



# Computer-Aided Design of Graphical Review Lessons for Audio-Tutorial College Algebra and Trigonometry\*

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## Abstract

An audio-tutorial minicourse system of instruction is being developed for the college algebra and trigonometry courses offered at Indiana University-Purdue University at Fort Wayne. This system of instruction is patterned after the one developed by S. Postlethwait. At present a college algebra course, MA 153, is being taught using audio-tutorial minicourses. In the spring semester two trigonometry courses, MA 112 and MA 154, are planned to be taught using this system of instruction. Also, a fourth course, MA 111, will be taught using this system in the spring, 1980, semester.

Much mathematics is more easily understood by using graphs. In many minicourses students are required to draw graphs. It is difficult to convey how one graph is "shifted" to obtain another graph using printed materials. As a result, a computer program was developed to create audio-visual review lessons of graphing concepts covered by the audio-tutorial minicourses. A review lesson may contain not only simple concepts such as point plotting, but also dynamic displays of graphs and their transformations. Creation of a review lesson is divided into four steps: (1) coordinate system definition, (2) function definition, (3) example definition, and (4) example viewing. The user is guided through each step via a series of menus. A sophisticated ten minute review lesson may take only 15 to 30 minutes to create. The program is written in TUTOR and runs on the University of Illinois PLATO system.

## INTRODUCTION

This is a preliminary report on the on-going college algebra and trigonometry audio-tutorial minicourse project at Indiana University-Purdue University at Fort Wayne[1,2]. In this paper we present a brief summary of the scope of the minicourse project and discuss the development of a PLATO-based graphics system for the design of audio-visual review lessons.

## THE AUDIO-TUTORIAL MINICOURSE SYSTEM IN MATHEMATICS

The Department of Mathematical Sciences at Indiana University-Purdue University at Fort Wayne currently offers six college algebra and trigonometry courses. Over the last four years class sizes have tripled in some of the courses. Due to the limited resources of the University, this resulted in increased class sizes in all classes. Because these are service courses they are populated by students of extremely diverse backgrounds, motivations, and aptitudes. While these students need individual attention, increased class sizes have resulted in the students receiving less and less individual instruction. We felt that a change from the traditional lecture-recitation system of instruction

was needed. We sought an individualized system; however, we also desired to include deadlines for the completion of work and personal contact between the instructor and the students.

We decided to adapt the audio-tutorial minicourse system of instruction to the college algebra and trigonometry courses. This system of instruction was developed by Samuel N. Postlethwait to teach an elementary botany course at Purdue University[3].

The first course to be adapted to the audio-tutorial system of instruction was MA 153. It was divided into 31 units of material called minicourses. Each minicourse consists of a study guide and an audio tape. These materials were developed by a Minicourse Project Team in the Summer of 1977[2].

Mathematics lends itself well to the audio-tutorial system of teaching. The learning of mathematics requires the understanding of a sequence of facts which lead to an important mathematical principle. The minicourses present these facts by means of a variety of learning activities. Students can accurately measure their progress since they are provided with self-testing

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exercises in the study guide. Because of the nature of mathematics these exercises have either right or wrong answers, and the students immediately know how well they understand the material.

Up to the present time all of the mathematical ideas have been presented using the printed materials in the study guide. For learning many algebraic skills this is sufficient. However, much of mathematics is more easily understood by using graphs. In many minicourses students are required to draw graphs. We desired to reinforce and summarize the graphing concepts such as plotting points, relating an equation to its graph, and locating intercepts, asymptotes, foci, vertices, symmetries, and other geometrical properties of the graphs. Also, it is often difficult to convey how one graph is shifted, compressed, or stretched to obtain another graph using printed materials. As a result, a computer program was developed to create audio-visual review lessons of graphing concepts covered by the minicourse.

#### THE MINICOURSE GRAPHING AIDS SYSTEM MiniGAS

The main objective of the Minicourse Graphing Aids System is to supply mathematics instructors with a tool to create audio-visual review lessons of graphing concepts presented in the minicourses. To be a useful tool we felt that MiniGAS should be human-engineered.

Many of the minicourse instructors are not familiar with computer operations, therefore a simple interface between the system and the user is a must. Two other features were considered desirable: a procedure for one-take videotaping/filming of the review lesson to minimize the time-consuming task of editing; and the flexibility to accommodate review lessons for minicourses planned in calculus and analytic geometry.

The PLATO system was an obvious choice on which to implement MiniGAS. PLATO supplies a wealth of special function keys (such as -NEXT- and -HELP-) which not only facilitate user response but also simplify programming. In addition, the TUTOR language possesses powerful graphics features. Like many PLATO programs, the MiniGAS interface consists of a series of menus describing possible actions the user may take. The user is always prompted for a response, and this response usually takes the form of pressing a letter, a digit or a function key.

