Algorithms

J. G. HERRIOT, Editor

ALGORITHM 334
NORMAL RANDOM DEVIATES [G5]

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KEY WORDS AND PHRASES: normal deviates, normal distribution, random number, random number generator, simulation, probability distribution, frequency distribution, random
CR CATEGORIES: 5.5, 5.13

procedure *norm* (*D*1, *D*2);
  real *D*1, *D*2;
comment This procedure generates pairs of independent normal random deviates with mean zero and standard deviation one. The output parameters *D*1 and *D*2 are normally distributed on the interval $(-\infty, +\infty)$. The method is exact even in the tails.

This algorithm is one of a class of normal deviate generators, which we shall call "chi-squared projections" [1, 2]. An algorithm of this class has two stages. The first stage selects a random number L from a χ_2^2-distribution. The second stage calculates the sine and cosine of a random angle θ. The generated normal deviates are given by $L \sin(\theta)$ and $L \cos(\theta)$.

The two stages can be altered independently. In particular, as better χ_2^2 random generators are developed, they can replace the first stage. (The negative exponential distribution is the same as that of χ_2^2.)

The fastest exact method previously published is Algorithm 267 [4], which includes a comparison with earlier algorithms. It is a straight chi-squared projection. Our algorithm differs from it by using von Neumann rejection to generate sin (ϕ) and cos (ϕ), $[\phi = 2\theta]$, without generating ϕ explicitly [3]. This significantly enhances speed by eliminating the calls to the sin and cos functions.

The author wishes to express his gratitude to Professor George Forsythe for his help in developing the algorithm.

REFERENCES

1. BOX, G., AND MULLER, M. A note on the generation of normal deviates. *Ann. Math. Stat. 28*, (1958), 610.
2. MULLER, M. E. A comparison of methods for generating normal deviates on digital computers. *J. ACM*, *6* (July 1959), 376–383.
3. VON NEUMANN, J. Various techniques used in connection with random digits. In Nat. Bur. of Standards Appl. Math. Ser. 12, 1959, p. 36.
4. PIKE, M. C. Algorithm 267, Random Normal Deviate. *Comm. ACM*, *8* (Oct. 1965), 606.;

comment *R* is any parameterless procedure returning a random number uniformly distributed on the interval from zero to one. A suitable procedure is given by Algorithm 266, Pseudo-Random Numbers [*Comm. ACM*, *8* (Oct. 1965), 605] if one chooses $a = 0$, $b = 1$, and initializes y to some large odd number, such as $y = 13421773$.;

```
begin
  real X, Y, XX, YY, S, L;
  comment von Neumann rejection for choosing a random
    angle φ = 2θ, θ = tan⁻¹ (Y/X);
A:  X := R;  Y := 2 × R − 1;
  XX := X ↑ 2;  YY := Y ↑ 2;
  S := XX + YY;
  if  S > 1 then go to A;
  comment chooses L randomly from a χ₂²-distribution and
    normalizes with S;
L := sqrt (−2×ln(R))/S;
  comment computes deviates as L × sin (φ) and L × cos (φ);
D1 := (XX−YY) × L;
D2 := 2 × X × Y × L;
end norm;
```

REMARK ON ALGORITHM 178 [E4]
DIRECT SEARCH [Arthur F. Kaupe, Jr., *Comm. ACM* *6* (June 1963), 313]
[as revised by M. Bell and M. C. Pike, *Comm. ACM 9* (Sept. 1966), 684]

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CR CATEGORIES: 5.19

The procedure does not exit, as specified, after *maxeval* (the maximum number of) function evaluations.

The 3 statements *eval* := *eval* +1 should be interchanged with the immediately preceding statement and replaced by a call to the procedure *test eval* defined below. The statement labeled 2 should be deleted.

```
procedure test eval;
  if eval < maxeval then eval := eval +1
  else begin converge := false;
  go to EXIT
end test eval
```

REMARK ON ALGORITHM 272
PROCEDURE FOR THE NORMAL DISTRIBUTION FUNCTIONS [S15] [M. D. MacLaren, *Comm. ACM 8* (Dec. 1965), 789]

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CR CATEGORIES: 5.5, 5.12

In [1] Hill and Joyce report that the value produced by Algorithm 272 for the argument $a = 0.8$ is correct only to 5 decimal places, although the algorithm specifies an accuracy of 2×10^{-8}. Upon checking we have found that the source of this inaccuracy is a typographical error in the section beginning "**begin comment** initialize **own** variables;" The statement initializing *C*[3] should be changed to "*C*[3] = .54674530." With this change the published algorithm is, as far as we know, accurate within the specified error limit of 2×10^{-8}.

In the first comment of the algorithm the lower limit of the first integral should be minus infinity and not merely a minus sign.

REFERENCE:

1. HILL, I. D., AND JOYCE, S. A. Remark on algorithm 123. *Comm. ACM 10* (June 1967), 377.