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ISBN
9781581135213

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Publication Date
2002-07-01

DOI
10.1145/566570.566579

Graphical Modeling and Animation of Ductile Fracture

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Abstract

In this paper, we describe a method for realistically animating ductile fracture in common solid materials such as plastics and metals. The effects that characterize ductile fracture occur due to interaction between plastic yielding and the fracture process. By modeling this interaction, our ductile fracture method can generate realistic motion for a much wider range of materials than could be realized with a purely brittle model. This method directly extends our prior work on brittle fracture [O'Brien and Hodgins, SIGGRAPH 99]. We show that adapting that method to ductile as well as brittle materials requires only a simple to implement modification that is computationally inexpensive. This paper describes this modification and presents results demonstrating some of the effects that may be realized with it.

CR Categories: I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling—Physically based modeling; I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Animation; I.6.8 [Simulation and Modeling]: Types of Simulation—Animation

Keywords: Animation techniques, physically based modeling, simulation, dynamics, fracture, cracking, deformation, finite element method, ductile fracture, plasticity.

1 Introduction

As techniques for generating photorealistic computer rendered images have improved, the use of physically based animation to generate special effects in film, television, and games has become increasingly common. Physically based animation techniques have proven to be particularly useful for violent or destructive effects that would be impractical or expensive to achieve using other methods. For example, when creating effects for the film *Pearl Harbor*, Industrial Light and Magic made extensive use of simulation methods for modeling the destruction of ships, planes, and other structures [Duncan, 2001].

Animating objects as they break, crack, tear, or in general fracture appears to be an obvious place where physically based modeling should be useful, particularly if the object is expensive, irreplaceable, or if breaking it would be hazardous. However even the most general of current techniques for animating fracture are limited to modeling only brittle materials.

The term *brittle* does not mean that a material is fragile. It means that the material experiences only elastic deformation before fracture. Few real materials are truly brittle. In contrast, *ductile* materials behave elastically up to a point and then experience some amount of plastic deformation before fracture. When brittle materials fracture, they shatter. However, ductile materials demonstrate a much wider range of fracture behaviors. (See figures 1 and 3.) This wider range of behaviors arises due to the interaction of plastic energy absorption with the fracture process.

This paper describes a method suitable for modeling ductile fracture in common solid materials such as plastics or metals. This method directly extends our prior technique presented in [O'Brien and Hodgins, 1999] for modeling brittle fracture. Adapting that technique to ductile as well as brittle materials requires only a simple to implement and computationally inexpensive modification. This extension dramatically expands the range of materials that may be modeled. For the sake of brevity, this paper describes only this modification and presents results demonstrating some of the effects that may be realized with it.

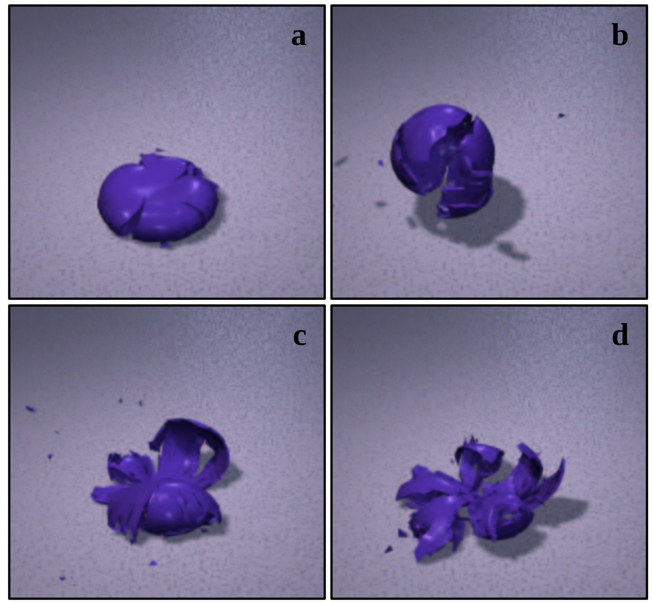

Figure 1: Four hollow balls that have been dropped onto a hard surface. The ball in (a) flattens out and visibly demonstrates plastic yielding. The other three do not show an appreciable amount of plastic deformation, but the manner in which they split and tear, as opposed to shattering, arises from of the interaction between plastic yielding and the fracture process.

