Cognitive stress and learning economic order quantity inventory management: An experimental investigation

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Abstract

We use laboratory experiments to evaluate the effects of cognitive stress on inventory management decisions in a finite horizon economic order quantity (EOQ) model. We manipulate two sources of cognitive stress. First, we vary individuals' participation in a pin memorization task. This exogenously increases cognitive load. Second, we introduce an intervention to reduce cognitive stress by only allowing participants to order when inventory is depleted. This restricts the order choice set. Increases in cognitive load negatively impact earnings with and without the intervention, with these impacts largely occurring in the first attempt of the task. With repetition, participants' choices in all treatments trend to near optimal policy adoption. However, only in the intervention and low cognitive load treatment do the majority of choices reach the optimal policy. We estimate the learning dynamics of order decisions using a Markov learning model. Estimates suggest increased cognitive load reduces the probability of switching to more profitable policies. Choice set complexity increases biases for smaller order size adjustments, leading to greater policy lock-in.

Keywords: Cognitive load; Choice set complexity; Economic order quantity; Inventory management; Learning

JEL codes: C92, D83, M11

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1 Introduction

Best inventory management practices call for the solution of dynamic optimization problems. This requires inventory managers to parse complex sets of alternative solutions and to use their short-term memory to hold and process information about the past, present, and future values of key variables. Current workplace trends impose increasing demands upon these managers' cognitive resources (Ruderman et al., 2017). Some examples of these trends are increasing complexity of supply chains (Bode and Wagner, 2015), and widely accepted increasing rates and scale of natural disasters and global social upheavals (Guan et al., 2020; Li et al., 2021). We assess how increasing cognitive stress through the introduction of an additional task impacts decision-making quality, and how a strategic intervention can mitigate the impact of this stress.

An extensive literature shows that, even under the best of circumstances, individuals systematically make suboptimal inventory management decisions. Decision-making biases and strategic considerations are often key factors diminishing individual performances in these tasks (Niranjan et al., 2011). When managing the inventory of a perishable good with uncertain demand, i.e. the newsvendor problem, decision makers neither follow the optimal risk neutral nor averse policies consistently in experimental studies. Feng et al. (2011) observed the "pull-to-center" effect in a cross-national laboratory study, using the newsvendor problem with a participant pool of Chinese and American decision makers. When there is a multi-level supply chain for a non-perishable good and certain demand, participants generate large bullwhip effects in beer game experiments. Research has shown key factors driving the excessive inventory levels and variance include strategic uncertainty regarding other decision makers (Croson et al., 2014), limited level two thinking (Narayanan and Moritz, 2015) and failure to fully take account of the future deliveries of past orders. In the setting of a durable good with uncertain demand, optimal inventory management follows the (S,s) policy. Experimental studies (Magnani et al., 2016; Khaw et al., 2017) demonstrate that individuals take time to find the optimal policy, their policy adaptations are idiosyncratic and often participants abandon the optimal policy once found. Despite all being important inventory management environments, none are ideal to begin an evaluation of how cognitive stress diminishes decision-making quality. The reason is that decision makers' performances are already suboptimal in their respective most favourable experimental conditions.

A more suitable inventory management environment should have two properties: the optimal policy is invariant to a decision maker's individual preferences and the majority of decision makers can find the optimal policy after repetitions. The finite horizon deterministic EOQ environment potentially possesses these properties. We choose the parameters of our environment such that the optimal inventory policy of the finite horizon matches that of the infinite horizon; when inventory is depleted, the manager orders an optimal quantity that is the multiple of the monthly demand for the good (Schwarz, 1972). We refer to this multiple as an EOQ cycle length. While eliminating some common complications of inventory environments in practice, this EOQ environment has several favourable features for our research question: participants have a relatively good chance of finding the optimal policy; the solution is invariant to a decision

¹See Katok et al. (2018) for an introduction and partial survey of this literature.

maker's risk attitude; and, it is an individual decision problem absent of strategic considerations.

The EOQ solution in our environment is dynamic, as the manager doesn't make the same decision at each point in time. This gives us an opportunity to observe pure learning behavior in a dynamic problem. In most behavioral supply chain studies, participants do not determine when to act. In our "Unrestricted" treatment, participants can order additional inventory each month regardless of the current inventory level. Our intervention, the "Zero Only" treatment, removes the possibility of violating the optimal inventory policy by forbidding participants from ordering when there is a positive level of inventory. The other aspect of our experimental design is the presence of an additional task competing for the participants' short term memory resources - we call this our "High" treatment. Our "Low" treatment doesn't involve this competing task. The crossing of the intervention treatments and cognitive load treatments constitute our 2×2 experimental design.

There is an a priori belief that our intervention will yield economically significant improvements. A growing literature in economics, e.g., Caplin et al. (2011); Masatlioglu et al. (2012); Abeler and Jäger (2015); Lleras et al. (2017), examines and measures how individual choices are increasingly suboptimal as their choice sets increase in complexity. Bolton and Katok (2008) find that reducing the number of order options does not necessarily result in better performance for newsvendor decisions. However, Feng et al. (2011) observe that thinning the set of order options in a way that the optimal order quantity is not an extreme option in the choice set does lead to better performance. Our Unrestricted treatment corresponds to the case of an unsupported inventory manager, while the simplified choice set of the Zero Only treatment corresponds to active management intervention. This allows our experiment to provide evidence on the value of this practice.

