A SIMULATION OF THE OPERATIONS OF AN ILLEGAL NUMBERS GAME

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The question of legalized gambling has arisen in recent years to pique the interest of the American public. Most proposals for legalization stem from beliefs that existing prohibitions against gambling are not enforced and are perhaps unenforceable, and that the public will accept heavy taxes on such a venture, as they accept "sin" taxes on cigarettes, liquor, and horse racing.

The topic at issue in this paper is a legalized numbers game (1). The numbers game is an illegal lottery, played in all major and many minor cities in the United States and believed to be played primarily by the poor. In a common variant of the game the bettor selects a three-digit number from 000 to 999 and places a small bet on that number, typically less than one dollar. Later the same day the winning number is chosen by some randomizing device; say, the last three digits of the total pari-mutuel handle at a nearby race track. Typically the gross payout to the winner is 600:1 and the net payout 599:1, because the bettor's original bet is not returned to him with his winnings.

The banker can be seen as engaged in a game described by a simple probability function. To him the expected value of the game before all other expenses except prizes, E(G), is

$$E(G) = xp - 599xq$$

where:

$p = .999$, the probability of winning

$q = .001$, the probability of losing

x = the mean amount wagered per number.

Therefore, $E(G) = +.4x$. The variance, Var(G), however, is high, which makes the game exciting for the players and risky for the banker. Assuming all numbers pay gross 600:1, $Var(G) = 359.8x^2$.

The game is believed to be enormously profitable. However, I have argued elsewhere that many numbers games do in fact go bankrupt and others lose money for periods of time (2). Moreover, profitability can not be viewed apart from risk factors.

This paper treats one aspect of the operations of a numbers game, its risk characteristics. Risks come from two sources, those internal to the bank such as poor control of overhead expenses and infelicitous relations with the police, and those external to the bank. The most serious external risk is clumping of bets. To understand this it is necessary to describe briefly the distribution of bets.

The simplest possible <u>a priori</u> model for the numbers game is the assumption that bets on any given day will be distributed uniformly, with

$$E(x) = (a+b)/2 = 499.5, \text{ and}$$

$$Var(x) = (a^2 + b^2)/12 = 8.32 \times 10^4,$$

and that the winning number will also be distributed uniformly with the same mean and variance.

Only one piece of published evidence explores the distribution of winning numbers; it presents evidence that winning numbers are distributed uniformly at the 5% confidence level but, surprisingly, not at a 1% level.(3)

The evidence for nonuniformity in the distribution of numbers bet is more persuasive, although still exiguous. A study by the author has shown distinct nonuniformity in numbers bet, but data were available only for one day. More persuasive is the fact that certain numbers are superstitiously "popular" and are bet far more heavily than other numbers. Bankers respond to this phenomenon by cutting the net payout odds, typically from 599:1 to 399:1. And due to the influence of public events on bettors' superstitions, bettors choose numbers in a manner that does not show statistical independence. For example a spectacular robbery or assassination of a public figure on, say, May 3 is likely to bring betting on 503 and 530 for several weeks or months afterward. Bettors also place combination bets, in which all distinguishable permutations of three digits are bet simultaneously. The distribution of bets, then, is distinctly nonuniform and is not characterized by statistical independence.

The simulation model was designed to explore the influence of the bank's reserves on the probability of bankruptcy. "Reserves" is defined as cash on hand plus unused capacity to borrow to meet short-term cash needs (4). A reserve level of zero means no cash <u>and</u> exhaustion of debt capacity; thus it is the border between solvency and ruin.

<u>A priori</u> it was hypothesized that the probability of ruin decreases as the reserve level increases, and increases as the variability of the payout increases. This latter is a function of the degree of "clumping" of bets. Because only one empirical bet distribution was available the degree of clumping was fixed, and therefore the variance of the payout. A third factor also influences the probability of ruin. The net profitability of the game, but this too is fixed given the unchanging structure of the game.

The Simulation

Figure 1 shows the flow chart for the simulation of one cycle of operation. This cycle was repeated twenty-five times for each of eleven reserve levels, \$0, \$2500, \$5000, ..., \$25,000. Figure 1 uses the following symbols:

