, otherwise it returns the length minus two. This is a possible general case for a recursive solution where it checks the

 $^{^6{\}rm The}$ full source code for HOTGP can be downloaded from: https://github.com/mcf1110/hotgp.

Table 2: Successful solutions found for each problem (% of executions) considering the training (Tr) and test (Te) data sets. HOTGP * lists the results obtained with HOTGP after the simplification procedure. The best values for the test data sets of each problem are highlighted. The *checksum*, *collatz-numbers*, *string-differences*, *wallis-pi* and *word-stats* problems are ommitted as no algorithm was able to find results for those problems.

	HOTGP HOTGP *		PushGP	G3P	G3P+		CBGP	P G3Phs		G3Ppy			
Benchmark	Tr	Te	Tr	Te	Te	Te	Tr	Te	Te	Tr	Te	Tr	Te
compare-string-lengths	100	100	100	100	7	2	96	0	22	94	5	12	0
count-odds	46	46	50	50	8	12	4	3	0	-	-	-	-
digits	-	_	-	-	<u>7</u>	0	0	0	0	-	_	-	-
double-letters	0	0	0	0	<u>6</u>	0	0	0	-	-	-	-	-
even-squares	0	0	0	0	<u>2</u>	1	0	0	-	-	-	-	-
for-loop-index	73	39	73	<u>59</u>	1	8	9	6	0	-	_	-	-
grade	37	32	39	<u>37</u>	4	31	63	31	-	-	-	-	-
last-index-of-zero	0	0	0	0	21	22	97	<u>44</u>	10	0	0	2	2
median	82	73	100	<u>99</u>	45	79	99	59	98	100	96	39	21
mirror-image	1	1	1	1	78	0	89	25	100	-	_	-	-
negative-to-zero	100	100	100	100	45	63	24	13	99	0	0	68	66
number-io	100	100	100	100	98	94	95	83	100	100	99	100	100
pig-latin	-	-	-	-	0	0	4	<u>3</u>	-	-	-	-	-
replace-space-with-newline	38	38	38	38	<u>51</u>	0	29	16	0	-	-	-	-
scrabble-score	-	-	-	-	<u>2</u>	2	1	1	-	-	-	-	-
small-or-large	28	<u>59</u>	28	<u>59</u>	5	7	39	9	0	30	4	0	0
smallest	98	95	100	100	81	94	100	73	100	100	100	99	89
string-lengths-backwards	87	87	89	<u>89</u>	66	68	20	18	-	0	0	35	34
sum-of-squares	1	1	1	1	<u>6</u>	3	5	5	-	-	_	-	-
super-anagrams	-	-	-	-	0	21	43	0	-	30	5	51	<u>38</u>
syllables	0	0	0	0	18	0	53	<u>39</u>	-	-	-	-	-
vector-average	78	79	80	80	16	5	0	0	88	67	4	0	0
vectors-summed	34	34	37	37	1	91	28	21	100	100	68	0	0
x-word-lines	-	-	-	-	<u>8</u>	0	0	0	-	-	-	-	-
# of Best Results		4		<u>9</u>	7	2		3	5		1		2
= 100%		3		<u>4</u>	0	0		0	<u>4</u>		1		1
≥ 75%		6		7	3	4		1	<u>7</u>		3		2
≥ 50%		8		<u>10</u>	5	6		3	7		4		3

last element and, if it is not zero, recurses with the remainder of the list.

As described in Section 3.3, the code refinement step always produces an equal or better solution. These improvements are more noticeable on the *median* and *for-loop-index* problems. This is due to the fact that code refinement is sometimes capable of discarding misused numerical constants. For example, one solution to the *median* problem with 99% of accuracy on the training set was max -96 (min (max x1 x2) (max (min x1 x2) x0)) that only works if the median of the three arguments is greater than -96, otherwise it will always return a constant value. After the code refinements, HOTGP finds the final and correct solution: min (max (min x2 x1) x0) (max x1 x2).

Another benefit of code refinement is reducing the program size, which can improve the readability of the generated program. Figure 1 shows the rate of decrease in the program size after refinements, with a geometric mean of 52%. The refinement process effectiveness varies depending on the nature of the solutions of the problem. For most problems, the end of the upper quartile is well within the > 75% reduction mark, meaning it was not unusual for some solutions to get largely simplified. However, more evident results are yielded in problems such as *counts-odds, even-squares*, and *sum-of-squares*, which dealt with fewer types (and thus a reduced grammar) and usually reached the maximum evaluation count, therefore were more susceptible to bloat. Notably, *number-io* and *negative-to-zero* had almost no reduction, showing the algorithm could directly find a perfect and near minimal solution.

