basic understanding of linguistic theory and human language. Of course, we should exploit machines wherever we can; and that is one function of computational linguistics, which, as Titus points out, badly needs an accretion in numbers of good and well prepared scholars. If such studies ultimately point the way to an acceptable MT, among other things, well and good. But if they do, it is already clear on theoretical grounds that this will happen on a basis that is different from that used during past strivings for MT.

It is true, as both Titus and the ALPAC report mention, that we have learned substantive lessons from the failures in the search for MT. No sensible person or committee would advocate cutting off basic research on an interesting problem. But no amount of tinkering can rectify a basic theory that can be shown to be inadequate. The obvious course is to offer the broadest possible scope to research on basic theory without tying such research to one limited goal, e.g., MT.

Titus speaks of abandonment of support of MT "after only twelve brief years" as if it meant utter relinquishment of all approaches. As a member of ALPAC (though in this letter throughout I can pretend only to speak for myself), I conceived my task as one of inspecting evidence with a view to encouraging support for investigators to seek out the currently most promising avenues of approach. Whether or not they include MT would itself constitute a capital contribution.

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On "Numerical Integration of a Function That Has a Pole"

Editor:

The paper by E. Eisner [*Comm. ACM 10*, 4 (April 1967), 239] describes a method for evaluating integrals when the integrand has a singularity outside the range of integration by determining weights which depend on the order and location of the singularity. An alternate approach described by Krylov [1] for dealing with such problems subtracts the singularity from the integrand and uses a conventional formula to evaluate the transformed integral. This approach appears to involve less work and to be directly applicable to multiple singularities or singularities which are not on the real axis. Even when experimental data is involved it should be possible to estimate the coefficient of the poles if its order and location are known.

For the example given by Eisner,

$$\int_0^X \sec^2 \pi X \, dX = \frac{4\pi^{-2}X}{1-2X} + \int_0^X \left(\sec^2 \pi X - \frac{4\pi^{-2}}{(1-2X)^2}\right) dX.$$

The midpoint rule was used to evaluate the integral on the right side. The resulting absolute error $\times 10^6$ for several step sizes, h, were

x/h	1/30	1/60	1/120	1/240
.1	33	8	2	.5
.2	50	13	3	.8
.3	61	15	4	1.0
.4	69	17	4	1.1
$(.5 - h)$	73	19	7	9.8

This is comparable to Eisner's results.

Reference:

1. Krylov, V. I. *Approximate Calculations of Integrals.* Macmillan, New York, 1962.

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Dr. Eisner's Reply

This is in reference to Squire's comments, [1] drawing attention to an approach by Kantorovich [2] that provides an interesting alternative to that described in my recent paper. [3] A proper comparison of the merits of the two methods can probably be made only after varied experience in using them both. I shall merely point out a source of inaccuracy in Kantorovich's method that has no counterpart in mine, without attempting to evaluate its importance. I shall restrict the discussion to the integration of a function $R(X)$ that has a single pole, of order n, at $X = T$.

Kantorovich takes the first $(k+1)$ terms $(k \geq n)$ of the Taylor series about T of $[(X-T)^n R(X)]$. This enables him to split $R(X)$ into two parts, A and B, such that A is singular at $X = T$ but can be integrated analytically, while B must be integrated numerically, but has no singularity. B can therefore be integrated by conventional methods, but it must be evaluated with care, since it is a difference of two nearly-equal quantities.

As Squire points out, both my method and Kantorovich's require the order, n, and location, T, of the pole to be accurately known, but Kantorovich's requires the $(k+1)$ coefficients in the Taylor series as well. Errors in these coefficients will appear direct in the integral of the analytical part, A, of the integrand. However, it may be more serious that such errors leave singularities in the nominally singularity-free part, B. The conventional formulae used to integrate B will therefore be inaccurate. If tabular values of $R(X)$ are to be used to estimate the Taylor coefficients by curve fitting and extrapolation, we have a procedure similar to that which underlies my method, but much less direct in use.

It would be interesting to know how important this source of error is. My guess is that Kantorovich's method may be simpler (though less automatic) when $R(X)$ is an analytical expression for which the Taylor coefficients can be found exactly, while my method is simpler and more accurate where $R(X)$ contains computed or experimental data (which was the case in the problem that gave rise to my work [4]). The potential user should not be deterred by the fairly complicated-looking formulae of my method: they are really very straightforward to program.

References:

1. Squire, W. Letter on "numerical integration of a function that has a pole." *Comm. ACM 10*, 10 (Oct. 1967) 608.
2. Kantorovich, L. V. On approximate calculation of certain types of definite integrals and other approximations of the method of removal of singularities. *Mat. Sbornik, Ser. 1. 41*, (1934), 235–245 (in Russian, with French summary); English translation deposited with the Special Libraries Translation Pool, John Crerar Library, Chicago. Described in part by V. I. Krylov, *Priblizhennoe Vychislenie Integralov* (Approximate Calculation of Integrals) (in Russian) Gos. Izd. Fiz.-Mat. Lit., Moscow, 1959, English translation by A. H. Stroud, Macmillan, New York, 1962; Chapter 11, "Increasing the Precision of Quadrature Formulas," especially Section 11.2, pp. 202–206 (English edition), "Weakening the Singularity of the Integrand." [Kantorovich's very interesting paper deals with many things I have not considered in [3], only some of which are described by Krylov. For instance, he considers in detail, multiple singularites, logarithmic singularities, and the application of his method to singular (ordinary and partial) differential equations, and integral equations.]
3. Eisner, E. Numerical integration of a function that has a pole. *Comm. ACM 10*, 4 (April 1967), 239–243.
4. Eisner, E. Complete solutions of the 'Webster' horn equation," *J. Acoust. Soc. Am. 41* (1967), 1126–1146.

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