CR = cumulative reserve of liquid stocks and unused debt capacity held by banker

R = initial reserve level

i = 000, 001, ..., 999, the numbers on which bets are placed

W = winning number

b_i = average bet on i^{th} number

b_w = average bet on winning number

N_i = number of bettors playing i^{th} number

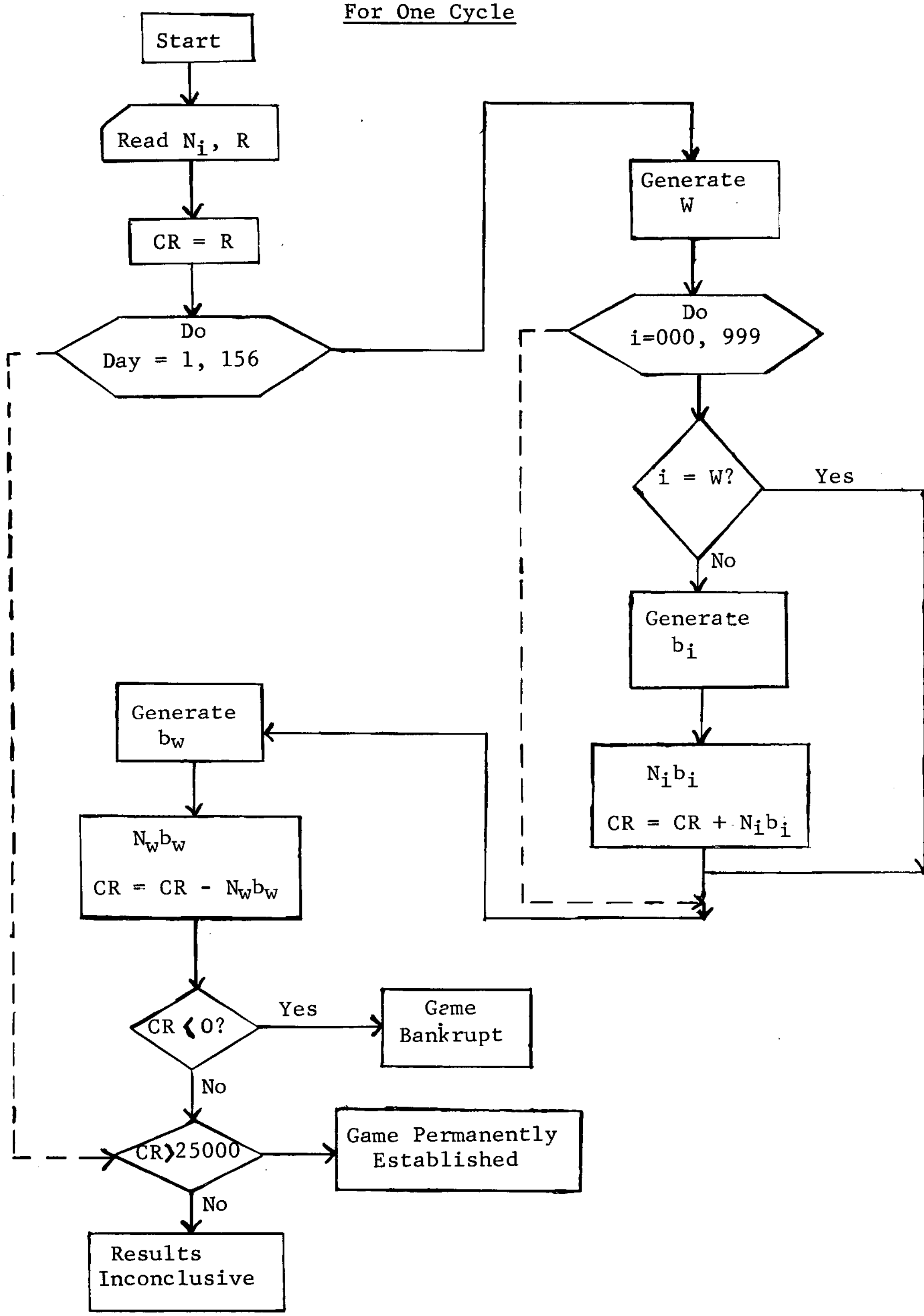
N_w = number of bettors playing winning number

Day = discrete time variable

The model used is straightforward. It simulates the operating of a numbers bank with 6,068 bettors, by randomly generating wagers from a truncated normal distribution with μ = \$.60 and σ = \$.56. The truncation forces all bets to be $\geq$ \$.10. The set $\{N_i\}$ was determined empirically in another study (5) and is fixed for all replications of the model. $\{N_i\}$ can be described as follows: number of elements in the set 6068, mean 431.2, standard deviation 260.0, median 389.5; the hypothesis of uniformity was tested by a chi-squared test and rejected at the 1% level (with 99 degrees of freedom).

Figure 1

Flow Chart of the Simulation Model, For One Cycle



The choice of an upper limit was unfortunately required to be somewhat arbitrary. Determination that $25,000 in cumulative reserves was equivalent to permanent establishment of the bank is not completely defensible except when limitations of computer time are taken into account. In early testing of the simulation one cycle of operation ultimately went bankrupt on the 180th day, but on more than half of the preceding days the reserve level exceeded $25,000. In another early test even $52,000 reserve level was not enough to prevent ultimate bankruptcy. Given the variance in payouts, that is, $25,000 is somewhat too low and will bias downward estimates of the probability of ruin.

The simulation ran for one-half year of simulated time, or 26x6=156 days, since the game operates everyday but Sundays, when the horses do not race.

Results

The basic measure of risk was the probability of ruin, computed as a statistic from the simulation results. Each sample was of size 25.

