Backlash Compensation for Plants With Saturating Actuators

DOI:
10.1177/0959651817692471

Document Version
Accepted author manuscript

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Journal Title
XX(X):1–17

DOI: 10.1177/ToBeAssigned
www.sagepub.com/
SAGE

Ma. del Carmen Rodríguez-Liñán[1] and William Heath[2]

Abstract

A compensation scheme is proposed for plants subject to both saturation and backlash in series at the actuator. It is shown that the series connection of a backlash inverse, saturation and the backlash itself is equivalent to a saturation function at a different level to the original saturation; we call this the 'saturation equivalence'. Therefore it is proposed to include such a backlash inverse in the compensator. This is shown to be optimal in a sense defined in the paper. It is then straightforward to devise a compensator scheme based on traditional anti-windup. This is illustrated in both simulation and on a laboratory scale rig with severe backlash. A correction for the chattering observed in the control signal due to the discontinuous nature of the nonlinearity inverse is also presented.

Keywords

Control system design, non-linear control, backlash compensation, backlash inverse, anti-windup, mechanical control systems

1 Introduction

Physical plants are commonly affected by nonlinearities such as backlash, while actuator saturation is always present. The occurrence of such nonlinearities in a given plant usually causes a degradation of the closed-loop system's performance; without appropriate compensation mechanisms (such as inversion or anti-windup) such

[1] Cátedra CONACyT at Instituto Tecnológico de Ensenada.

[2] School of Electrical and Electronic Engineering, University of Manchester.

Corresponding author:

Ma. del Carmen Rodríguez-Liñán, Instituto Tecnológico de Ensenada,

degradation can be catastrophic. In this paper a novel compensation scheme is proposed for plants that suffer from both saturation and a backlash nonlinearity in the actuator, as in Figure 1.

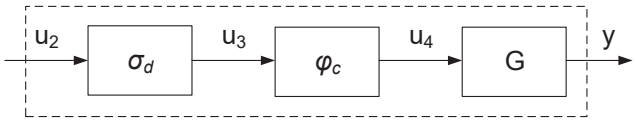

Figure 1. Studied plant dynamics, represented by three blocks in series. The block σ_d represents actuator saturation and φ_c represents the backlash nonlinearity. The block G represents the remaining dynamics, assumed to be linear. Here u_2 represents the signal delivered to the plant, while u_3 and u_4 are internal states.

1.1 Compensation strategies for backlash

Backlash is a dynamic nonlinearity. It can produce delays in the output with respect to the input and loss of information. Typical examples of backlash are mechanical gears, robotic arms, servo-lenses etc, as discussed by Tao and Kokotović [1] whose treatment has greatly influenced the control literature for such nonlinearities when they occur in actuators. They propose the construction of an adaptive inverse, such that the effects of backlash are cancelled. Then, a control structure is used to achieve the required performance.

Several applications of this idea can be found in the literature. Zhou et al. in [2] construct a backlash inverse together with a backstepping adaptive controller. Agrawal et al. [3] provide a different version of a backlash inverse and use it in a robotic arm. In [4] Selmic and Lewis use neural networks and backstepping to calculate and compensate for the backlash. Consideration of plants with either backlash or saturation can be found in [5] using PID fractional algorithms and describing function analysis. Nordin and Gutman in [6] provide a detailed survey on compensation strategies for backlash.

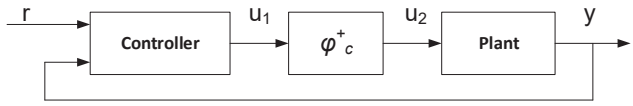

Figure 2. Compensation for plants with actuator backlash. As in Figure 1, u_2 represents the signal delivered to the plant. The block φ_c^+ represents a backlash right inverse. The controller is then designed for the linearised dynamics from u_1 to y.

1.2 Saturation and anti-windup

Physical systems are always bounded by actuator constraints; these could be limits on speed, voltage, torque, etc. If the signal commanded by the controller exceeds the limits of the actuator, then actuator saturation takes place and the efficiency of the controller will be reduced. The saturation characteristic can be described as a memoryless nonlinearity that maps small input signals linearly to the

Boulevard Tecnológico 150, Ex–Ejido Chapultepec, 22780, Ensenada, Baja California, Mexico.

Email: macaroli270@yahoo.com.mx

output. When the input exceeds a limit value $\pm d$, the output remains constant and equal to the limit, referred to as the saturation level. When this happens, the action of the controller is restricted leading to loss of performance and possibly instability of the system.

A 'quick fix' to the saturation problem would be to enlarge the actuators so that the limits are never exceeded by the control action. However, this is usually an impractical and unrealistic solution [7]. More efficient is to design the controllers with the saturations in mind to exploit the capacity of the actuator in full. However, this does not come freely because it can give rise to the 'windup' problem. This refers to the damage to the controller's performance when the constraints are exceeded, [7],[8]. Strategies that are meant to solve the 'windup' problem are traditionally called 'anti-windup' compensators [7].

The basic structure of an anti-windup augmentation is shown in Figure 3, where u_0 is the output of the unconstrained controller and $u_1 = \sigma_d u_0$. When the actuator is not saturated, the controller with anti-windup augmentation should yield the same response as the unconstrained controller. Conversely, when the saturation is active the output should approximate the unconstrained response in the best possible way. These two properties, called respectively *small signal preservation* and *large signal recovery*, are the basic properties any anti-windup augmentation must satisfy, [7],[8].

