discussions related to this work. We would also like to thank Ed Ashcroft, Edsger Dijkstra, Jim King and Wolfgang Polak for their careful critical reading of the manuscript.

Received June 1976; revised May 1977

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Graphics and Image Processing

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Editor

Some New Methods of Detecting Step Edges in Digital Pictures

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This note describes two operators that respond to step edges, but not to ramps. The first is similar to the digital Laplacian, but uses the max, rather than the sum, of the x and y second differences. The second uses the difference between the mean and median gray levels in a neighborhood. The outputs obtained from these operators applied to a set of test pictures are compared with each other and with the standard digital Laplacian and gradient. A third operator, which uses the distance between the center and centroid of a neighborhood as an edge value, is also briefly considered; it turns out to be equivalent to one of the standard digital approximations to the gradient.

Key Words and Phrases: image processing, pattern recognition, edge detection

CR Category: 3.63

The support of the Directorate of Mathematical and Information Sciences, U.S. Air Force Office of Scientific Research, under Contract F44620-72C-0062, is gratefully acknowledged, as is the help of Shelly Rowe in preparing this paper. Authors' address: Computer Science Center, University of Maryland, College Park, MD 20712.

1. Introduction

Many edge detection techniques have been described in the literature on image processing and pattern recognition; see [1, 2] for reviews. This note describes two new edge detection operators. Like the digital Laplacian (see Section 2), these operators respond to steps, but not to ramps[1]; they also seem to be less sensitive to noise than the Laplacian. They are simple to compute and under some circumstances might even be computationally advantageous—e.g. if one were doing median filtering, the mean-median difference operator (Section 3) might be a preferred edge detector.

The first operator, which we call the pseudoLaplacian, is a minor variation on the standard digital Laplacian, but appears to be less noise sensitive; it is described in Section 2. The second operator uses the difference between the mean and median gray levels in a neighborhood as an edge value; it is quite insensitive to noise (Section 3). A third operator, which uses the distance between the center and centroid of a neighborhood as an edge value, is discussed in Section 4; it turns out to be mathematically equivalent to one of the standard digital approximations to the gradient.

2. PseudoLaplacian

Two derivative operators that have commonly been used as edge detectors are the magnitude of the gradient

$$\left(\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right)^{\frac{1}{2}}$$

and the Laplacian

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}.$$

For digital pictures, finite differences are used instead of derivatives, i.e.

$$\Delta_x f = f(x+1, y) - f(x, y) \quad \text{for } \partial f/\partial x \text{ and}$$
$$\Delta_x^2 f = f(x+1, y) + f(x-1, y) - 2f(x, y) \quad \text{for } \partial^2 f/\partial x^2$$

and similarly for the y-derivatives. For computational simplicity, in the digital "gradient," the sum or max of $|\Delta_x f|$ and $|\Delta_y f|$ are often used instead of the square root of the sum of their squares.

In general, the gradient is a better edge detector than the Laplacian because the latter responds more strongly to isolated points than it does to edges. Indeed, it is easily verified that the response of both operators to a horizontal or vertical step edge of unit height is 1; but for an isolated spike of unit height, the gradient response is 1 while the Laplacian response is 4. For a "ramp" such as

$$\begin{matrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{matrix}$$

(i.e. a portion of the picture with constant, nonzero gradient) the Laplacian response is zero.

We define the *pseudoLaplacian* as the max, rather than the sum, of the second differences, i.e.

$$\max(\Delta_x^2 f, \Delta_y^2 f).$$

This has response 2, rather than 4, to a spike of unit height, and still has zero response to a ramp.

Figures 1, 2(a) and 2(b) compare the responses of several of these operators applied to a set of test pictures. (Some of these pictures were also used in [2] to illustrate the performance of other edge detector techniques.) The operators tested are:

(1) The digital "gradient" $\max(|\Delta_x f|, |\Delta_y f|)$.
(2) The digital "gradient" $|\Delta_x f| + |\Delta_y f|$.
(3) The positive digital Laplacian $\max[0, \Delta_x^2 f + \Delta_y^2 f]$.
(4) The absolute digital Laplacian $|\Delta_x^2 f + \Delta_y^2 f|$.
(5) The positive pseudoLaplacian $\max[0, \Delta_x^2, \Delta_y^2 f]$.
(6) The absolute pseudoLaplacian $\max[|\Delta_x^2 f|, |\Delta_y^2 f|]$.

Note that for a horizontal or vertical step edge of unit height, these operators all have value +1; their outputs have therefore all been scaled alike for display. (Specifically, the values have all been multiplied by 2, with values beyond the top of the grayscale truncated to the maximum gray level (such values were rare).)

