

Algorithms

H. J. WEGSTEIN, Editor

ALGORITHM 93
GENERAL ORDER ARITHMETIC
MILLARD H. PERSTEIN
Control Data Corp., Palo Alto, Calif.

procedure arithmetic (*a*, *b*, *c*, *op*);
integer *a*, *b*, *c*, *op*;
comment This procedure will perform different order arithmetic operations with *b* and *c*, putting the result in *a*. The order of the operation is given by *op*. For *op* = 1 addition is performed. For *op* = 2 multiplication, repeated addition, is done. Beyond these the operations are non-commutative. For *op* = 3 exponentiation, repeated multiplication, is done, raising *b* to the power *c*. Beyond these the question of grouping is important. The innermost implied parentheses are at the right. The hyper-exponent is always *c*. For *op* = 4 tetration, repeated exponentiation, is done. For *op* = 5, 6, 7, etc., the procedure performs pentation, hexation, heptation, etc., respectively.

The routine was originally programmed in FORTRAN for the Control Data 160 desk-size computer. The original program was limited to tetration because subroutine recursiveness in Control Data 160 FORTRAN has been held down to four levels in the interests of economy.

The input parameter, *b*, *c*, and *op*, must be positive integers, not zero;

```
begin own integer d, e, f, drop;
  if op = 1 then
    begin a := b + c; go to 1
  end if op = 2 then d := 0;
  else d := 1; e := c; drop := op - 1;
  for f := 1 step 1 until e do
    begin arithmetic (a, b, d, drop);
      d := a
    end;
1: end arithmetic
```

ALGORITHM 94
COMBINATION

JEROME KURTZBERG
Burroughs Corp., Burroughs Laboratories, Paoli, Pa.

procedure COMBINATION (*J*, *N*, *K*); **value** *N*, *K*; **integer** **array** *J*; **integer** *N*, *K*;
comment This procedure generates the next combination of *N* integers taken *K* at a time upon being given *N*, *K* and the previous combination. The *K* integers in the vector *J*(1) ... *J*(*K*) range in value from 0 to *N* - 1, and are always monotonically strictly increasing with respect to themselves in input and output format. If the vector *J* is set equal to zero, the first combination produced is *N* - *K*, ..., *N* - 1. That initial combination is also produced after 0, 1, ..., *N* - 1, the last value in that cycle;

```
begin integer B, L;
  B := 1;
```

```
mainbody: if J(B) ≥ B then begin A := J(B) - B - 1;
  for L := 1 step 1 until B do J(L) := L + A;
  go to exit end;
if B = K then go to initiate;
B := B + 1; go to mainbody;
initiate: for B := 1 step 1 until K do J(B) := N - K - 1 + B
exit: end COMBINATION
```

ALGORITHM 95
GENERATION OF PARTITIONS IN PART-COUNT FORM

FRANK STOCKMAL
System Development Corp., Santa Monica, Calif.

procedure partgen(*c*, *N*, *K*, *G*); **integer** *N*, *K*; **integer** **array** *c*; **Boolean** *G*;
comment This procedure operates on a given partition of the positive integer *N* into parts ≤ *K*, to produce a consequent partition if one exists. Each partition is represented by the integers *c*[1] thru *c*[*K*], where *c*[*j*] is the number of parts of the partition equal to the integer *j*. If entry is made with *G* = **false**, procedure ignores the input array *c*, sets *G* = **true**, and produces the first partition of *N* ones. Upon each successive entry with *G* = **true**, a consequent partition is stored in *c*[1] thru *c*[*K*]. For *N* = *KX*, the final partition is *c*[*K*] = *X*. For *N* = *KX* + *r*, 1 ≤ *r* ≤ *K* - 1, final partition is *c*[*K*] = *X*, *c*[*r*] = 1. When entry is made with **array** *c* = final partition, *c* is left unchanged and *G* is reset to **false**;

```
begin integer a, i, j;
  if ¬ G then go to first;
  j := 2;
  a := c[1];
test: if a < j then go to B;
  c[j] := 1 + c[j];
  c[1] := a - j;
zero: for i := 2 step 1 until j - 1
  do c[i] := 0;
  go to EXIT;
B: if j = K then go to last;
  a := a + j × c[j];
  j := j + 1;
  go to test;
first: G := true;
  c[1] := N;
  j := K + 1;
  go to zero;
last: G := false;
EXIT: end partgen
```