2 Related Work

The primary contribution of this paper is extending [O'Brien and Hodgins, 1999] to include ductile fracture by adding a plasticity model to the underlying finite-element method. The plasticity model we describe is not novel. It consists of the von Mises yield criterion, simple kinematic work hardening, and a finite yield limit [Fung, 1965]. This plasticity model is similar to the one used in [Terzopoulos and Fleischer, 1988a] and [Terzopoulos and Fleischer, 1988b]. The primary differences between their model and the one presented here are that this model realistically preserves vol-

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ume and it includes a second elastic regime once a limit on the amount of plastic flow has been exceeded.

Although ductile fracture has not been widely addressed in the graphics literature, several other graphics researchers have investigated brittle fracture. In [Terzopoulos and Fleischer, 1988a] and [Terzopoulos and Fleischer, 1988b] a finite differencing scheme was used to model tearing sheets of cloth-like material. Work by [Norton et al., 1991] used a mass/spring system to model a breaking teapot. Fracture in the context of explosions was explored by [Mazarak et al., 1999], [Neff and Fiume, 1999], and [Yngve et al., 2000]. Most recently [Smith et al., 2001] used constraint-based methods for modeling brittle fracture.

Outside the graphics literature, both brittle and ductile fracture have been investigated extensively. A comprehensive review of this work can be found in [Anderson, 1995].

3 Ductile *vs.* Brittle Fracture

The common usage of the terms *elastic* and *brittle* differs substantially from their technical meanings. For example, *elastic* is often used incorrectly as a synonym for *flexible*, and the term *brittle* as a synonym for *fragile* [Merriam-Webster, 1998]. The technically correct definition of an elastic material refers to a material that returns to its original configuration when deforming forces have been removed. The ratio between the magnitude of a force and the amount of deformation it induces, that is how easily the material deforms, is the *compliance* of the material and it is irrelevant to whether or not the material is elastic. Although no real material is perfectly elastic, both natural rubber and common glass are examples of nearly elastic materials. Rubber's elastic behavior is obvious while glass appears to be rigid. A brittle material is simply one that behaves elastically up until the point where it fractures.

In contrast to an elastic material, a plastic material will not return to its original configuration once deforming forces have been removed. When a material, such as lead, bends and then holds its new shape, it demonstrates plastic behavior. As previously stated, real materials do not behave perfectly elastically. Real materials can be deformed only to a limited extent before they will no longer return to their original configuration. This limit is known as the material's elastic limit or yield point. When the elastic limit has been exceeded, the material enters a plastic regime and begins to experience plastic flow. Eventually, at the failure threshold, it fractures.

The terms *brittle* and *ductile* relate to the relative values of the elastic limit and failure threshold. If the failure threshold nearly coincides with the elastic limit, then the material will experience only negligible plastic deformation before fracture. The term *brittle* refers to such a material. In contrast, for a *ductile* material the failure threshold is significantly larger than the elastic limit so that as the material deforms it experiences an elastic regime, followed a plastic regime, and then finally fracture.

The significance of the distinction between ductile and brittle materials arises because elastic deformation stores energy whereas plastic deformation dissipates it. When a brittle material is deformed to its failure threshold, the majority of the energy used to deform it has been stored as elastic potential. When fracture occurs, the energy is released and it tends to drive the fracture further into the material. Thus, even though a large or small force may be required to start a crack in a brittle material (depending on its toughness), once the crack is started only a small amount of energy is required to push it further. In contrast, a ductile material requires significantly more work to propagate a crack because energy is being absorbed by plastic deformation. As a result, brittle materials tend to shatter, whereas ductile ones tend to tear.