The second factor we investigate is the presence of a concurrent task that competes for the inventory manager's cognitive resources. Tokar et al. (2012) finds experimental evidence of cognitive overload with an increased quantity of information. In practice this would involve the introduction of inventory management responsibilities of additional product lines. However, managing the inventory of an additional product line typically introduces cross-demand impacts and potential synergies for inventory costs reductions. To control for the "costs" and "benefits" of successful inventory management we introduce an additional task unrelated to the inventory management one.

This concurrent task is the memorization of a PIN code at the beginning of each inventory year, and successful recall at the end of the year earns a monetary reward. The PIN task was first introduced by Miller (1956), and has been successively used in economics and psychology to exogenously shock cognitive load. Some examples of its application are in food choice (Shiv and Fedorikhin, 1999), generosity (Roch et al., 2000), strategic games (Allred et al., 2016; Duffy and Smith, 2014) and intertemporal choice (Hinson et al., 2003). Deck and Jahedi (2015) surveys the use of PIN task in economic experiments with financial incentives as well as reporting new experiments, one of which finds increasing PIN length reduces individual numeracy. To the best of our knowledge, we are the first to use this technique in behavioral operations management. Correspondingly, this allows our experiment to evaluate the impact of asking inventory managers

to multitask.

Our results show that experimental participants earn less when there is a competing task or when the intervention is absent. We observe there is a trend that participants learned to adopt near optimal EOQ policies in general. The restriction of managers to only place orders when inventories are exhausted and the alleviation of the competing task improved the chance for decision makers to reach the optimal inventory policy. It should be noted that these performance differences and suboptimal choices largely occur in the first three iterations of our environment.

As performance improves rapidly across treatments, we attempt to characterize the individual-level learning driving this trend. In a simple framework that models an individual using an Elimination-By-Aspect (EBA) (Tversky, 1972) choice architecture, we first estimate the propensity to follow the basic characteristics of EOQ solutions: avoiding stockouts or carrying excess inventories. We find that iterations of the task quickly diminish the probability of making such choices and, surprisingly, imposing high cognitive loads doesn't affect these probabilities. Once participants follow EOQ types of actions, we model the number of monthly demand orders requested, the EOQ cycle length, using a Markov learning model (Shachat and Zhang, 2017), which quantifies the low rationality of simply increasing the likelihood of choosing payoff improving choices, an essence of Learning Direction (LD) theory (Selten and Stoecker, 1986). Our estimates of the model suggest that under high cognitive load participants are less likely to switch to EOQ cycle lengths that increase payoffs. The estimates also suggest that with more complicated policy choice sets participants are more reluctant to make large changes in EOQ cycle length leading to greater sub-optimal policy lock-in.

We have found limited other experimental research examining behavior in finite horizon EOQ environments.² Pan et al. (2020) build upon the theoretical framework and experimental design we introduce here. They examine individual differences in a stationary limited horizon EOQ setting with a distinctive sample of participants and a similar EOQ finite horizon setting with different parameter values. Further, absent of the cognitive stress interventions, their experimental instrument incorporates our Unrestricted-Low treatment. Pan et al. (2020) focus on individual heterogeneity in terms of cognitive reflection, which is measured by the Cognitive Reflection Test (CRT). They find participants with higher CRT scores tend to choose more effective inventory management policies. However, the performance gap is transitory as participants with lower CRT scores exhibit faster learning. Pan et al. (2020) study learning in the same framework as ours. They find that individuals with higher cognitive ability are more likely to switch to more profitable actions and also exhibit less policy lock-in.

2 Experiment

2.1 Inventory decision task

In the core decision-making part of our experiment, participants complete a series of six discrete dynamic inventory management tasks. We refer to each task as a year, indexed zero to five, and each year consists of twelve months, indexed by t. We use the following context to describe

²We found two experimental studies which examine infinite horizon EOQ models: Stangl and Thonemann (2017) and Chen and Wu (2017).

these tasks to a participant.

The participant manages the enterprise 'S-store' which sells coffee makers with a constant demand rate (D) of 10 units per month. S-store sells a new model of coffee maker every year. Coffee maker orders are placed prior to the start of a month, an integer amount denoted q_t , and arrive without lag, hence are included in the calculation of a month's opening inventory. The participant chooses the quantity of each monthly order.

Monthly orders and demand determine the changing inventory levels. Let I_t denote the closing inventory for month t. The initial inventory of coffee makers prior to month one is zero, so the first month's opening inventory is the amount of the first month's coffee maker order, i.e. $I_0 + q_1 = q_1$. In general, the opening inventory of coffee makers in month t is $I_{t-1} + q_t$. This inventory is drawn down by monthly sales, the lesser of the monthly order flow of 10 or the opening inventory (i.e. a stockout). This results in the closing inventory of $I_t = I_{t-1} + q_t - \min\{10, I_{t-1} + q_t\}$. When the model life cycle concludes at the end of month 12, any remaining inventory is disposed at no cost but also generates no revenue. Further, we limit a participant's monthly order by its annual demand, i.e., $q_t \in \{0, 1, 2, ..., 120\}$.