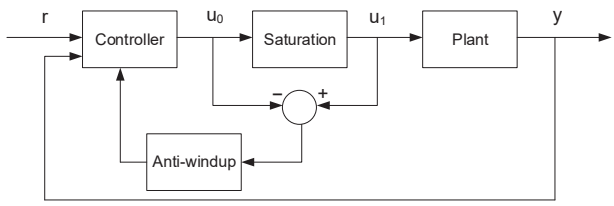

Figure 3. Basic anti-windup structure, r represents the reference signal, y is the output of the plant, u_1 is the output of the unconstrained controller, and $u_1 = sat(u_0)$ is the signal delivered to the plant.

Different anti-windup strategies, ranging from the very simple [9],[10] to the more sophisticated, [11],[7],[8], can be found in the literature.

1.3 Compensation for combined backlash and saturation

Design recommendations for plants exhibiting both saturation and backlash are more rare. Zabiri and Samyudia in [12]-[13] propose using MPC (model predictive control) for a plant subject to backlash and saturation. They use constraints to characterize the backlash inverse and the saturation. These constraints are included in the MPC design requiring the online solution of a mixed-integer quadratic program. A stability analysis for control systems with such nonlinearities can be found by [14], [15] using Lyapunov functions and LMIs.

Utilizing generalized sector conditions, stability for systems with backlash and saturation is studied in [16]. Specifically, the focus is on systems with saturation in the

input and backlash in the output. The authors obtain a Lyapunov function to derive local or global stability of the equilibrium.

1.4 Contribution

This paper proposes a novel compensation strategy for plants that exhibit both backlash and saturation at the actuator. It is shown that the series interconnection of a backlash inverse, a saturation and the backlash itself results in a simple saturation, albeit with different bounds to the original saturation; this property is defined as "saturation equivalence". Furthermore the implementation of such a backlash inverse is shown to be optimal in a sense defined in the paper. The resulting nonlinearity can be compensated with a controller suitable for actuator saturation such as anti-windup. We illustrate the idea with simple PID anti-windup in both simulation and applied to a laboratory rig suffering from severe backlash.

The right inverse backlash is a discontinuous function everywhere. This means that chattering can appear in the control signal. This, however, is overcome using a spatial regularization technique similar to those discussed by Johansson et al. in[17].

Some preliminary results were presented by Rodríguez-Liñán and Heath in [18] where a similar control strategy was also proposed for plants with both deadzone and saturation. An MPC design based on the same results was proposed by the same authors in [19].

2 Description of nonlinearities

2.1 Backlash operator

Following [1] let φ_c be the backlash operator mapping u to v, characterized as

$$\dot{v}(t) = \begin{cases} \dot{u}(t), & \text{if } \dot{u}(t) > 0 \text{ and } v(t) = u(t) - c, \\ & \text{or } \dot{u}(t) < 0 \text{ and } v(t) = u(t) + c, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

with $c > 0$, where $\dot{u}(t), \dot{v}(t)$ represent the time–derivatives of $u(t)$ and $v(t)$, respectively. Figure 4 illustrates (1).

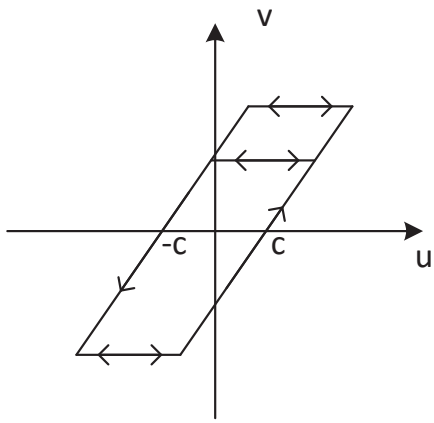

Figure 4. Backlash, φ_c.

2.2 Inverse backlash

Again following [1] we define the right inverse of an operator in the following sense.

Definition 1. *Let φ and φ^+ be operators such that*

$$v = \varphi^+(u) \text{ and } w = \varphi(v) \text{ for some } u, v \text{ and } w. \quad (2)$$

Then φ^+ is the right inverse of φ if

$$w(t) = u(t) \text{ for all } t \geq t_0 \text{ whenever } w(t_0) = u(t_0). \quad (3)$$

Following [1], let φ_c^+ be the operator mapping u to v, characterised as

$$\dot{v}(t) = \begin{cases} \dot{u}(t), & \text{if } \dot{u}(t) > 0 \text{ and } v(t) = u(t) + c, \\ & \text{or } \dot{u}(t) < 0 \text{ and } v(t) = u(t) - c, \\ 0, & \text{if } \dot{u}(t) = 0, \\ g(t,t), & \text{if } \dot{u}(t) > 0 \text{ and } v(t) = u(t) - c, \\ -g(t,t), & \text{if } \dot{u}(t) < 0 \text{ and } v(t) = u(t) + c, \end{cases} \quad (4)$$

where

$$g(\tau,t) = 2c\delta(\tau - t), \quad (5)$$

with $\delta(t)$ as the Dirac δ–function. Thus, $g(t,t)$ in equation (4) indicates that $g(\tau,t)$ is illustrated with $\tau = t$.

The operator φ_c^+ is illustrated in Figure 5.