In general the underlying causes of plasticity are fairly complicated and they give rise to a number of phenomena. For example, the energy absorbed by plastic deformation does not simply vanish and it may result in effects such as fatigue weakening. However for the purposes of animating failure events that occur over relatively short periods of time, the most significant effect of plasticity is how it directly effects fracture propagation, and the methods discussed here focus on modeling those effects efficiently. Additional information about mathematical models of deformation and plasticity can be found in [Fung, 1965; Fung, 1969] and [Han and Reddy, 1999]. Additional information concerning both brittle and ductile fracture may be found in [Anderson, 1995].

4 Modeling Ductility

The dynamic fracture propagation technique described in [O'Brien and Hodgins, 1999] models the fracture process using a simple tetrahedral finite-element method, rules for fracture initiation and propagation, and procedures for automatic remeshing as a crack advances. The quality of the results produced with that method is sufficient for graphical applications, and the only limitation that makes it unsuitable for modeling fracture in ductile materials is that the continuum model does not account for plastic deformation.

Extending that model to account for plasticity may be accomplished by simply redefining the strain metric used to compute element stresses. This change has only a local impact on the fracture algorithm, and so we will not repeat the details of the method which appear in [O'Brien and Hodgins, 1999]. Instead we describe only the modifications that should be made to the algorithm:

- The elastic strain, $\boldsymbol{\epsilon}^e$ defined in section 4.1 of this paper, takes the place of the total strain, $\boldsymbol{\epsilon}$, when computing the elastic stress.
- A routine for updating the plastic strain, described in section 4.2 of this paper, must be called during every integration step.

Even though this extension requires only incremental modifications to the previous method, it significantly extends the range of materials that may be realistically modeled. Furthermore, as our examples will demonstrate, small amounts of plastic yielding can dramatically effect the overall appearance of fracture patterns in a material, even though the plastic deformation itself cannot be observed directly. We feel that the significant relationship between plasticity and the appearance of fracture in most materials makes modeling plasticity a required component of any general system for animating fracture.

4.1 Decomposing Strain

The first step towards modeling plastic deformation requires separating the strain into two components:

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^p + \boldsymbol{\epsilon}^e \tag{1}$$

where $\boldsymbol{\epsilon}$ is the total strain, $\boldsymbol{\epsilon}^p$ is the strain due to plastic deformation, and $\boldsymbol{\epsilon}^e$ is the strain due to elastic deformation. The total strain is a purely geometric measure, it indicates how much the local shape of an object has changed from some initial reference configuration and it may be computed from the material's current configuration. (See [O'Brien and Hodgins, 1999] for computation of Green's strain.) The plastic strain reflects how the material's rest shape has been permanently distorted and it is part of the material's state. Initially, the plastic strain is zero[1] and it will evolve according to an update rule as the simulation progresses. Because the total and plastic strains are known at any given time, equation (1) may be used to compute the elastic strain.

4.2 Plastic Update

The algorithm for modeling the evolution of the plastic strain consists of a yield condition that must be met before plastic deformation occurs and a rule for computing plastic flow once the yield criterion has been met. We employ von Mises's yield criterion for the condition under which plastic flow will begin [Fung, 1965]. Our method for updating the plastic strain assumes that the rate of plastic flow in the material is close enough to its rate of deformation so that plastic flow can be updated instantaneously. This assumption precludes modeling phenomena such as creep and relaxation, but

[1] A non-zero initial value for the plastic strain could be used to model an object that has already experienced plastic deformation.