To provide further insights into how minimal the correct solutions actually are, and how susceptible to bloat each problem is, Table 3 takes the smallest correct solution that HOTGP could find for each problem, and compares them to the handwritten solutions crafted by the authors. Even before the refinement procedure, most of the solutions have the same node count as a the handwritten ones, and nearly all of them are reasonably close. The *sum-of-squares* was initially much larger than the manual solution, but after refinement the size reduction is notable. The only correct solution we found for

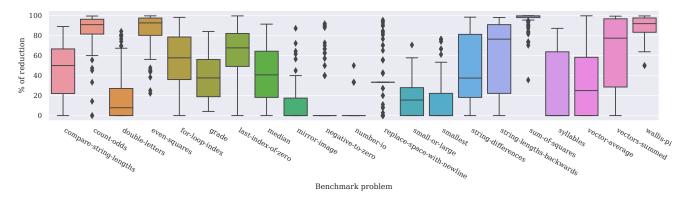


Figure 1: Percentage of reduction in the number of nodes caused by the refinement process.

mirror-image also has the biggest reduction of the batch, showing a 87% reduction overall.

Table 3: Node count of the hand-crafted solutions (HC) and the smallest correct solutions found by HOTGP and HOTGP*. The node count relative to HC is shown in parenthesis.

Benchmark	HC	HOTGP	HOTGP *
compare-string-lengths	11	11 (1.0x)	11 (1.0x)
count-odds	4	4 (1.0x)	4 (1.0x)
for-loop-index	7	25 (3.6x)	9 (1.3x)
grade	27	45 (1.7x)	29 (1.1x)
median	9	9 (1.0x)	9 (1.0x)
mirror-image	10	102 (10.2x)	13 (1.3x)
negative-to-zero	3	3 (1.0x)	3 (1.0x)
number-io	4	4 (1.0x)	4 (1.0x)
replace-space-with-newline	8	8 (1.0x)	8 (1.0x)
small-or-large	11	12 (1.1x)	12 (1.1x)
smallest	7	7 (1.0x)	7 (1.0x)
string-lengths-backwards	4	5 (1.2x)	5 (1.2x)
sum-of-squares	7	163 (23.3x)	30 (4.3x)
vector-average	6	6 (1.0x)	6 (1.0x)
vectors-summed	5	5 (1.0x)	5 (1.0x)

5 CONCLUSION

This paper presents HOTGP, a GP algorithm that supports higherorder functions, λ -functions, polymorphic types, and the use of type information to constrain the search space. It also sports a grammar based on the Haskell language using only pure functions in the nonterminals set. Our main arguments in favor of this approach are: i) limiting our programs to pure functions avoids undesirable behaviors; ii) using type-level information and parametric polymorphism reduces the search space directing the GP algorithm towards the correct solution; iii) higher-order functions eliminate the need of several imperative-style constructs (*e.g.*, for loops).

HOTGP differs from most GP implementations as it actively uses the information of input and output types to constrain the candidate terminals and nonterminals while creating new solutions or modifying existing ones, and to select feasible points of recombination. We have evaluated our approach with 29 benchmark problems and compared the results with 6 state-of-the-art algorithms from the literature. Overall, we got favorable results, consistently returning a correct program most of the time for 10 problems, a mark that was not met by any of the tested methods. Moreover, HOTGP achieved the highest success rates more often than the state of art.

We also applied code refinements to the best solution found by the algorithm to reduce the occurrence of *bloat* code. This procedure leads to further improvements in the results while at the same time improving the readability of the final program.

Even though we achieved competitive results, we observed that there are still possible improvements. First, our nonterminals set is much smaller than some of the state-of-the-art algorithms (*e.g.*, PushGP). Future work includes carefully examining the impact of adding new functions to the grammar. This inclusion might further simplify the PS or allow us find solutions that are not currently being found. On the other hand, including new functions also expands the search space and can hinder some of our current results.

Our approach could also benefit from a more modular perspective for PS. In a modular approach, the problem is first divided into simpler tasks which are solved independently and then combined to create the complete synthesized program. This approach will require support to different forms of functional composition and the modification of the benchmark to create training data for the different subtasks. Such a synthesizer could also be coupled with Wingman⁷ (the current implementation of advanced Haskell code generation), which can either synthesize the whole program or guide the process using only the type information, and code holes.

Further research is also warranted concerning more advanced type-level information such as Generalized Algebraic Data Types (GADTs), Type Families, Refinement Types and Dependent Types. More type information could further constrain the search space and, in some situations, provide additional hints to the synthesis of the correct program. Clearly, this must be accompanied by a modification of the current benchmarks and the inclusion of new benchmarks that provides this high-level information about the desired program.

⁷https://haskellwingman.dev/

HOTGP-Higher-Order Typed Genetic Programming

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