Lemma 1. [1]. *The operator φ_c^+ is the right inverse of φ_c.*

2.3 Saturation

Let σ_d be the operator mapping u to v, characterised as

$$v(t) = \text{sat}_d(u(t)) \quad (6)$$

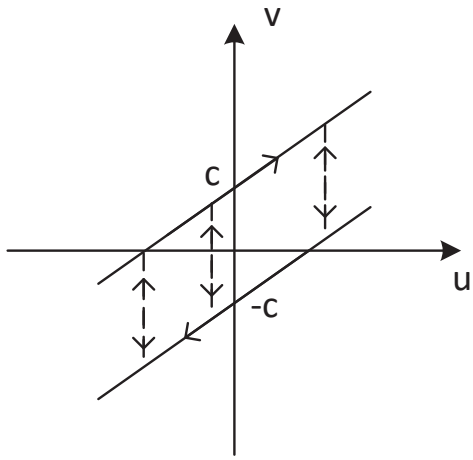

Figure 5. Right inverse of backlash, φ_c^+.

where

$$\text{sat}_d(u(t)) = \begin{cases} d, & \text{if } u(t) > d, \\ u(t), & \text{if } |u(t)| \leq d, \\ -d, & \text{if } u(t) < -d, \end{cases} \quad (7)$$

with $d > 0$.

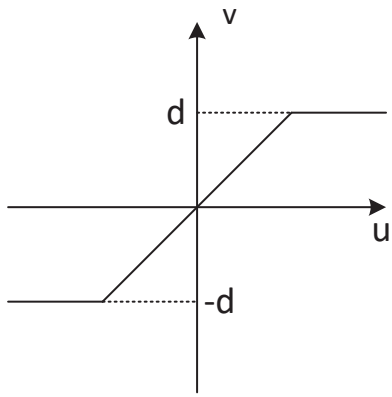

Figure 6. Saturation.

3 Proposed control structure

3.1 Saturation equivalence

The main technical result of this paper is to show that the combination of the inverse nonlinearity with a saturation and the nonlinearity itself results in a saturation at a different level.

Theorem 1. *Suppose for all continuous and differentiable signals u*

$$y = \varphi_c(w),\ w = \sigma_d(v) \text{ and } v = \varphi_c^+(u),\ \text{with } d > c > 0. \tag{8}$$

Then if

$$y(t_0) = sat_{d-c}(u(t_0)) \text{ for some } t_0, \tag{9}$$

then

$$y(t) = sat_{d-c}(u(t)) \text{ for all } t \geq t_0. \tag{10}$$

Proof. Three cases are considered.

Case 1. Suppose $|u(t)| \leq d-c$ for some interval $[t_1,t_2]$. Then $w(t) = v(t)$ for all $t \in [t_1,t_2]$. It follows that if $y(t_1) = u(t_1)$ then $y(t) = u(t)$ for all $t \in [t_1,t_2]$.

Case 2. Suppose $u(t) \geq d-c$ for some interval $[t_1,t_2]$. Then $v(t) \geq d-2c$ and $d-2c \leq w(t) \leq d$ for all $t \in [t_1,t_2]$. If $y(t_1) = d-c$ then $\dot{y}(t) = 0$ for all $t \in [t_1,t_2]$ and hence $y(t) = d-c$ for all $t \in [t_1,t_2]$.

Case 3. Suppose $u(t) \leq -(d-c)$ for some interval $[t_1,t_2]$. Then similar to Case 2, $y(t) = -(d-c)$ for all $t \in [t_1,t_2]$.

To complete the proof the transitions between levels must be considered. Only transitions between when $|u(t)| \leq d-c$ and $u(t) \geq d-c$ (in both directions) will be considered. Conditions for transitions between when $|u(t)| \leq d-c$ and $u(t) \geq -(d-c)$ follow by symmetry. Since $u(t)$ is continuous, these are sufficient to show that (9) $\Rightarrow$ (10).

Transition 1. Suppose $|u(t)| \leq d-c$ for some interval $[t_1,t_2]$ and $u(t) \geq d-c$ for some interval $[t_2,t_3]$. If $y(t_1) = u(t_1)$ then $y(t) = u(t)$ for all $t \in [t_1,t_2]$; in particular $y(t_2) = d-c$. Hence $y(t) = d-c$ for all $t \in [t_2,t_3]$.

Transition 2. Suppose $u(t) \geq d-c$ for some interval $[t_1,t_2]$ and $|u(t)| \leq d-c$ for some interval $[t_2,t_3]$. If $y(t_1) = d-c$ then $y(t) = d-c$ for all $t \in [t_1,t_2]$; in particular $y(t_2) = u(t_2)$. Hence $y(t) = u(t)$ for all $t \in [t_2,t_3]$.

With some slight abuse of notation, it can be said that

$$\varphi_c \sigma_d \varphi_c^+ = \sigma_{d-c}. \tag{11}$$

Since

$$\sigma_{d-c}\sigma_{d-c} = \sigma_{d-c}. \tag{12}$$

it follows immediately that one can say, with the same abuse of notation,

$$\varphi_c \sigma_d \varphi_c^+ \sigma_{d-c} = \sigma_{d-c}. \tag{13}$$

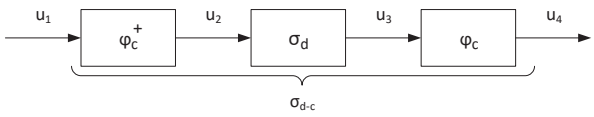

Figure 7. Series interconnection of the right inverse φ_c^+ of the nonlinearity φ_c, a saturation and the nonlinear operator φ_c. This is equivalent to a saturation σ_{d-c}.

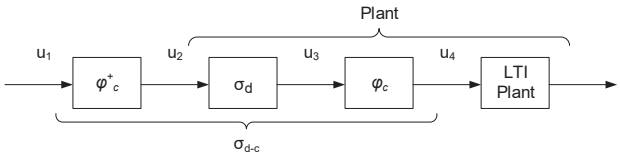

Figure 8. Backlash right inverse φ_c^+ is applied to the plant input.

