TOWARD A SIMULATION MODEL OF THE STOCK MARKET

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Summary

A simulation model of the stock market is proposed here. While it is not practical (nor desirable) to include all aspects of the stock market in our model, we do hope that the proposed market mechanism has captured a sufficient number of essential features of basic forces underlying the stock market that it can serve as a first step toward an operational framework for studying the behavior of stock prices.

I.

After reviewing the literature on the stock market one gets the feeling that the prevailing view held by the researchers in the field of stock market behavior is that the movement of stock prices follows a random walk model. That is, if we let p_t be the price (or the logarithm of the price) of a stock on the t-th day, and $\varepsilon_t = p_t - p_{t-1}$, then $\{\varepsilon_t\}$ forms a sequence of independent random variables with zero mean.

The first important study of the stock market was done by Bachieler[2] at the turn of the century. In this pioneering work, Bachieler proposed a model which was a special case of the above in that the ε_t had a Gaussian distribution. Over the years, numerous investigators have questioned the Gaussian assumption, but the contention of independence has rather well withstood the test of empirical evidence. Such evidence can be found in the work of Kendall[6], who analyzed twenty-two economic series and found virtually no correlation between the successive ε_t's. Some researchers such as Alexander[1] have attempted to refute the random walk hypothesis by devising trading strategies which would be profitable. Under careful examination, all such strategies proposed to date have failed to prove themselves to be significantly better than a random trading strategy.

A more radical model was proposed by Mandelbrot[7]. Upon investigation into the distribution of the ε_t, he found a large number of observations in the tails of the distribution. As he increased the size of his sample, he found the estimate of the variance of ε_t getting larger and larger. This led him to the conclusion that ε_t has a stable Paretian distribution. What is so radical about this assumption is that, if true, it would mean that most of the standard statistical techniques such as regression and spectral analysis would be useless because they depend upon the assumption of finite variance which, with the exception of the Gaussian distribution, the stable Paretian distributions do not possess.

It appears to us that two observations can be made:

1. If we are trying to discover a universal law which will govern the behavior of stock prices from the first day the stock market came into existence to a time far in the indefinite future, then it would be natural for us to conclude that the behavior of stock prices follows a random walk model when we observe that $\{p_t - p_{t-1}\}$ forms a sequence of independent random variables with zero mean, and that the distribution of $p_t - p_{t-1}$ is a stable Paretian when we observe that the estimate of the variance of $p_t - p_{t-1}$ becomes larger and larger when we increase the sample size. While it may be true that we have not been successful in predicting tomorrow's prices with the information about today's prices, we have not exhausted testing all the possibilities of predicting future price patterns based on past price patterns. Since the stock market is one aspect of overall economic activity, it is reasonable to expect that in the long run the stock price movement tends to be related to overall economic activity. Therefore, it is rather unlikely that we can construct a good model of the long-run behavior of the stock market without first constructing a good long-run model of the economy as a whole. Furthermore, the stock market as an economic institution is continually evolving. It has undergone and will undergo various changes in the regulations concerning its transactions; for example, it is entirely possible that sometime in the future there may be legal maximum limits on the daily price fluctuation for certain stocks. As long as this kind of possibility exists, is it reasonable to think that the law, whatever it may be, that governs the behavior of stock prices so far will continue to do so, say, a thousand years from now? It thus appears to us that if there is any justification for studying the stock market it is more interesting to examine the short-run behavior of stock prices.

2. Most studies on the stock market have confined themselves mainly to the examination of the stock price series. Where the volume series was dealt with, it was treated as a series independent of the price series. The first serious attempt to relate price and volume in the stock market was made by Granger and Morgenstern[4]. After applying spectral analysis to price and volume data from the New York Stock Exchange, they found that there was no connection between the price series and the corresponding volume series. In a subsequent paper, Godfrey, Granger and Morgenstern[3] extended their previous investigation. Although they found that the volume series tends to be a quarter cycle out of phase with the series of lows, the corresponding coherence was too low to attach any significance to this result. Again they concluded that there is no correlation between stock price changes and the volume of transactions.