ALGORITHM 96
ANCESTOR

ROBERT W. FLOYD
Armour Research Foundation, Chicago, Ill

procedure ancestor (*m*, *n*); **value** *n*; **integer** *n*; **Boolean** **array** *m*;

comment Initially $m[i, j]$ is **true** if individual i is a parent of individual j . At completion, $m[i, j]$ is **true** if individual i is an ancestor of individual j . That is, at completion $m[i, j]$ is **true** if there are k, l , etc. such that initially $m[i, k], m[k, l], \dots, m[p, j]$ are all **true**. Reference: WARSHALL, S. A theorem on Boolean matrices, *J. ACM* 9(1962), 11-12;

```
begin
integer i, j, k;
for i := 1 step 1 until n do
for j := 1 step 1 until n do
if m[j, i] then
for k := 1 step 1 until n do
if m[i, k] then
m[j, k] := true
end ancestor
```

ALGORITHM 97 SHORTEST PATH

ROBERT W. FLOYD

Armour Research Foundation, Chicago, Ill.

procedure shortest path (m, n); **value** n; **integer** n; **array** m;
comment Initially $m[i, j]$ is the length of a direct link from point i of a network to point j . If no direct link exists, $m[i, j]$ is initially ∞ . At completion, $m[i, j]$ is the length of the shortest path from i to j . If none exists, $m[i, j]$ is ∞ . Reference: WARSHALL, S. A theorem on Boolean matrices, *J. ACM* 9(1962), 11-12;

```
begin
integer i, j, k; real inf, s; inf :=  $\infty$ ;
for i := 1 step 1 until n do
for j := 1 step 1 until n do
if m[j, i] < inf then
for k := 1 step 1 until n do
if m[i, k] < inf then
begin s := m[j, i] + m[i, k];
if s < m[j, k] then m[j, k] := s
end
end shortest path
```

Contributions to this department must be in the form stated in the Algorithms Department policy statement (*Communications*, February, 1960) except that ALGOL 60 notation should be used (see *Communications*, May 1960). Contributions should be sent in duplicate to J. H. Wegstein, Computation Laboratory, National Bureau of Standards, Washington 25, D. C. Algorithms should be in the Reference form of ALGOL 60 and written in a style patterned after the most recent algorithms appearing in this department. For the convenience of the printer, please underline words that are delimiters to appear in boldface type.

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ALGORITHM 98

EVALUATION OF DEFINITE COMPLEX LINE INTEGRALS

JOHN L. PFALTZ

Syracuse University Computing Center, Syracuse, N. Y.