3.2 Anti-windup control structure

One can take advantage of Theorem 1 and (13) to compensate for saturation and backlash at the input of a linear plant. If an appropriate inverse to the backlash can be constructed, then the overall system can be compensated using standard anti-windup techniques. However care must be taken to set the saturation compensation at the appropriate level.

The compensation scheme proposed in this paper is illustrated in Figure 9. The plant has saturation and backlash at the input so its dynamics are given as

$$y = (G\varphi_c\sigma_d)u_2, \tag{14}$$

where G represents the linear dynamics and u_2 is the signal delivered to the plant. If u_2 is given as

$$u_2 = \varphi_c^+ u_1 \tag{15}$$

for some u_1 then by Theorem 1

$$u_4 = \sigma_{d-c}u_1. \tag{16}$$

Hence the proposal is to generate u_1 using a standard linear compensator together with anti-windup set at level $d-c$. The overall control consists of a right backlash inverse and a linear compensator with anti-windup.

3.3 Optimality of compensation

In the proposed control scheme the input delivered to the plant u_2 is generated using a backlash inverse $u_2 = \varphi_c^+ u_1$, even though the plant itself has both saturation and backlash at its input. The linear controller and anti-windup can then be designed arbitrarily based on the linear plant dynamics G and the saturation level $d-c$. In Section 4 this is illustrated with PID control and simple anti-windup. Nevertheless the structure is optimal in the sense that it ensures the best possible match between u_1 and u_4. Specifically

Theorem 2. *Let φ_c, φ_c^+ and σ_d be defined as in Section 2. Suppose $u_1 \in L_p[0,\infty)$ with $p \geq 1$ and let*

$$u_4 = (\varphi_c\sigma_d\varphi_c^+)u_1 \text{ with } u_4(0) = sat_{d-c}u_1(0). \tag{17}$$

Then for any causal ψ such that $(\varphi_c\sigma_d\psi)u_1 \in L_p[0,\infty)$,

$$\|u_1 - (\varphi_c\sigma_d\psi)u_1\|_p \geq \|u_1 - u_4\|_p. \tag{18}$$

Proof. Define $\tilde{u}_3 = (\sigma_d\psi)u_1$ and $\tilde{u}_4 = (\varphi_c\sigma_d\psi)u_1$. When $p < \infty$, since both u_1 and $\tilde{u}_4$ are Lebesgue measurable, one

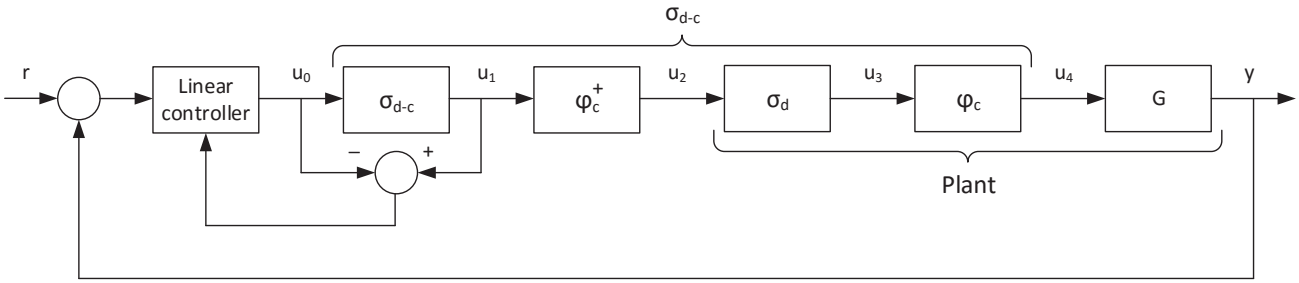

Figure 9. Proposed controller for a system subject to a nonlinearity φ_c and saturation σ_d in the input using a linear controller with back-calculation anti-windup and backlash right inverse. The anti-windup saturation σ_{d-c} has a lower level than the plant saturation σ_d.

may write

$$(\|u_1-\tilde{u}_4\|_p)^p = \int_{\mathscr{T}_0} |u_1(t)-\tilde{u}_4(t)|^p\,dt + \int_{\mathscr{T}_-} |u_1(t)-\tilde{u}_4(t)|^p\,dt + \int_{\mathscr{T}_+} |u_1(t)-\tilde{u}_4(t)|^p\,dt, \quad (19)$$

where $\mathscr{T}_0$, $\mathscr{T}_-$ and $\mathscr{T}_+$ are the Lebesgue measurable subsets of $[0,\infty)$ where $|u_1(t)| \le d-c, u_1(t) < -(d-c)$ and $u_1(t) > d-c$ respectively. Hence

$$(\|u_1-\tilde{u}_4\|_p)^p \ge \int_{\mathscr{T}_-} |u_1(t)-\tilde{u}_4(t)|^p\,dt + \int_{\mathscr{T}_+} |u_1(t)-\tilde{u}_4(t)|^p\,dt. \quad (20)$$

But $|\tilde{u}_4(t)| \le d-c$ for all t (since $|\tilde{u}_3(t)| \le d$ for all t), so

$$(\|u_1-\tilde{u}_4\|_p)^p \ge \int_{\mathscr{T}_-} |u_1(t)+d-c|^p\,dt + \int_{\mathscr{T}_+} |u_1(t)-d+c|^p\,dt. \quad (21)$$

If $u_4 = (\varphi_c^+\sigma_d\varphi_c)u_1$ and $u_4(0) = \mathrm{sat}_{d-c}u_1(0)$ then by Theorem 1, $u_4(t) = \mathrm{sat}_{d-c}u_1(t)$ for all t. It follows that if we u_4 is substituted for $\tilde{u}_4$ then (21) holds with equality. Hence

$$\|u_1-\tilde{u}_4\|_p \ge \|u_1-u_4\|_p, \text{ for any } \tilde{u}_4 = (\varphi_c\sigma_d\psi)u_1. \quad (22)$$

A similar result also follows immediately if we let $p \to \infty$.

