A STOCK MARKET SIMULATION MODEL FOR PRICING ASSET VALUE GUARANTEES

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ABSTRACT

To compute the cost of an asset value guarantee for a variable annuity product, a stock market simulation model was used. The model was based on the assumption that the time series represented by a stock market index could be broken into alternating bull and bear half cycles, with each half cycle determined by a duration, a growth rate, a within cycle noise factor and a between cycle noise factor. Each of thousands of generated market cycles were used on a monthly basis by the model for purchase of stock, removal of asset charges, crediting dividends and other transactions.

I. INTRODUCTION

Traditionally, insurance products such as annuities and whole life insurance have had a fixed value at each instant in time. In a dynamic economy such as ours, however, where inflation is eroding fixed savings, there is an increasing demand for products that offer the hope of a hedge against inflation. To help meet this demand, insurance companies have developed equity based products whereby the investor, by assuming some risk, has a potential for greater gain. Simply stated, these "variable" products (e.g. variable life insurance, variable annuities) base the value of the product on the performance of an investment fund managed by the insurance company. Since the investor has assumed some of the investment risk, it is reasonable that he look to insurance companies (whose business is risk) to minimize his possibility of loss.

In essence then, insurance companies are being asked to insure the performance of a portfolio of common stocks. The Hartford Insurance Group is contemplating offering such a guarantee as a Minimum Living Benefit (MLB) rider to a Variable Annuity Contract. The MLB rider will guarantee to the purchaser that at a specified future date, the value of his annuity will be at least equal to his total investment.

The premium needed to cover such a benefit is a function of many variables including duration of contract, volatility of portfolio, dividend yield of portfolio, lapse rates, interest earned on reserves, loading and fees, etc; but the primary factor in determining cost is the performance of the stock market. Unfortunately it is difficult to account for future stock market performance.

II. REASONS FOR USING A SIMULATION

Insurance rates are based on the law of large numbers. For insurance such as Life Insurance or Auto Insurance this means that rates are based on the historical experience of a large number of lives or autos. Therefore to insure an m-year period of stock market performance, we would need a <u>large</u> number of independent m-year periods on which to base our rate. This necessity presents a problem, for although many lives or autos can be sampled simultaneously, time periods must be sampled consecutively. Use of historical data, therefore, has three major drawbacks:

1) There are too few m-year periods on which to determine a statistically valid rate. (Valid stock market indices go back less than 100 years).

2) Use of historical data makes the assumption that the future will exactly mirror the past.

3) To investigate different portfolio types would require gathering a new data base for each portfolio.

In order to overcome the drawbacks of historical data and to be able to validly compute an adequate premium for a variety of portfolio types, a simulation model was used. The overall model included many routines related to variables inherent in the variable annuity product, but the heart of the simulation was the stock market model. It is with this stock market model that the remainder of this paper is concerned.

III. THE MODEL

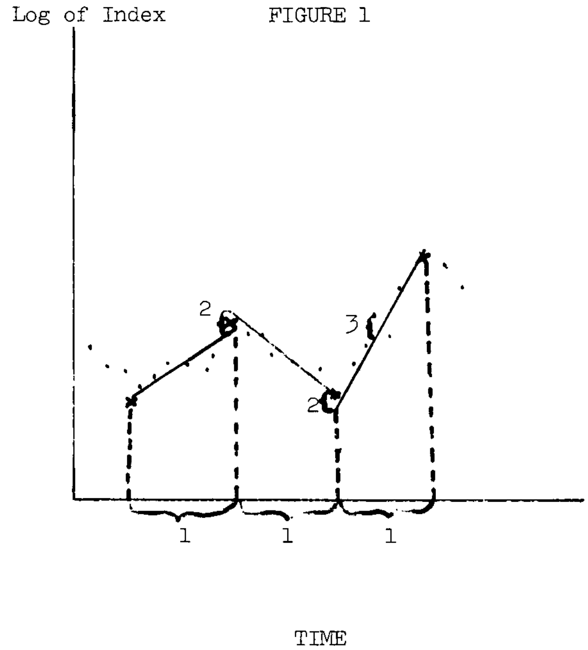
To structure our stock market model we first analyzed a time series of stock prices to determine identifiable components of the series. The components were then used as building blocks to construct a large variety of possible futures. In this way we were able to make use of data from the past without assuming that the exact same sequence would occur. It is important to keep in mind that we were not attempting to predict the course of the stock market at any future point in time. Our object was to generate stock market price series for a large number of possible future m-year periods.

The first step in the analysis was to divide the time series into bull and bear half-cycles. For each half-cycle a least squares line was fitted to the log of the market index. As a result of the fit, frequency distributions of the following characteristics for both bull and bear half-cycles were formed:

1) duration
2) growth rate (slope)
3) variance of within cycle noise
4) between cycle noise

These characteristics are illustrated in figure 1.

FIGURE 1



Sample Market Index

extreme points marked x
all other points .

1 is duration
2 is between cycle error
3 is within cycle error

By fitting a line to the log of the time series we were in fact assuming that within each half-cycle the price series followed the equations:

$$P_{N+1} = P_N \times (1+B \times R)^D \times e$$

where

P_i = the price at time i

B = volatility index

R = growth index

D = duration

e = within cycle noise

The above equation besides yielding a good fit to the data, is consistent in its form with the idea that the market grows in a manner similar to money invested at compound interest. Successive half-cycles were assumed to be independent with the restriction that bull and bear half-cycles alternate. Since duration and growth rate are not independent, growth rates were separated into classes by duration before distributions were formed.