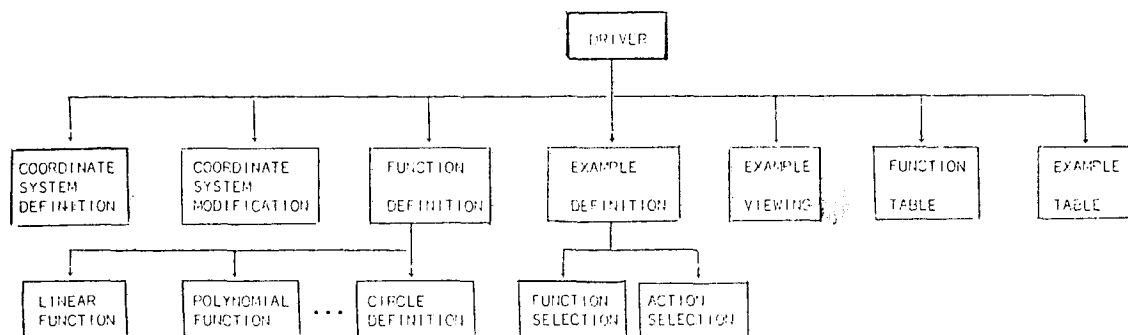
The overall structure, as characterized by the menus, is diagrammed below.

The specific features of the MiniGAS system are described in the remainder of the paper.

A MiniGAS review lesson consists of a series of one or more examples each of which contains at least one function. All functions are plotted on a predefined coordinate system. The user interactively creates each example and function used in the review, and may redefine the coordinate system or alter the review lesson at any time. Four basic steps are required to create a review lesson: coordinate system definition, function definition, example definition and example viewing/taping. In general, subject to a few obvious restrictions, these steps may be taken in any order and may be repeated as often as necessary. A detailed explanation of each step is given below.

Coordinate System Definition. To draw a graph, an appropriate coordinate system must be defined. MiniGAS is designed to plot functions in either the Cartesian coordinate system or the polar coordinate system; however, only the former system has been implemented. For viewing purposes, the Cartesian coordinate system is defined by five parameters: maximum x-coordinate, maximum y-coordinate, x-axis label, y-axis label, and a grid flag. If the grid flag is "on" then a coordinate grid is displayed (a nice feature to use when students are plotting on graph paper); otherwise only the axes are drawn. MiniGAS supplies default values to all five parameters which may be selectively changed by the user at any time.

Function Definition. All examples involve plotting graphs of functions or conic sections. MiniGAS presently supports four function types (linear, polynomial, exponential and logarithmic) and all the conic sections (circle, parabola, hyperbola and ellipse). Two additional function types, trigonometric and rational, are planned for implementation in the near future. For a single review lesson a maximum of 20 functions may be defined. Function definition is quite simple; the user first selects the type of function desired, then either supplies an appropriate equation for the function or supplies parameters which uniquely define the function. For example, a line may be defined by supplying the coefficients of the general linear equation  $Ax + By + C = 0$  or by specifying the slope and y-intercept.



MiniGAS assigns a unique function identifier to each function defined. The function identifier is an integer and is assigned in the same order as functions are defined; i.e., the third function defined receives the function identifier "3". These identifiers are used while creating an example to designate which functions are used in that particular example. The user need not memorize the order in which the functions are defined since MiniGAS maintains a function definition table accessible to the user at all times. The table contains the function identifier, type and equation for all functions previously defined.

Example Definition. The basic unit of a graphing review lesson is the example. An example contains one or more functions along with a sequence of optional actions for each function. Example definition consists of two parts: selection of all functions contained in the example, and

selection of the action sequence for each function. Up to five actions may be specified for each function. These include displaying an xy-coordinate table, plotting individual points from this table, selecting pertinent display information (such as the equation of the function), plotting the function, and plotting transformations of the function. Any combination of actions may be chosen with one exception--an xy-coordinate table must be defined if the user desires to plot points from such a table.

Some additional information must be supplied for each action the user selects. For example, the user must designate the x-coordinate for each point in the xy-coordinate table. For all actions, the user is prompted for the necessary information. A table of additional information or options is given below for each action.

ACTION	ADDITIONAL INFORMATION REQUIRED
xy-coordinate table	x-coordinate of each point
plot points from table	plotting character select option pertaining to display of coordinates as each point is plotted
plot function	select line type
display function information	select desired information for function such as equation, asymptotes, intercepts, etc.
plot transformations	enter function identifier representing function transformation for each transformation desired.