A participant's compensation, excluding a fixed show-up fee, is proportional to S-store's profits, which are expressed - as are all further monetary quantities - in experiment currency units (denoted P). Each coffee maker sells at a price of P7. So revenue in month t is $7 \cdot \min\{10, I_{t-1} + q_t\}$. S-store's cost has two components: a fixed ordering cost, S, of P45 whenever she places a strictly positive order; and a constant per-unit monthly inventory holding cost. The monthly inventory holding costs is calculated by multiplying the average inventory of coffee makers held in t, specifically $\frac{(I_{t-1}+q_t+I_t)}{2}$, and the monthly holding cost, h, of P1 per unit. The monthly profit of S-store is the difference between the revenue and costs, and is calculated

$$\pi_t(q_t, I_{t-1}) = \begin{cases} 7 \cdot 10 - S \cdot \mathbb{1}_{q_t > 0} - \frac{I_{t-1} + q_t + I_t}{2} \cdot 1 & \text{if } I_{t-1} + q_t \ge 10\\ 7 \cdot (I_{t-1} + q_t) - S \cdot \mathbb{1}_{q_t > 0} - \frac{I_{t-1} + q_t}{2} \cdot 1 & \text{if } I_{t-1} + q_t < 10 \end{cases}$$

where, 1 is the indicator function.

A participant i's inventory policy for year a is the sequence of the twelve monthly quantity orders, $Q_{i,a} = (q_{i,1}, q_{i,2}, \dots, q_{i,12})$. For a given inventory policy S-store's annual profits are,

$$\Pi_{i,a}(Q_{i,a}) = \sum_{t=1}^{12} \pi_t.$$

In the supply chain literature, the set of EOQ policies is the subset of inventory policies which only place a quantity order once inventory reaches zero with no stockouts allowed. In our dynamic decision-making environment, stockouts can occur if a non-optimal policy was chosen previously. Correspondingly, we adjust the definition of an EOQ policy to classify choices at these points off the optimal path.

Definition 1. An EOQ action is a temporal inventory management decision satisfying the following conditions:

- (1). A participant only orders when the closing inventory of the previous period is less than 10 units, i.e., $q_t > 0$ when $I_{t-1} < 10$;
- (2). A participant doesn't order when the closing inventory of the previous period is at least 10 units, i.e., $q_t = 0$ when $I_{t-1} \ge 10$;
- (3). Participant's order guarantees no stockouts in t, i.e., $I_{t-1} + q_t \ge 10$.

Definition 2. An EOQ policy is a inventory management policy that consists only of EOQ actions.

The original EOQ model solution is derived assuming an infinite demand horizon, in which the average cost minimizing EOQ policy is to order the following quantity whenever the closing inventory of the previous period is zero,

$$q^* = \sqrt{\frac{2DS}{h}}. (1)$$

In our context, the cost minimizing policy would be to order 30 coffee makers, an EOQ cycle length of three months, whenever closing inventory of the previous period is zero. This would also be the profit maximizing policy as average revenue is constant, up to the monthly demand capacity, and is greater than the minimum average cost. In our finite horizon setting the optimal policy does not change. However, if an inventory manager deviates from this policy early in the year, the optimal course can involve alternative EOQ actions later in the year.

Schwarz (1972) characterizes the optimal EOQ policies for the finite horizon of T months. First, we note the result that average total cost minimizing policy is to order according to Equation 1 if T is an integer multiple of the $\frac{q^*}{D}$. As simply following the EOQ policy of ordering 10 units each period is profitable in our environment, profit maximization will call for satisfying the full annual demand. The EOQ policy of always taking the EOQ action of 30 when inventory is depleted maximizes profit in addition to minimizing average cost.

As individuals do fail to act optimally, we now consider alternative decision horizons (i.e. shorter in this case). Let C(T) be total incremental cost over the finite time interval T. We restrict our attention to policies which only place orders when inventory is zero. An EOQ cycle length is the interval of months between such orders, denoted by s_k , which is the interval between the (k-1)th and the kth order. Let $C(s_k)$ be the total incremental cost for an EOQ cycle, and n be the number of orders over T. We can formulate the problem as

min
$$C(T) = \sum_{k=1}^{n} C(s_k)$$
 s.t. $\sum_{k=1}^{n} s_k = T$

where

$$C(s_k) = S + hDt^2/2.$$

From the quadratic formulation, it is clear that in the optimal solution all of the s_k are of the same length. An EOQ constant inventory policy, denoted \bar{Q}^{s_k} , is one with a constant cycle length.

Let $C^n(T)$ be the total incremental cost for the interval T given n orders,

$$C^n(T) = nS + hDT^2/2n.$$

Minimising $C^n(T)$ gives

$$n^* = \sqrt{\frac{hDT^2}{2S}}.$$

Notice for the first month in our task, i.e. T=12, this yields the same solution as the infinite horizon formulation, $n^*=4$ and $s_k^*=3.^3$ Further investigations on situations when the horizon T is sufficiently small reveals that the optimal number of orders, n^* , is the smallest integer satisfying $n(n+1) \geq \frac{hDT^2}{2S}$. With the parameter values in our task, Table 1 gives an overview of the optimal solutions for different values of T.

