Estimates of Distributions of Random Variables for Certain Computer Communications Traffic Models

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A study of multiaccess computer communications has characterized the distributions underlying an elementary model of the user-computer interactive process. The model used is elementary in the sense that many of the random variables that generally are of interest in computer communications studies can be decomposed into the elements of this model. Data were examined from four operational multiaccess systems, and the model is shown to be robust; that is, each of the variables of the model has the same distribution independent of which of the four systems is being examined. It is shown that the gamma distribution can be used to describe each of the continuous variables of the model, and that the geometric distribution can be used to describe the discrete variables. Approximations to the gamma distribution by the exponential distribution are discussed for the systems studied.

KEY WORDS AND PHRASES: computer communications, time-sharing, operating systems, optimization models
CR CATEGORIES: 3.80, 3.81, 6.20

Introduction

Since time-sharing burst on the world some six or seven years ago, many analytical studies have been published of the behavior of such systems [1, 3, 7, 10, 14–17, 21]. In general, the completion of an analytical study of a real process requires several steps to be performed: Construction of a process model, analysis of the model, estimation of the model parameters, and verification of the results. It is sad to report that in almost all of the published studies, the last two steps are omitted.[1] It is evident that the basic reasons for these omissions are: (1) the difficulties encountered in the collection of necessary data due to the complexity of requisite simulations, the potential impairment of the efficiency of real systems by the measurement process, and the problem of avoiding violation of the proprietary constraints of systems applications; (2) the costs in time and dollars for conducting such studies; and (3) the questionable utility of such data in light of the rapid evolution of system capabilities and user characteristics. Nevertheless, as was first pointed out by Sackman [25] in 1967, inferences drawn from such models for the design of systems without empirical determination of parameter values and without testing of the model with the estimated parameters rest on extremely shaky foundations.

Clearly, the third reason is the most difficult to respond to. Many systems are changing so rapidly that a detailed characterization of any one will probably be outdated before it is completed. However, the architecture of computer communication systems has matured to the point that the potential for insight gained from analysis of operational systems for testing models and for forming a basis for research aimed at improvement far outweigh the drawback of obsolescence. Indeed, this situation calls for continued study and review.

If analytical models are to be of value in the design of systems, then the first two problems should be resolved. Efforts have been underway for some time at Bell Telephone Laboratories to model the user-computer interaction process in on-line multiaccess computer systems as an aid in the development of new computer communication systems and services. The studies include extensive efforts at the collection of data from representative working systems to obtain realistic estimates of the parameters of the models. In every case these data are obtained on the premises of the computer service provider and with his full permission and cooperation. To ensure the privacy of the four systems under discussion, however, they are not identified by name.

In a previous paper [13], Jackson and Stubbs reported some of the results of these efforts; specifically, a data stream model of the interaction process was presented, together with estimates of the average values of the basic random variables of the model as obtained from measurements on working systems. In this paper, we report additional results. First, we present the results of goodness of fit tests in which standard probability density functions are fitted to the empirical estimates of the distributions of the random variables of the model. Second, we examine the

[1] The pioneering work of Alan Scherr [26] was supported by extensive measurements on the MIT Project MAC CTSS System, and his results were verified by simulations. Other investigations which were supported by measurements were undertaken for the JOSS system at RAND Corp. [2], the Q-32 Time Sharing System at S.D.C. [6, 18, 19, 27], and additional investigations at Project MAC [11]. Each of these investigations was performed for a specific problem for the system at hand, with no attempt at generalization. However, the results of these studies have been quoted in lieu of measurements by authors of more general studies. An excellent summary and comparisons of these investigations may be found in [25].

significance levels of the fits for the various probability density functions and find that analytically tractable probability density functions can be used for the variables with reasonable significance levels. Third, we note a consistency between systems of widely varying types and applications in characterization of key variables and comment on the significance of this consistency.

In a small way, these analyses are analogous to the early studies of Erlang [8] and others over 70 years ago,[2] in which representative examples of traffic data were collected for the purpose of characterizing local and toll telephone system behavior. The Poisson arrival rate process and exponential interarrival time distribution were the results of some of the earliest of these studies. It is interesting to note that the validity of these characterizations has been retained throughout the years despite the many technological changes in telephone systems and sociological changes in telephone usage.

To provide a framework for presentation of the results, we first give an overview of the study methods and review the data stream model presented in [13]. We then discuss the techniques employed to characterize the variables. Finally, we present the results of the study.

