Fast and Distributed Power Control Algorithm in Underlay Cognitive Radio Networks

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Abstract — The power allocation in an underlay cognitive radio network rises up several challenges due to rapid utilization of the available spectrum hole. In this paper, we propose a non-cooperative power-game algorithm to solve the power allocation problem in an underlay cognitive radio network with two main objectives: (i) to provide good quality of service to cognitive radio nodes, and (ii) to protect the transmission of primary users from the interference generated by nearby cognitive radio. These objectives have been assured by including the following constraints: transmit power on each cognitive node, acceptable aggregate interference at the primary' receiver and quality of service at the cognitive radio' receiver. Simulation results show that the proposed algorithm is more convenient in the distributed manner because of its faster convergence in terms of power and the received SINR. Cheating scenario has been tested as well. Moreover, existence and uniqueness for the Nash Equilibrium have been proved mathematically and by simulation as well.

Index Terms—Underlay transmission, cognitive radio, power allocation, Nash equilibrium, pricing theory.

I. Introduction

Recent studies supported by the Federal Communication Commission (FCC) have shown that conventional fixed spectrum allocation policy is becoming insufficient in addressing today's rapidly developing wireless communications, and there is a call for open spectrum access. One attempt of achieving this goal is by introducing dynamic spectrum access and cognitive radio technology which has recently gained great attentiveness by researchers (see [1] for a comprehensive review). Cognitive Radio termed as (CR) is technique in which wireless devices are intelligent enough to sense and discover a specific range from frequency spectrum to identify unused band (also called spectrum hole) by primary user (PUs) in order to execute its tasks efficiently without interfering with the licensed users (i.e., PU).

The analysis of power allocation in cognitive radio networks (CRNs) bring to the researchers some challenges due to the fact of two systems (CRs, and PUs), in conflict of interests, interacting with each other aiming to maximize their own objective. Game theory, on the other hand, shown to be one of the most effective mathematical tools that can break up the challenges associated with power allocation in CRNs.

There are two main resource allocation scenarios in CRNs based on game theory: (i) overlay spectrum sharing, in which cognitive radio users can take the chance and accessing the spectrum rationally when it is not being occupied by the PUs, and thus a spectrum sensing procedure is required to avoid potential collision with the licensed users and (ii) underlay resource sharing where both PUs and (CRs) co-exist on the same geographical area and using the same spectrum band. In the latter case, the transmit parameters for CRs should be allocated carefully to achieve two main objectives: firstly, to maintain the generated interference to PU within an acceptable level. Secondly is to ensure that the quality of service for CRs is at good level.

Disturbing PUs, in contrast, is one of the drawbacks of underlay scenario since CRs coexistence and transmitting together with PUs on the same band. One way to punish CRs from disturbing the transmission of PUs is by using pricing technique. Pricing mechanism (also called punishment mechanism) based on non-cooperative game theory has shown to be very effectual technique for allocating power in CRNs by minimizing the generated interference to the owner of the spectrum (i.e., PUs). Moreover, pricing ability to guide selfish user toward more efficient operating point is another advantages of using pricing mechanism in CRNs [2], [3].

In this work, we address the problem of distributed power allocation in an underlay CRN using non-cooperative game theory based on pricing technique to achieve interference-free environment between CRs and PUs. In order to provide good quality of service (QoS) for CRs, meanwhile protecting the transmission of PUs from the generated interference, we have adopted two interference constraints [4] as follows: (i) local power constraints and (ii) global interference constraints. In the former constraint, the total transmission power for all CRs is limited by the maximum transmission power to be within an upper threshold, while in the latter one, each primary receiver can accept a figure of interference from active nearby cognitive user equal to some defined threshold.

A. Review of the literature

Two main scenarios can be found in the literature for the power allocation problem in CRNs. Firstly, is an

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underlay scenario (e.g., [5]). Secondly, is opportunistic (also called overlay) access scenario (e.g., [6]). The problem of power control based on game theory has widely been explored as in [6]-[16].

In [6], interference temperature is adopted as a main constraint to limit the generated interference to the ongoing PUs. The problem has been formulated and solved using potential game theory which is the most interesting point in this study because the convergence of the NE is guaranteed in potential game. However, the authors in [6] ignore the uniqueness of NE. Non-cooperative power control model based on game theory has been proposed in an underlay spectrum-sharing problem [7]. In ordered to restrict the interference to the licensed users the authors proposed an exponential pricing function to offer practical and fair spectrum sharing among CRs and to minimize the generated interference to PUs. Similarly, authors in [8] proposed algorithm based on non-cooperative game with non-linear pricing for power control in MC-CDMA CR system to offer fair communication environment between CR networks and PU networks. Adopting super-modular game is the main noticeable advantage in [8] where pure strategy equilibrium is achieved. However, the difficulty associated with adopting non-liner pricing function is one of the drawback behind [7] and [8]. Moreover, missing of convergence of the NE is another drawback in both [7&8].