Theorem 2 lays the optimality of the construction introduced by Theorem 1 and proposed in Section 3.2. To appreciate this, consider Figure 7 and let $u_4 = (\varphi_c\sigma_d\varphi_c^+)u_1$. Theorem 2 refers to the relationship between u_1 and u_4. In general, one would wish the designed controller to make $u_1 = u_4$ (that is, to 'remove' the nonlinearities of the plant). However, as Theorem 2 shows, when a saturation and a nonlinearity, such as backlash, are present, this may not be possible. The best approximation of u_4 to u_1 is a saturation function σ_{d-c} (i.e., $u_4 = \sigma_{d-c}\mathrm{u}_1$). Notice that the saturation σ_{d-c} is more restrictive than the original plant saturation σ_d.

4 Experimental Results

The results are demonstrated in an HP Deskjet 3000 printer, both in simulation and experimentally. This control kit was developed by [20] at The University of Manchester. The objective of the experiment is to control the position of the printer head, which is driven by a DC motor. The head position is sent to an ELVIS Power Interface Daughter Board through a digital encoder. The Daughter Board sends this signal to the PC using an ELVIS Board. In the PC, the control signal is calculated using LabVIEW, and then sent back to the printer via the ELVIS Board. The experimental setup is shown in Figure 10.

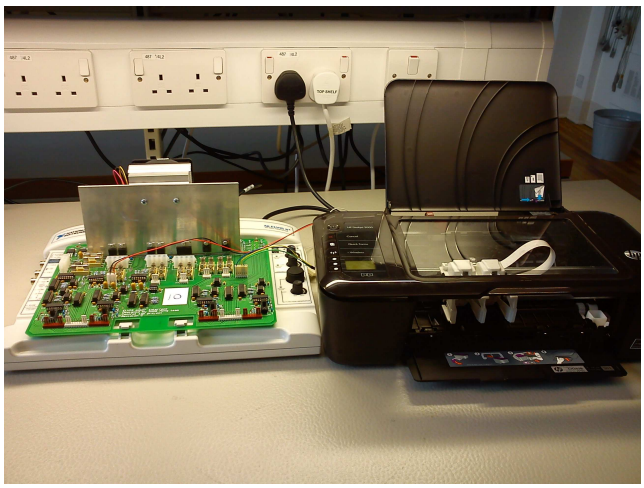

Figure 10. Printer and ELVIS and ELVIS Power Interface Daughter Boards.

The identification of the printer's model yields the following model

$$\hat{G}(s) = \frac{1144.9}{s(s+78.6)}. \tag{23}$$

The system's structure is that shown in Figure 9. Simple PID suffices for the linear controller since the printer model is second order.

4.1 Simulation results

The simulations are run for backlash with saturation to correspond with the experimental case. Three simulations are performed using different variations of the controller, namely:

Scheme 1

(a) PID (saturation active).

(b) PID with anti-windup (saturation active).

Scheme 2

(a) Saturation equivalence with PID with antiwindup (saturation active).

(b) Saturation equivalence with PID with antiwindup (saturation active) and anti–chattering.

Schemes 1(a) and 1(b), do not include a specific compensation for the nonlinearity. They are constructed to compare their response against the proposed controller (saturation equivalence).

The gains of the PID are chosen as $K_p = 2.5, K_i = \frac{K_p}{T_i} = 5$, $K_d = K_p T_d = 0.00025 \approx 0$, with the gain in the anti-windup feedback set as $T_a = T_i$. The anti-windup augmentation compensates for the equivalent saturation sat_{d-c}, like in

Figure 9. The parameter that describes backlash is set to $c = 0.75$ and the saturation level is assumed to be at $d = 3$.

Remark: In the simulations and experiments, the reference $r(t)$ and the output $y(t)$ represent the desired and actual position of the printer head. They are scaled from 0 to 5, where 5 corresponds to the end of the rail. The rail itself is 38 cm, while the printer head measures 8 cm, which means that there is 30 cm movement end-to-end.

Scheme 1(a): The output of the input saturated system using a standard PID is shown in the top part of Figure 11. In this case, the output not only presents an overshoot caused by the saturation in the actuator which winds the integral term of the PID up, it can also be observed that the response oscillates around the setpoint due to the presence of backlash. The control signal in this case appears at the bottom of Figure 11.

Scheme 1(b): Now, the input saturated plant is controlled using a PID with anti-windup. As Figure 12 illustrates, the anti-windup augmentation reduces the overshoot in the response, but it does nothing to compensate for the backlash nonlinearity. The control signal in this case appears at the bottom of the same figure.

Scheme 2(a): Next, the proposed controller, using the inverse backlash and the PID with anti-windup, adjusted to the saturation equivalence σ_{d-c} is built. In this case, the oscillations caused by the backlash nonlinearity are eliminated, obtaining a nice tracking of the reference signal. This is presented in Figure 13 (top). The control signal u, which corresponds to u_2 in Figure 9, is plotted at the bottom of the same figure.