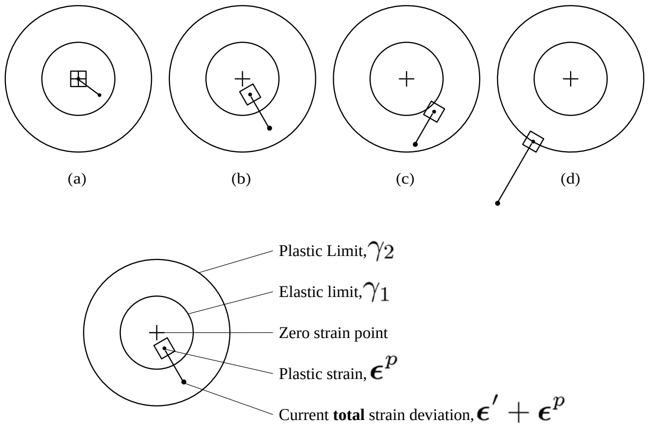

Figure 2: These diagrams illustrate the behavior of the plasticity model. **(a)** Elastic deformation. **(b)** and **(c)** Plastic deformation. **(d)** Limit of plastic yield. (See explanation in the text.)

under most circumstances these phenomena do not significantly effect fracture behavior. We also ignore the weakening of a material due to repeated plastic deformation known as fatigue. While fatigue often plays a significant role in the failure of mechanisms and structures, a previously fatigued object may be modeled by locally adjusting its toughness and plastic limits.

The von Mises yield criterion is based on the deviation of the elastic strain given by

$$\boldsymbol{\epsilon}' = \boldsymbol{\epsilon}^e - \frac{\mathrm{Tr}\left(\boldsymbol{\epsilon}^e\right)}{3}\boldsymbol{I} \tag{2}$$

where $\mathrm{Tr}\left(\cdot\right)$ is the trace of a matrix and $\boldsymbol{I}$ is the identity matrix. By averaging out the sum of the diagonal terms, the elastic strain deviation reflects only the portion of the elastic strain that is due to shape distortion and it excludes dilation. Excluding dilation makes the plastic deformation insensitive to hydrostatic pressure and will prevent the material from changing its volume which would generate unnatural behavior.

The yield criterion compares the magnitude of the elastic strain deviation (Frobenius norm) to a material constant, γ_1:

$$\gamma_1 < ||\boldsymbol{\epsilon}'|| \ . \tag{3}$$

Together equations (2) and (3) define the von Mises yield criterion [Fung, 1965]. If this condition is met then plastic deformation will occur. We compute the base change in plastic deformation according to:

$$\Delta\boldsymbol{\epsilon}^p = \frac{||\boldsymbol{\epsilon}'|| - \gamma_1}{||\boldsymbol{\epsilon}'||}\boldsymbol{\epsilon}' \ . \tag{4}$$

A limit on the total amount of plastic deformation that can be withstood by the material, γ_2, is enforced by updating the plastic strain at every time-step according to:

$$\boldsymbol{\epsilon}^p := \left(\boldsymbol{\epsilon}^p + \Delta\boldsymbol{\epsilon}^p\right)\min\left(1, \frac{\gamma_2}{||\boldsymbol{\epsilon}^p + \Delta\boldsymbol{\epsilon}^p||}\right) \ . \tag{5}$$

The behavior of this plasticity model is illustrated by figure 2. The image plane represents a two-dimensional projection of the five-dimensional space of strain deviations.[2] The plastic strain behaves as if it were being dragged by the total strain using a rope of length γ_1. The difference between the plastic strain's and the total

[2] For three-dimensional objects, strain is a 3×3 symmetric tensor with nine components. Because of symmetry, only six of these components are independent. Equation (2) removes one degree of freedom, leaving five.