II.

In a paper by one of the authors[8], we examined the New York Stock Market daily volume series and the Standard and Poor 500 composite stocks daily price series from 1957 to 1962. It was found, among other things, that:

1. a small volume on a given trading day tends to be accompanied by a fall in price on that trading day,

2. a large volume on a given trading day tends to be accompanied by a rise in price on that trading day,

3. a large increase in volume on a given trading day tends to be accompanied by either a large rise in price or a large fall in price on that trading day,

4. a large volume on a given trading day tends to be followed by a rise in price on the following trading day,

5. if the volume has been decreasing consecutively for a period of five trading days, then there will be a tendency for the price to fall over the next four trading days, and

6. if the volume has been increasing consecutively for a period of five trading days, then there will be a tendency for the price to rise over the next four trading days.

Although some people questioned the validity of our findings (for example, Granger and Morgenstern[5]), we find it difficult to accept the proposition that there is one process that determines the price and another (presumably different) that determines the volume. As a matter of fact, it is not difficult to show that both the price and the volume are joint products of a single market mechanism. The following simple example can serve as an illustration:

Suppose that there are only four traders (denoted by Mr. A, Mr. B, Mr. C and Mr. D respectively) and two shares of stock outstanding at time t. Let us define the demand price for each trader as follows: A trader will hold one share of stock if the market price is less than or equal to his demand price, and he will hold no share of stock if the market price is greater than his demand price. Assume that the price of a stock is measured to the nearest dollar; then a distribution of the demand price among individual traders at time t may be as follows:

Individual Trader	Demand Price
Mr. A	$48
Mr. B	49
Mr. C	50
Mr. D	51

Let the market equilibrium price be defined to be the market price at which the total number of shares outstanding are held by, and only by, those traders whose demand prices are equal to or above the market price; then the market equilibrium price of the stock at time t will be $50, and Mr. C and Mr. D will each hold one share of stock.

Transactions take place and prices fluctuate over time because individual traders change their demand prices over time (according to the various market information they receive and the various decision rules they adopt). Suppose that at time t + 1, Mr. A changes his demand price from $48 to $49, Mr. B from $49 to $51, Mr. C from $50 to $49 and Mr. D remains at $51. The distribution of demand prices at time t + 1 is:

Individual Trader	Demand Price
Mr. A	$49
Mr. C	49
Mr. B	51
Mr. D	51

and the market equilibrium price of the stock at time t + 1 will be $51. It resulted in a rise in price of $1 and a volume of transactions of one share (Mr. B buying one share from Mr. C).

The above situation can be more conveniently described by the following table. We shall call it the Market Table of the t + 1-th Trading Day.

Market Table 1

t \ t+1	$48	$49	$50	$51
$48		1		
$49				1
$50		1		
$51				1

A "1" in the first row and the second column indicates the fact that there is one individual trader (i.e., Mr. A) who changes his demand price from $48 (at time t) to $49 (at time t + 1). To find the equilibrium market price at time t, add the number of 1's in the rows starting from the last row until the number of 1's equals the total number of shares outstanding at time t; then the price corresponding to that row will be the market equilibrium price at time t. To find the equilibrium market price at time t + 1, add the number of 1's in the columns starting from the last column until the number of 1's equals the total number of shares outstanding at time t + 1; then the price corresponding to that column will be the market equilibrium price at time t + 1. The volume of sales is equal to the sum of 1's which represent traders whose demand prices were below the market equilibrium price at time t but are equal to or above the market equilibrium price at time t + 1; since stocks bought are equal to stocks sold, the volume of sales is also equal to the sum of 1's which represent traders whose demand prices were equal to or above the market equilibrium price at time t but are below the market equilibrium price at time t + 1.

The following are a few examples of such tables describing different changes in demand prices.

Market Table 2

t \ t+1	$48	$49	$50	$51
$48	1			
$49			1	
$50		1		
$51				1

Table 2 describes a situation where there is no change in market equilibrium price from time t to time t + 1, and there is a volume of sales of one share at time t + 1.