procedure COMPLINEINTGRL(A, B, N, RSSUM);
value A, B, N; **real** A, B, N; **array** RSSUM;
comment COMPLINEINTGRL approximates the complex line integral by evaluating the partial Riemann-Stieltjes sum $\sum_{i=1}^n f(z_k)[z_i - z_{i-1}]$ where $a \leq t \leq b$ and $z_k \in (z_{i-1}, z_i)$. The programmer must provide 1) the procedures GAMMA(T, Z) to calculate $z(t)$ on Γ , and FUNCT(Z, F) to calculate function values, and 2) the end points A and B of the parametric interval and N the number of subintervals into which $[a, b]$ is to be partitioned;

```
begin integer I; real T, DELT; real array ZT, ZTL, DELZ,
ZK, PART[1:2]; RSSUM[1] := 0.0; RSSUM[2] := 0.0;
DELT := (B - A)/N; T := A;
```

```
line: GAMMA(T, ZT);
if T = A then go to next;
for I := 1 step 1 until 2 do
begin
DELTZ[I] := ZT[I] - ZTL[I]; end;
for I := 1 step 1 until 2 do
begin
ZK[I] := ZTL[I] + DELTZ[I]/2.0; end;
FUNCT(ZK, FZ);
PART[1] := FZ[1]  $\times$  DELTZ[1] - FZ[2]  $\times$  DELTZ[2];
PART[2] := FZ[1]  $\times$  DELTZ[2] + FZ[2]  $\times$  DELTZ[1];
for I := 1 step 1 until 2 do
begin
RSSUM[I] := RSSUM[I] + PART[I]; end;
if T < B - (0.25  $\times$  DELT) then go to next else go to
exit;
```

```
next: for I := 1 step 1 until 2 do
begin
ZTL[I] := ZT[I]; end;
T := T + DELT;
go to line;
exit: end COMPLINEINTGRL.
```

ALGORITHM 99

EVALUATION OF JACOBI SYMBOL

STEPHEN J. GARLAND AND ANTHONY W. KNAPP

Dartmouth College, Hanover, N. H.

procedure Jacobi (n, m, r); **value** n, m;

integer n, m, r;

comment Jacobi computes the value of the Jacobi symbol (n/m) , where m is odd, by the law of quadratic reciprocity. The parameter r is assigned one of the values $-1, 0$, or 1 if m is odd. If m is even, the symbol is undefined and r is assigned the value 2 . For odd m the routine provides a test of whether m and n are relatively prime. The value of r is 0 if and only if m and n have a nontrivial common factor. In the special case where m is prime, $r = -1$ if and only if n is a quadratic nonresidue of m ;

begin

integer s;

Boolean p, q;

Boolean procedure parity (x); **value** x; **integer** x;

comment The value of the function parity is **true** if x is odd, **false** if x is even;

begin

parity := $x \div 2 \times 2 \neq x$

end parity;

```

if  $\neg$  parity (m) then begin r := 2; go to exit end;
p := true;
loop: n := n - n  $\div$  m  $\times$  m;
    q := false;
    if n  $\leq$  1 then go to done;
even: if  $\neg$  parity (n) then
    begin
        q :=  $\neg$  q;
        n := n  $\div$  2;
        go to even
    end n now odd;
if q then if parity ((m2 - 1)  $\div$  8) then p :=  $\neg$  p;
if n = 1 then go to done;
if parity ((m-1)  $\times$  (n-1)  $\div$  4) then p :=  $\neg$  p;
    s := m; m := n; n := s; go to loop;
done: r := if n = 0 then 0 else if p then 1 else -1;
exit: end Jacobi

```

ALGORITHM 100

ADD ITEM TO CHAIN-LINKED LIST

PHILIP J. KIVIAT

United States Steel Corp., Appl. Research Lab., Monroeville, Penn.

