SUMMARY

The Bending of Rectangular Plates with Opposite Edges Simply Supported

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There are many engineering design problems in which plates under load are encountered. Among these are reinforced and prestressed concrete slabs under lateral loadings and steel plates used for ship hulls which are exposed to water-pressure loadings. Recently a plate simply supported on two edges was analyzed at Battelle.

After a literature survey, it became apparent that the derivation of the deflection equation to meet these conditions would have to be made from the basic equations. These derivations are presented in this paper because it is felt that they may be useful to others. One possible application of this paper might be in refining the computational method now used in analyzing such structures as a one-span prestressed concrete-slab bridge simply supported.

In discussing the bending of plates, we are limited to problems in which one dimension, the thickness, of the plate can be considered small relative to the other dimensions. The differential equation representing the deflection surface of a plate due to a load acting normal to its surface is:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \qquad (1)$$

where w = Deflection of plate in z coordinate.
q = Intensity of a continuously distributed load.
D = Flexural rigidity of a plate.

Equation (1) is based on the assumption that the sheering forces along the x and y coordinates, and the compressive stress along the z coordinate produced by a load q, have little direct effect on the deflections of the plate. This assumption is valid, provided the thickness of the plate is small relative to the dimensions in its plane.

The plates considered in this paper are homogeneous thin plates simply supported at two opposite edges, the other two opposite edges being free. The plates are loaded uniformly over a rectangular area which is smaller than the area of the plate itself. Two cases are considered as follows:

(I) The center of the load lies on a line through the center of the plate, the line being perpendicular to the simply supported edges.

(II) The center of the load lies on a line through the center of the plate, the line being parallel to the simply supported edges.

For Case I, the loadings are represented by a Fourier series of the form:

$$q\ (x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \qquad (2)$$

where:

$$A_{mn} = \frac{4}{ab} \int_{0}^{a} \int_{-\frac{b}{2}}^{\frac{b}{2}} q\ (x,y) \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b}\, dxdy \qquad (3)$$

where a and b are the dimensions of the plate and the loading q (x,y) is represented analytically as a function of the loading area.

The deflection curves are developed from Equation (2) which yields the following:

$$w = \sum_{m=1}^{\infty} \left\{ \left[A_m \cosh \left(\frac{m\pi y}{a} \right) + B_m \left(\frac{m\pi y}{a} \right) \text{Sinh} \left(\frac{m\pi y}{a} \right) \right] + \xi_m \sum_{n=1}^{\infty} \frac{\text{Sin} \left(\frac{n\pi B}{2b} \right) \cos \left(\frac{n\pi y}{b} \right)}{n \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right]^2} \right\} \sin \left(\frac{m\pi x}{a} \right) \qquad (4)$$

where

$$\xi_m = \left[\frac{16\, q_o}{Dm\pi^2} \sin \left(\frac{m\pi\, x_o}{a} \right) \sin \left(\frac{m\pi A}{2a} \right) \right] \qquad (5)$$

From the boundary conditions, the following pair of simultaneous equations is obtained:

$$\left\{ A_m \cosh \alpha_m + B_m \left[\alpha_m \sinh \alpha_m + \frac{2}{(1-\nu)} \cosh \alpha_m \right] \right\} = \frac{m}{(1-\nu)\left(\frac{m\pi}{a} \right)^2} \sum_{n=1}^{\infty} \frac{\left[\left(\frac{n\pi}{b} \right)^2 + \nu \left(\frac{m\pi}{a} \right)^2 \right]}{n \left[\left(\frac{m\pi}{b} \right)^2 + \left(\frac{m\pi}{a} \right)^2 \right]^2} \sin \left(\frac{n\pi B}{2b} \right) \cos \left(\frac{n\pi}{2} \right) \qquad (6)$$

$$\left\{ A_m \sinh \alpha_m + B_m \left[\alpha_m \cosh \alpha_m - \left(\frac{1+\nu}{1-\nu} \right) \sinh \alpha_m \right] \right\} = \frac{m}{(1-\nu)\left(\frac{m\pi}{a} \right)^3} \sum_{n=1}^{\infty} \frac{\left(\frac{n\pi}{b} \right) \left[\left(\frac{n\pi}{b} \right)^2 + (2-\nu) \left(\frac{m\pi}{a} \right)^2 \right]}{n \left[\left(\frac{n\pi}{b} \right)^2 + \left(\frac{m\pi}{a} \right)^2 \right]^2} \sin \left(\frac{n\pi B}{2b} \right) \sin \left(\frac{n\pi}{c\,2} \right) \qquad (7)$$

Equations (6) and (7) are solved for A_m and B_m, which, when substituted into Equation (4), yield the complete deflection surface for this type of loading.

A similar procedure is used in solving for the deflections in Case II. This case is somewhat more complex because the bending of the plate does not possess the symmetry which existed in the first case.

For Case I, Equation (2) was evaluated for each position of x_o desired. This equation consists of a series approximation to be evaluated over m which in itself contains a series to be evaluated over n for each m. The constants of integration, A_m and B_m, are defined by Equations (6) and (7), each of which contains series approximations which must be evaluated for each m. It is fortunate that these series converge rapidly, although not all with the same rapidity.

An IBM Type 650 computer with automatic floating-point device, index registers, high-speed memory, and four tape units was used for solving this problem. The computations were performed in floating-point mode, and truncation for each series was based on the change in exponent in each succeeding term. Six different loading positions were used for each case. The results are given as two families of curves, one for each case.