A non-cooperative power control with dynamic pricing algorithm in CDMA CR system has been projected in [9]. An equivalent bandwidth criterion was used to evaluate the interference among CRs and then dynamically adjust the price to be taken from each CR node. However, the uniqueness of the NE has been ignored in this study. A Stackelberg game theory has been applied to design joint pricing and power allocation model in [10] in which both PU and CRs share the spectrum in an underlay manner. Unlike [7], [8] and [9] the authors in [10] proposed an algorithm with less complexity in reaching NE. however, the cheating scenario has been ignored in [10]. A new power control algorithm has been proposed in [11]. The authors proposed a new non-cooperative power algorithm in CRNs with a new cost function that addresses both SINR requirement and the effect of power thresholds in CR nodes. Similar to [10] the cheating scenario has been ignored in [11]. In [12], authors proposed a time-varying price using unique interference equilibrium where the prices are accurately fixed such that all CR users can modify their power dynamically. The novelty of this work is that the pricing is taken from CRs if they produce higher level of interference. Otherwise the pricing function is setting to zero. However, the convergence of the proposed algorithm is slower compared to [9-11]. Novel approach using Variational Inequality (VI) was adopted in [13] for power control to solve the complex constrained problem and to reach a unique NE. However, the convergence of the proposed algorithm is slower compared with those studies in [9-12]. A joint non-cooperative rate, modulation and power control algorithm in an uplink scenario has been proposed in [14]. The authors propose an iterative algorithm to allocate the best power and rate values to the chosen modulation which shows to converge to unique NE. However, the interference among players and the cheating scenario have been ignored in this work. Hence, the proposed algorithm is not suitable for practical cognitive scenario in its current format. An energy-efficient algorithm for power and spectrum allocation in an uplink underlay cognitive radio scenario has been proposed in [15]. The authors proposed a linear and simple pricing function to punish the CRs with high interference and to improve the efficiency of the NE as well. The authors claim that the proposed algorithm converge to unique NE. However, there is no evidence for the uniqueness property in the simulation. Moreover, the cheating scenario has been ignored in this study as well. Finally, cooperative game theory has been adopted in [16] to design a distributed power control algorithm for CRNs. Obviously, the cheating scenario has been ignored in this study because the authors adopting cooperative game theory where the players help each other to allocate their resources.

B. Contributions

Following the presented work in [16], we proposed an efficient distributed power control algorithm in underlay CRNs based on non-cooperative game with controlling pricing. We have emphasized on single channel scenario only. However, the proposed algorithm can be extended to situation in which there is multichannel scenario. This work is driven by the following questions: There are many challenges that must be addressed to implement a cognitive radio in an underlay manner including: (i) how CRs compete with each other to get access with transmission power to the licensed spectrum?; (ii) how to achieve good QoS for cognitive radio network and keep the generated interference to PUs within an acceptable level?; (iii) is there a Nash equilibrium exist? If so is it unique? We answer these questions by making the following contributions:

1) Price based game theory: we have proposed a simple and fast non-cooperative power control algorithm with simple pricing function that resulted in a fast convergence for both power and received SINR.
2) Existence and uniqueness: we derived a distributed-based iterative algorithm to determine the NE in non-cooperative power game. Existence and uniqueness have mathematically been examined.
3) Cheating scenario: to prevent cheating in underlay CRNs, we have proposed a modified version of SINR balancing power control algorithm (SBPC) [17]. The modified version is called *controlling based SBPC* termed as (C-SBPC) algorithm to make sure that all players follow the networks policies.
4) Locally-Joint computed algorithm: the proposed algorithm jointly allocates power with pricing function. Meanwhile, each CRs in the proposed algorithm requires only local information (e.g., its

own utility function and channel gain, nearby channel gain, pricing function and SINR).

Our work is different from that in [16] in the following: (1) work in [16] adopted cooperative game theory and we have adopted non-cooperative game theory. Hence the assumption of strategy spaces and the derivative of existence and uniqueness of the NE are not the same; and (2) we have proposed a new power control algorithm that is differ from [16] to be suitable with the non-cooperative concept and the selfishness of CRs in underlay manner.

C. *Organization of the paper*

The paper is organized as follows. In Section 2, we show the system model including cognitive radio scenario, system setting, interference analysis and QoS analysis. In Section 3, we construct our problem formulation using non-cooperative game theory, set up the pricing function and illustrate the proposed distributed algorithm. The Nash equilibrium concept including existence and uniqueness properties are discussed in Section 4. Simulation results of the proposed algorithm and a comparison study are provided in Section 5. Finally, we conclude this paper with some discussion in section 6.

II. SYSTEM MODEL

2.1 Cognitive Radio Model and system setting

Spectrum sharing scenario in an underlay CRNs involves the following network units:

1) PUs: licensed user which has a license to operate in a certain spectrum band with interference-free environment [18].
2) PUBS: Primary user base station (PUBS) that offer licensed spectrum to PUs. CRs can take the advantages of the unused spectrum band.
3) CRs: Cognitive Radio users that coexist with PUs in an underlay manner and have no spectrum license. They must achieve channel sensing in order to access the available spectrum/channel.

In this work we consider an underlay spectrum-sharing scenario in (CRNs), composed of N CR nodes denoted by $\{CR_i\}_{i=1}^{N}$. Let $\ell_N = \{1,2,...,N\}$ denote the available set of CR links, and $\{f_k\}_{k=1}^{K}$ denotes the available channel. Assuming that a subset of active CR sources ($S_{i\in\ell_N}$), wish to communicate with subset of active CR destination ($D_{i\in\ell_N}$) with the aid of PUBS. The CRs distributed randomly and coexist with PUs in an underlay manner, located in the same area and sharing the same band. The primary user network (PUN), on the other hand, consisting of M active PUs denoted by $\{PU_m\}_{m=1}^{M}$. When CRs and PUs transmits in an underlay manner, mutual interference may arise as shown in Fig. 1. Thus, CR nodes have to firmly control their power to avoid any serious interference with active PUs. The hard lines in Fig.1 specify intended communication links among nodes in CR and nodes in PUNs, while the spotted lines denoted the interference links. Without loss of generality, we assume that no interference generated among PUs.

