PROGRAMMING TECHNIQUES

A Number System for the Permutations

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Key Words and Phrases: permutation, ordering, number, number system, p-number, combinatorial
CR Categories: 4.49, 5.39

A number system described here is particularly suitable for numbering the permutations. An algorithm is then presented by which the permutation associated with a number of this kind may be obtained. Since arithmetical operations, such as the addition of 1, are easily performed in the number system, one may make use of the algorithm to generate the permutations one at a time, using a minimal amount of work space. Employed for this purpose, it provides an alternative to the method given in [1].

An n-ary *p-number* is a sequence of integral digits $a[n]\cdots a[1]$ such that $1 \le a[i] \le i$ for $1 \le i \le n$. If $a[t] < t$ for some t, the number which "follows" $a[n]\cdots a[1]$, i.e. "$a[n]\cdots a[1] + 1$", is $b[n]\cdots b[1]$ where, if k is the least number i such that $a[i] < i$, then $b[i] = 1$ for $1 \le i < k$, $b[i] = a[i]$ for $k < i \le n$, and $b[k] = a[k] + 1$. Hence, except for the range of digits employed, for each i the ith significant digit in a p-number behaves as if it were of base i. Note that there are exactly $n!$ distinct n-ary p-numbers.

A permutation of $1, \cdots, n$ to which $a[n]\cdots a[1]$ uniquely corresponds may be obtained using the following ALGOL statement:

```
for i := 2 step 1 until n do
for j := 1 step 1 until i - 1 do
if a[j] ≥ a[i] then a[j] := a[j] + 1;
```

It can be shown furthermore that the permutation will be even or odd according to whether $\sum_{i=1}^{n} a[i]$ is even or odd.

The following is an example of successive 3-ary p-numbers and their corresponding permutations.

	P-NUMBER	PERMUTATION
1	1 1 1	1 2 3
2	1 2 1	1 3 2
3	2 1 1	2 1 3
4	2 2 1	2 3 1
5	3 1 1	3 1 2
6	3 2 1	3 2 1

REFERENCE:
1. LANGDON, G. G., JR. An algorithm for generating permutations. *Comm. ACM 10*, 5 (May, 1967), 298–299.

* Information Sciences Program. The present work was supported by the National Science Foundation under Grant GJ-596. A proof of the algorithm appeared as a part of a 1959 EMI Technical report by the author.

Another Method of Converting from Hexadecimal to Decimal

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Key Words and Phrases: binary-decimal conversion, computer arithmetic
CR Categories: 1.5, 4.0, 6.2

There is a simple paper-and-pencil method of converting a hexadecimal number N to decimal.

1. Convert the leftmost digit of N to decimal and write the result under N with its low order digit aligned with the leftmost digit of N.
2. If there are no more digits left in N, you have the answer. Otherwise, underline the most recent result and bring down the next digit from N to the right of the underlined number.
3. Multiply the underlined portion by six and place the result with its units digit aligned to the hexadecimal digit brought down in step 2.
4. Add the two bottom lines and write the result below. Go to step 2.

Example: Convert $(\mathrm{ABCDE})_{16}$ to decimal.

```
  A B C D E
1 0 B
  6 0
1 7 1 C
1 0 2 6
2 7 4 8 D
1 6 4 8 8
4 3 9 8 1 E
2 6 3 8 8 6
7 0 3 7 1 0   Answer:   (703710)10
```

A very similar method for converting octal numbers to decimal numbers is found in [1].

REFERENCES:
1. KNUTH, D. E. *The Art of Computer Programming, Vol. 2.* Addison-Wesley, Reading, Mass., 1968, pp. 284–285.

OPERATING SYSTEMS

Comment on a Paging Anomaly

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Key Words and Phrases: paging machines, demand paging, replacement algorithm
CR Categories: 4.30

A recent paper by L. A. Belady, R. A. Nelson, and G. S. Shedler [1] demonstrates that anomalous behavior can result from the FIFO (first in first out) page replacement algorithm in a virtual storage computer system with a