Market Table 3

t \ t+1	$48	$49	$50	$51
$48		1		
$49			1	
$50	1			
$51	1			

Table 3 describes a situation where there is a fall in market equilibrium price from $50 (at time t) to $49 (at time t + 1) and a volume of sales of two shares.

From the results of the above examples, the following things in general can be said:

1. If the traders change their demand prices upwards (or downwards) in similar fashion, then there will be a rise (or fall) in price (hereafter, by price we mean the market equilibrium price) with relatively small volume;

2. If the traders change their demand prices in opposite directions by a large (or small) amount, then there will be a large (or small) volume with relatively little change in price.

Let us now examine briefly some of the factors that determine a trader's demand price.

1. The first factor (important especially for a short-term trader) is his expectation about the future price movements. A trader forms his expectation based on the market information he received from various sources available to him. By market information we mean all relevant information concerning the market that may affect a trader's expectation. The sources of such information include price and volume statistics, company financial reports, newspapers, journals, bulletins or letters from investment advisory services and brokerage firms, etc.

2. The second factor is his financial position. For example, a trader with a strong financial position will not set his demand price at a low level in order to be able to sell his stocks if he believes that the anticipated price fall is of a short-run nature. On the other hand, such an anticipation will significantly affect the market behavior of a trader whose financial position is weak.

3. The third factor is the trader's motive for holding stocks. The price a trader is willing to pay (or sell for) depends on whether his motive is to obtain long-term gains by receiving dividends and providing a protection against inflation, or to obtain short-term capital gains through price fluctuations.

4. The fourth factor is emergency needs. A trader may be forced to liquidate some or all of his stock holdings as a result of certain emergency needs, such as meeting his financial obligations or the requirements of law.

The last two factors are determined normally by forces outside of the stock market; therefore, they are not usually affected by short-run price movements. The financial position of a trader can be affected by price fluctuations; for instance, a large profit resulting from a sharp price rise will strengthen a trader's financial position, and a large loss resulting from a sharp price fall will weaken a trader's financial position. However, in order for the change in price to have a significant effect on a trader's financial condition, either the price change has to be, in the trader's opinion, "permanent" (that is, the change in price is once and for all), or the change in his financial condition (as a result of a price change) has to be relatively unaffected by future price changes. Also, since for every trader who is strengthened (or weakened) by a price change there is another trader (or traders) who is weakened (or strengthened) by such a price change, the aggregate effect of a price change on the market (aside from its effect on the trader's expectations) is rather uncertain.

Regarding the factor of expectation, a point to be emphasized is that the market information only forms a working basis for a trader's expectation; how such information affects his expectation depends on how he utilizes it. Therefore, the most critical and often mysterious element in the process of expectation formation is the trader's "market psychology." Since this element is highly subjective and in general differs from individual to individual, it accounts, to a large degree, for the divergences among traders' demand prices, without which tradings in the stock market cannot occur.

As mentioned before, the market information includes sales and earnings of various companies, forecasts of general economic conditions, and many other things, but one of the most important items for the short-term traders is the values of past market prices and volumes of transactions. It is from these figures that a trader gets a feeling of what the other traders' "market psychologies" are and how they are changing.

Why do traders change their demand prices from time to time? Because the factors that determine their demand prices change from time to time. How frequently a trader changes his demand price depends on how rapidly the determining factors change, how closely he observes these changes, and how sensitively he reacts to them.

III.