For diagonal edges, the operators based on sums have twice the response of those based on maxima; this can be seen by comparing Figures 1(b), 1(c), and 1(d) with Figures 1(a), 2(a), and 2(b). The "absolute" operators (Figures 1(d) and 2(b)) yield thicker edges than the "positive" operators (Figures 1(c), 2(a)) since they detect edges in two positions. For example, the absolute Laplacian has value 1 at both of the underlined points adjacent to the edge

$$.\,.\,0\underline{0}\underline{1}1\,.\,.\,.\,,$$

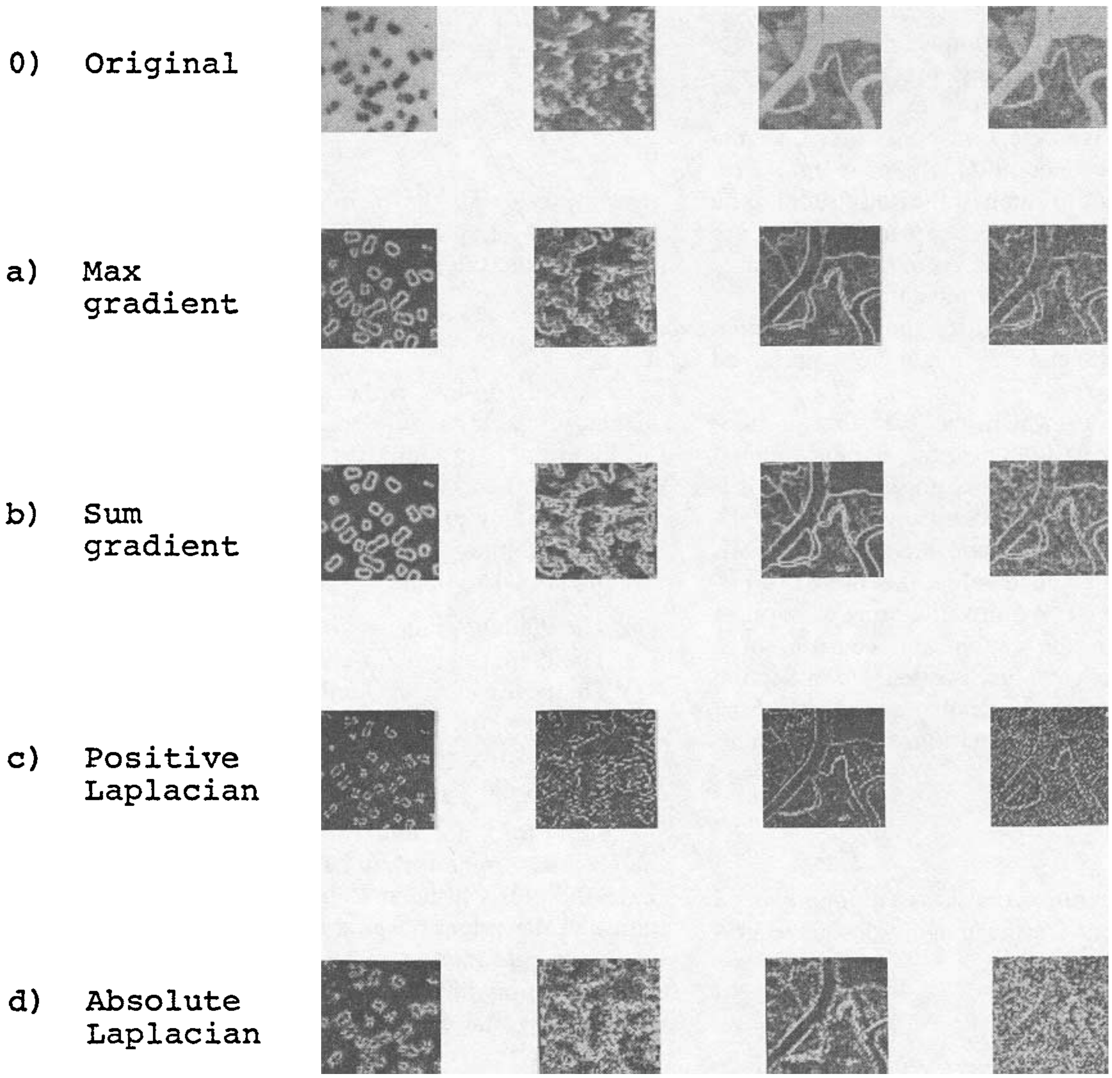
whereas the positive Laplacian has value 1 only at the underlined $\underline{0}$ since the Laplacian is negative at the underlined $\underline{1}$.