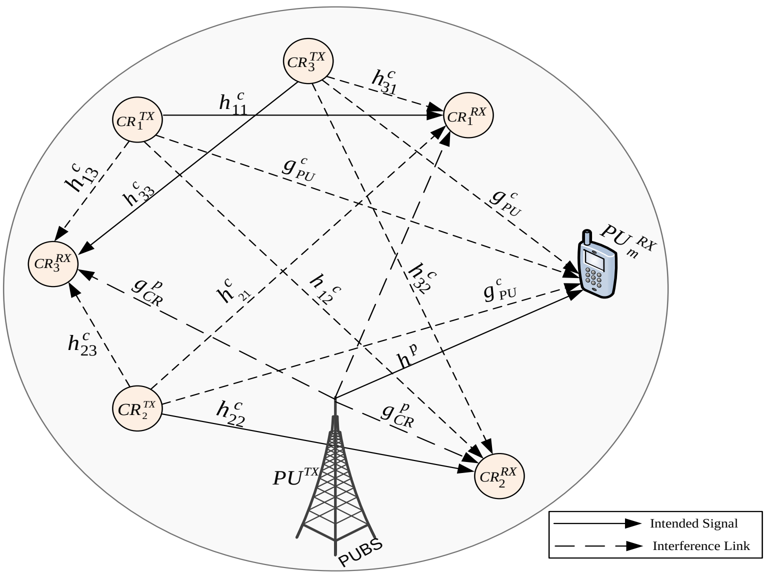

Fig. 1. An example of underlay CRNs with three CR users and one PU [19].

2.2 Interference Characterization and Modeling of QoS

A. *Protecting Primary User*

In underlay scenario, CRs are permitted to utilize the spectrum with PUs. However, this permission resulted in degradation in the PU's performance which can be avoided by introducing interference constraints that uses as a global interference limits for CR' transmission power, (e.g., interference temperature limits [1], [4]) and by adopting pricing technique. Let us denote

$F = \{f_k : k = 1,2,...,K\}$ the set of orthogonal frequency channels in the network, and the transmit power of the *CR* transmitter of the *i*-th link on channel f_k denoted by $P_i^c(f_k)$, where superscripts *c* refer to the transmission from the active CR users.

Let I_m^{th} represent the maximum tolerable interference of *m*-th primary user. Assuming that g_{im} is the channel gain between the transmitter of the *i*-th secondary user and the receiver of the *m*-th PU, and $P_i^{\max}$ is the maximum transmission power constraint for CR_i.

Definition 1: The aggregate interference at the receiver of PU can mathematically be stated according to (1) [4], [12] [13] and [16]. This means that that, each primary receiver can accept a figure of interference from nearby CR but less than the defined threshold (I_m^{th}) in order to keep good QoS for PUs.

$$\sum_{i=1}^{N} P_i^c(f_k) g_{im} \le I_m^{th} \qquad m = 1,...,M \tag{1}$$

Definition 2: The total power for all CRs in the networks must be less than the defined upper threshold (i.e., $P_i^{\max}$) and can be represented mathematically according to (2) [20]. While the total power constraint for each CR user can be written according to

$$\sum_{i=1}^{N} P_i^c(f_k) \le P_i^{\max} \qquad \forall i \in N \tag{2}$$

Since the interference assumption in (1) and the power constraint in (2) are satisfied, the primary network has no cause to prevent the users of cognitive radio network from using the available spectrum holes

B. Modelling of QoS for Cognitive Radio User

In an underlay CRNs, PUs also generates some interference to CRs that cause poverty in the received signal. The generated interference form PU-to-CR can be described according to following definition:

Definition 3: The received SINR for all CRs should be above some defined threshold (e.g. $\bar{\gamma}$) in order to provide reliable transmission opportunities among CR nodes with good QoS.

Following the above definition, let the transmit power of PU_m on channel f_k denoted by $P_m^p(f_k)$, where the superscripts (^{p}) refer to the transmission from the corresponding PU. Assuming that $\mathbf{P} = (P_1^c, P_2^c, ..., P_N^c)$ is a vector of the CR users' transmission power. The SINR of link i on channel (f_k) is defined as

$$\gamma_i^{f_k}(\mathbf{P}) = \frac{P_i^c(f_k) h_i^c}{I(\mathbf{P}_{-i}^c) + \sum_{m=1}^{M} P_m^P(f_k) g_{mi}} \tag{3}$$

Assuming h_i^c is the channel gain between the *i*-th CR transmitter to the intended receiver, the thermal noise denoted by $\eta_i^{f_k}$ and the subscript $(-i)$ used to indicate the interference generated from all CRs except that from *i*-th user. Moreover, the interference model $I(\mathbf{P}_{-i}^c)$ given by

$$I(\mathbf{P}_{-i}^c) = \sum_{j \in 1_N, j \ne i} p_j^c(f_k) h_j^c + \eta_i^{f_k} \tag{4}$$

where the second term in the denominator represented the interference generated from PUs-to-CRs. To simplify the mathematical representation we denote the total interference to CR *i* on channel (f_k) as $I_i^{total}(f_k)$ which includes the interference from competing CR nodes, the interference generated from active PU and the thermal noise. It can equivalently be denoted as follow

$$\gamma_i^{f_k}(\mathbf{P}) = \frac{P_i^c(f_k) h_i^c}{I_i^{total}(f_k)} \tag{5}$$

Not that, the aim of ***definition 3*** is to ensure that no CRs' SINR (γ_i^{fk}) falls below its given threshold ($\bar{\gamma}$) chosen to ensure sufficient QoS requirement, i.e., to maintain

$$\gamma_i^{fk} \ge \bar{\gamma} \quad \forall i \in N \tag{6}$$

III. PROBLEM FORMULATION

In the deployment scenario shown in Fig.1, each user' quality of service is quantified by a utility function $\{u_i^N\}$, which reflects the efficiency of spectrum utilization. Let $\mathbf{P}_i^c = \left[p_i^c(f_1), ..., p_i^c(f_K) \right]^T$ denote the power profile of user CR_i. Furthermore assuming that the strategy space $\hat{P}_i$ of link *i* is a compact convex set bounded with minimum and maximum power denoted by $P_i^{\min}$ and, $P_i^{\max}$ respectively and denoted according to

$$\hat{P}_i = \left\{ \mathbf{P}_i^c : \sum_{i=1}^{N} p_i^c(f_k) \le P_i^{\max},\ P_i^{\min} \le p_i^c(f_k) \le, P_i^{\max}, \forall k \in K \right\} \tag{7}$$

which denote the users' strategy set of allowable powers. Without loss of generality we assume that ($P_i^{\min} = 0,$) for all CR nodes.