procedure inlist (t,info,m,list,n,first,flag,addr,listfull);
integer n,m,first,flag,t; **integer array** info,list,addr;
comment inlist adds the information pair {t,info} to the chain-link structured matrix list (i,j), where t is an order key ≥ 0 , and info(k) an information vector associated with t. info(k) has dimension m, list(i,j) has dimensions (n \times (m+3)). flag denotes the head and tail of list(i,j), and first contains the address of the first (lowest order) entry in list(i,j). addr(k) is a vector containing the addresses of available (empty) rows in list(i,j). Initialization: list(i,m+2) = flag, for some i \leq n. If list(i,j) is filled exit is to listfull;

```

begin integer i, j, link1, link2;
0: if addr [1] = 0; then go to listfull; i := 1;
1: if list [i,1]  $\leq$  t
    then begin if list [i,2]  $\neq$  0 then begin link1 := m+2;
        link2 := m+3; go to 2 end; else begin if
        list [i,m+2] = flag then begin i := flag;
        link1 := m+3; link2 := m+2; go to 3 end;
        else begin i := i+1; go to 1 end end end;
    else begin link1 := m+3; link2 := m+2 end;
2: if list [i,link2]  $\neq$  flag
    then begin k := i; i := list [i,link2];
        if (link2 = m+2  $\wedge$  list [i,1]  $\leq$  t)  $\vee$ 
        (link2  $\neq$  m+2  $\wedge$  list [i,1] > t) then go to 4;
        else go to 1 end;
    else begin list [i,link2] := addr [1] end;
3: j := addr [1]; list [j,link1] := i;
    list [j,link2] := flag; if link2 = m+2 then
    first := addr [1]; go to 5;
4: j := addr [1]; list [j,link1] := list [i,link1];
    list [i,link1] := list [k,link2] := addr [1];
    list [j,link2] := i;
5: list [j,1] := t; for i := 1 step 1 until m do
    list [j,i+1] := info [i]; for i := 1 step 1 until n-1 do
    addr [i] := addr [i+1]; addr [n] := 0
end inlist

```

ALGORITHM 101

REMOVE ITEM FROM CHAIN-LINKED LIST

PHILIP J. KIVIAT

United States Steel Corp., Appl. Res. Lab., Monroeville, Penn.

```

procedure outlist (vector,m,list,n,first,flag,addr);
integer n,m,first,flag; integer array vector,list,addr;
comment outlist removes the first entry (information pair with
    lowest order key) from list(i,j) and puts it in vector(k);
begin integer i;
for i := 1 step 1 until m+1 do vector[i] := list [first,i];
for i := n-1 step -1 until 1 do addr [i+1] := addr [i];
addr [1] := first;
if list [first,m+3] = flag then
    begin list [1,m+2] := flag; first := 1;
        for i := 1 step 1 until n do addr [i] := i end;
    else begin first := list [first,m+3];
        list [first,m+2] := flag end;
for i := 1 step 1 until m+3 do list [addr [1], i] := 0
end outlist

```

ALGORITHM 102

PERMUTATION IN LEXICOGRAPHICAL ORDER

G. F. SCHRACK AND M. SHIMRAT

University of Alberta, Calgary, Alberta, Canada

```

procedure PERMULEX(n,p);
integer n; integer array p;
comment Successive calls of the procedure will generate all
    permutations p of 1,2,3,...,n in lexicographical order. Before the
    first call, the non-local Boolean variable 'flag' must be set to
    true. If after an execution of PERMULEX 'flag' is false,
    additional calls will generate further permutations—if true, all
    permutations have been obtained;
begin integer array q[1:n]; integer i, k, t; Boolean flag2;
if flag then
    begin for i := 1 step 1 until n do
        p[i] := i; flag2 := true; flag := false;
    go to EXIT
    end initialize;
if flag2 then
    begin t := p[n]; p[n] := p[n-1]; p[n-1] := t;
        flag2 := false; go to EXIT
    end bypass;
flag2 := true; for i := n-2 step -1 until 1 do
    if p[i] < p[i+1] then go to A;
    flag := true; go to EXIT;
A: for k := 1 step 1 until n do q[k] := 0;
    for k := i step 1 until n do q[p[k]] := p[k];
    for k := p[i] + 1 step 1 until n do
    if q[k]  $\neq$  0 then go to B;
B: p[i] := k; q[k] := 0;
    for k := 1 step 1 until n do
    if q[k]  $\neq$  0 then begin i := i + 1; p[i] := q[k] end
    else if i  $\geq$  n then go to EXIT;
EXIT:
end PERMULEX

```

ALGORITHM 103
 SIMPSON'S RULE INTEGRATOR
 GUY F. KUNCIR
 UNIVAC Division, Sperry Rand Corp., San Diego, Calif.