Month	Т	$\frac{hDT^2}{2S}$	$n^*(n^*+1)$	The optimal order number (n^*)	The optimal EOQ cycle length (s_k^*) sequence	The optimal order size (q_k^*)
12	1	0.111	2	1	{1}	{10}
11	2	0.444	2	1	$\{2\}$	{20}
10	3	1	2	1	{3}	{30}
9	4	1.778	2	1	{4}	{40}
8	5	2.778	6	2	$\{3, 2\}$	{30, 20}
7	6	4	6	2	$\{3, 3\}$	${30, 30}$
6	7	5.444	6	2	$\{3, 4\}$	$\{30, 40\}$
5	8	7.111	12	3	$\{3, 3, 2\}$	$\{30, 30, 20\}$
4	9	9	12	3	$\{3, 3, 3\}$	$\{30, 30, 30\}$
3	10	11.111	12	3	$\{3, 3, 4\}$	$\{30, 30, 40\}$
2	11	13.444	20	4	$\{3, 3, 3, 2\}$	{30, 30, 30, 20}
1	12	16	20	4	$\{3, 3, 3, 3\}$	{30, 30, 30, 30}

Table 1: Optimal solutions for different T in our task

With our finite horizon of one year, the following set of constant EOQ cycles $s_k = \{1, 2, 3, 4, 6, 12\}$ and the corresponding constant EOQ policies are of particular interest. Table 2 shows for these EOQ constant policies the corresponding annual profits, the number of orders placed annually and the percentage of maximum potential annual profits, i.e. efficiency. Notice that EOQ constant 2 and 4 both generate over 93% of the potential annual profits. Given the minimal loss incurred by adopting these policies we define an alternative decision quality benchmark. When a participant chooses $s_k = \{2,4\}$ we call this "near optimal" performance.

2.2 Experimental design

Our experimental design has two treatment variables, each of which has two categories. This generates a 2×2 factorial experimental design. We adopt a between subject design; a participant only experiences one of the four possible treatment cells.

The first treatment variable is the feasible set of inventory policies a participant can follow. The first category is called "Unrestricted", where a participant can choose any quantity they wish

³Pan et al. (2020) use the same inventory management decision problem as we do but with different parameter values. In their setting, the monthly demand is 20 units, price is P5 per unit, the fixed ordering cost is P80, and the constant monthly holding cost is P0.5 per unit. These parameters yield a different optimal solution, i.e., $n^* = 3$ and $s_k^* = 4$.

Table 2: Alternative EOQ constant strategies which do not generate stockouts or positive closing inventories in month 12 and their respective performance properties.

\bar{Q}^{s_k}	The number of orders per year	Constant order size	Profit per EOQ cycle	Annual profit	Efficiency
12	1	120	75	75	15.63%
6	2	60	195	390	81.25%
4	3	40	155	465	96.88%
3	4	30	120	480	100.00%
2	6	20	75	450	93.75%
1	12	10	20	240	50.00%

each month as long as the quantity does not exceed 120. The second category is called "Zero Only", where participants are restricted to ordering only once the inventory level is zero. We expect that the larger set of alternatives in the unrestricted category presents participants with a more difficult learning task.

The second treatment variable is the level of exogenous cognitive load burden we induce by introducing a competing task. In the "Low" cognitive load category participants complete the inventory tasks without distractions. In the "High" cognitive load we introduce an incentivized PIN task that is completed alongside the inventory management task and requires the utilization of short term memory. At the start of each year, a participant is given 15 seconds to memorise a random 6-digit PIN. The PIN is case sensitive, consisting of numbers, upper and lower case letters.⁴ After the completion of the year, a participant is prompted to enter the PIN. Entering the correct PIN unlocks an extra reward of P300. A participant only has one attempt at the PIN task. If a participant actively tries to complete the PIN task successfully, we expect the diminished access to short term memory to reduce decision-making quality and the speed of any learning.

Table 3 summarizes our experimental design and provides summary statistics on the demographics of the participants. We designate treatment cells by the word pairs x-y, where x is feasible set of policies category and y is category of the cognitive load.

2.3 Experimental procedures

Seven sessions were conducted at Newcastle University Business School experimental economics laboratory during May and July 2017. 162 participants⁵ were recruited via random selection for invitation from a participant pool database of the Behavioural Economics Northeast Cluster. All participants were students from Newcastle University except for three who were from Northumbria University.

 $^{^4{\}rm The~PIN}$ is the same for all participants across each year to ensure control.

 $^{^5}$ We excluded five participants from our data analysis and the participant counts are given in Table 3. One participant, in the Zero Only-Low treatment, always submitted the random slider starting position when inventory reached zero. Two other participants, in the Zero Only-High treatment, grossly took advantage of the limited liability rule. The final two excluded participants attended the last session and demonstrated behaviour that they had been briefed about the content of the experiment; they clicked through the instructions without reading them and subsequently provided the solution \bar{Q}^3 for all years - even though this was not optimal for the practice year.

Table 3: Summary of the demographic information of participants for each treatment

Treatment cell	Participants	Average age	Male	Postgrad	$\begin{array}{c} {\rm STEM} \\ {\rm subjects}^1 \end{array}$	Average $level^2$	math
Unrestricted-Low	41	25	34%	49%	37%	3.68	
Unrestricted-High	41	25	37%	44%	56%	3.20	
Zero Only-Low	39	25	23%	47%	34%	3.26	
Zero Only-High	36	28	50%	56%	28%	3.53	

¹ STEM subjects include Engineering & Technology, Life Sciences & Medicine and Natural Sciences. Non-STEM subjects include Arts & Humanities and Social Sciences & Management.

Each session lasted no more than sixty minutes, with strict procedures to limit the access to any aides that would provide assistance in calculations or remembering PIN codes. Participants were signed in individually and instructed to leave their personal belongings, including any writing instruments, in the reception area before being escorted to a computer desk placed in a privacy carrel. Each participant was then provided with a pen and two copies of an informed consent document, which they read and signed if they wished to continue their participation. The pen and signed forms were then collected by a monitor. Participants were sternly informed thereafter that no electronic devices - such as mobile phones, calculator, smart watches, etc. - could be used until their session was completed. They were further instructed that the rest of the experiment only using their mouse. Prior to participants entering the laboratory, all computer keyboards were concealed under a thick opaque cover. This aimed to diminish any access to mnemonic devices for remembering PIN codes. These measures were taken in all sessions to provide control between High and Low cognitive load treatments.