A. Game Theoretic Framework Formulation

The fundamental component of game theory is made up of three main components and mathematically given by $G = \langle N, A, \{u_i\} \rangle$ where $N = \{1, 2, ..., N\}$ represent the finite set of decision makers $A = A_1 \times A_2 \times ... \times A_N$ denote the action space available to each CR node and $u_i : A \to R$ represent the utility function.

Definition 4: Non-Cooperative Game Power Allocation problem in an underlay CRNs termed NCGPA defined as $\text{NCGPA} = \left\langle 1_N, \{\hat{P}_i\}_{i \in 1_N}, \{u_i\}_{i \in 1_N} \right\rangle$, in which CR nodes are the players in the network; power level on each link represent the strategy for each player, while, the utility function in NCGPA has been chosen to ensure good QoS for CRs. Mathematically speaking; the utility function can be stated according to [16]

$$u_i(p_i^c, \mathbf{p}_{-i}^c) = \theta_i \sum_{i=1}^{N} \log(\boldsymbol{\gamma}_i) \quad (8)$$

where $\boldsymbol{\gamma}_i = (\gamma_i^{f_k}(\mathbf{P}) - \overline{\gamma})$ [16], $\gamma_i^{f_k}$ is the obtained SINR for link i, $\theta_i > 0$ is a user dependent priority parameter [21]. Thus, by using interference constraints defined in (1) and (2) in addition to QoS constraint defined in (6), the optimization problem can be modeled according to

$$\max_{\{\mathbf{P}: p_i^c \in \hat{P}_i\}} \sum_{i=1}^{N} u_i(\bullet) \quad s.t. \begin{cases} \sum_{i=1}^{N} p_i^c g_{im} \le I_m^{th}, \ m = 1, \ldots, M \\ \sum_{i=1}^{N} p_i^c \le P_i^{\max}, \ \forall i \in N \\ \gamma_i^{f_k} \ge \overline{\gamma}, \ \forall i \in N \end{cases} \quad (\text{P1})$$

B. Pricing Function: Setting and Formulation

In self-organized network, such as underlay CRN, users may behave selfishly aiming to maximize their own objective function by increasing their own power. A common technique to manage the selfishness in CRNs is through pricing mechanism. Accordingly, in this work, pricing technique has been adopted for the following reasons: (1) to encourage CR players to use the network resources (i.e., power) more efficiently by punished those users that behave selfishly or generating more interference to PUs and (2) to reduce the complexity of (P1) by releasing the global interference constraint (C1) as in [16] because the pricing function forcing CRs to reduce their power to be with their strategy space.

Definition 5: pricing function based on non-cooperative power game is termed by P-NCGPA reflect the punishment that player takes if he generate amount of interference to PUs. Mathematically speaking, utility function with pricing can be modeled as

$$u_i^{\Pr}(p_i^c, \mathbf{p}_{-i}^c) = \theta_i \sum_{i=1}^{N} \log\left((\boldsymbol{\gamma}_i) - \Theta_i\right) \quad (9)$$

where $u_i^{\Pr}(\bullet)$ is the net utility function (or the surplus function) for CR user i that can be defined as the difference between its utility function and the payment function $\Theta_i = \sum_{m=1}^{M} p_i^c(f_k).g_{im}$. The superscript ($^{\Pr}$) refers to the pricing utility function. Putting altogether, the net optimization problem with a pricing function is mathematically formulated as

$$\max_{\mathbf{P}_i} u_i^{\Pr}(p_i^c, \mathbf{p}_{-i}^c), \ \forall i \in \ell_N, \text{s.t.} \begin{cases} \sum_{i=1}^{N} p_i^c \le P_i^{\max}, \ \forall i \in N \\ \gamma_i^{f_k} \ge \overline{\gamma}, \ \forall i \in N \end{cases} \quad (\text{P2})$$

C. Proposed power control algorithm

Optimal power allocation strategies are required for CR nodes such that (P2) can be maximized subject to maximum power and QoS constraints. Hence, we have the following proposition:

Proposition 1: if all CRs maximize their surplus function according to (P2), the optimal power strategy is given by

$$p_i^c = \min\left\{\left(\omega_i \frac{p_i^{c(t)}\overline{\gamma}}{\gamma_i^{f_k(t)}}\right), \max\left(0, \ \frac{\overline{\gamma}}{\zeta_i^t} + \frac{\theta_i}{\alpha^t - \beta_i^t \dfrac{\gamma_i^{f_k^{(t)}}}{p_i^{c(t)}}}\right)\right\} \quad (10)$$
$$\forall i \in \ell_N$$

