On a "Counter - Example"

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In [1] Deb discusses the thesis of Shapiro [2] and gives a "counter-example" to a result found therein. Deb misunderstood the generality of the theorems and his counter-example is not valid. The questions is about skewing schemes, viz. functions which map array elements to parallel memory modules. Shapiro claims "for this important case . . . if there does not exist a linear skewing scheme using N memory modules . . . then there is no valid skewing scheme of any type whatsoever." A linear skewing scheme is a function which maps the ijth element of a matrix to the memory module $(ai + bj) \bmod N$, where a and b are appropriate constants.

The "important case" is a skewing scheme which for each row, column and diagonal maps the elements into distinct modules, allowing conflict-free access to those "lines." This pertains not only to the two main diagonals but to all the "wrap-around" diagonals as well. The concept of a wrap-around diagonal can be reinforced by imagining the right and left edges of the matrix to be identified, as well as the top and bottom edges [2, p.48]. Deb did not consider these wrap-around diagonals and his counter-example is not valid for these "lines."

It is possible, however, that for a particular problem the wrap-around diagonals are not relevent and we desire a skewing scheme which is valid only for the main diagonals of the matrix. Deb indicated he would investigate this problem but it has been solved as a combinatorics problem. Wlog assumes the dimension of the matrix is $N \times N$. Construct another $N \times N$ matrix S such that $s_{ij} = k$ if the ijth matrix element is assigned to module k, i.e. S is the skewing scheme. If the skewing scheme is valid for each row and column then S is a latin square. If further it is valid for the main diagonals, S is a "diagonal" latin square. Denes and Keedwell [3, p.203] report the following theorem which has several independent proofs:

"Theorem 6.1.3 For $n \geqslant 4$ there exists at least one diagonal latin square of order n."

If further, the wrap-around diagonals are valid then S is a "pandiagonal" latin square. Shapiro [4] reports on this aspect.

The particular example Deb gave is a famous arrangement of mutually orthogonal diagonal latin squares (e.g.[5] p. 191) but, superficially, there is no relationship to the ideas presented here.

REFERENCES:

[1] Ashoke Deb, Conflict-Free Access of Arrays-a Counter-Example, Computer Architecture News, V. 7, N. 11, Dec. 1979, pp.14-5.

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[3] J. Denes and A.D. Keedwell, Latin Squares and their Applications, Academic Press, 1974.

[4] Henry D. Shapiro, Generalized Latin Squares on the Torus, Discrete Mathematics, V. 24, 1978, pp. 63-77.

[5] W.W.R. Ball and H.S.M. Coxeter, Mathematical Recreations and Essays, University of Toronto Press, 1974.