III. THE MODEL (continued)

Because neighboring least squares line are in general not concurrent at the end points, an additional error factor was used in the model. This "between cycle error" was accumulated into frequency distributions for both types of junctures (bull-bear, and bear-bull).

The volatility index, B, is a measure of the sensitivity of the portfolio to changes in the overall stock market. The value of B can be calculated for a known portfolio as the regression coefficient of the log of the portfolio performance regressed against the log of a stock market index. In using beta, two more portfolio indicators could appropriately be used. These are alpha (an index of performance), and gamma (a measure of the variability of beta). Because recent studies of portfolio management have cast doubt on managers' ability to consistently out-perform the market, alpha was assumed to be zero. Gamma, besides being difficult to estimate, tended to obscure the effect of beta and added little to overall results. For simplicity reasons, therefore, both are omitted from the above model.

IV. DIVIDENDS

In the variable annuity plan under investigation dividends are automatically reinvested for the policyholder and help to meet the guaranteed minimum level. Dividends, however, are a function of several factors including market price and portfolio volatility. Although dividends in general keep track with long term growth, short term fluctuations play havoc with rates due to a relativley constant dollar dividend yield being related to a fluctuating price.

We found a relatively simple model which yielded an excellent fit to past dividends. This model involved choosing an average dividend rate based on the beta value used. An eighteen month moving average was calculated and used to compute the dollar yield that would produce (over the 18 months) the average rate chosen. The ratio of the computed dollar yield to the latest monthly price gave the monthly dividend yield.

Although prices could fluctuate substantially month to month, the dollar yield was tied to the 18 month average and was fairly stable over time.

V. THE SIMULATION

In the simulation each half-cycle was generated by choosing its attributes from the appropriate frequency distribution. Under the assumption that our distributions represented a discrete sample from a continuous density function, the generated attributes were made continuous by interpolating within the distribution.

Each generated half-cycle was divided into monthly values by interpolating and applying within cycle noise. It was at each monthly value that transactions such as buying stock, crediting dividends, removing asset charges, etc. were handled. In our simulation we ran 5000 fifteen year periods and used a variety of assumptions. Enclosed is a sample of output from a run of 2000 fifteen year periods. Although certain information has been blocked out the nature of results is well illustrated. Of importance is not just the premium figures on Table1 but the distribution of final equities on Table 2. The distribution not only aids understanding, but gives probabilities of catastrophic loss.

VI. CONCLUSION

The simulation was run using a variety of assumptions. Our model fully lived up to standards set for it and exceeded many expectations. Without the simulation it would not have been feasible to evaluate the difference between an asset charge and a premium load for obtaining premium for the MLB; it would not have been possible to test the effects of different portfolio types on the cost of the MLB; and there would have been no adequate way of answering capacity questions (how much to sell without jeopardizing earnings). The simulation enabled us to evaluate the probabities of both small loss and catasprophic loss and to test the sensitivity of each to changes in investment assumptions. Simulation, therefore, proved to be an appropriate and unique tool in evaluating equity based products and not only gave us reasonable final answers, but enabled us to answer many questions that would otherwise have been unanswerable.

ACKNOWLEDGMENTS

I would be remiss if I did not acknowledge the contributions of James Smith who conveived the basic model and Louis Mizel, Arthur Jacoby, and John Parker who contributed to its early testing.

HARTFORD VARIABLE ANNUITY LIFE INSURANCE COMPANY
MINIMUM LIVING BENEFIT SIMULATION
PHASE I - FINAL RUN
THIS RUN BASED ON STANDARD & POOR'S 500 INDEX (1957-1972)

PERIODIC PURCHASE PLAN PURE PREMIUM CALCULATION

MONTHLY INVESTMENT $
LIMIT OF LIABILITY $
INITIAL PRICE/SHARE $
POLICY TERM (MONTHS)
ACTUAL INVESTED SUM $

PURE PREMIUM (EXPECTED LOSS) $	321.9668
ST.DEV. OF PURE PREMIUM	871.6865
NUMBER OF REPLICATIONS	2000
NUMBER OF LOSSES	370
TOTAL AMOUNT LOST	$643933.8125

DESCRIPTION	TABLE REFERENCE	INTEREST	LAPSES	NET PREMIUM	NET PREM 98.00 CONFIDENCE
UNADJUSTED NET PREMIUM	NONE	NONE	NONE	3.577	4.002
NET PREM WITH NONFORFEITURE VALUES & CONSTANT INTEREST	4	CONSTANT	NONE	2.737	3.062
NET PREM WITH NONFORFEITURE VALUES & VARIED INTEREST	3	VARIED	NONE	2.329	2.605
NET PREM-CONSTANT INTEREST	2	CONSTANT	VARIED	1.672	1.870
NET PREM-VARIED INTEREST	1	VARIED	VARIED	1.386	1.551

(THE ABOVE PREMIUM FIGURES ARE EXPRESSED AS PERCENTS)

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7000	74
8000	112
9000	123
10000	151
11000	145
12000	143
13000	156
14000	142
15000	111
16000	99
17000	104
18000	88
19000	61
20000	69
21000	55
22000	54
23000	42
24000	32
25000	20
26000	18
27000	16
28000	16
29000	13
30000	12
31000	7
32000	7
33000	7
34000	7
35000	8
36000	4
37000	6
38000	8
39000	2
40000	1
41000	5
42000	5
43000	1
44000	3
45000	1
46000	1
47000	0
48000	0
49000	2
OVER	6

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