The experiment itself was conducted using a self-contained program developed in oTree (Chen et al., 2016). Access was restricted to other programs on the computer. The sum of these measures eliminated many of the tools participants commonly used to perform mathematical calculations. This dismal work environment was applied to all four treatment cells.

Once instructed to start by the monitor, participants read through the instructions⁶ at their own pace. After reading the instructions, participants were asked to complete seven multiple choice questions designed to ensure that they understand the calculation of costs and profits. Participants who provided more than two incorrect answers had to review the mistaken questions with one of the experimenters before proceeding to the decision tasks.

Participants then took part in the six year decision task sequence, followed by a short post-experiment survey which collected demographic information. Year 0 was a practice round which used an alternative set of cost parameters⁷ from those of Years 1 through 5, and the performance

 $^{^2}$ Math Level was self-assessed, and was categorised into 6 levels. 1 = "Below GCSE", 2 = "GCSE", 3 = "A Level", 4 = "Undergraduate", 5 = "Postgraduate", 6 = "Above Postgraduate". Note that GCSE (General Certificate of Secondary Education) is an academic qualification in a specific subject typically taken by school students aged 14-16 of the UK (except Scotland), at a level below A level.

⁶In the supplement appendix, we provide a complete set of instructions.

⁷In the practice year the order costs were P45 and the holding costs were P0.5.

in this task did not affect a participant's total earnings. The purpose of the practice year was to help familiarize the participants with the task and the decision screen. Orders were entered by moving a slider whose value range was zero to one hundred and twenty. The initial point of the slider was random each month, and in the case of a Zero Only treatment it was greyed out if there is a positive opening inventory. The decision screen included a table providing the entire history of a participant's monthly ordering choices, as well as opening inventory, units sold, closing inventory, sales revenue, ordering costs, holding costs and profits. For participants who experienced the High cognitive load treatment, we provided an opportunity to practice the PIN task in the practice year.

Participants then completed the Years 1 through 5 decision tasks. Participants were paid for their accumulated earnings from these decision tasks, at the conversion rate of P300 = £1, as well as a £5 show-up fee. There was limited liability; to ensure the motivation to make profits would not be affected by large negative earnings made in a particular year, any negative profits made in a year will be treated as 0 earnings. The average earnings across all treatments were £13.37 per participant, including the participation fee. 10

One last important aspect of the experiment was the fixed length of time a participant had to complete the inventory management task for a year. We required that a participant spend exactly four minutes completing each task in Years 1 through 5. This was designed to prevent participants from racing through the monthly decisions in order to reduce the cognitive cost of remembering their PIN. If a participant completed their twelve monthly decisions early they could not advance to the next period (or enter the PIN) until the four minutes expired. If they failed to complete the twelve tasks before the time expired, the computer program executed the remaining months' sales with the existing inventory stock.

3 Empirical evaluation of treatment effects

We evaluate the treatment effects of restricted inventory policy choice sets and increased cognitive load by considering their impacts upon participant's earnings in the inventory management tasks, the propensity to choose optimal inventory policies, and then the efficacy of the PIN task and whether performance in that task is correlated with inventory performance.

3.1 Hypotheses

Our motivation of treatment variables leads to several natural hypotheses. Naturally, better performance leads to greater average annual earnings and is indicated by greater percentage of participants adopting optimal (near-optimal) inventories. Increases in cognitive load reduce short term memory capacity and lead to diminished performance in both the Zero Only and Unrestricted policy choice sets, giving the following hypotheses:

Hypothesis 1.a. Participants perform better in the Zero Only-Low treatment than the Zero Only-High treatment. This will be reflected in two measures: average annual earnings and the

⁸We provide screen captures of these interfaces in the supplement appendix.

⁹This limited liability only affected the earnings of five participants in five different years.

¹⁰The average earnings of Low treatment (without PIN task) was £11.58 per participant, while the average earnings of High treatment (with PIN task) was £15.23 per participant.

percentage of participants who adopt optimal (near-optimal) inventories.

Hypothesis 1.b. Participants perform better in the Unrestricted-Low treatment than the Unrestricted-High treatment. This will be reflected in two measures: average annual earnings and the percentage of participants who adopt optimal (near-optimal) inventories.

The set of inventory policies in the Unrestricted is much larger than and only adds suboptimal alternatives to the Zero Only restricted set of policy choices. The reduced focalness of EOQ strategies and greatly complicated participants' choice sets in the Unrestricted treatments leads to our next set of hypotheses:

Hypothesis 2.a. Participants perform better in the Zero Only-Low treatment than the Unrestricted-Low treatment. This will be reflected in two measures: average annual earnings and the percentage of participants who adopt optimal (near-optimal) inventories.

Hypothesis 2.b. Participants perform better in the Zero Only-High treatment than the Unrestricted-High treatment. This will be reflected in two measures: average annual earnings and the percentage of participants who adopt optimal (near-optimal) inventories.