Proof: The Lagrangian relaxation and the (K.K.T) conditions have been adopted to drive the optimal solution of (P2) in a selfish environment. Thus (P2) can be rewritten according to

$$\begin{aligned} \varsigma(\mathrm{p}_i^c, \alpha, \beta_i) = \theta_i \sum_{i=1}^{N} \log\left((\gamma_i) - p_i^c \sum_{m=1}^{M} g_{im}\right) - \\ \alpha\left(\sum_{i=1}^{N} p_i^c - p_i^{\max}\right) - \sum_{i=1}^{N} \beta_i\left(\overline{\gamma} - \gamma_i^{f_k}\right) \end{aligned} \quad (11)$$

$$\alpha \ge 0 \ , \quad \beta_i \ge 0, \ \forall i \in \ell_N$$

where, α, and β_i are the Lagrangian multipliers that can be obtained using subgradient scheme as shown in APPENDIX A. By taking the partial derivative of (11) with respect to (p_i^c and α), $\forall i \in N$, setting the result to zero we have

$$\frac{\partial \varsigma}{\partial p_i^c} = \theta_i \bullet \frac{\left(\dfrac{\partial \gamma_i^{f_k}}{\partial p_i^c} - \sum_{m=1}^{M} g_{im}\right)}{\gamma_i^{f_k} - \overline{\gamma} - p_i^c \sum_{m=1}^{M} g_{im}} - \alpha + \beta_i \frac{\partial \gamma_i^{f_k}}{\partial p_i^c} = 0 \quad (12)$$

$$\frac{\partial \varsigma}{\partial \alpha} = P_i^{\max} - \sum_{i=1}^{N} p_i^c \quad (13)$$

From (13), we note that the power for CR nodes (p_i^c) is constrained by the maximum power ($P_i^{\max}$). Assuming that

$$(\nabla_i = \frac{\gamma_i^{f_k}}{p_i^c} - \sum_{m=1}^{M} g_{im})$$

then by rearranging (12) we can obtain (p_i^c) according to

$$p_i^c = \frac{\overline{\gamma}}{\nabla_i} + \frac{\theta_i}{\alpha - \beta_i\left(\gamma_i^{f_k} / p_i^c\right)} \quad (14)$$

Equation (14) is slightly different compared to that in [16], which comes with user' bargaining powers to reflect the cooperation among nodes. In our case, we have ignored that term from our formulation because our problem based on non-cooperative game and there is no means of cooperation among CR users.

The optimal power can be gained such that each CR maximizes its own surplus function iteratively. Therefore, the iterative technique to update the power for CR nodes has been derived using fixed point method as in [16]. Hence we have

$$p_i^{c(t+1)} = \max\left[0,\ \ \frac{\bar{\gamma}}{\nabla_i^t} + \frac{\theta_i}{\alpha^t - \beta_i^t\left(\gamma_i^{f_k^{(t)}} \middle/ p_i^{c(t)}\right)}\right]^+ \tag{15}$$

Note that, using (15) in non-cooperative CRNs game is not strict enough since the users behave selfishly in the game and some CR players can cheat once the game played by allocating their power somewhat higher than the allowable one. To overcome this crisis, which is common in non-cooperative game, we have modified the SINR balancing power control algorithm (SBPC) in [17] and integrated with our algorithm in (15) to make sure that no CR player can cheat by increasing his own power abruptly and that all CR players are satisfies and their power level is below the defined threshold. The modified algorithm is then called as controlling based SBPC termed as C-SBPC. Mathematically speaking, the C-SBPC can be implemented according to

$$\Delta_i^{c(t)} = \omega_i \frac{p_i^{c(t)}\bar{\gamma}}{\gamma_i^{f_k(t)}} \tag{16}$$

where ω_i is the power controlling factor that force players with cheating behavior to reduce their powers.
Using (15) together with (16) the proposed power allocation strategy can be mathematically writing as

$$p_i^c = \min\left\{(\Delta_i^{c(t)}), (\Omega_i^{c(t)})\right\} \qquad \forall i \in \ell_N \tag{17}$$

where, (Ω_i^c) is known as in (15)

By using (17), the convergence of NE can be guaranteed without any cheating scenario in the game from any CR nodes.

IV. Nash Equilibrium in P-NCGPA

In this section we provide mathematical description of the NE' properties (i.e., existence and uniqueness) as follows:

Definition 6: Nash equilibrium in P-NCGPA can be defined as power vector (e.g., $\mathbf{P}_i^c = [\, p_i^c(1),..., p_i^c(K)\,]$) at which no player can improve its utility function $u_i(p_i^c, \mathbf{p}_{-i}^c)$ by unilaterally changing its own strategy (i.e, p_i^c). Mathematically speaking, Nash equilibrium can be modeled as

$$u_i^{\mathrm{Pr}}(p_i^c, \mathbf{p}_{-i}^c) \geq u_i^{\mathrm{Pr}}(\bar{p}_i^c, \mathbf{p}_{-i}^c), \quad \forall p_i^c \in \hat{P}_i,\ \forall i \in \ell_N \tag{18}$$

A. Existence of NE in P-NCGPA

The Nash equilibrium in P-NCGPA offers a predictable, stable outcome of a game where multiple CR users with conflicting interests compete and reach a point where no CR player requests to change its strategy profile.

Theorem 1: A Nash equilibrium exists in the $\text{P-NCGPA} = \left\langle \mathrm{l}_N, \left\{\hat{P}_i\right\}_{i\in \mathrm{l}_N}, \left\{u_i^p\right\}_{i\in \mathrm{l}_N} \right\rangle$, if it satisfies the following conditions $\forall i \in \ell_N$.

1) The action strategy profile (i.e., p_i^c) is a nonempty, convex, and compact subset of some Euclidean space.
2) The utility function $u_i^{\mathrm{Pr}}(\gamma_i(\mathbf{P}))$ is a continuous and quasi-concave function over the strategy set of the players.

Proof: this can be achieved by showing the two conditions offered in ***theorem 1*** are met in P-NCGPA and the poof can be shown according to:

- Since each CR user has a strategy profile (as defined in (7) that is defined by a minimum power and a maximum power, thus the first condition is readily satisfied.
- To prove the second condition is also satisfied, we have to show that the given price based utility function is quasi-concave in p_i^c, $\forall i \in \ell_N$.

