AN ALTERNATE CHARACTERIZATION OF PREORDER, INORDER, AND POSTORDER

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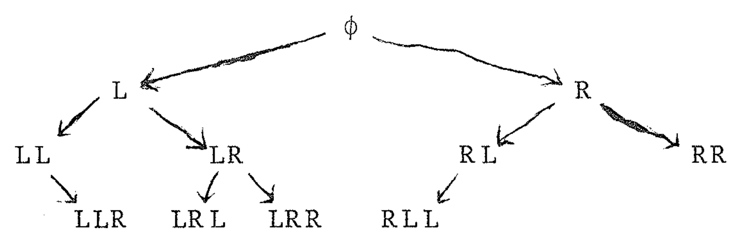
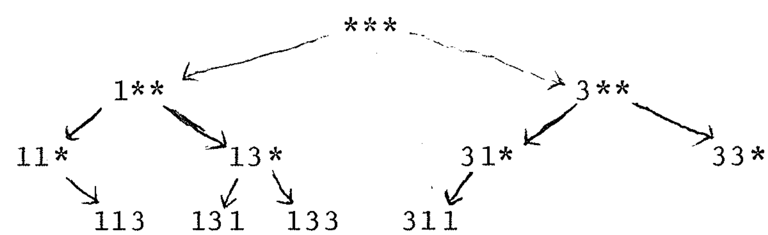
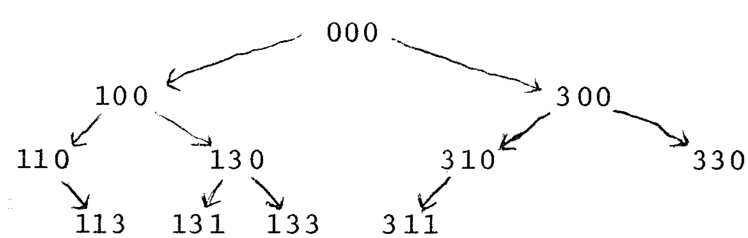
With every node in a binary tree we may associate a sequence of L's and R's representing the decisions (left or right) which are made while traversing the path to that node from the root. A typical tree with its associated sequences is shown below:



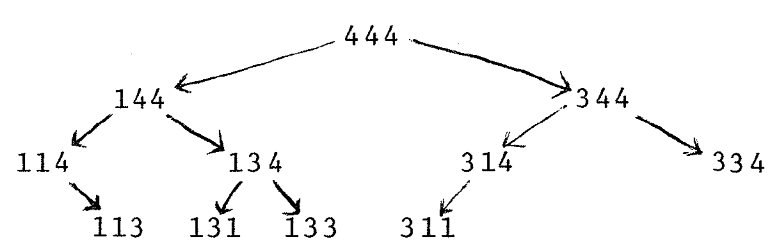
Now replace L and R by 1 and 3, and append asterisks to each sequence until all the sequences are the same length:



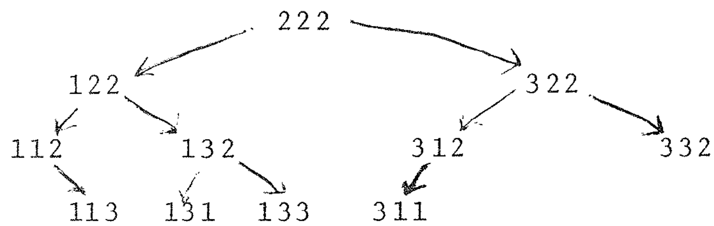
The preorder, inorder, and postorder can now be read off, using three variations on a single idea. To obtain the preorder, replace * by 0 (a digit that comes <u>before</u> 1 and 3):



and the ordinary numerical order becomes the preorder: 000, 100, 110, 113, 130, 131, 133, 300, 310, 311, 330. To obtain the postorder, replace * by 4 (a digit that comes <u>after</u> 1 and 3):



and the ordinary numerical order becomes the postorder: 113, 114, 131, 133, 134, 144, 311, 314, 334, 344, 444. Finally, to obtain the inorder, replace * by 2 (a digit that comes between 1 and 3):



and the ordinary numerical order becomes the inorder: 112, 113, 122, 131, 132, 133, 222, 311, 312, 322, 332.

One may easily verify that these properties hold for arbitrary binary trees. In fact, consider the left subtree in the example. The strings for this subtree, as a tree, are obtained by deleting the first digit (1) from each string as shown. This corresponds to subtracting 100 (or, in general, 10^{d-1}, where d is the string length) from each string -- an operation which clearly preserves the numerical order. By induction, the preorder (inorder, postorder) within the left subtree may be obtained in the manner described above, and similarly for the right subtree (here it is $3 \cdot 10^{d-1}$ which is subtracted). It remains only to show that the root comes before (between, after) the subtrees in numerical order; but this follows immediately from the choice of the digit 0 (2, 4), completing the proof.