Example Viewing. Example viewing is the final step in the creation of a review lesson. It serves two purposes; first, to allow the user to examine and properly sequence the examples contained in the review; and secondly, to permit one-take taping/filming of the lesson. Prior to the actual viewing, the user creates a viewing list containing all the examples he wishes to see. Each entry in this list is an example identifier. Example

identifiers are completely analogous to function identifiers. Also, an example table is maintained by MiniGAS. This table contains an entry for each example consisting of the identifier and a function list. Immediately after the example list is completed the actual display begins.

An outline of the example display process is given below.

```

start (DISPLAY-EXAMPLE)
erase screen
for each example in display list
  display axes
  for each function in example
    obtain next action
    case action of
      1. xy-coordinate table: draw xy-table
                           display function equation
                           display coordinates for each point in the table
      2. plot points: individually plot points from xy-coordinate table
      3. plot function: draw graph of function using line type
                       and intervals previously selected
      4. display function: display selected function information
      5. plot transformation: for each transformation
                           display transformation equation
                           plot transformation using broken line
                           pause
                           erase graph of transformation
    end case
  end for
end
erase screen
end
end (DISPLAY-EXAMPLE)

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Although MiniGAS has been successfully implemented and sample review lessons created, several technical difficulties have hampered the integration of these visual review lessons with the current audio-tutorial minicourses. Video-tape was chosen as the final display medium for two reasons. First, color enhancement and audio could be added with the equipment available on campus. Secondly, video tape cassettes are easily handled by the large number of students served by the minicourses and a number of video tape cassette players, each of which may be used by a single student or a group of students, are available on campus.

During the implementation of MiniGAS, several videotaping trials were performed with limited success. A major difficulty lies in the poor resolution and low light level of the PLATO screen. These problems may be solved by limiting the area of the screen being taped, either by zooming in on portions of the screen or by squeezing the entire display into a much smaller area. Both solutions have proved undesirable. Zooming makes one-take taping much more difficult since the camera operator must know the exact sequence and location of information in the display. Also, the camera must be repositioned continually to follow the action in the display. Jamming the entire display into a restricted area of the screen causes unpleasant distortions in the graphs. These distortions are highly undesirable since the main objective of a review lesson is to give the student a clear notion of the relationship between a function and its graph. In addition, the density of the information increases significantly to the point where one can't see the forest for the trees.

Two other alternatives are being explored. There has been much success in taking slides of the PLATO screen. In order to explore this alternative, a pause facility was added to MiniGAS. This facility can be used to indefinitely halt the display during the viewing phase. Using this alternative, 20 or 30 slides of the review lesson are taken. These slides are then videotaped. A videotape slide presentation offers one major disadvantage: the animation of the plotting process is lost. It's one thing to see an equation and its corresponding graph; it's another to see the points being plotted, followed by a curve passing through those points.

Another alternative which may prove more fruitful is filming the screen. Using 8mm film, several successful films have been made in the past. A problem does exist with graininess, but overall picture quality is acceptable. Employing this alternative regains the one-take objective, but adds one more step to lesson creation since the film must be videotaped and probably edited.

We feel certain that one of these alternatives can be used to overcome the technical problems previously described.

We envision future algebra and trigonometry minicourses to present a multimedia approach to graphing topics. Each minicourse will be divided into two parts: an audio-tutorial presentation of

of the topic as described in the first section of the paper; and an audio-visual review of the topic which would contain animated examples of exercises contained in the student study guide, and descriptive comments emphasizing the major objectives of the lesson. Each review lesson would be 5-10 minutes long.

#### SUMMARY

Indiana University-Purdue University at Fort Wayne has developed a series of minicourses for algebra and trigonometry students. Presentation of the material is based on Postlethwait's methods of individualized instruction. A need was seen for visual augmentation of those minicourses which covered graphing topics. As a result, a PLATO-based graphics system was developed to permit mathematics instructors to create audio-visual review lessons. These review lessons are integrated with the minicourses to offer an enhanced audio-tutorial system of instruction for mathematics.

#### BIBLIOGRAPHY

- [1] Svoboda, Rudy G., "The Adaption of the Postlethwait Audio-Tutorial Minicourse System to College Algebra," presented at the Fourth National Conference of the Center for Personalized Instruction, San Francisco, California, April 15-17, 1977.
- [2] Svoboda, Rudy G., "Audio-Tutorial Minicourse for College Algebra and Trigonometry: A Progress Report," presented at the Ninth Annual Conference of the International Congress for Individualized Instruction, West Lafayette, Indiana, November 6-8, 1977.
- [3] Postlethwait, S. N., Novak, J., and Murray, Jr., H. T., The Audio-Tutorial Approach to Learning, Third Edition, Burgess Publishing Co., 1972.