ALGORITHM 669
BRKF45: A FORTRAN Subroutine for Solving First-Order Systems of Nonstiff Initial Value Problems for Ordinary Differential Equations

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1. INTRODUCTION

This package complements [1], where the mathematical and algorithmic details are given. Also given in [1] are some results using the well-known DETEST test set [2]. Complete results for the whole test set may be obtained from the author. As an additional aid to implementation, we give a simple driver program and the results obtained in single precision on a CDC6600. The problem solved is

$$\frac{dy}{dx} = -y, \qquad y(0) = 1, \qquad 0 \le x \le 20,$$

where an absolute accuracy of 10^{-6} is sought and output is required at $x = 1, 2, 3, \ldots, 20$. The results from the test program are given in Table I.

2. ALGORITHM

Both single-precision and double-precision versions of the algorithm are available from the author.

```
*** Double precision, test driver ***
     y'=-y
```

T	Y	DY/DT	ERROR IN Y	ERROR IN DY/DT
1.0000	3.6787947981E-01	-3.6787984571E-01	3.86E-08	-4.05E-07
2.0000	1.3533532348E-01	-1.3533561814E-01	4.02E-08	-3.35E-07
3.0000	4.9787165559E-02	-4.9787173578E-02	9.72E-08	-1.05E-07
4.0000	1.8315660778E-02	-1.8315684970E-02	2.19E-08	-4.61E-08
5.0000	6.7380022889E-03	-6.7375876186E-03	5.53E-08	3.59E-07
6.0000	2.4788389141E-03	-2.4783533978E-03	8.67E-08	3.99E-07
7.0000	9.1194030270E-04	-9.1157009341E-04	5.83E-08	3.12E-07
8.0000	3.3547823885E-04	-3.3545447781E-04	1.56E-08	8.15E-09
9.0000	1.2348192176E-04	-1.2360598622E-04	7.21E-08	-1.96E-07
10.0000	4.5485467397E-05	-4.5130876357E-05	8.55E-08	2.69E-07
11.0000	1.6739385659E-05	-1.6829136520E-05	3.77E-08	-1.27E-07
12.0000	6.2076827736E-06	-5.9479481358E-06	6.35E-08	1.96E-07
13.0000	2.3325490270E-06	-2.3745408674E-06	7.22E-08	-1.14E-07
14.0000	8.3022899836E-07	-8.4012289847E-07	-1.30E-09	-8.59E-09
15.0000	3.8313475831E-07	-1.7703786622E-07	7.72E-08	1.29E-07
16.0000	1.7897077748E-07	-1.9281591594E-07	6.64E-08	-8.03E-08
17.0000	4.8119912719E-08	-8.0400553531E-08	6.72E-09	-3.90E-08
18.0000	1.1303315288E-08	6.4705401405E-09	-3.93E-09	2.17E-08
19.0000	5.1904463978E-08	6.0926843403E-08	4.63E-08	6.65E-08
20.0000	7.2258265741E-08	-7.2258265741E-08	7.02E-08	-7.02E-08

ACKNOWLEDGMENT

REFERENCES

1. CASH, J. R.   A block 6(4) Runge-Kutta formula for nonstiff initial value problems. *ACM Trans. Math. Softw. 15*, 1 (Mar. 1989), 15-28.
2. ENRIGHT, W. H., AND PRYCE, J. D.   Two FORTRAN packages for assessing initial value methods. *ACM Trans. Math. Softw. 13*, 1 (Mar. 1987), 1-27.