Methods and Models

During the study, we investigated several selected multiaccess computer communication systems in detail. These systems were selected on the basis that they are representative of the advanced state of the art, that the providers of the particular system are knowledgeable in communications, that the systems are fully operational with the initial break-in period accomplished, and that the computer service providers are willing to participate in the study. More detail on the selection procedure is given in [13].

The data which are utilized in the results reported here are the detailed relationships of the flow of message characters to and from users and computers during on-line transactions. The model describes the communications process in terms of random variables which give intercharacter times and the sizes of clusters of characters as they are transmitted through the communication interface. The raw data, then, were collected at the computer ports of active multiaccess computer systems. The model did not require nor did we collect data from internal computer processes such as the length of various internal queues. It is apparent that a model which portrays the interplay of the internal computer processes, such as memory management and processor time scheduling algorithms, with the communication processes would be more satisfactory for joint optimization of computer and communication performance. However, acquisition of data describing the former processes was not within the scope of this study.

[2] Molina [21] reports that G. T. Blood of the AT&T Co in 1898 found a close agreement between the terms of a binomial expansion and the results of the observations on the distribution of busy telephone calls. This is the earliest reference that we have been able to find to empirical studies aimed at verification of assumptions employed in telephone traffic modeling.

Figure 1 illustrates the data stream model. A "call" (or a connect-disconnect time period) is represented as the summation of a sequence of time periods during which the user sends characters without receiving, interleaved with time periods during which he receives characters without sending. (This implies half-duplex operation. Simple modifications to the model would allow the accommodation of full-duplex operation.) The periods during which the user is sending characters to the computer are defined as user

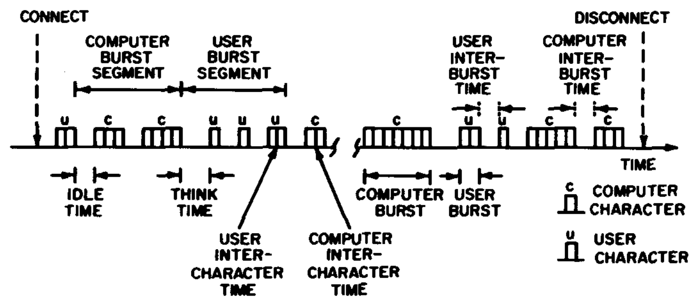

FIG. 1. The data stream model

burst segments. The periods during which he is receiving characters sent from the computer are computer burst segments. A user burst segment, by definition, begins at the end of the last character of the previous computer burst segment. Similarly, a computer burst segment begins at the end of the last character sent by the user. The first burst segment of a call begins when the call is established, and the last burst segment ends when the call is terminated as measured at the computer interface.

Within a given burst segment, there are periods of line activity and of line inactivity. The first inactive period of a user burst segment is defined as think time. That is, think time is the time that elapses from the end of the last previous computer character until the beginning of the first user character in that burst segment. In most cases, think time is employed by the user to finish reading the previous computer output and to think about what to do next. The corresponding inactive period in a computer burst segment is called idle time. In some systems idle time represents time during which the user waits for the return of "line feed," after sending "carriage return"; in other systems, idle time represents the time during which the user's program is being processed or is in queue. The remaining inactive periods within a burst segment are called intercharacter times and interburst times. A prerequisite for their definition is the definition of a burst.

Two consecutive characters are defined as belonging to the same burst if the period of inactivity between the characters is less than one-half character width. Thus each burst is the longest string of consecutive characters where the period of inactivity between any two consecutive characters is less than one-half character width. All of the characters in a burst must, of course, be transmitted from the same party (user or computer). For example, every character of an unbroken string of characters sent at line rate is in the same burst.

For characters within the same user burst, an inactive time between two consecutive characters is called a user intercharacter time. The corresponding variable for computer bursts is computer intercharacter time. For bursts within the same user (computer) burst segment, the inactive time between two consecutive bursts is called a user (computer) interburst time. Five final variables of the data stream model are number of user bursts per burst segment, number of computer bursts per burst segment, number of characters per user burst, number of characters per computer burst, and temporal character width (time from start to end of one character).