Comparison of Figures 1(d) and 2(b) indicates that the absolute pseudoLaplacian outputs are at least as noisy as the absolute Laplacian outputs, but they respond somewhat better to blurred edges (e.g. the large chromosomes in the third column), though not as strongly as the gradient operators (Figures 1(a) and 1(b)). On the other hand, comparison of Figures 1(c) and 2(a) shows that the outputs of the positive pseudoLaplacian appear to be appreciably less noisy than those of the positive Laplacian.

The comparisons made in these figures could have

[1] More precisely, they respond at the top and bottom of a ramp, but not within the ramp. Thus they detect blurred edges (at their "shoulders"), but they do not respond to linear variations in illumination (which do not give rise to shoulders).

Fig. 1. Digital gradients and Laplacians: (0) original, (a) max gradient, (b) sum gradient, (c) positive Laplacian, (d) absolute Laplacian.



been done in a number of other ways. For example, each column could have been scaled individually to best bring out the edges in each image, but it was felt that uniform scaling of all columns would be preferable for the purpose of comparing results for each detector over a variety of images. Another possibility would have been to double the output values of the positive operators relative to those of the absolute operators. But here again it was felt that this would interfere with an objective comparison. It would be desirable to have an objective method of comparing edge detector outputs, rather than relying on visual comparison, but the development of such a method is beyond the scope of this note. (The authors are indebted to one of the referees for these suggestions.)

3. Mean-Median Difference

Suppose that a point is just adjacent to a step edge so that its 3×3 neighborhood looks like, for example,

$$\begin{matrix} z & z & w \\ z & z & w \\ z & z & w \end{matrix} \quad \text{or} \quad \begin{matrix} z & w & w \\ z & z & w \\ z & z & z \end{matrix} \quad \text{(or rotations of these).}$$

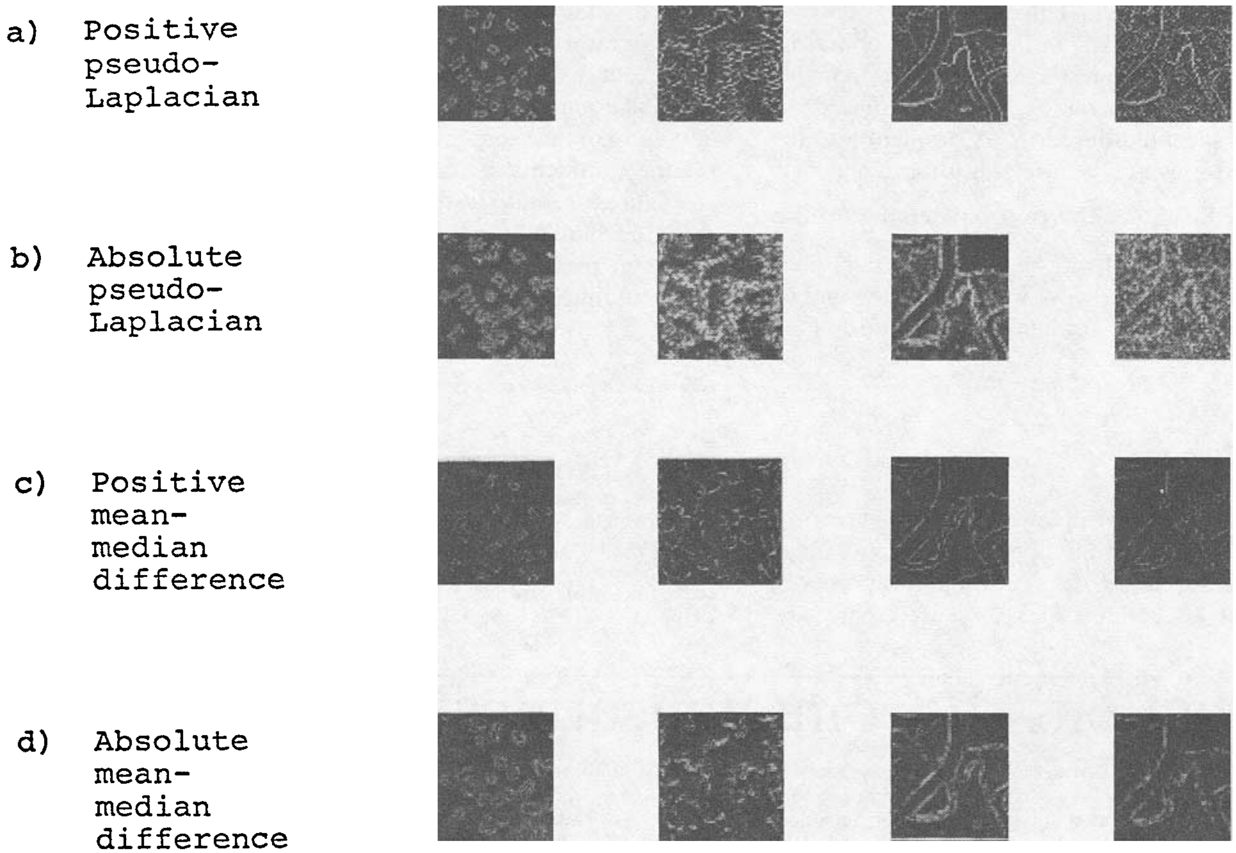
Then the mean μ of the gray levels in such a neighborhood is $(6z + 3w)/9 = (2/3)z + (1/3)w$, whereas the median m of these gray levels is z. The mean and median thus differ by $z - ((2/3)z + (1/3)w) = (1/3)(z - w)$, which is proportional to the contrast of the edge.

