Computational Geometry Column 5

Joseph O'Rourke
Department of Computer Science
Smith College, Northampton, MA 01063
jorourke@smith.bitnet

1 1988 Symposium

The Fourth ACM Symposium on Computational Geometry [CG88] was held June 6-8, 1988 on the University of Illinois campus in Urbana, Champaign. Herbert Edelsbrunner, the conference chair, estimates attendance at 168, with 37% of the attendees students (and 63% non-students), and about 22% of the attendees from industry (with 78% affiliated with universities). The conference chair Bernard Chazelle reports that 106 papers were submitted, and 39 (37%) accepted. Although the number of attendees and paper submissions have remained approximately constant over the four-year history of this symposium (175 attendees and 100 papers), there was a widespread feeling that this was the first year a large number of excellent papers could not be accommodated. This situation may be relieved next summer when there are two conferences: the Fifth ACM symposium, to be held at the Universtät des Saarlandes in Saarbrücken, Germany, June 5-7, 1989 (paper deadline 15 Dec 1988), and the First Canadian Computational Geometry Conference, to be held at McGill University in Montreal, Canada, August 21-25, 1989 (paper deadline 1 Dec 1988).

2 Trends

Every field has its hot topics, and the symposium evinced several trends.

Random sampling is rampant, resulting in "Las Vegas" algorithms for triangulating a simple polygon of n vertices in $O(n \log^* n)$ time [CTV88], finding the diameter of n points in three dimensions in $O(n \log n)$ time [CS88], and constructing the convex hull in three dimensions in $O(n \log h)$ time [Cla88], where h is the number of vertices of the output hull. These times are expected time complexities, but they do not depend on assumptions about the input distribution: rather the "expectation is over the random behavior of the algorithm." One exciting aspect of these algorithms is that many are simpler to implement than the fastest deterministic algorithms.

A half dozen papers struggled with *robust computation*, an issue of increasing importance as algorithms migrate from theory towards implementation. For example, Edelsbrunner and Mücke offered a technique for breaking geometric degeneracies (the bane of geometry programming) by a symbolic perturbation of the input data [EM88].

Combinatorics in general, and *Davenport-Schinzel* theory in particular, continue to play a central role in the complexity analysis of many geometric algorithms. A quarter of all papers presented at the conference used $\alpha(n)$, the near-constant inverse of Ackerman's function that arises in the analysis of Davenport-Schinzel sequences. See problem 4 below for an extension of these sequences to two dimensions.

Two output-size sensitive *hidden surface removal* algorithms were reported: Bern's $O(n \log n \log\log n + k \log n)$ algorithm for overhead orthogonal projection of rectangles oriented parallel to the axes [Ber88], and Reif and Sen's $O((n+k) \log n \log\log n)$ algorithm for polyhedral "terrains" [RS88]. In both algorithms, n is the input size, and k the output size, the number of edges in the final image.

The *motion planning* frontier seems to have shifted from the pure problems of the past to more realistic situations: movable obstacles [Wil88] [Nat88], coordination of several motions [SS88], planning under uncertainty [Don88], and implementation issues [KS88].

3 Open Problems

In the belief that open problems attract newcomers and spur oldtimers, I offer here are a collection of problems culled from the proceedings (or from papers referenced therein).

1. [*Empty convex sets in 3d*]. Dobkin, Edelsbrunner, and Overmars presented an algorithm [DEO88] that finds the largest empty convex polygon whose vertices are drawn from a set S of n points in the plane, in time $O(t(S))$, where $t(S)$ is the number of empty triangles determined by S. A polygon is empty if it contains no points of S in its interior. The size of a convex polygon is measured by the number of its vertices. Clearly $t(S)$ is $O(n^3)$, and its expected value is $O(n^2)$

for points chosen uniformly from a unit square. The problem is surprisingly more difficult in higher dimensions, and is not even known to be polynomial. To be specific, the following is open: "Is there a polynomial-time algorithm for finding the largest empty convex set of n points in three dimensions?" Here a "convex set" is a polytope, and its size is again measured by the number of its vertices.

2. [*Shortest guard tour*]. Chin and Ntafos define a *shortest guard route* [1] in a simple polygon P as a shortest path from a fixed point s on the boundary that returns to s, such that every point x in the interior of the polygon is visible from some point y along the path, that is, xy is nowhere exterior to P. They have developed an $O(n^4 \log\log n)$ algorithm to find such a route [CN87] for simple polygons of n vertices, and proved that the problem is NP-hard for polygons with holes. But the problem remains open if no starting point s is given, and the goal is the shortest route overall, a *shortest guard tour.* Here only the case of simple orthogonal polygons has been solved, with an $O(n \log\log n)$ algorithm [CN86]. For simple non-orthogonal polygons, no polynomial-time algorithm for finding the shortest guard tour is known.

3. [*Moving a directed stick*]. Fortune and Wilfong [FW88] define a *directed stick* to be a line segment in the plane directed from tail endpoint B to head endpoint A, which can move either in the direction it is pointing, $A - B$, or rotate about its head A. Despite the superficial similarity between this and the problem of moving a "ladder" (an undirected stick), a problem that can be solved in $O(n^2 \log n)$ time [LS87], it is an open problem to plan collision-free motion of a directed stick in an environment cluttered with polygonal obstacles: not even a decision procedure is known.

4. [*One cell of an arrangement of triangles*]. Aranov and Sharir conjecture [AS88] that the worst-case combinatorial complexity of a single non-convex cell in an arrangement of triangles in three dimensions, is $O(n^2\alpha(n))$. Examples are known with cells of complexity $\Omega(n^2\alpha(n))$, but the best general result is that the maximum complexity of *all* non-convex cells is $O(n^{7/3+\delta})$ for any $\delta > 0$. Their conjecture is motivated by motion planning problems, which often may be reduced to the problem of constructing just one cell of the configuration "free space," and

[1] They call it a "shortest watchman route."

the maximum complexity of one cell is clearly a lower bound on any algorithm that constructs it.

5. [*Pairwise-visible faces*]. Dobkin posed a problem related to ray tracing: enumerate all pairs of mutually visible faces in a collection of bounded polyhedra in three dimensions. Two faces are visible to one another if there is a line segment with endpoints on each that does not intersect the interior of any polyhedron. For an environment of a total of n faces, McKenna and I offered an $O(n^4\alpha(n))$ time and $O(n^2)$ space algorithm [MO88], but because the output is of size $O(n^2)$, we suspect a faster algorithm is possible.

6. [*Separating convex polytopes*]. Dawson showed several years ago [Daw84] the counterintuitive result that there are a collection of (12) convex polytopes in three dimensions such that no one polytope can be moved (translated along a fixed direction to infinity) without disturbing (intersecting) the others. Perhaps this is counterintuitive because in any collection of spheres, one *can* always be moved without disturbing the others. Natarajan conjectures [Nat88] that any finite collection of polytopes S can always be partitioned into two (non-empty) sets of polytopes S_1 and S_2 such that S_1 can be moved as a unit along some direction without disturbing S_2, that is, without any member of S_1 intersecting a member of S_2. This would imply, in Natarajan's nomenclature, that a composite of convex polytopes may be assembled with two hands.

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