Scheme 2(b): It can be observed from Figure 13, that the signal u shows chattering. The backlash inverse presents discontinuities everywhere, which means that the chattering can occur anywhere. However, the chattering can be avoided, as in the case of deadzone, using an anti-chattering solution which can be classified as spatial regularization, see [17]. The algorithm for such regulator is given in Algorithm 1. In the algorithm, *uap* is a variable indicating the sign of $\dot{u}_1$, and u_s indicates at which side of the backlash inverse the output should be located, depending on the sign of *uap*.

Algorithm 1: Anti-chattering regulator for backlash inverse.

```
Data: uap,usp,c,error
Result: us
if abs(error)≥0.00001 then
    if uap<0 then us=-c;
    if uap>0 then us=c;
    else us=usp;
else
    us=usp;
end
```

Figure 14 (bottom) presents the control signal u when same figure it is clear that the anti-chattering compensator using an anti-chattering solution. From the top plot of the

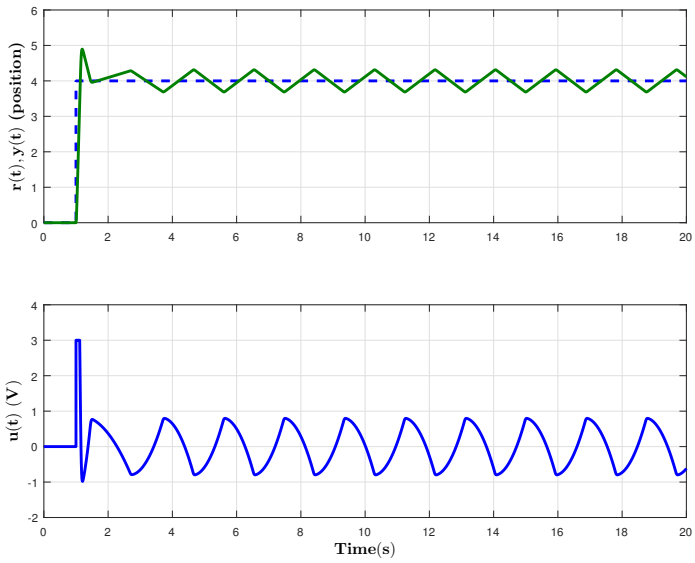

Figure 11. **Scheme 1(a)**. System response for a plant with saturation and backlash in the input, using a PID controller (top) and the corresponding control signal u representing u_2 in Figure 9 (bottom).

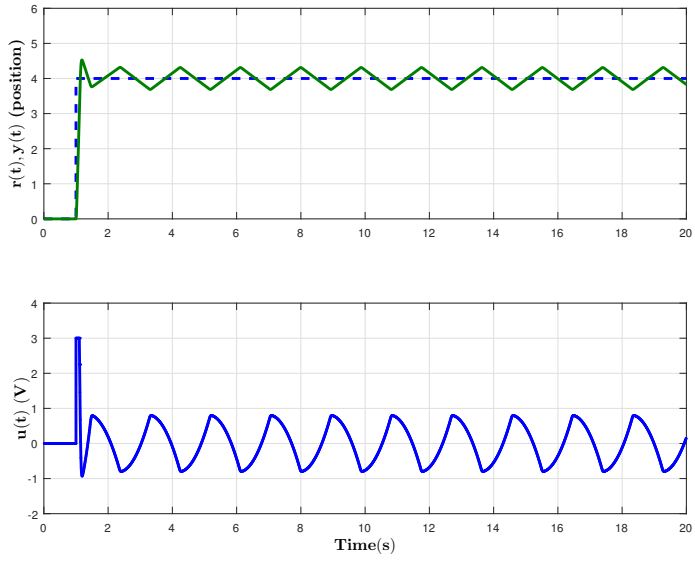

Figure 12. **Scheme 1(b):** System response for a plant with saturation and backlash in the input using a PID controller with anti-windup augmentation (top), and control signal u (which corresponds to u_2 in Figure 9 (bottom).

described in Algorithm 1 does not affect the response of the system.

The example above illustrates the applicability of Theorem 1. In this case, anti-windup is used to exploit the implications of the theorem, though other more specific anti-windup compensators can be used, as described by Tarbouriech et al. in[11] and by Zaccarian and Teel in [8].

4.2 Laboratory results

The results in this section correspond to the experiments carried out on the printer shown in Figure 10 using an anti-windup controller. The plant is first run in open-loop to identify the type of nonlinearity it is subject to. From this first experiment it is concluded that the affecting nonlinearity is a backlash with deadband of size $2c = 1.5$, as can be appreciated in Figure 15. Moreover, the control signal is saturated by a limit in the DC motor voltage of $\pm 3V$. Then, from Theorem 1, the nonlinearities acting on the plant correspond to a saturation given by $d - c = 2.25$.

A set of experiments were performed, first using a standard PID controller with anti-windup, as in Figure 3 and then using the results from section 3. as in Figure 9.

For each controller a reference signal $r(t) = A\sin(\omega t)$, with $A = 1$ and $\omega = 2\pi \text{rad}/s$ was assumed. The PID gains in both cases were set as $K_p = 2.5$, $K_i = 5$ and $K_d = 0.00001$. Note that the derivative actions is practically zero.