TABLE I. Characteristics of Systems Studied

	Systems			
	A	*B*	*C*	*D*
Computer type	Brand X	Brand X	Brand Y	Brand Z
Transmission speed (characters/second)	10	10	15	10
Primary application	Scientific	Scientific	Business	Inquiry/ response
Load*	Moderate	Heavy	Moderate	Light/ moderate

* The term load denotes the relative occupancy of the processor due to on-line demands and background batch work (if any); nothing is implied directly as to the load on the communication channel.

Collected Data and Analysis

During the study, data have been collected for a large number of transactions for each of several multiaccess computer systems. Data from four of the systems are discussed in this paper. These systems are labeled A, B, C, and D. Systems A and B have the same computer equipment and basically the same mix of computer applications (scientific/engineering programming and problem solving), although the average loads supported by the two systems during the study periods were quite different. System C has computer equipment different from each of the others and its mix of user applications is oriented toward business problem solving. System D also has computer equipment different from each of the others, and its applications are data collection and data dissemination in an inquiry/ response method of operation. All four systems serve low-speed, half-duplex, teletypewriter-like terminals and fall within the category of "long-holding time systems" as defined by Jackson and Stubbs, i.e. with average holding times or call durations on the order of 20 to 30 minutes. Table I summarizes the salient characteristics of these systems.

The random variables of Figure 1 are of two types. Some are discrete, such as the number of characters per burst. Others are continuous, such as think time. Modeling techniques most commonly used in computer communications studies include queueing processes, renewal processes, birth-death models, Markov processes, and to a limited extent, flow models. Most key random parameters of models used in computer-communication studies are either interevent times such as times between arrivals at a server or burst length counts such as the number of arrivals in a batch arrival process. In solving these types of models, only a very few random functions are tractable and in some cases allowable. In the category of desirable functional forms fall the Poisson, geometric, and binomial distributions for discrete processes, and the gamma distribution family for continuous processes. Hence, we are interested in the extent to which the key variables of such models can be described by these few desirable distributions.

Data collected from the communication lines at the computer ports of the four operating systems described above were used to seek desirable distributions to describe each of the random variables of the data stream model. These data were laundered to remove ambiguities and were then partitioned into sets describing each of the variables. For each set of data for each variable for each system, goodness-of-fit tests were performed to ascertain which standard probability functions could be used to describe the variables. As the existing tests for goodness-of-fit were not satisfactory for our purposes because of their low power or excessive computation time, a new test was devised [12].

The set of distributions used for goodness-of-fit tests included the normal, Cauchy, Laplace, chi-square, exponential, hyperexponential, gamma, and lognormal distributions for continuous variables and the geometric (with and without mass at the origin), uniform, Poisson, compound Poisson and binomial distributions for discrete variables. For each variable, a compound goodness-of-fit test was performed where the parameters of the hypothesized distributions (those being tested) were adjusted so that the mean and variance for a two-parameter distribution were the same as the sample mean and sample variance. For a single-parameter distribution, the mean of the distribution was equated to the sample mean.

Results of the goodness-of-fit tests are shown in Table II. From Table II, we see that the geometric distribution can be used to describe every discrete process but one (the single exception is an impulse function which is a degenerate form of the geometric distribution). Similarly, each of the continuous random variables of the model can be described by the gamma distribution, and the think times, idle times, and interburst times can be described additionally by the lognormal distribution. These results are significant for two reasons.

First, the data stream model, which is elementary in the sense that many of the variables that generally are of in-

TABLE II. Results of Goodness of Fit Tests Acceptable* Distributions†

Random variable	*Systems* A	B	C	D
No. of burst segments per call	G	G	G	G,CP
Think time	L,Γ	L,Γ	L,Γ	L,Γ
User interburst time	L,Γ	L,Γ	L,Γ	L,Γ
Computer interburst time	L,Γ	L,Γ	L,Γ	L,Γ
No. bursts/user burst segment	G	G	G	G
No. bursts/computer burst segment	G	G	G	G
No. characters/user burst.	G	G	I	G,CP
No. characters/computer burst	G	G	G	G
User intercharacter time	Γ	Γ	N/A	Γ
Computer intercharacter time	Γ	Γ	Γ	Γ

* Acceptable at the 5 percent level.
† Γ—gamma distribution, L—lognormal distribution G—geometric distribution, CP—compound Poisson distribution, I—constant at X = 1.