procedure SIMPSON (a, b, f, I, i, eps, N);
value a, b, eps, N; **integer** N;
real a, b, I, i, eps; **real procedure** f;
comment This procedure integrates the function $f(x)$ using a modified Simpson's Rule quadrature formula. The quadrature is performed over j subintervals of $[a, b]$ forming the total area I . Convergence in each subinterval of length $(b-a)/2^n$ is indicated when the relative difference between successive three-point and five-point area approximations

$$A_{3,j} = (b-a)(g_0 + 4g_2 + g_4)/(3 \cdot 2^{n+1})$$

$$A_{5,j} = (b-a)(g_0 + 4g_1 + 2g_2 + 4g_3 + g_4)/(3 \cdot 2^{n+2})$$

is less than or equal to an appropriate portion of the over-all tolerance eps (i.e., $|A_{5,j} - A_{3,j}|/A_{5,j} \leq \text{eps}/2^n$ with $n \leq N$). SIMPSON will reduce the size of each interval until this condition is satisfied.

Complete integration over $[a, b]$ is indicated by $i = b$. A value $a \leq i < b$ indicates that the integration was terminated, leaving I the true area under f in $[a, i]$. Further integration over $[i, b]$ will necessitate either the assignment of a larger N , a larger eps, or an integral substitution reducing the slope of the integrand in that interval. It is recommended that this procedure be used between known integrand maxima and minima.;

begin integer m, n; **real** d, h; **array** g[0:4], A[0:2], S[1:N, 1:3];
 I := i := m := n := 0;
 g[0] := f(a);
 g[2] := f((a + b)/2);
 g[4] := f(b);
 A[0] := (b - a) × (g[0] + 4 × g[2] + g[4])/2;
 AA: d := 2↑n; h := (b - a)/4/d;
 g[1] := f(a + h × (4 × m + 1));
 g[3] := f(a + h × (4 × m + 3));
 A[1] := h × (g[0] + 4 × g[1] + g[2]);
 A[2] := h × (g[2] + 4 × g[3] + g[4]);
if abs (((A[1] + A[2]) - A[0])/(A[1] + A[2])) > eps/d
 then begin m := 2 × m; n := n + 1;
 if n > N **then go to** CC;
 A[0] := A[1]; S[n,1] := A[2];
 S[n,2] := g[3]; S[n,3] := g[4];
 g[4] := g[2]; g[2] := g[1]; **go to** AA
 end
else begin I := I + (A[1] + A[2])/3;
 m := m + 1; i := a + m × (b - a)/d;
 BB: **if** m = 2 × (m ÷ 2) **then**
 begin m := m ÷ 2; n := n - 1; **go to** BB **end**
 if (m ≠ 1) ∨ (n ≠ 0) **then**
 begin A[0] := S[n,1]; g[0] := g[4];
 g[2] := S[n,2]; g[4] := S[n,3]; **go to** AA **end**
 end
 CC: **end** SIMPSON

REMARK ON ALGORITHM 19
 BINOMIAL COEFFICIENTS (Richard R. Kenyon,
Comm. ACM, Oct. 1960)
 RICHARD STECK
 Armour Research Foundation, Chicago 16, Ill.

The **for** clause of Algorithm 19 should read:

for i := 0 **step** 1 **until** b-1 **do**

With this correction the algorithm was certified on the Armour Research Foundation UNIVAC 1105.

The recursion formula stated in the **comment** should read:

$$C_{i+1}^n = (n-i) C_i^n / (i+1).$$

CERTIFICATION OF ALGORITHM 46
 EXPONENTIAL OF A COMPLEX NUMBER (J. R.
 Herndon, *Comm. ACM* 4 (Apr., 1961), 178)
 A. P. RELPH
 Atomic Power Div., The English Electric Co., Whetstone,
 England

Algorithm 46 was translated using the DEUCE ALGOL compiler, no corrections being required, and gave satisfactory results.