Definition 7 [13], [22]: The utility function $\{u_{i=1}^N\}$ defined on the convex set $\hat{P}_i$ is quasi-concave in P_i^c if and only if the following condition is satisfied

$$\begin{aligned} &u_i(\lambda p_i^c + (1-\lambda)p_i', \mathbf{p}_{-i}) \geq \min(u_i(p_i, \mathbf{p}_{-i}), u_i(p_i', \mathbf{p}_{-i})), \\ &\forall p_i^c, p_i' \in \hat{P}_i \quad \text{and}\, \lambda \in [0,1] \end{aligned} \tag{19}$$

To show this condition is true we have to solve the following set of equations: $\frac{\partial^2 u_i^p}{\partial^2 p_i^c} < 0$, $\forall i$. In the following we have

Lemma 1: The priced-based utility function given in (9) is concave in p_i^c, $\forall i \in \mathrm{l}_N$.

Proof: in order to show that the two variable priced-based utility function shown in (9) is a concave, it is necessary to show that its Hessian matrix $H(\theta_i, p_i) < 0$, thus the first-order necessary optimality condition is known according to

$$\frac{\partial u_i^p(p_i, \mathbf{p}_{-i})}{\partial p_i^c} = 0. \tag{20}$$

Then,

$$\begin{pmatrix} \dfrac{\partial^2 u_i^p}{\partial^2 p_i^c} & \dfrac{\partial^2 u_i^p}{\partial p_i^c \partial \theta_i} \\ \dfrac{\partial^2 u_i^p}{\partial p_i^c \partial \theta_i} & \dfrac{\partial^2 u_i^p}{\partial^2 \theta_i} \end{pmatrix} = \begin{pmatrix} -\theta_i\left(\dfrac{\partial \gamma_i^{f_k}}{\partial p_i^c}\right)^2 & 0 \\ 0 & 0 \end{pmatrix} \tag{21}$$

Since the first element from Hessian matrix is negative, therefore, $u_i^{\mathrm{Pr}}(\gamma_i(\mathbf{P}))$ is continuous and quasi-concave in the strategy profile (p_i^c).

Given that the two conditions of ***theorem 1*** are met, then P-NCGPA is a concave n-player game in which there should be at least one NE exists in the game. ■

B. Uniqueness of NE in P-NCGPA

By ***theorem 1*** we prove that utility function is continuous and quasi-concave in the strategy profile of the players and the strategy profile of each CR player is non-empty, compact and convex, as shown in (7). Thus, NE exists in P-NCGPA. However, a natural question may arises at this point, is the existed NE unique? In the following we prove the uniqueness of the NE.

Definition 8: the best response strategy is an alternative definition of the NE that can be defined according to

$$BR(\mathbf{p}_{-i}) = \left\{ p_i^c \in \hat{P}_i : u_i^p(p_i^c, \mathbf{p}_{-i}^c) \geq u_i^p(\overline{p}_i^c, \mathbf{p}_{-i}^c),\ \forall \overline{p}_i^c \in \hat{P}_i \right\} \quad (22)$$

Which is a set that contain only and only one optimal point that maximize the objective function and can be mathematically represented according to

$$p_i^c = \arg\max_{\mathrm{P_i}} u_i^{\mathrm{Pr}} \quad (23)$$

Furthermore, the second derivative is less than zero (i.e., $\frac{\partial^2 u_i^{\mathrm{Pr}}}{\partial^2 p_i^c} < 0$) as shown in (21) which means that the maximum is unique.

Theorem 2*:* The NE of the non-cooperative game $\text{P-NCGPA} = \left\langle \mathrm{l}_N, \left\{ \hat{P}_i \right\}_{i \in \mathrm{l}_N}, \left\{ u_i^p \right\}_{i \in \mathrm{l}_N} \right\rangle$ is unique.

Proof: The key feature of NE' uniqueness is to prove that the best response function is a standard function .

For the given game $\text{P-NCGPA} = \left\langle \mathrm{l}_N, \left\{ \hat{P}_i \right\}_{i \in \mathrm{l}_N}, \left\{ u_i^p \right\}_{i \in \mathrm{l}_N} \right\rangle$, the best response of the *i*th user, given the power strategy of others (i.e., $\mathbf{p}_{-i}$) is mathematically given by

$$BR(\mathbf{p}_{-i}) = \min(\Delta_i^c, \Omega_i^c), \quad \forall i \in \mathrm{l}_N \quad (24)$$

To show that the NE is unique, the best response function must be a standard function and also have to assure the subsequent properties [23]

1- Positivity: $BR(\mathbf{p}_{-i}) > 0$.
2- Monotonicity: given $\mathbf{p} \geq \tilde{\mathbf{p}}$ then $BR(\mathbf{p}_{-i}) \geq BR(\tilde{\mathbf{p}}_{-i})$.
3- Scalability: given, for all $\varepsilon > 1$, then $\varepsilon \mathrm{BR}(\mathbf{p}_{-i}) > \mathrm{BR}(\varepsilon \mathbf{p}_{-i})$.