3.2 Annual inventory profits

We test the differences in average annual profit for different treatment groups using two-sided t-tests and non-parametric Wilcoxon rank-sum tests. As participants have the best performance with the intervention and without the cognitive load, we use Zero Only-Low treatment as a reference point to demonstrate profit loss resulting from the presence of more complicated choice sets and a shock to their cognitive load. We report the results of these hypotheses tests in Table 4. The first two rows indicate that both the absence of the intervention and shocking their cognitive load negatively impact average annual profits both statistically and economically. More complicated policy choices cause more profit loss than High cognitive load.

When we examine the effect of exogenously increasing a participant's cognitive load conditional on the policy choice set we find mixed support for Hypothesis 1. There is a statistically significant reduction on average earnings in the Zero Only treatment, but not in the Unrestricted treatment. We do find stronger evidence in support of Hypothesis 2, as we find limiting participant's choices to EOQ restricted policies leads to statistically greater average earnings in both Low and High cognitive load settings.

A disaggregated view of the average annual profits permits insight into learning over time and how our treatments impact it. Figure 1 presents these time trends for each of the four treatments. There are several prominent features of this figure which provide refined insights into our hypotheses results on the average profit levels. First, performance gains are mostly achieved in Years 1 through 3. Second, average earnings are around 90% of the possible earnings in the last two years; except for the Unrestricted-High treatment which are around 5-10% lower. Third, High cognitive load and Unrestricted policy choice sets both cause the greatest negative performance impact in Year 1.

We quantify and assess these remarks by conducting a series of dummy variable linear regressions using random effects estimators with standard errors clustered at the level of the individuals.

Table 4: Average annual profits by treatment and hypotheses tests for differences in average annual earnings

Panel A: Annual profits by treatment

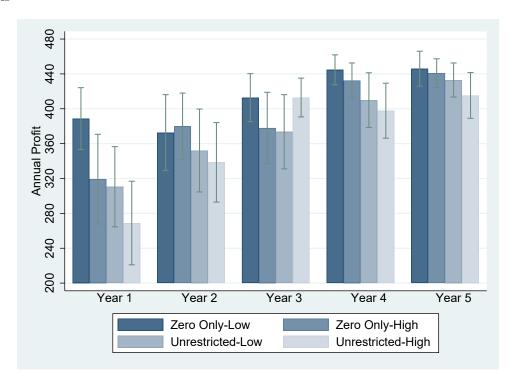
	Unrestricted-Low	Unrestricted-High	Zero Only-Low	Zero Only-High
Average	375.85	366.70	412.94	390.10
Stand. Dev.	129.38	126.87	97.35	113.78

Panel B: Hypotheses tests for differences in average annual profits (p-values reported)

Treatment Comparison	Difference	Profit loss (%)	Γ wo-sided t -tests	Wilcoxon rank-sum
Zero Only vs Unrestricted Low vs High	30.71 16.29	7.64% 4.14%	0.000 0.055	0.001 0.003
Zero Only-Low vs Zero Only-High ¹	22.84	5.53%	0.038	0.012
Unrestricted-Low vs Unrestricted-High	2 9.15	2.43%	0.470	0.124
Zero Only-Low vs Unrestricted-Low ³ Zero Only-High vs Unrestricted-High ⁴	37.09 23.40	$8.98\% \ 6.00\%$	$0.001 \\ 0.059$	$0.012 \\ 0.052$

¹ Strong Evidence for Hypothesis 1a

Figure 1: Annual Profits over individual Years and by treatment: Averages and 95% confidence intervals



We report these results in Table 5.

In model (1), we simply regress annual profit on a constant and dummy variables for Years 1 through 4, rendering Year 5 the base level. In model (2) we introduce dummy variables for the Zero Only and High treatment categories. In this case the constant reflects the average profit level for Year 5 in the Unrestricted-Low treatment; and the Year 1 through 4 dummy

² Weak Evidence for Hypothesis 1b

 $^{^3}$ Strong Evidence for Hypothesis 2a

⁴ Strong Evidence for Hypothesis 2b

Table 5: Dummy variable regressions for annual profit: random effects panel data (n=785)

	(1)	(2)	(3)	(4)
	Annual Profit	Annual Profit	Annual Profit	Annual Profit
Year 1	-112.27***	-112.80***	-122.44***	-122.44***
	(11.24)	(18.34)	(20.77)	(20.83)
Zero Only*Year 1		45.45**	65.22**	63.91**
II: -1-*W 1		(21.97)	(26.77)	(27.09)
High*Year 1		-43.20^* (22.17)	-23.91 (32.23)	-23.91 (32.32)
Zero Only*High*Year 1		(22.11)	-40.40	-39.09
Zero Omy Ingli Teal 1			(44.19)	(44.46)
Year 2	-73.39***	-82.86***	-80.89***	-80.89***
	(10.02)	(19.08)	(22.92)	(22.98)
Zero Only*Year 2	,	11.56	7.53	$\stackrel{\cdot}{5.07}^{'}$
		(20.05)	(29.81)	(30.10)
High*Year 2		8.04	4.11	4.11
		(20.13)	(29.94)	(30.02)
Zero Only*High*Year 2			8.24	10.69
3 V 9	90.01***	90 75**	(40.13)	(40.40)
Year 3	-38.81***	-38.75**	-59.41***	-59.41*** (18.20)
Zero Only*Year 3	(7.78)	(15.39) -16.24	$(18.34) \\ 26.15$	$(18.39) \\ 28.35$
Zero Omy Tear 3		(15.78)	(22.69)	(22.85)
High*Year 3		15.70	57.02***	57.02***
111811 1001 0		(15.65)	(19.90)	(19.95)
Zero Only*High*Year 3		()	-86.59***	-88.79***
v S			(30.82)	(30.97)
Year 4	-12.85***	-20.08**	-23.09**	-23.09**
	(4.61)	(9.40)	(11.57)	(11.61)
Zero Only*Year 4		15.59*	21.75*	21.90*
TT: 1 437		(9.03)	(12.16)	(12.23)
High*Year 4		-0.45	5.56	5.56
Zero Only*High*Year 4		(9.25)	(15.57) -12.59	$(15.62) \\ -12.74$
Zero Omy mgn Tear 4			(18.12)	(18.19)
Zero Only		19.12*	12.96	14.30
Zero Omy		(10.31)	(13.87)	(15.17)
High		-11.70	-17.71	-22.49
3		(10.45)	(16.25)	(16.77)
Zero Only*High		, ,	$12.58^{'}$	[15.31]
			(20.71)	(22.68)
Male				15.04
D 1				(11.85)
Postgrad				-14.88
CTIVA				(12.03)
STEM				12.18 (11.47)
Math Level				-2.69
1,10011 120 (01				(5.13)
Constant	433.40***	430.01***	433.01***	440.60***
	(5.28)	(8.56)	(9.70)	(20.84)
Wald χ^2	183.28	210.84	228.77	239.23
vvaiu X	100.40	410.04	440.11	∠ <i>ე</i> შ.∠ე