CERTIFICATION OF ALGORITHM 48
 LOGARITHM OF A COMPLEX NUMBER (J. R.
 Herndon, *Comm. ACM* 4 (Apr., 1961), 179)
 A. P. RELPH
 Atomic Power Div., The English Electric Co., Whetstone,
 England

Algorithm 48 was translated using the DEUCE ALGOL compiler, after certain modifications had been incorporated, and then gave satisfactory results.

The original version will fail if $a = 0$ when the procedure for arctan is entered. It also assumes that $-\pi/2 < d < 3\pi/2$, whereas the principal value for logarithm of a complex number assumes $-\pi < d \leq \pi$.

Incidentally, the ALGOL 60 identifier for natural logarithm is **ln**, not **log**.

The modified procedure is as follows:

procedure LOGC (a,b,c,d); **value** a,b; **real** a,b,c,d;
comment This procedure computes the number $c + di$ which is equal to the principal value of $\log_e(a + bi)$. If $a = 0$ then c is put equal to -1047 which is used to represent " $-\infty$ ";
begin integer m,n
 m := sign (a); n := sign (b);
if a = 0 **then begin** c := -1047;
 d := 1.5707963 × n;
 go to k
end;
 c := sq rt(a × a + b × b);
 c := ln (c);
 d := 1.5707963 × (1-m) × (1+n-n×n) + arctan (b/a);
 k: **end** LOGC;

CERTIFICATION OF ALGORITHM 58
 MATRIX INVERSION (Donald Cohen, *Comm. ACM* 4,
 May 1961)
 RICHARD A. CONGER
 Yalem Computer Center, St. Louis University, St.
 Louis, Mo.

Invert was hand-coded in FORTRAN for the IBM 1620. The following corrections were found necessary:

The statement $a_{k,j} := a_{k,i} - b_j \times c_k$ *should be*

$$a_{k,j} := a_{k,j} - b_j \times c_k$$

The statement **go to** back *should be changed to*

i := z_k; z_k := z_j; z_j := i; **go to** back

After these corrections were made, the program was checked by inverting a 6 × 6 matrix and then inverting the result. The second result was equal to the original matrix within round-off.

CERTIFICATION OF ALGORITHM 66

INVR (J. Caffrey, *Comm. ACM*, July 1961)

JOHN CAFFREY
Palo Alto Unified School District, Palo Alto, California

INVR was translated using the Burroughs 220 Algebraic Computer (BALCOM) at Stanford University, using 8-digit floating-point arithmetic. The misprint noted by Randell and Broyden (*Comm. ACM*, Jan. 1962, p. 50) was corrected, and the same example (Wilson's 4×4 matrix) was used as a test case. The resulting inverse was:

68.0000	-41.0000	-17.0000	10.0000
	25.0000	10.0000	-6.0000
		5.0000	-3.0000
			2.0000

It may also be useful to note that the determinant of the matrix may be obtained as the successive product of the pivots. That is, if $t_i (=T(1, 1))$ is the i th pivot of a matrix of order n ,

$$\text{determinant} = \prod_i^n t_i.$$

For the above *input* example,
determinant = 1.0

Randell and Broyden's observation concerning the *apparent* limitation of INVR to positive definite cases is correct: That is, any nonsingular real symmetric matrix (positive, indefinite, or negative) may be inverted using this algorithm. The original INVR should therefore be modified as follows:

if pivot = 0 **then go to** singular;

Randell and Broyden's second example (of order 5) was also used as a test case, with the resulting inverse:

-.0000	.9999	.0000	.0000	.9999
1.5333	-.7333	-.1333	.7999	
	-.8666	-1.0666	-.5999	
		-1.4666	-.1999	
			.2000	

determinant = -14.999999

An attempt to invert the *inverse* of the 4×4 segment of the Hilbert matrix, as presented by Randell (*Comm. ACM*, Jan. 1962, p. 50), yielded the following results:

.9999	.4999	.3333	.2499
	.3333	.2499	.1999
		.1999	.1666
			.1428

determinant = 6048020.6

CERTIFICATION OF ALGORITHM 67

CRAM (J. Caffrey, *Comm. ACM* 4 (July 1961), 322)