To be slightly more general, the idea contained in our simple example can be formulated as follows:

Let $\{P_j\}$, $j = 1,\ldots,m$ be the set of possible values that the price of our stock can take; let $N_j(t)$ be the number of traders with demand price P_j at time t; let $Q(t)$ be the total number of shares of the stock outstanding at time t; and let $P_e(t)$ be the market equilibrium price of the stock at time t. A market equilibrium price is defined as a price at which the market is cleared; i.e., the aggregate demand for the stock equals the total shares of the stock outstanding. Since we only allow certain discrete values of the stock price, we do not require that the market be completely cleared at all times. Then,

$$P_e(t) = P_{j^*(t)}$$

where $1 \leqq j^*(t) \leqq m$ such that

$$\sum_{i=0}^{m-j^*(t)} N_{m-i}(t) \leqq Q(t)$$

and

$$\sum_{i=0}^{m-j^*(t)+1} N_{m-i}(t) > Q(t).$$

Suppose that for $1 \leqq j \leqq m$, $y_j(t)$ percent of the traders with demand price P_j at time t will change their demand price at time $t + 1$ in the following fashion:

Let the random variable $x_j(t)$, $j = 1,\ldots,m$ assume only discrete values from 1 to m; let $pr\{x_j(t) = k\}$ be the probability that $y_j(t)$ percent of the traders with demand price p_j at time t will change their demand price to p_k at time $t + 1$; then,

$$N_k(t + 1) = [1 - y_k(t)]\, N_k(t) + \sum_{i=1}^{m} \delta_{x_i(t)k}\, y_i(t) N_i(t)$$

where $\delta_{x_i(t)k} = 1$ if $x_i(t) = k$

and $\delta_{x_i(t)k} = 0$ if $x_i(t) \neq k$.

Let $Q(t + 1)$ be the total number of shares of stock outstanding at time $t + 1$. Then the market equilibrium price of the stock at time $t + 1$ will be determined as before; i.e.,

$$P_e(t + 1) = P_{j^*(t + 1)}$$

where $1 \leqq j^*(t + 1) \leqq m$ such that

$$\sum_{i=0}^{m-j^*(t + 1)} N_{m-i}(t + 1) \leqq Q(t + 1)$$

$$\sum_{i=0}^{m-j^*(t + 1) + 1} N_{m-i}(t + 1) > Q(t + 1)$$

The volume of sales at time $t + 1$, denoted by $V(t + 1)$, will be determined as follows:

Let $W_k(t + 1) = \sum_{i=1}^{j^*(t) - 1} \delta_{x_i(t)k}\, y_i(t) N_i(t)$;

then $V(t + 1) = \sum_{j=j^*(t + 1)}^{m} W_j(t + 1)$

if $j^*(t + 1) \geqq j^*(t)$,

$$V(t + 1) = \sum_{j=j^*(t + 1)}^{j^*(t)-1} [1 - y_j(t)]\, N_j(t) + \sum_{j=j^*(t + 1)}^{m} W_j(t + 1)$$

if $j^*(t + 1) < j^*(t)$. This is true because $V(t + 1)$ is the number of traders (or shares) whose demand prices at time t were less than $P_e(t) = P_{j^*(t)}$ (therefore none of them held any share of stock at time t) and whose demand prices at time $t + 1$ are equal to or greater than $P_e(t + 1) = P_{j^*(t + 1)}$ (therefore each of them now holds one share of stock).

By specifying the distributions of $x_j(t)$ and $y_j(t)$ for $t = 1,\ldots,T$ we can generate, by computer simulation, a sequence of values of market equilibrium prices and volumes of sales which can be compared with the actual data over a certain period of time (for which we believe that the underlying process is relatively stable). If we construct a certain metric to measure the deviation between the generated data and the actual data, we can choose one set of distributions among the possible ones which give the minimum deviation.

Some of the problems that confront us in performing the proposed simulation include the following:

1. Which values should we choose as the set of possible values that our stock can assume? The choice depends on the following things: (a) How closely we wish the generated price movements to resemble the actual price movements; the more closely we wish them to resemble each other, the finer the divisions of our price have to be. (b) How widely the actual price fluctuated during the period under investigation; the more widely the actual price fluctuated, the wider the range of our price values has to be. (c) How much computer time we can use; the more computer time we have, the finer the divisions and the wider the ranges of our price values can be.