The first two properties can be easily verified by the power strategy space given in (7) and by ***theorem 1***. To show the third property, we apply the third condition to (Ω_i^c) only since (Δ_i^c) is a special case and can be easily verified as well, in the following we have

$$\varepsilon(\Omega_i^c) > (\varepsilon \Omega_i^c) = \varepsilon \left[\frac{\overline{\gamma}}{\frac{\gamma_i^{f_k(t)}}{p_i^{c(t)}} - \sum_{m=1}^{M} g_{im}} + \frac{1}{\alpha^t - \beta_i^t \frac{\gamma_i^{f_k^{(t)}}}{p_i^{c(t)}}} \right]^+$$
$$> \left[\frac{\overline{\gamma}}{\frac{\gamma_i^{f_k(t)}}{\varepsilon p_i^{c(t)}} - \sum_{m=1}^{M} g_{im}} + \frac{1}{\alpha^t - \beta_i^t \frac{\gamma_i^{f_k^{(t)}}}{\varepsilon p_i^{c(t)}}} \right]^+ \quad (25)$$

Thus, for $\varepsilon > 1$, then $\varepsilon \mathrm{BR}(\mathbf{p}_{-i}) > \mathrm{BR}(\varepsilon \mathbf{p}_{-i})$ ■

V. Simulation Results And Conclusion

A. Simulation setting

Is this section we show the convergence of the NE for both power and SINR. We compare our results (i.e., convergence of NE in both power and SINR) with the data of [11] and [17] to demonstrate the advantages of our new power algorithm. The simulation scenario is as shown in Fig.1. We assume that 20 CRs randomly positioned around the PUBs in underlay manner as shown in Fig.2.

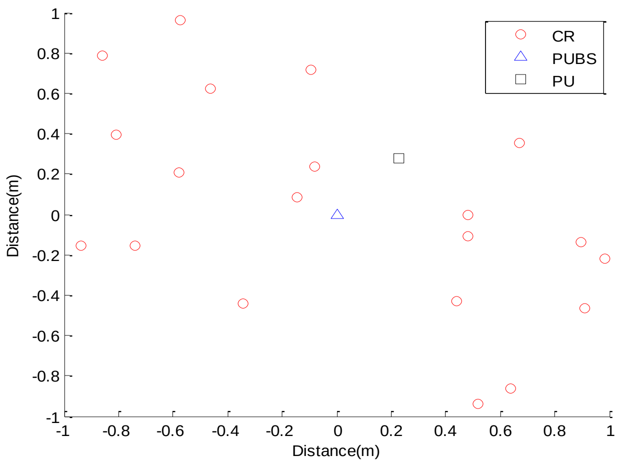

Fig.2 Deployment scenario of 20-CRs and 1-PU in underlay manner

Thermal noise are assumed to be 2×10^{-13} mW and the channel gain is generated according to (v / d_{ij}^{α}) where $d_{i,j}$ is the distance between *i*th player and the intended receivers. Path loss exponent ($\alpha = 2$) and $v = 10^{-11}$ is a constant which is related to channel attenuation [17]. The user priority parameters (θ_i) is set to (1) for all users. In addition, we set the interference threshold power to $p^{th} = 10\text{mW}$ where CRs should not increase their power above the defined interference threshold in order to keep free interference environment. The minimum SINR for CRs has been selected to be 8dB (i.e., $\overline{\gamma} = 8\text{dB}$).

B. Convergence Verification and Comparison Study

In this section, for simplicity, we assume that there is only one PU and 20-CRs that communicate in an underlay single-hop fashion. Convergence of our algorithm for both SINR and power has been illustrated in Fig. 3 and Fig. 4 respectively.

Fig.3 proves the convergent condition of SINR with 20 lines that indicate the SINR conditions for 20 CRs in the network. Contrast to the convergence results in [11] and [17], our algorithm shows better performance since it converge faster and takes only (4) iterations to converge compared with (10) and (20) iterations in [11] and [17] respectively. Figs.3 shows that the value of SINR of our algorithm can also meet the threshold requirement since all CR users achieve their SINR which are higher than the threshold value. However, SINR data in [11] shows better values of SINR in general and this resulted in better QoS among CRs.

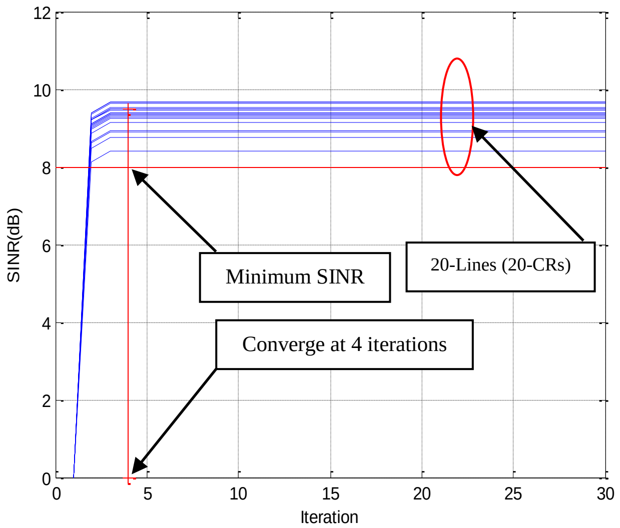

Fig.3 Convergence of SINR in the proposed algorithm

In contrast, Fig.4 proves the convergent conditions of power control algorithm for 20 CRs in the network. Compared to the results in [11] and [17], our algorithm also shows better performance since it is faster and takes only (4) iterations to converge compared with almost (10) iterations in [11] and (29) iterations in [17].

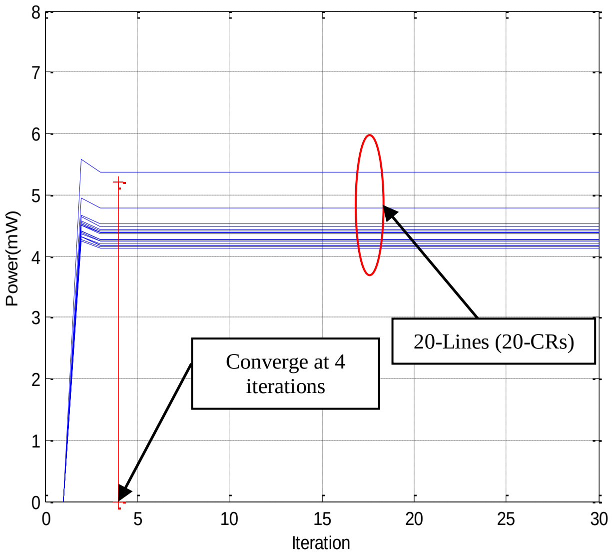

Fig.4 Convergence of the proposed power algorithm

However, the power values of [11] are slightly less than that in our work. Unlike [11] and [17] where the algorithm convergence with the aid of the correlation factor termed as (*b*, and *c*), our algorithm converge to a stable point without restriction which is another advantage that can be observed in our algorithm.