Reserve Level ($, in thousands)	P (Ruin)	P (Permanently Established)
0	.96	.04
2.5	.64	.36
5.0	.64	.36
7.5	.68	.32
10.0	.28	.72
12.5	.10	.90
15.0	.36	.64
17.5	.075	.925
20.0	.15	.85
22.5	.025	.975
25.0	.0	1.0

These results can be described by a simple linear equation,

$$P = .8 - .034R$$
$$(.05)\ (.006)$$

where:

P = probability of ruin

R = reserve level, in thousands of dollars: $0 \leq R \leq 25$

and the standard errors of the regression coefficients are in parentheses. For this equation $R^2 = .84$ and the standard error of the estimate = .12.

These results support the basic hypothesis that probability of ruin decreases with reserve level. It would also be expected that time to ruin would tend to increase as reserve level increased. The results support this conclusion but not so clearly.

Reserve Level ($, in thousands)	Mean Time to Ruin (in days)	n
0	6.9	24
2.5	12.7	16
5.0	9.2	16
7.5	13.2	17
10.0	15.1	7
12.5	8.8	4
15.0	11.1	9
17.5	29.3	3
20.0	26.1	6
22.5	46.0	1
25.0		0

Again these results can be described as follows:

$$\text{Log}(T) = 2 + .065R$$
$$(.18)\ (.02)$$

where T is mean time to ruin. For this equation $R^2 = .67$ and the standard error of the estimate is .331. As above it is necessary that $0 \leq R \leq 25$.

Finally one further view of the importance of the clumping of bets can be explored by observing the mean number of players on the winning number, when that winning number is the one whose payoffs ruin the bank. The number of winning players at ruin, N, is hypothesized to be an increasing function of R, the reserve level, because in general the higher the initial reserve level, the higher the cumulative reserve level that must be offset by a high N.

Here the results are somewhat more inconclusive.

Reserve Level ($, in thousands)	E(N)
0	15
2.5	15.6
5.0	14.7
7.5	14.5
10.0	26.9
12.5	19.0
15.0	27.0
17.5	54.0
20.0	26.8
22.5	32.0
25.0	

There is some tendency of mean N, E(N), to increase with R but the pattern contains a considerable amount of variance. The results are described as follows:

$$E(N) = 12 + 1.1R$$
$$(3.5)\ (.5)$$

with $R^2 = .47$ and standard error of the estimate = 8.4.

It should be noted that the mean number of bettors per number is 6.07 with standard deviation 3.52; hence E(N) of the order 15 or more means that E(N) lies typically at least 2.5 standard deviations above the mean. This confirms the belief that ruin is most likely to occur when the winning number has a large clump of bets on it.

Conclusions

The simulation provides some evidence for the general conclusion that illegal numbers games are properly called "risk businesses." For a small to medium game, such as the one simulated here, there is a definite likelihood of ruin if the game for any reason is under-capitalized. The inherent riskiness of the game can be further illustrated by citing descriptive statistics on the net contribution per day, which is total revenue from bets less winnings. A random sample of 125 days of operation yielded the following statistics for net contribution per day:

Mean:	$244.94
Standard deviation:	$3113.50
Coefficient of variation:	12.71
Skewness:	-2.99
Range:	$20,755.40

These clearly describe a game of rapid changes of fortune, although one with a positive expected monetary value.

Two cautions must be entered in considering these results. First the simulation is not the real thing. In particular the number of bettors per number was fixed throughout the simulation. Moreover in a real game some numbers will not pay at 600:1, slightly reducing the variance of the cumulative reserve level over time. Second, real bankers have a set of dilatory tactics to enable them to survive liquidity crises. They may pay winners over several days instead of all at once; they may palter and refuse to pay at all on some pretext; they may cut payout odds after the winning number has been determined but before winnings are paid; or they may tip off the police as to the location of their bank and then tell winners that the betting slips were arrested and, under normal operating procedures, can not be paid. (Such police raids, by the way, always seize betting slips but rarely cash). Thus what has been defined as ruin in this paper may not be in a real numbers game.

The major question with which this paper is unable to deal is the influence of the size of the numbers game on risk. There is no evidence, but it is possible that large games are able to spread their risks over a larger number of bettors and thus increase the degree of stochastic independence in the distribution of bets. If so, the conclusions in this paper concerning the probability of ruin would have to be modified.

Footnotes

1. Two detailed proposals for legalization of the numbers are Off Track Betting Corporation, "Legalized Numbers," February 1973; and Fund for the City of New York, "Legal Gambling in New York," November 1972.
2. Rados, David L., "Finite Numbers," The Columbia Forum, II, (New Series), Spring 1973, 2 - 7.
3. Off Track Betting Corporation, op. cit.
4. This usage follows Sear, Hilary L., Stochastic Theory of a Risk Business, (New York: Wiley, 1969), p. 3.
5. Rados, David L., "The Numbers Game: An Economic and Competitive Analysis," in preparation.

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