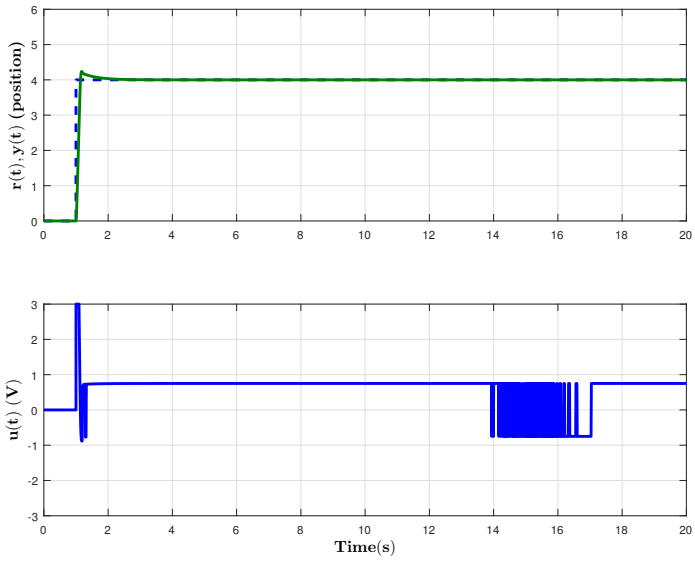

Figure 13. Scheme 2 (a): System response for a plant with saturation and backlash in the input using the proposed saturation equivalence, combined with a PID with antiwindup controller (top), and the corresponding control signal (bottom). Note that chattering in the control signal u, induced by the inverse backlash.

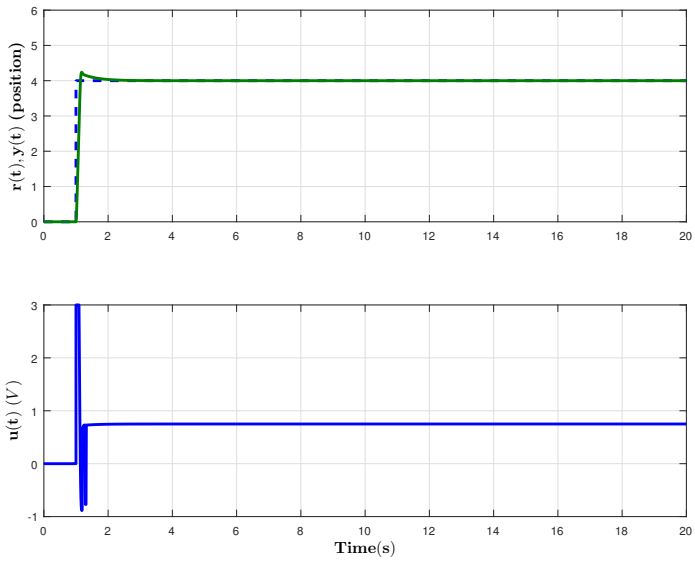

Figure 14. **Scheme 2 (b):**System response for a plant with saturation and backlash in the input using the proposed 'saturation equivalence' controller with anti-chattering regulation (top), and the corresponding control signal (bottom).

The response obtained with the standard PID and the proposed controller are shown in figs. 16 and 17, respectively. From the figures it is easy to see that the inverse backlash compensator offers a better response compared to the standard PID controller. The presence of the backlash nonlinearity could call for a more complex control strategy, however the results of Theorem 1 allow us to use a simple anti-windup scheme that offers improved results.

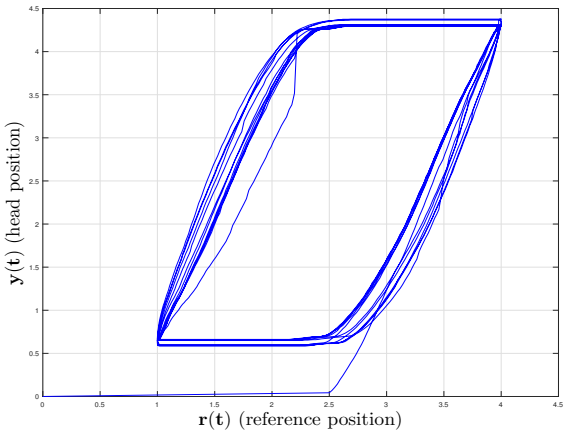

Figure 15. Input versus output plot of the printer in open-loop.

The step response of the closed-loop system using the standard PID with anti-windup is shown at the top part of Figure 18. The control signal in this case is presented at the bottom of the same figure.

Similarly, Figure 19 shows the response to the step using the proposed saturation equivalence controller, combined with a PID with anti-windup. The proposed controller provides better results by removing the oscillations that are evident in the standard PID plus anti-windup case. Note that

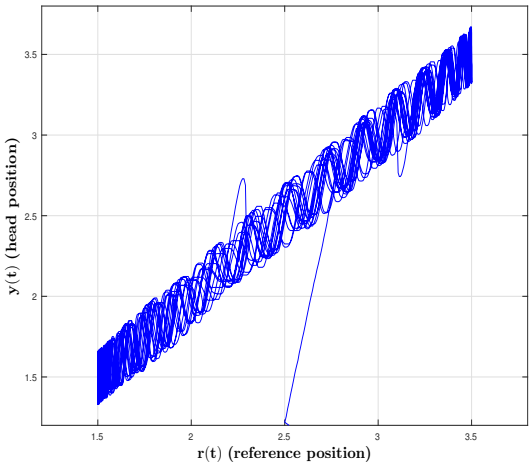

Figure 16. Input versus output plot of the printer in closed-loop using a standard PID with anti-windup compensation.

