## COLOR SPACES FOR COMPUTER GRAPHICS

by
George H. Joblove and Donald Greenberg
Program of Computer Graphics
Cornell University

## ABSTRACT

Normal human color perception is a product of three independent sensory systems. By mirroring this mechanism, full-color display devices create colors as mixtures of three primaries. Any displayable color can be described by the corresponding values of these primaries. Frequently it is more convenient to define various other color spaces, or coordinate systems, for color representation or manipulation. Several such color spaces are presented which are suitable for applications involving user specification of color, along with the defining equations and illustrations. The use of special color spaces for particular kinds of color computations is discussed.

KEYWORDS: Computer Graphics, Color, Color Displays, Color Space, Image Synthesis
COMPUTING REVIEWS CLASSIFICATION: 8.2

## Introduction

The normal human visual system is comprised of three components which produce color sensation. Fullcolor display devices used in computer graphics take advantage of this phenomenon and produce their entire range of color simply by mixing three primary colors. This paper describes the concept of color space, wherein color is regarded as occupying a three-dimensional volume. One such space is inherently defined by the primary colors of a display system; others may be used provided that locations within them may ultimately be transformed to three proper input parameters for the display device.

Following a review of the principles of color, several color spaces will be presented which are useful for the purpose of communicating color specifications between a computer graphics system and a user. Other color mappings which are primarily suitable for internal use by the graphics system for color computations are discussed.

## Light, Color, and Perception

Visible light is electromagnetic radiation in the range of wavelengths to which the normal human visual system is sensitive, about 400 to 700 nanometers. The wavelength determines the perceived hue of the light; the amplitude is interpreted as intensity.

In general, light is not of a single wavelength but. a continuous mixture of all the wavelengths across some part or all of the visible spectrum. The perceived color is then a function of the spectral distribution of amplitude by wavelength. The shape of the distribution determines what is perceived as the chromaticity (color without regard to intensity) of the light. For a given chromaticity, the abso-
lute power level (or height of the distribution) determines what is perceived as the intensity. Two kinds of light-sensitive bodies in the retina, rods and cones, send signals to the brain. The rods function mostly in very dim light to which the cones are insensitive. At normal light levels, virtually all visual information is provided by the cones. Three types of these receptors, each sensitive to different wavelengths, together are responsible for color discrimination. The strengths of the resulting three separate signals are interpreted as a particular chromaticity. Although the sensitivities of the three receptors overlap, one is sensitive particularly in the blue area of the spectrum, one in the green, and one in the red. ${ }^{1}$

Different spectral distributions can produce the same color sensation, or receptor responses. This phenomenon is called metamerism; "different" colors which appear the same as a result are called metamers. ${ }^{2}$ If light has a flat, even spectral distribution, or any other distribution that stimulates all three receptors equally, it is seen as black, gray, or white, the achromatic colors. Any other distribution produces the sensation of a chromatic color, one which has a distinguishable hue.

Any three wavelengths of light can be mixed in varying proportions to create many different colors. Some sets of three wavelengths can produce more colors than others, but the particular characteristics of the three human receptor systems make it impossible for any such set of three primary colors to duplicate all colors. The three primaries which can be mixed to produce the greatest number of colors are particular wavelengths of red, green and blue. For this reason, most color display systems are based on three light sources which are as close
to these colors as possible.
Chromaticity is a function of the ratios between the primary colors. If their relative values are expressed as proportions of the sum of all three, chromaticity may then be plotted as a function of any two primary colors as shown in Figure 1. In this chromaticity diagram, the locations of the three primaries are the vertices of a color triangle which is the boundary of the area representing all colors which they can reproduce. The horse-shoe-shaped curve is the locus of spectral colors and therefore bounds all the visible colors. The colors on the straight line connecting the endpoints of the spectral locus are the pure nonspectral hues. The colors lying within the spectral locus but outside the color triangle are those which cannot be produced by the primaries because, in terms of the diagram, they would each require one primary component to assume a negative value. The primaries are "optimum" in that the area representing the nonreproducible colors is minimum.

U1timately, any color to be displayed by a red/ green/blue component system must be specified by those components. All colors which can be created can therefore be represented within a cubic volume in the all-positive octant of an orthogonal threespace whose axes are the rgb primaries (Figure 2). For many applications, it is convenient to regard a color as a point in this rgb color space. For many others, however, it may be preferable to specify colors as points in some other color space. If this alternative space has been defined parametrically in terms of $r, g$, and $b$ and a function exists to transform its coordinate system to an rgb coordinate system, the result of such a transformation can be fed to the display device.

