COMPUTERIZED SELECTIVE MODELING FOR THE GOLDBACH CONJECTURE PROOF

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ABSTRACT

The branch of Number Theory in the field of mathematics deals primarily with integer or whole number relationships. One such relationship classifies the integers on the basis of their divisibility into two classes of numbers, the even numbers and the odd numbers. Divisibility criteria can also be used to classify integers as being either prime or composite. This paper addresses a problem concerning these attributes of the integers.[1]

INTRODUCTION

In the middle of the eighteenth century, a relatively obscure English mathematician, Charles Goldbach, advanced two conjectures concerning prime numbers.

(1) Every even number $N \geq 6$ is the sum of two odd prime numbers.
(2) Every odd number $N \geq 9$ is the sum of three odd prime numbers.

The second conjecture can easily be shown to be true if the first conjecture is assumed to be true. However, since Goldbach's time, all attempts to prove the validity of the first conjecture have been unsuccessful. In recent years it has been shown that every even integer can be written as the sum of two positive odd integers each of which is the product of four or fewer prime factors. However, this result still falls short of Goldbach's first conjecture.[2,3,4]

This paper describes a computerized algorithm to validate Goldbach's first conjecture for a randomly selected set of even integers. This method *will not* prove the Goldbach conjecture since it deals only with a finite set of even numbers. Any approach to prove conclusively the Goldbach conjecture must apply to *all* integers. This approach, however, is the Goldbach conjecture. All computer programs are written in FORTRAN for execution on an IBM 370 computer.

The following algorithm is used to test a candidate even number for Goldbach attributes:

Step 1. Let M denote the even integer being tested.

Step 2. Set N1 = 3 and N2 = M - 3 so that M = N1 + N2.

Step 3. If both N1 and N2 are prime, print these out along with M and terminate the algorithm for this even number. If N1 or N2 is not a prime number, proceed to *Step 4*.

Step 4. Set N1 = N1 + 2 and N2 = N2 - 2. If N1 > N2, then the conjecture is not valid, print all possible odd integer addends N1 and N2 such that M = N1 + N2 and show in each case that either N1 or N2 is not a prime number. If N1 ≤ N2 then proceed to *Step 3*.

MODEL PROGRAM LOGIC

In examining an integer M to determine if it is a prime number, use is made of the fact that if the integer M is not divisible by any prime (in this paper odd prime since we are dealing only with odd integer addends) less than or equal to the square root of M, then M is a prime number. For this reason the prime number generator was developed to compute all consecutive odd prime numbers less than 100,000. This array of odd prime numbers is sufficient to test any number less than 10^{10} to determine if it is prime; hence, the validation program is capable of testing any odd integer in the range of the IBM 370 computer ($2^{31} - 1 < 10^{10}$). The complete table of primes (3 to 99991) is read as input by the Goldbach Validation Program. The Prime Number Generator program is not considered to be a part of the Goldbach Validation Program. Each of the computer programs shown in the schematic diagram (Figure 1) of the Goldbach Validation Program is discussed below.

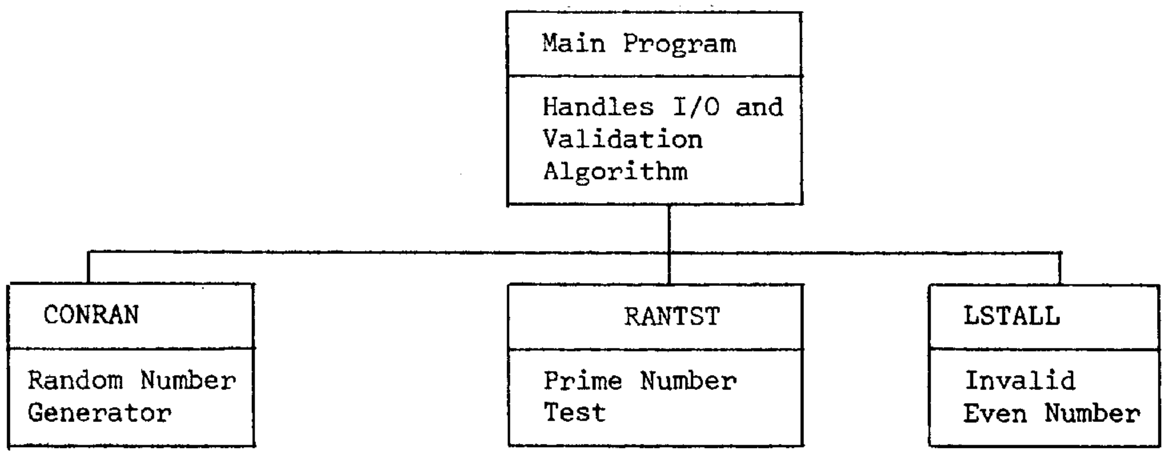

Figure 1. Goldbach Validation Program

MAIN PROGRAM

The main program of the Goldbach Validation Program performs the following functions:

(1) Reads a data card containing the lower (ILOW) and upper (IUP) limits that determine the range of even numbers to be considered during the run, and a "seed", ISEED, for the random number generator. The program then reads an array, KPRM, of odd prime numbers for use in testing odd number addends in the RANTST routine. Prior to reading the array of odd prime numbers, the maximum number, IMAX, of entries in the KPRM array is input as shown in Figure 2. These cards constitute all input requirements for the Goldbach Validation Program.

(2) Since time limitations precluded the examination of all possible even numbers to see if they conformed to the Goldbach conjecture, a finite set of randomly selected even number candidates was generated in the following fashion:

- (a) A random number X, $0 \le X < 1$, containing eight significant digits was generated by CONRAN.
- (b) The random number X was transformed to an integer IX, $0 \le IX < 1,000,000$ by multiplying X by 1,000,000.
- (c) An even number in the interval (ILOW-1)* 2,000,000 + 2 * IX was selected for testing. This step essentially partitions the set of integers less than $2^{31} - 1$ (the maximum for the IBM 370) into distinct even number subsets, each containing 1,000,000 even integers

except for the last subset which consists only of those even integers greater than 2,145,999,998 and less than $2^{31} - 1$. In this way 1,074 candidate even numbers were constructed and examined to see if they conformed to the Goldbach conjecture.

(3) Logic is built into the main program to transfer control to the ISTALL program in the event an even number is examined and found to invalidate the Goldbach Conjecture.

Data Element	Card Column	Format
	CARD 1	
ISEED	1-10	I10
ILOW	11-15	I5
IUP	16-20	I5
	CARD 2	
IMAX	1-5	I5
	CARD 3 AND ALL OTHERS	
KPRM (1)	1-5	I5
KPRM (2)	6-10	I5
KPRM (16)	76-80	I5

Note: For data card 4 begin reading KPRM (17) KPRM (18), etc. In general, for data card M begin reading KPRM((M-1)* 16 + 1)). Each card fills 16 positions in the KPRM array.

Figure 2. Data Card Input for Main Program

CONRAD

This subroutine generates pseudorandom numbers based on the formula:

$$X_n + 1 = (B*X_n + A) \text{ MOD } (M).$$

The initial values for the seed, X_n, the constants A and B, and the modulus, M, are those recommended by Maisel and Gnugnoli.[5] A gap test using the digit 3 was performed on 2,000 8-digit random numbers generated by this subroutine and the results are shown in Figure 3. The chi-square statistic of 4.407 indicates that CONRAD generated random numbers exhibit "randomness' above the 95% confidence level.

RANST

This subroutine accepts as input two odd integers N1 and N2, which are the odd number addends of a candidate even number, M, examines them to determine if they are prime, and outputs indicators 11, 12, and ITEST as follows:

(1) If N1 and N2 are both prime, ITEST is set to 1; otherwise, ITEST = 0.

(2) If either N1 is composite or N2 is composite (ITEST = 0) then 11 and 12 are set as follows:

(a) For N1 composite, 12 is set to the smallest prime factor that divides N1.
(b) For N2 composite, 12 is set to the smallest prime factor that divides N2.

The outputs 11 and 12 are used by the LSTALL subroutine only in the event an even number is discovered that does not conform to the Goldbach criteria.

Length of Gap	Probability	Theoretical Frequency (TF)	Actual Frequency (AF)	(AF=TF)**2/TF
0, 1, 2	0.1 + 0.09 + 0.081	320.05	288	3.210
3, 4, 5, or 6	+0.0729 + 0.06561	296.08	308	0.480
	+0.05 049			
	+0.0531441			
Greater than 6	0.4783	564.87	585	0.717
TOTALS	1.0	1181	1181	X^2 = 4.407

Figure 3. Chi-Square Calculations Based on Gap Lengths

LSTALL

This subroutine will be executed only if an even number, M, is examined that does not conform to the Goldbach conjecture. In this case, LSTALL will produce a listing which shows all odd numbers N1 and N2 such that M = N1 + N2 and in each instance shows that either N1 or N2 is composite by producing two factors of N1 or N2. The examination of the 1,074 even numbers during this analysis did not produce an even number which invalidated the Goldbach conjecture.

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