Standard errors in parentheses adjusted for clusters in individuals

variable coefficients reflect the average annual profits across participants in the Unrestricted-Low treatment. In model (3), we add interaction dummy variables for the Zero Only and High treatment categories to examine if their joint imposition leads to super- or sub-additive impact on annual profit. In model (4), we add individual characteristic dummy variables to examine individual differences.

Our treatment effects for Zero Only and High are largely generated by their Year 1 impacts,

^{*} p < 0.10, ** p < 0.05, *** p < 0.01

suggested by their individually significant coefficients in models (2) and (3). We conduct a Chow test, for which the null is model (1) versus the alternative of model (2), i.e. the joint differences of the two treatments are significant. The resulting χ^2 -stat is 27.32, and has a p-value of 0.002. We conduct a second Chow test to compare the veracity of model (3) versus model (2). The resulting χ^2 -stat in this case is 8.73, and has a p-value of 0.120.

Our analyses of annual profits leads us to our first set of results.

Result 1. Reducing the participants' policy choice sets to EOQ restricted ones leads to higher profits. However, these gains predominantly occur in Year 1 - when the participants face the inventory decision problem for the first time.

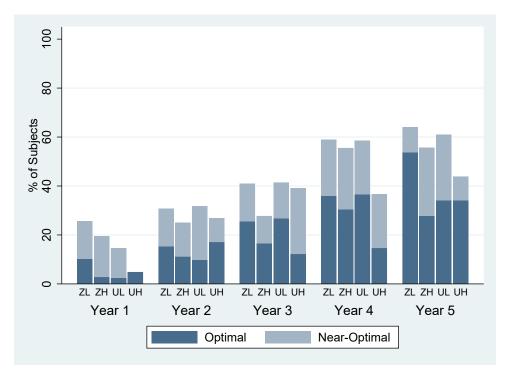
Result 2. Exogenously increasing participants' cognitive load leads to lower profits. However, these losses predominantly occur in Year 1 - when the participants face the inventory decision problem for the first time.

Result 3. There is no super- or sub-additive effect of simultaneously exposing participants to the Zero Only and High treatment categories.

3.3 Inventory management policy choices

We turn our analysis towards the inventory policy choices of participants. For each participant we evaluate each of the annual inventory policies, $Q_{i,a}$, for whether it is optimal, \bar{Q}^3 , or if it is near-optimal, and EOQ constant strategy of either \bar{Q}^2 or \bar{Q}^4 . Figure 2 depicts the evolution across years of the percentages of participants following optimal and near-optimal policies in each treatment. Inspection of this figure reveals our next set of results.

Figure 2: Stacked graph of the percentage of participants following optimal and near-optimal EOQ constant strategies: by Year and treatment



Result 4. There is a trend in all treatments for increasing use of optimal and near-optimal policies from Year 1 to Year 4.

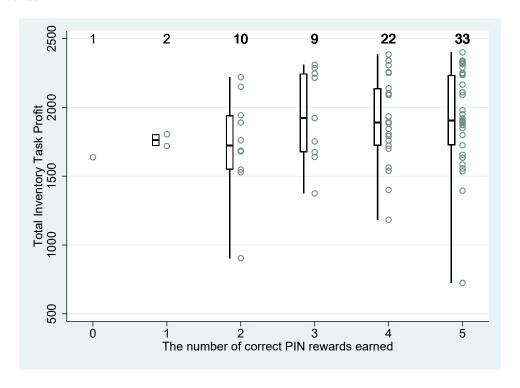
Result 5. High cognitive load leads to lower percentage use of these policies for both Zero Only and Unrestricted in all five Years.

3.4 Efficacy of the PIN reward procedure

Next, we evaluate the efficacy of procedure for exogenously increasing the cognitive load. Our experimental design faces a challenging balancing act. If the PIN reward procedure is too simple, participants will always collect the reward utilizing minimal short run memory resources, and if it is too difficult they could either decide to forgo the mental costs of trying to commit the PIN to short term memory or forgo effort in the inventory management tasks. A second concern is that raw intelligence is an omitted variable in our analysis which would manifest itself in a strong positive correlation between a participant's performances in the PIN reward and the inventory management task.