Color Spaces for User Interaction
Because most people are not familiar with additive primary mixing, rgb space is not always suitable for applications when the user is composing colors. However, many people who are artistically inclined are quite proficient at seeing a color as a mixture of the "subtractive primaries" cyan, magenta, and yellow. A color space so defined is just the inverse of the rgb space, with white at the origin (Figure 3). The correspondingly simple transformation to rgb space is given by:

$$
\left[\begin{array}{ll}
\mathrm{r} & \mathrm{~g}
\end{array} \mathrm{~b}\right]=\left[\begin{array}{lll}
1 & 1 & 1
\end{array}\right]-\left[\begin{array}{lll}
\mathrm{c} & \mathrm{~m} & \mathrm{y} \tag{eq.1}
\end{array}\right]
$$

where $r, g, b$ are the red, green, and blue components, $c, m, y$ are the cyan, magenta, and yellow components, all scaled in the range $\left[\begin{array}{lll}0 & 0 & 0\end{array}\right]$ to $\left[\begin{array}{lll}1 & 1 & 1\end{array}\right]$.

Most viewers, though, first notice a color's hue and then characteristics which might be described as lightness, brightness, brilliance, strength, saturation, vividness, purity, etc., many of which are interrelated. For interactive purposes, then, a system of specifying colors based on such qualities would be most natural for a naive user.

Since hue is a circular or modular quality, while all the other characteristics mentioned imply the existence of minimum and maximum values, points in any color space related to these qualities may be specified by cylindrical coordinates. The scaling of the angular component (that is, the precise function relating hue to the angle) is somewhat arbitrary for these applications. The remaining two axes can be defined in a variety of ways.

One convenient arrangement of hue is an equidistant


Figure 1. Chromaticity diagram, with color triangle, spectral locus, and "line of purples,"
for monochromatic primaries of wavelengths 650 nanometers (red), 530 nanometers (green), and 460 nanometers (blue).


Figure 2. RGB color space, and color solid.


Figure 3. Cyan/magenta/yellow color space, and color solid.
spacing of the primaries and their complements in their natural order, with other hues located by linear interpolation as graphed in Figure 4a. The rgb-component ratios are computed from the hue angle as follows:

$$
\begin{align*}
& r=\left\{\begin{array}{l}
1 \text { if } 0 \leqslant h \leqslant 1 / 6 \text { or } 5 / 6 \leqslant \mathrm{~h}<1 \\
2-6 \mathrm{~h} \text { if } 1 / 6 \leqslant \mathrm{~h} \leqslant 2 / 6 \\
0 \text { if } 2 / 6 \leqslant \mathrm{~h} \leqslant 4 / 6 \\
6 \mathrm{~h}-4 \text { if } 4 / 6 \leqslant \mathrm{~h} \leqslant 5 / 6
\end{array}\right.  \tag{eq.2a}\\
& g=\left\{\begin{array}{l}
6 \mathrm{~h} \text { if } 0 \leqslant \mathrm{~h} \leqslant 1 / 6 \\
1 \text { if } 1 / 6 \leqslant \mathrm{~h} \leqslant 3 / 6 \\
4-6 \mathrm{~h} \text { if } 3 / 6 \leqslant \mathrm{~h} \leqslant 4 / 6 \\
0 \text { if } 4 / 6 \leqslant \mathrm{~h}<1
\end{array}\right.  \tag{eq.2b}\\
& \mathrm{b}=\left\{\begin{array}{l}
0 \text { if } 0 \leqslant \mathrm{~h} \leqslant 2 / 6 \\
6 \mathrm{~h}-2 \text { if } 2 / 6 \leqslant \mathrm{~h} \leqslant 3 / 6 \\
1 \text { if } 3 / 6 \leqslant \mathrm{~h} \leqslant 5 / 6 \\
6-6 h \text { if } 5 / 6 \leqslant \mathrm{~h}<1
\end{array}\right.
\end{align*}
$$

where $h$ is the hue angle in revolutions modulo 1 (with equivalent wavelength decreasing from red with positive revolution from zero).

Because this function is non-continuous, hue series will exhibit Mach banding (the illusion of overly light or dark areas) at the discontinuities due to the tendency of the human visual system to enhance such variations in luminance for any of the three receptor systems ${ }^{3}$ (see Figure $4 a$ ). These discontinuties can be eliminated by using a sinusoidal interpolation instead of the linear one, as illustrated by Figure 4b, in which

$$
\left[\begin{array}{lll}
r & g & b
\end{array}\right]=\left(\left[\begin{array}{lll}
1 & 1 & 1
\end{array}\right]+\cos \left(\left(\left[\begin{array}{lll}
1 & 1 & 1
\end{array}\right]-\left[r^{\prime} g^{\prime} b^{\prime}\right]\right) \pi\right)\right) / 2
$$

$$
(e q .3)
$$

where $r^{\prime}, g^{\prime}, b^{\prime}$ are computed using equations $2 a, 2 b$, and 2c.