C. Cheating scenario

Cheating scenario is a common behavior in non-cooperative game theory where some players choosing their strategies (i.e., power in our study) somewhat higher than the allowable one and this will mainly affect the performance of PUs since the transmitted power of those players is above the permitted threshold given by PUs for safe environment. Moreover, the QoS of CRs will be affected in cheating scenario as well by receiving poor SINR.

To illustrate the cheating scenario, we have released the ***C-SBPC*** term from our algorithm and let the players chooses their power without restrictions. The results of cheating scenario are as shown in Fig.5 and Fig.6 respectively.

According to Fig.5, almost half of the players chooses their strategy space (i.e., power) higher than the defined threshold. This behavior will mainly affect the performance of the PUs because the generated interference from nearby CR, in cheating scenario, to the PU will increases accordingly. Moreover, the convergence of power strategy space, in cheating scenario, has been increased to almost 10 iterations compared to 4 iterations only with C-BSPC algorithm.

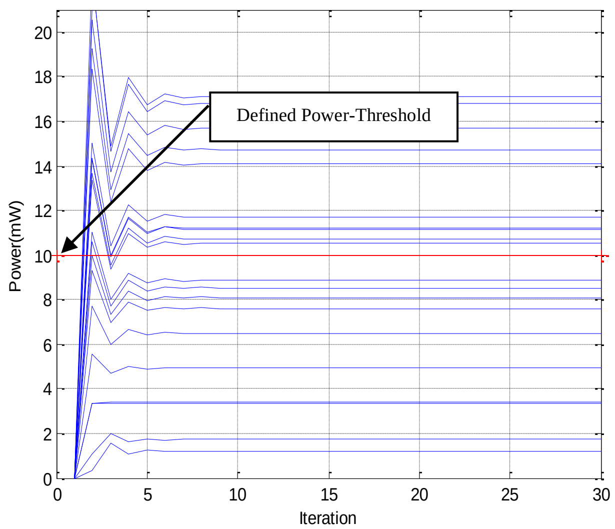

Fig.5 Behaviour of CRs (transmitted power) in cheating scenario

In addition, the QoS of CRNs will be affected as well and resulted in poor SINR because the PU punish those CR players transmitted with higher power than the defined threshold through pricing which resulted in poor SINR as shown in Fig.6.

According to Fig.6, almost half of the players failed to maintain their good SINR because of the pricing cost in the surplus function. Other players, in contrast, received lower SINR than that with C-SBPC algorithm which resulted in poor QoS in the entire CRNs.

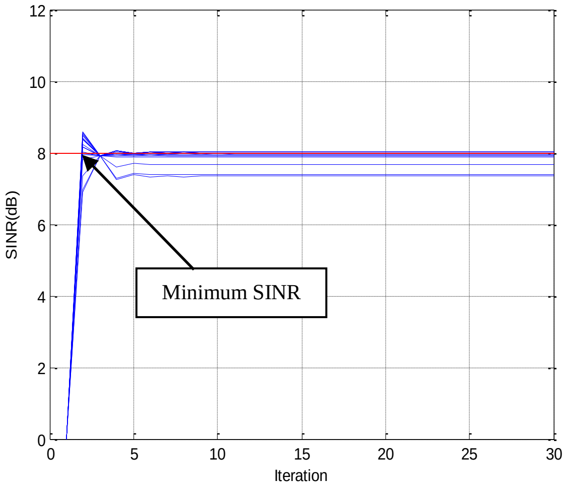

Fig.6 Behaviour of CRs (received SINR) in cheating scenario

Similar to the convergence of power strategy, the SINR in cheating scenario converge to unique solution in almost 10 iterations.

VI. Conclusions

In this paper we have proposed a novel algorithm, controlling based SBPC, termed as C-SBPC in an underlay CRNs based on non-cooperative game-theory. The convergence of the proposed algorithm has been verified mathematically and by simulation as well. Existence and uniqueness of the NE have been proven mathematically. One of the obvious advantages of the proposed power allocation algorithm is the quick convergence. Hence, the proposed algorithm is more suitable for practical distributed application. Moreover, pricing is an essential technique in CRNs to prevent cheating behavior of the players. Thus, better performance for both PUN and CRN can be achieved.

Appendix A Subgradient Method for Lagrangian Multipliers

The Subgradient method has been adopted in order to calculate the Lagrangian multipliers similar to [24]. Then the Lagrangian multipliers (α and β_i) can be written with respect to power and QoS constraints respectively. That is we have

$$\alpha^{(t)} = \left[\alpha^{(t-1)} - \delta^{(t)} \left(p_i^{\max} - \sum_{i=1}^{N} p_i^{c(t)} \right) \right]_0^+ \qquad 26$$

$$\beta_i^{(t)} = \left[\beta_i^{(t-1)} - \delta^{(t)} \left(\gamma_i^{f_k(t)} - \bar{\gamma} \right) \right]_0^+ \qquad 27$$

where $[\psi]_0^+ = \psi$ if $\psi > 0$, and $[\psi]_0^+ = 0$ if $\psi \le 0$. Furthermore, δ^t is the step size which can be chosen according to [24]

$$\delta^{(t)} = 10/t \qquad 28$$

$$\delta^{(t)} = 10/\sqrt{t} \qquad 29$$

However, in our study we have chosen equation (28) to determine the step size.

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