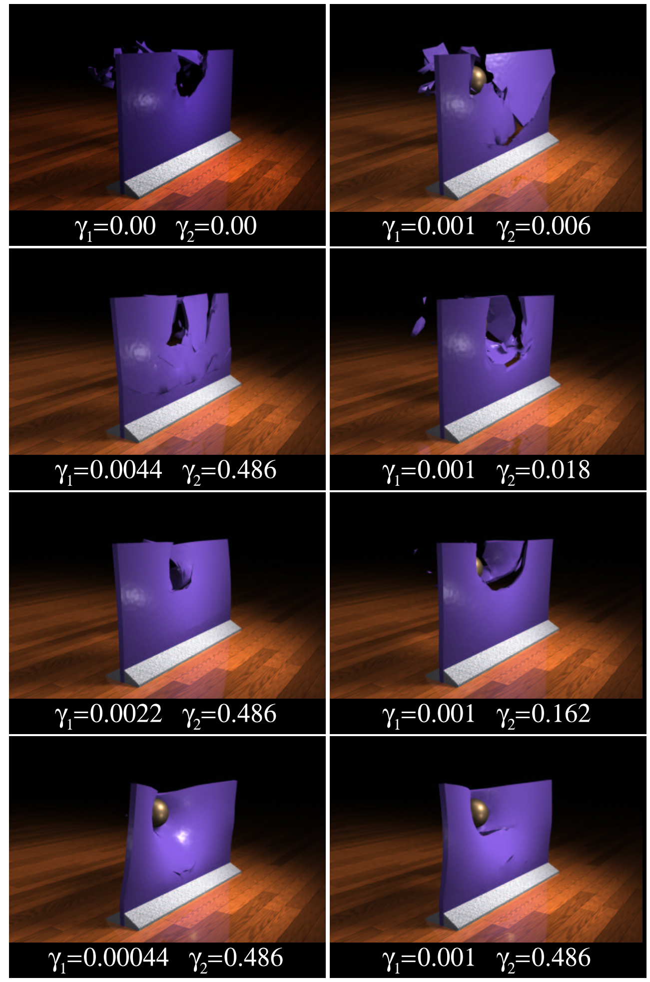

Figure 3: Images showing the results of simulating a set of eight thin walls with different material parameters as they are each struck by a heavy projectile. A purely brittle material is shown in the top-left. The others images demonstrate how varying the plasticity of the material can produce a range of effects.

strain's locations represents the current elastic strain. A barrier at radius γ_2 restricts the motion of the plastic strain, but not the total strain. An elastic force (stress) attracts the total strain to the plastic strain, but not the plastic strain to the total strain. As shown in figure 2.c, the plastic deformation will depend on the history of the total strain's movement.

5 Results and Discussion

Figure 3 shows a set of thin walls that have been struck by a heavy weight. The walls are clamped at the bottom, and they experience collision forces due to contacts with the ground plane, the weight, and self-collisions. The top-left image in figure 3 with ($\gamma_1 = \gamma_2 = 0$) shows the behavior of a purely brittle material. The other images in figure 3 show some examples that demonstrate the effects of different plastic parameter values. In the left column γ_1 has been varied while γ_2 was held fixed. The right column demonstrates the result of varying γ_2 while γ_1 was held fixed. Some of the images, such as the bottom-right with ($\gamma_1 = 0.001, \gamma_2 = 0.486$), demonstrate obvious amounts of plastic yielding. However, plasticity also plays a significant role in the images where plastic yielding is not obviously visible. For example, ($\gamma_1 = 0.001, \gamma_2 = 0.162$) shows only a small part of the wall being torn away largely intact, and ($\gamma_1 = 0.001, \gamma_2 = 0.006$) shows the wall breaking into several large pieces. Both of these behaviors demonstrate how the fracture

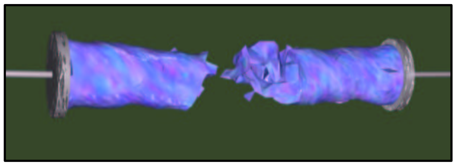

Figure 4: A solid cylinder that experiences ductile fracture when it is pulled apart.