2. Which initial set of $\{N(i)\}$ should we choose? If we are short of computer time (so that we cannot afford to let it run until the "equilibrium" values of $\{N(i)\}$ are reached), then the values of the initial $\{N(i)\}$ will definitely affect the outcomes of the simulation.

3. Which metric should we use as a proper measure of the deviation between the actual data and generated data?

4. How should we modify our model, based on the value of the observed deviation between the actual data and the generated data, so as to make the process converge?

IV.

In what follows, we shall describe our first attempt to carry out the proposed simulation model of the stock market.

To begin with, we shall assume that there are two kinds of forces that affect the market. One kind of force affects the traders collectively; it will be called the market force of the first kind and can be classified into two categories: (1) fundamental factors and (2) technical factors. The fundamental factors include the performance and outlook of the economy, the movement of interest rates, the flow of funds into and out of the market, the earnings prospects of a company, etc. These are assumed to be largely reflected by monthly changes in stock price. The technical factors include the commonly held notions of "over-bought" and "over-sold" conditions which tend to result in "profit taking" and "bargain hunting" in the market. The technical factors are assumed to be best reflected by weekly changes in stock price. The other kind of force affects the traders individually. It will be called the market force of the second kind and includes things such as advance inside information concerning the company, sudden emergency needs for cash, seeking investment opportunities as a result of large monetary windfalls, brokers' sales pressure, hot tips, rumors, etc.

For simplicity, we shall assume that the market force of the first kind affects all the traders uniformly. In what follows we shall specify how the fundamental and the technical factors affect the traders' demand prices.

1. Fundamental Factors

Let α be the fundamental coefficient; then at the beginning of the t-th month, all the traders will increase their demand prices by $\alpha g(t)$ percent each trading day over the next 20 trading days (there are approximately 20 trading days in a month), where $g(t)$ is a random variable whose probability distribution depends on the monthly closing prices of the previous three months. Let $S_f(t) = dP_u(t) - dP_u(t-1)$,

$$\text{where } dP_u(t) = \frac{P_u(t) - P_u(t-1)}{P_u(t-1)}$$

and $P_u(t)$ is the closing price of the t-th month; then the conditional probability distribution of $g(t)$ will be described by the following matrix:

	g(t) -2%	-1%	0%	1%	2.5%
$S_f(t-1)<-50\%$	0.3	0.3	0.2	0.1	0.1
$-50\% \leq S_f(t-1)<-15\%$	0.2	0.3	0.2	0.2	0.1
$-15\% \leq S_f(t-1)<20\%$	0.1	0.3	0.2	0.3	0.1
$20\% \leq S_f(t-1)<100\%$	0.1	0.2	0.2	0.3	0.2
$S_f(t-1) \geq 100\%$	0.1	0.1	0.2	0.3	0.3

That is, if $-50\% \leq S_f(t-1) < -15\%$, $g(t)$ will assume the value -2% with probability 0.2, the value -1% with probability 0.3, etc.

2. Technical Factors

Let β be the technical coefficient; then at the beginning of the t-th week, all the traders will increase their demand prices by $\beta h(t)$ percent each trading day over the next five trading days (there are generally five trading days in a week), where $h(t)$ is a random variable whose probability distribution depends on the weekly closing prices of the previous three weeks.

$$\text{Let } S_c(t) = dP_w(t) - dP_w(t-1),$$

$$\text{where } dP_w(t) = \frac{P_w(t) - P_w(t-1)}{P_w(t-1)},$$

and $P_w(t)$ is the closing price of the t-th week; then the conditional probability distribution of $h(t)$ will be described by the following matrix:

	h(t) -10%	-8%	0%	10%	12%
$S_c(t)<-20\%$	0.4	0.2	0.2	0.1	0.1
$-20\% \leq S_c(t)<-8\%$	0.2	0.4	0.2	0.1	0.1
$-8\% \leq S_c(t)<10\%$	0.1	0.2	0.4	0.2	0.1
$10\% \leq S_c(t)<25\%$	0.1	0.1	0.2	0.4	0.2
$S_c(t) \geq 25\%$	0.1	0.1	0.2	0.2	0.4

Let $y_i(t)$ be the percentage of traders with demand price P_i on the t-th trading day who will change their demand price. We shall assume that the probability distribution of $y_i(t)$ depends on (a) how active the traders with demand price P_i are (the more active the traders, the more likely the value of $y_i(t)$ will be high); (b) the percentage change of the daily volume of sales on the previous trading day (the greater the percentage change of the daily volume of sales on the previous trading day the more likely the value of $y_i(t)$ will be high); and (c) the expectation of the daily price movement (to be explained later).