TABLE III. Average Parameter Values

Random Variable	$\hat{\mu}$	$\sigma_{\hat{\mu}}$
No. of burst segments per call	81.	31.
Think time (seconds)	5.0	3.1
Idle time (seconds)	0.81	0.44
User interburst time (seconds)	1.5	0.83
Computer interburst time (seconds)	13.	21.
No. of bursts/user burst segment	11.	2.8
No. of bursts/computer burst segment	2.9	2.4
No. of characters/user burst	1.1	0.1
No. of characters/computer burst	41.	24.
Character width	0.092	
User intercharacter time	0.0	
Computer intercharacter time	0.0	

terest in computer communication studies can be decomposed into the elements of the model, is shown to be robust; that is, each of the variables of the model is described by the same distribution independent of the computer system being examined.[3] These results were obtained in spite of the fact that three different computer types and operating systems were investigated. In addition, the computer loads and programming applications were different.

Table III summarizes the measured values of the model parameters. To insure the privacy of the four systems under discussion, these values are not shown on a per system basis. Rather, for each parameter, a grand mean $\hat{\mu}$ is given where $\hat{\mu}$ is the mean of the four system means for the given parameter. Each number in the column headed σ_μ is the standard deviation of the four numbers (system means) averaged to find $\hat{\mu}$. As the character widths are treated in the model as random variables, they are included in Table III for completeness. In the four systems discussed, however, they were constant as can be derived from Table I. The intercharacter times are all zero as the characters occur at line rate. Infrequent, small departures from zero were measured for intercharacter times, but these measurements are attributable to clock slippage between the computer's clock and our clock.

Jackson and Stubbs [13] have examined the mean values of the model variables for the first three systems and make the following observations:

(1) Delays introduced by the computer (primarily idle time and computer interburst delay) can be a large component of holding time and are affected by the number of simultaneous users on the system, probably by the computer scheduling algorithm, and by the characteristics of the communications control unit.

(2) The average number of characters sent by the computer to the user is an order of magnitude greater than the number of characters sent by the user to the computer.

(3) Delays introduced by the user are a significant contributor to average holding time and are remarkably close in absolute values for the four systems studied.

These three observations are examples of information that may be employed by system designers in investigations into improved communications for multiaccess computers. In modeling (probabilistically) the behavior of present and proposed systems to determine their sensitivity to particular elements of the data stream model, the parameters of the distributions need only be changed and not the distributions themselves. These data are equally valuable for investigations into computer operating systems. For example, one might investigate changes in computer scheduling algorithms as reflected in changes in idle time and interburst delay parameters, changes in transmission speed from computer to user and the converse, and changes in terminal characteristics which may influence (hopefully reduce) user delays. Indeed, recently there have been reported many investigations into the performance of scheduling algorithms as measured by response time [1, 3, 7, 10, 14–17, 20]. Almost without exception, these investigations hypothesize arrival rates of requests for CPU time without the support of measurements. Since such arrivals can be approximated from the variables of the data stream model, the above observations as to the efficacy of the results reported in this paper are demonstrated.

Second, the particular distributions obtained in Table II are tractable and are useful in further analytical studies. For example, the geometric distribution was obtained for the discrete distributions and the gamma family for the continuous distributions.

Although the gamma distribution is tractable for some

[3] Although the truth of the statement for the "number of characters per user burst" is artificial, it is made because even for that case the same distributional form can be used in practice with no operational difficulty by choosing appropriate parameters for the distribution.

types of analyses, it is interesting to investigate to what degree the gamma distributions obtained in Table II can be approximated by the exponential distribution. We investigated this approximation by examining the coefficients of variation of the gamma-distributed variables. We found that the arrival process is approximately Poisson and thus, assuming independent sources, the large body of queueing theory based upon the Poisson arrival process may be brought to bear for analyses of communications processes in time-sharing systems.

Table IV shows the coefficients of variation, V, for the continuous variables for the four computer systems investigated.[4] The exponential distribution belongs to the gamma distribution family and is the special case where $V = 1$. To illustrate the similarity between the exponential distribution and the gamma distribution with $1.0 \leq V \leq 1.8$, Figure 2 is included.

The error shown in Figure 2 is the difference between the cumulative distributions at the point given. It is apparent that the approximation becomes less accurate as V becomes larger and is less accurate for smaller values of the independent variable than for larger ones. We further observe that even close to the origin, the class of gamma distributions defined by $V \geq 1$ has the same general shape as the exponential distribution. For much analytical work, the behavior of the distribution function in the neighborhood of the mean and in the upper tail are of the most interest. For these types of problems, if errors of the magnitudes shown in Figure 2 are allowable (or alternatively if the relative coefficients of variation shown in Table IV are tolerable), then the exponential distribution may be used in place of the gamma distribution.