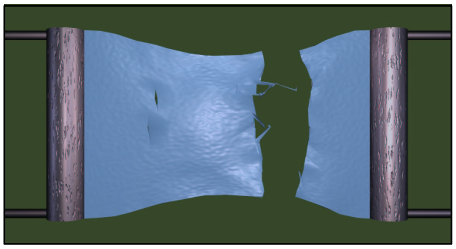

Figure 5: A thin sheet that has been torn apart.

process can be affected by otherwise unnoticeable amounts of plastic deformation. The proceedings DVD contains animations that further illustrate the behaviors depicted in figure 3 as well as the behaviors shown in the other figures.

Figure 4 shows a solid cylinder tearing as it is pulled and twisted apart. Figures 5 and 7 show the ductile fracture that results when other objects are ripped apart.

One way to assess the realism of an animation technique is by comparing it with the real world. Figure 6 shows a real clay slab that has been struck by a spherical projectile and a simulated slab of plastic material that has also be struck by a spherical projectile. Although the two images have obvious differences, the holes left by the projectiles demonstrate qualitative similarities.

While modifying the computation of the element stresses to use the elastic strain instead of the total strain requires only minor changes to an existing code, the change may also have an effect on the integration scheme. Our implementation uses an explicit integrator that takes adaptive time steps. The step size is determined by monitoring the total energy to ensure that the system is not going unstable. We compared the size of steps taken when simulating a purely elastic material to those taken when simulating a material that was identical except that the plasticity code had been enabled. During periods when collisions were occurring, both simulations took similar-sized integration steps. At other times, however, the average step size for the plastic material was approximately twice that of the purely elastic one. This result is not surprising because plastic deformation absorbs energy implying that it should tend to help stabilize the system, but it is only a single test on a single set of parameters and further tests would need to be done before any more general statement could be made.

The deformation model we implemented allows a regime of elastic deformation, followed by a plastic regime, and then possibly followed by a second elastic regime. While this model suffices for many materials, other materials, such as woven fabrics, may go through multiple cycles of elastic and plastic behavior. We have also worked only with a linear relationship between elastic strain and stress. While a linear model adequately describes many materials, other materials such as biological tissues demonstrate distinctly non-linear elastic behavior. Developing adequate graphical models for these types of materials remains an area for future work.

Acknowledgments

The authors thank Cindy Grimm for the use of the model shown in figure 7, and the members of the Berkeley Graphics Group for their helpful criticism and comments. This work was supported in part by Pixar Animation Studios, Intel Corporation, Sony Computer Entertainment America, a research grant from the Okawa Foundation, and NSF grant CCR-0204377.

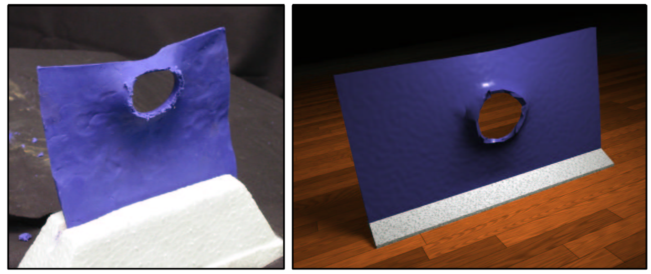

Figure 6: A comparison showing (left) a real clay slab that has been punctured by a spherical projectile and (right) a similar result generated with our method.

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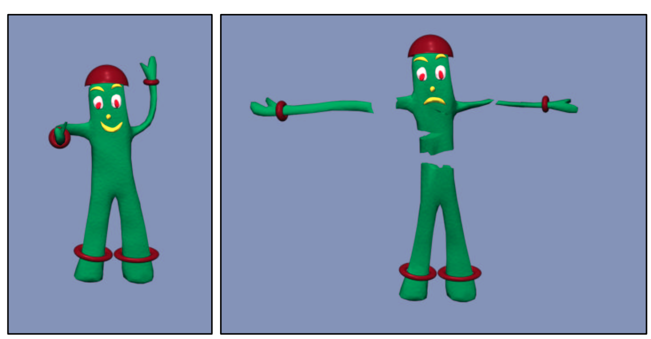

Figure 7: A cartoon character being dismembered by a red torture device.