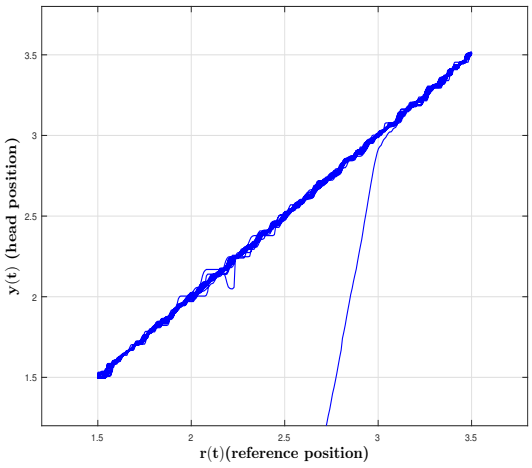

Figure 17. Input versus output plot of the printer in closed-loop using the proposed 'saturation equivalence' controller with anti-windup compensation.

when using only the PID with anti-windup the controller struggles to keep the printer head at the reference, while the proposed controller reaches the steady state effectively.

However, it is observed that when using the proposed controller the control signal u_2 presents chattering in steady-state, as shown in the bottom part of Figure 19.

As in the simulations, a spatial regularization was added to the backlash inverse to remove such chattering. The effects on the control signal u (i.e. u_2 in Figure 9) and the step response can be observed in Figure 20 (bottom and top plots, respectively). Notice that the anti-chattering does not negatively affect the response of the system compared with the case when chattering is present, see Figure 19

5 Conclusions

This paper is dedicated to the control of plants that exhibit saturation and backlash in the input. A result is obtained that shows that the linear interconnection of an inverse nonlinearity, a saturation, and a nonlinearity (backlash), is equivalent to a saturation whose parameters depend on the parameters of the original nonlinearities. It is shown, as well, that this solution is optimal in the sense that when saturation and backlash are present in a plant's input, the best one can expect is to compensate for a saturation σ_{d-c}, which is achieved by building an inverse for the affecting nonlinearity. The results are later demonstrated in simulation and experimental using a basic anti-windup controller to compensate for the obtained saturation.

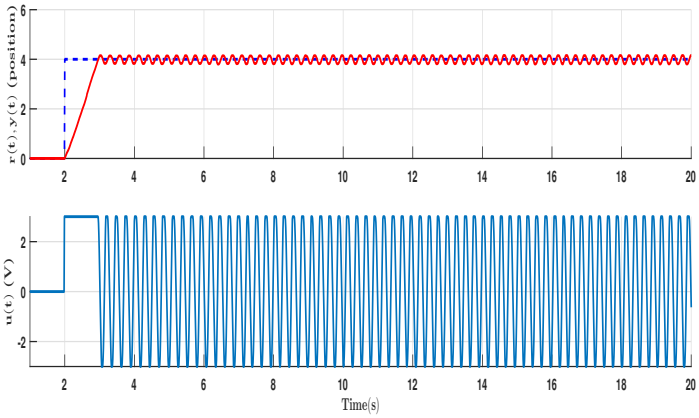

Figure 18. Step response of the plant using a standard PID with anti-windup control (top). Control signal u applied to the plant (bottom).

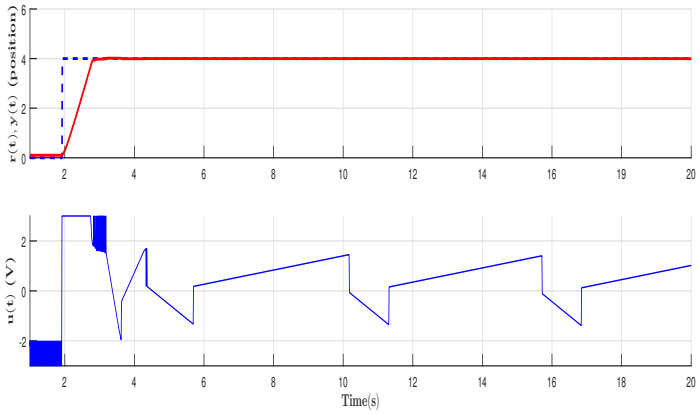

Figure 19. Step response of the system in closed-loop using the proposed saturation equivalence controller with a PID with anti-windup (top). Control signal u when using the saturation equivalence controller with anti-windup compensation (bottom). Note that the signal is affected by chattering in steady state.

Acknowledgements

Authors would like to thank CONACyT for the funding provided for the realization of this work.

Notes

1. Some of the work presented in this paper was done while the first author was at The University of Manchester, United Kingdom, and some while at Universidad Autónoma de San Luis Potosí, Mexico.

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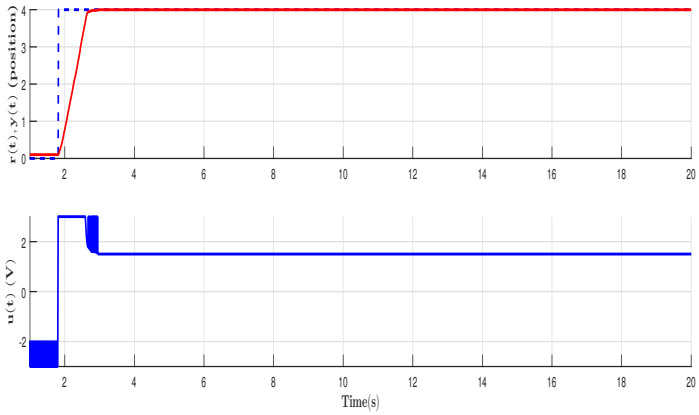

Figure 20. Step response of the system in closed-loop using the saturation equivalence controller with an anti-chattering strategy (top). Control signal u when using the proposed controller with anti-windup and an anti-chattering strategy (bottom).

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