We shall define an active trader to be a trader who follows the stock market closely. Thus an active trader's demand prices are usually close to the market equilibrium price. We shall measure how active a trader is by how close his demand price is to the market equilibrium price. If $P_i < P_e(t)$, then the degree of activeness of the traders with demand price P_i on the t-th trading day is given by

$$z_i(t) = \frac{P_e(t) - P_i}{P_e(t) - P_1}\ ; \text{ if } p_i \geq p_e(t), \text{ then}$$

$$z_i(t) = \frac{P_i - P_e(t)}{P_m - P_e(t)},$$

where P_1 is the smallest demand price, and P_m is the largest demand price. It is clear that the smaller the value of $z_i(t)$, the more active the traders with demand price P_i are on the t-th trading day.

Let $dV(t)$ be the percentage change of the daily volume of sales on the t-th trading day; then

$$dV(t) = \frac{V(t) - V(t-1)}{V(t-1)},$$

and the higher the value of $dV(t)$ the more likely the value of $y_i(t+1)$ will be high.

Suppose that we are at the beginning of the t-th trading day, the j-th month and the k-th week. First, we put into effect the market force of the first kind by setting

$$P_i(t) = [\alpha g(j) + \beta h(k)]P_i(t-1)$$

for $i = 1,\ldots,m$; where $P_i(t)$ is the i-th demand price defined on the t-th trading day. That is, all the traders with demand price $P_i(t-1)$ on the (t-1)-th trading day will change their demand price to $P_i(t) = [\alpha g(j) + \beta h(k)]P_i(t-1)$ on the t-th trading day. Let $P_s(t) = [\alpha g(j) + \beta h(k)]P_e(t-1)$; $P_s(t)$ can be interpreted as the transitory market equilibrium price for the t-th trading day resulting from the market force of the first kind. Let $r(t) = \alpha g(j) + \beta h(k)$; $r(t)$ will be used to serve as the indicator of the expected daily price movement.

The dependence of the probability distribution of $y_i(t-1)$ on $r(t)$ will be as follows: If $P_i(t-1) < P_e(t-1)$, then the higher the value of $r(t)$ the more likely the value of $y_i(t-1)$ will be high. This is so because of a built-in bullish psychology among the traders. If $P_i(t-1) \geq P_e(t-1)$, then $r(t)$ will affect the probability distribution of $y_i(t-1)$ if $r(t)$ has a large negative value, say -10% or less. This is so because most traders hate to take losses unless they are scared by sharp drops in the market price.

After determining what percentage of traders with demand price $P_i(t)$ on the t-th trading day will change their demand price, let us specify what demand price they will change to on the (t+1)-th trading day. The following will be assumed: If the demand price of a certain group of traders following the effect of the market force of the first kind is less than the transitory market equilibrium price, then a certain percentage of the traders in this group will further change their demand price to some demand price equal to or higher than $P_s(t+1)$; if the demand price of a certain group of traders following the effect of the market force of the first kind is equal to or more than the transitory market equilibrium price, then a certain percentage of the traders in this group will further change their demand price to some demand price less than $P_s(t+1)$.

Due to a shortage of time, we have not been able to incorporate all the features of our model as specified above. Although several sample runs have been made, we prefer not to include them here (in order to avoid giving any possible misleading impressions) until we have had a chance to make a reasonable amount of statistical analyses of our sample data and to make the appropriate modifications of our model.

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