The dependencies which exist between the random variables of the model are obvious from the definitions of the variables. In this study no attempt was made to characterize dependencies where they exist. An example of the dependencies which are generated by the computer-user interaction process is the number of characters per computer burst random variable. This variable is a function of such things as the program the user is running, the I/O habit patterns of the user, and the I/O load and CPU load on the computer. Other variables exhibit similar dependencies. No effort was undertaken to test the assumption of independence between sample points of the same process for all of the variables of the model. Rather, the assumption of independence was tested for certain of the variables where the property was required for particular studies.

Conclusions

In analyzing computer communication systems for time-sharing applications, the results of this work have shown that a variety of techniques can be applied to model the processes. Since the input traffic process has been characterized in terms that are usually tractable for analytical models, realistic results may be obtained using standard analytical techniques. In some models where the estimated distributional forms are not amenable to analysis, appropriate approximation techniques are available.

This work has shown that the communication process between a multiaccess computer and a user at a teletypewriter-like terminal can be represented by an elementary model from which more complex models may be constructed. Further, by using real data from operational multiaccess systems, we have shown that the model is robust and that the distributions obtained for each of the variables are tractable. In certain cases, the character arrival process can be approximated by a Poisson process. Thus, in modeling the communication process of long-holding-time multiaccess computer systems only the parameters of the distributions for the variables change for various computer types, applications and system loading.

These observations can be combined with the observa-

[4] For one system, the user terminal had an automatic response at the end of a computer burst segment rather than a true user "think time" response. For this system, the estimated value of V for the think time distribution was 0.72, close to that for the hyperexponential distribution ($V = 0.71$).

Table IV. Coefficient of Variation for Gamma Distributions

	Systems			
	A	*B*	*C*	*D*
Think time	1.56	1.64	0.72	1.61
Idle time	1.09	1.54	1.59	1.45
User interburst time	1.39	1.59	1.49	1.54
Computer interburst time	1.56	1.61	1.59	1.64
User intercharacter time	1.67	1.54	1.67	1.59
Computer intercharacter time	1.67	1.67	1.59	1.56

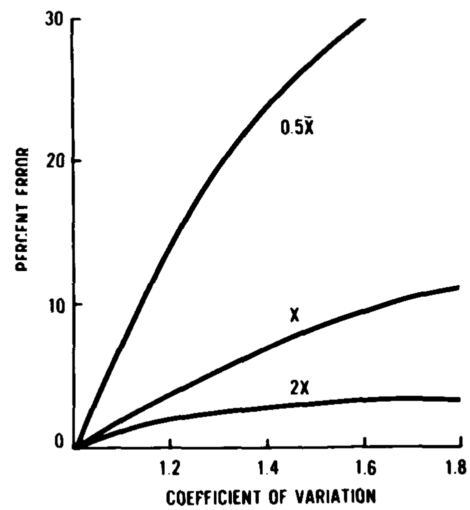

Fig. 2. Difference between gamma and exponential distributions

tions of Jackson and Stubbs [13] on computer-introduced delays, user-introduced delays, and the relative amounts of information flow in each direction on the communication line to give a comprehensive picture of the communication process. For example, these analyses support analytical and simulation studies at Bell Telephone Laboratories which seek solutions to computer access data communications problems, cf. Chu's, Meltzer's and Pilc's studies on asynchronous time division multiplexing [4, 5, 20, 23].

Several authors [9, 24, 28] have suggested that some computer parameters are hyperexponentially distributed or are the sum of exponential variates. In this study, no variable successfully passed the goodness-of-fit test with a significance level of 5 percent under the hyperexponential hypothesis although the coefficient of variation of the think time distribution for System C was close to the coefficient of variation for the hyperexponential distribution. Work is presently under way to investigate the hypothesis of a distribution which is the sum of exponential variates.

Studies of multiaccess computer communications are continuing. Data are being collected from systems with different terminal types, system configurations, average holding times, and user applications. Analyses of data for these new systems will expand our understanding of the computer-communication processes involved as we have a broader base from which to draw conclusions and make comparisons.

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