Several methods exist for describing the chromatic content of a color, which is the attribute of a visual sensation according to which an area appears to exhibit more or less chromatic color, or "colorfulness."4 Two words which are normally used somewhat loosely have been precisely defined as follows": "Chroma" is colorfulness judged with reference to white or the general level of surrounding illumination; the "saturation" of a color is its colorfulness judged with respect to its lightness. Thus pure monochromatic red light of low intensity or lightness has very high saturation but low chroma.

The Munsell Color System (1905) ${ }^{6}$, one of the earliest arrangements and means of describing color and one which is in common use today, maps colors according to a quantity closely related to chroma (called "Munsell chroma"), in addition to hue, and lightness relative to a reference white ("Munsell value"). For user interaction with a computer graphics system, a convenient adaptation of this arrangement may be defined as follows: The grays are linearly interpolated on a straight line (the cylindrical axis) from black to white; the colors
which are of maximum chroma for their respective hues (for the given rgb display space) are located on a circle centered on the gray axis and perpendicularly intersecting it halfway between the black point and the white point; and all other colors are linearly interpolated by rgb components between black, white and the colors on that circle of corresponding hues. All the colors that can be displayed define a volume (hereafter referred to as the color solid) which in this color space is biconical. The space and color solid are diagrammed in Figure 5. Figure 6 a is a radial (constant-hue) section of the color solid, and Figure $6 b$ is a section perpendicular to the cylindrical axis (con-stant-intensity). In this space, the lightness or intensity of a color is defined by the vertical coordinate of the horizontal plane which contains the color; the radius relates to its chroma.

It is usually more convenient to define the space so that the radius corresponds to the chroma relative to the maximum achievable chroma for the hue and intensity of the color. In this color space, the color solid is a cylinder, with black and white bases at which levels the relative chroma is undefined. This space and cylindrical color solid are diagrammed in Figure 7. Figure 8a is a projection of the surface of the cylinder, and Figure $8 b$ is a radial (constant-hue) section. The rgb components of a color described by its coordinates in this space are

$$
\begin{align*}
& {\left[\begin{array}{lll}
r & g & b
\end{array}\right]=} \\
& \left\{\begin{array}{l}
\left([.5 .5 .5]+c\left(r^{\prime} g^{\prime} b^{\prime}\right]-\left[\begin{array}{ll}
.5 & .5 .5
\end{array}\right]\right) \cdot 2 i \\
\quad \text { if } i \leqslant 1 / 2 \\
{[.5 .5 .5]+c\left(\left[r^{\prime} g^{\prime} b^{\prime}\right]-\left[\begin{array}{ll}
.5 .5 .5
\end{array}\right)+\right.} \\
\left([.5 .5 .5]-c\left(\left[r^{\prime} g^{\prime} b^{\prime}\right]-\left[\begin{array}{lll}
.5 & .5 & 5
\end{array}\right]\right) \cdot(2-2 i)\right. \\
\quad \text { if } i \geqslant 1 / 2
\end{array}\right. \tag{eq.4}
\end{align*}
$$

where $c$ is the relative chroma and i is the intensity (each on the range 0 to 1 ), and $\left[r^{\prime} g^{\prime} b^{\prime}\right]$ is computed using an equation such as 2 or 3 .

In a color space defined by hue, chroma, and intensity, all the colors of a given saturation define a conical surface whose apex is the black point (at which the saturation is undefined). A color space may be defined in which the radial component is directly related to saturation by letting that component be

$$
\mathrm{s}=(\max (\mathrm{r}, \mathrm{~g}, \mathrm{~b})-\min (\mathrm{r}, \mathrm{~g}, \mathrm{~b})) / \max (\mathrm{r}, \mathrm{~g}, \mathrm{~b})
$$

(eq. 5)
Furthermore, the axial component can be specified to correspond to that component of color which is equal (and maximal) for all the colors representing the maximum intensities for all chromaticities and zero for black. In other words, the circle of maximum-chroma colors can be located so its center intersects the cylindrical axis at the white point. This axial component is denoted here by the term "value." In this color space, the color solid is a right circular cylinder (Figure 9) whose "top" base is a circularized chromaticity diagram, whose "bottom" base is black, and whose cylindrical surface



Figure 5. Hue/chroma/intensity color space, and color solid.