A. P. RELPH

Atomic Power Div., The English Electric Co., Whetstone, England

CRAM was translated using the DEUCE ALGOL compiler with the following corrections:

$V[i] = S$ was changed to $V[i] := S$

$f[k,j] = V[k]$ was changed to $f[k,j] := V[k]$

It is quicker not to use the table of the $C[i]$ in the "load" sequence and instead use the following sequence:

load: $m := n \times (n+1)/2;$

for $i := 1$ **step** 1 **until** m **do** READ $(a[i]);$

REMARK ON ALGORITHM 76

SORTING PROCEDURES (Ivan Flores, *Comm. ACM* 5, Jan. 1962)

B. RANDELL

Atomic Power Div., The English Electric Co., Whetstone, England

The following types of errors have been found in the Sorting

Procedures:

1. Procedure declarations not starting with **procedure**.
 2. Bound pair list given with array specification.
 3. = used instead of :=, in assignment statements, and in a **for** clause.
 4. A large number of semicolons missing (usually after **end**).
 5. Expressions in bound pair lists in array declarations depending on local variables.
 6. Right parentheses missing in some procedure statements.
 7. Conditional statement following a **then**.
 8. No declarations for A , or z , which is presumably a misprint.
 9. In several procedures attempt is made to use the same identifier for two different quantities, and sometimes to declare an identifier twice in the same block head.
 10. In the Presort quadratic selection procedure an array, declared as having two dimensions, is used by a subscripted variable with only one subscript.
 11. At one point a subscripted variable is given as an actual parameter corresponding to a formal parameter specified as an array.
 12. In several of the procedures, identifiers used as formal parameters are redeclared, and still assumed to be available as parameters.
 13. In every procedure K is given in the specification part, with a parameter, whilst not given in the formal parameter list.
- No attempt has been made to translate, or even to understand the logic of these procedures. Indeed it is felt that such a grossly inaccurate attempt at ALGOL should never have appeared as an algorithm in the *Communications*.

CERTIFICATION OF ALGORITHM 77

AVINT (Paul E. Hennion, *Comm. ACM* 5, Feb., 1962)

VICTOR E. WHITTIER

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AVINT was transliterated into BAC-220 (a dialect of ALGOL-58) and was tested on the Burroughs 220 computer. The following minor errors were found:

1. The first statement following label L11 should read:
 $\text{dif} := 2 \times a \times \text{xarg} + b;$
2. The semicolon (;) at the end of the line beginning with the label L16 should be deleted.
3. There appears to be a confusion between "1" (numeric) and "I" (alphabetic) following label L12. This portion of the program should read:
L12: $\text{sum} := 0;$ $\text{syl} := \text{xlo};$ $\text{jul} := \text{nop} - 1;$ $\text{ib} := 2;$

After making the above corrections the procedure was tested for interpolation, differentiation, and integration using e^x , $\log X$, and $\sin X$ in the range $(1.0 \leq X \leq 5.0)$. Twenty-one values of each of these functions, evenly spaced with respect to X and accurate to at least 7 significant digits, were tabulated in the above range. Then the procedure was tested. The following table indicates approximately the accuracy obtained:

Function	Number of Significant Digits		
	Interpolation	Differentiation	Integration
e^x	$\geq 4^*$	≥ 2	≥ 4
$\log X$	$\geq 4^*$	≥ 2	≥ 3
$\sin X$	$\geq 4^*$	≥ 2	≥ 4

* Except for interpolation between the first two points in the table.

The above results are quite reasonable in view of the relatively large increment in X . Tests using smaller increments in X and uneven spacing of X were also satisfactory.

It was also discovered that for integration the following restrictions must be observed:

1. $\text{xlo} \leq \text{xa}$ (1).
2. $\text{xup} \geq \text{xa}$ (nop).