Note that for a linear ramp, e.g.

$$\begin{matrix} 1 & 2 & 3 \\ 1 & 2 & 3, \\ 1 & 2 & 3 \end{matrix}$$

the mean and median are the same ($=2$); thus this edge detection operator, like the Laplacian, responds to steps but not to ramps. For an isolated noise point

Fig. 2. PseudoLaplacians and mean-median differences: (a) positive pseudoLaplacian, (b) absolute pseudoLaplacian, (c) positive mean-median difference, (d) absolute mean-median difference.



$z\,z\,z$
$z\,w\,z$
$z\,z\,z$

the median is z, while the mean is close to z (namely, $(8z + w)/9 = z + (w - z)/9$); so the mean-median difference is only a third as great as it is for a step edge. Thus this operator should be quite insensitive to noise.

Figures 2(c) and 2(d) show positive and absolute mean-median differences, i.e. $\max[0, m - \mu]$ and $|m - \mu|$, for the same pictures as in Figure 1. As before, the absolute differences yield thicker edges than the positive difference. Since the response of these operators to a step edge of unit height is only 1/3, the output values have been scaled by a factor of 3 relative to the values shown in Figure 1. The responses are markedly less noisy than those of the Laplacians.

4. Center-Centroid Distance

We conclude by describing another edge detection operator whose definition also involves moments, but which turns out to be equivalent to one of the standard digital gradient operators. The idea for this operator was suggested by a method used by Zucker [3] to detect the edges of dot clusters.

In the neighborhood

$a\,b\,c$
$d\,e\,f$
$g\,h\,i$

if we take the center e of the neighborhood as the origin, then the coordinates of the neighborhood's centroid (ignoring the scale factor $a + b + c + d + e + f + g + h + i$) are

$$m_x = (c + f + i) - (a + d + g),$$
$$m_y = (a + b + c) - (g + h + i).$$

Thus the distance between the center of the neighborhood and its centroid is

$(m_x^2 + m_y^2)^{1/2}$ (Euclidean distance)

or

$|m_x| + |m_y|$ (city block distance)

or

$\max(|m_x|, |m_y|)$ (chessboard distance).

On the other hand, m_x and m_y are just the x- and

y-components of a commonly used digital gradient operator [2] which combines smoothing along the edge with differencing across it. Thus the center-centroid distance is just the magnitude of this gradient operator, or an approximation to that magnitude. Note that this measure does respond to ramps, since it is based on first rather than second differences. A simpler operator can be defined by using 2×2 neighborhoods; in fact, for $\begin{matrix} a & b \\ c & d \end{matrix}$ the coordinates of the centroid relative to the center are proportional to $(b + d) - (a + c)$ and $(a + b) - (c - d)$, respectively, which are the components of another standard gradient approximation (see [2, page 285, Figure 13]).

5. Concluding Remarks

This note suggests that many simple variations on the standard edge detection operators are possible. For most purposes, the sum or max gradient is probably the most useful edge detector. However, there are cases in which one may need to use a Laplacian-like operator, e.g. when strong brightness ramps are present. This note has shown that one can design Laplacian-like operators (e.g. the positive mean-median difference) that are less sensitive to noise than the standard Laplacian and yet that do not respond to ramps. (For detailed comparisons of the operators' performances, see the comments in Sections 2–4 on the results shown in Figures 1 and 2; these comparisons will not be repeated here.) It is hoped that these operators will find their place among the growing array of tools that are becoming available for image processing and analysis.

Received October 1976; revised January 1977

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