Figure 6a. Radial section of color solid
in hue/chroma/intensity space.
contains the maximum-saturation colors for all hues and lightnesses (Figure 10a). A radial (constanthue) section of this solid is shown in Figure 10b. For a color in this space,

$$
\left[\begin{array}{lll}
r & g & b
\end{array}\right]=\left(\left[\begin{array}{lll}
1 & 1 & 1
\end{array}\right]+s \cdot\left(\left[r^{\prime} g^{\prime} b^{\prime}\right]-\left[\begin{array}{lll}
1 & 1 & 1
\end{array}\right]\right) \cdot v\right.
$$

(eq. 6)
where $s$ is the saturation and $v$ is the "value" (each on the range 0 to 1 ), and $\left[r^{\prime} g^{\prime} b^{\prime}\right]$ is computed using equations 2 or 3 .

Alternative variants of this arrangement are possible. If the bases of the cylinder, instead of being circles, are triangles with the primaries at the vertices, any section corresponds to the color triangle in the chromaticity diagram of Figure 1. This suggests the possibility of defining a color space based on the CIE (Commission Internationale
 mination) $x, y$ chromaticity coordinates, the standard system established by that organization in 1931.7 Existing transformations could then be used to work in other CIE standard coordinate systems such as


Figure 6b. Section of color solid perpendicular to cylindrical axis in hue/chroma/intensity space.


Figure 7. Hue/relative chroma/intensity color space, and color solid.


Figure 8a. Projection of cylindrical surface of color solid in hue/relative chroma/intensity space.


Figure 8b. Radial section
of color solid in
hue/relative chroma/intensity space.


Figure 9. Hue/saturation/value color space, and color solid.


Figure 10a. Projection of cylindrical surface of color solid in hue/saturation/value space.


Figure 10 b . Radial section of color solid in
hue / saturation/value space.
u,v. CIE color solids, such as $x y Y, U^{*} V^{*} W^{*}, L^{*} u^{*} V^{*}$, etc., could even be used once the proper correspondences of the display primaries were determined. However, the usefulness of these spaces in computer graphics applications has not been demonstrated.

## Color Spaces for Computation

Suitable color spaces can also serve to facilitate certain color computations encountered in the generation of images. Some of these computations are trivial and can be done in the rgb space of the display system. For example, the shadow series for a color illuminated by a single light source is determined by linear interpolation between the color and black in the rgb space. For other computations, different color spaces car be more useful.

The effect of atmospheric scattering and haze on colors viewed at long distances may be approximated using a cylindrical color space. In this system, the changes that a particular color will undergo may be determined by computing a straight line from the original color to a bluish gray. Such a line in rgb space might not prove satisfactory in terms of the colors through which it passes.

The simulation of specular reflection by a chromatic object requires a series of colors starting at black, increasing in intensity with saturation in the direction of the particular hue, and then decreasing in saturation and ending at or near white. In any of the spaces discussed so far, such a color scale lies on a complex curve. If a color space can be defined in which the locus of these colors or, satisfactory approximations, is an easily calculated function, the computation of the image may be simplified.

A general class of computations which could greatly benefit from such a technique are those involving the simulation of chromatic alteration of chromatic colors, such as the reflection of colored lights by colored objects or the transmission of colored light by a colored transparent medium. Proper simulation of such effects requires the specification of reflection and transmission characteristics, not by rgb primary components, but rather by complete spectral distribution curves and computations over these spectra. However, this requires a very large and probably impractical amount of computation. A good approximation based on primary components could be very useful. Another problem is to simulate the effects of chromatic transparency or filtration. Several color spaces are presently being examined in which a linear interpolation between the incident color and the filter color yields a good approximation of a likely resultant transmitted color.

The concept of color space is also useful in the creation of synthetic-color images in which the variation of some parameter is represented as a variation in color. The colors define a path through color space. The intermediate colors obtained by computing a straight line in rgb space probably are not as visually preferable as those existing on such a line in one of the cylindrical color spaces discussed earlier. In any of the latter spaces, all of the intermediate colors could have the same saturation as the end colors; whereas in rgb space, they would be paler.

Summary
Color display devices used in computer graphics produce their range of colors by mixing the three primaries red, green, and blue. For many applications, the interactive use of this additive color mixing is not always suitable. By defining color spaces as three-dimensional volumes representing the available color spectrum, more appropriate methods for selecting colors can be obtained. This paper has described several of these useful color solids and their beneficial characteristics. The equations for transforming to these color spaces from rgb space have been presented. Of particular importance is the fact that linear paths through certain color spaces can accurately represent complex color transitions such as shadowing or filtration. The utilization of the appropriate color space can thus result in substantial savings in the computational time for the creation of synthetic images.

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