We provide visual evidence that our design successfully addresses this balancing act in Figure 3. First, we observe that only three out of the seventy-seven participants earned one or less PIN rewards; and at the same time thirty-three out of seventy-seven collected all five pin rewards. Second, there doesn't appear to be a clustering of poor inventory management performers, below the *ad hoc* threshold of P1500, on high or low numbers of earned PIN rewards. Third, there is little evident differences in the conditional means of total profits - suggesting the PIN and inventory management task performance are independent.

Figure 3: Participants' total inventory management task profits conditional on the number of PIN rewards earned and the corresponding whisker plots for the 50, 75, and 95% quantiles. The numbers across the top are the counts of participants who earned the corresponding number of PIN rewards.



We quantify the evidence of the independence of PIN and inventory management task performance by statistically measuring their correlation and testing its statistical significance. Table 6 reports these correlations and the p-values of the hypotheses tests that the correlation is zero. The left portion of the table addresses the correlation between the success of a PIN reward task and the corresponding annual inventory profit. The evidence is mixed. We do not find correlations significantly different from zero in four out of five years, but do find a highly significant positive correlation when we pool all of the years. This analysis suggests potential positive correlation between a correct PIN task and individual reward; however, this analysis does not allow for differences in participants' performances for the PIN task. To address this concern we evaluate the correlations between the total number of PIN rewards earned by a participant j and both j's annual profits and her total inventory tasks profit. We report these correlations in the right side of Table 6. In this analysis we find evidence in favor of no correlation. None of these correlations are statistically significant.

Table 6: Spearman correlations between PIN reward earned in Year a by participant j and j's corresponding Inventory task profit; Spearman and Pearson Rank correlations between a participant j's total number of earned PIN rewards and their Inventory task profits

<u> </u>		PIN reward eared in Year a	Number of	PIN reward earned
		Spearman Rank Corr.	Pearson Corr.	Spearman Rank Corr.
	Year 1	$0.08 \\ (0.512)$	0.13 (0.248)	0.11 (0.324)
Annual	Year 2	$\begin{pmatrix} 0.12 \\ (0.315) \end{pmatrix}$	$\begin{pmatrix} 0.08 \\ (0.507) \end{pmatrix}$	$\begin{pmatrix} 0.11 \\ (0.338) \end{pmatrix}$
Profit	Year 3	$\begin{pmatrix} 0.21 \\ (0.072) \end{pmatrix}$	$\begin{pmatrix} 0.10 \\ (0.391) \end{pmatrix}$	$\begin{pmatrix} 0.04 \\ (0.750) \end{pmatrix}$
	Year 4	$0.09 \\ (0.459)$	$ \begin{array}{c} 0.15 \\ (0.182) \end{array} $	$ \begin{array}{c} 0.04 \\ (0.725) \end{array} $
	Year 5	$0.14 \\ (0.226)$	$ \begin{array}{c} 0.10 \\ (0.379) \end{array} $	$ \begin{array}{c} 0.14 \\ (0.218) \end{array} $
	All Years	$0.18 \\ (0.001)$	N/A N/A	N/A N/A
Total Profit		N/A N/A	$ \begin{array}{c} 0.179 \\ (0.120) \end{array} $	0.182 (0.114)

^{1.} The p-values of the respective tests are reported in the parenthesis.

4 Learning Dynamics

The rapid behavioural adjustment over the first three annual iterations is a surprising feature of our data. We propose a behavioural process underpinning this rapid individual learning. This process is a Markov learning model in the spirit of Shachat and Zhang (2017), also adopted by Pan et al. (2020) for their similar inventory management task. The Markov learning model incorporates the Elimination-By-Aspect (EBA) choice model and specifies choice transitions by the Learning Direction (LD) theory.

The EBA is a framework for making choices among deterministic alternatives which are characterised by multiple attributes. Under the EBA choice model there is a hierarchy of attributes

^{2.} We don't report the correlations for Total Profit in column three because the calculation will include multiple repetitions of a participant's total inventory profit.

^{3.} We don't report the correlations for all Years in columns for and five because the calculation will include multiple repetitions of a participant's total number of PIN rewards.

and a decision maker sequentially refines the number of alternatives based on attribute values. In our two-tier hierarchical structure, the first tier sorts potential choices that are EOQ actions or not. The second tier chooses an EOQ cycle length. The choice process for EOQ cycle length accommodates the potential conflict of a choice set's natural ordering not having monotonicity in value.

The sequential feature of the EBA model is an alternative to standard comprehensive utility evaluation and can lead to suboptimal choices. This is a positive feature in our analysis as it allows us to establish an empirically justified degrees of irrationality by which learning occurs. While this is not a fully rational framework, it is not an uncommon one. McFadden (1980) generalised the EBA model to one of probabilistic choice and noting it has significant potential for econometric applications because it allowed complex patterns of substitutability among alternatives. For example, Daniel et al. (2018) conducted a discrete choice experiment eliciting Swedish households' willingness-to-accept compensation for restrictions on household electricity and heating use during peak hours and found more than half of the respondents use an EBA strategy. Fader and McAlister (1990) estimate an EBA model using UPC scanner data collected from purchases of ground coffee and find the model fits and predicts the data well. EBA has also been employed as a descriptive model of choice in transportation investment and demand analysis, residential choice, and career choice (Khraibani et al., 2016; Kato and Kosuda, 2008; Young, 1984; Gati et al., 1995).

We find an overwhelming proportion of choices are EOQ actions from the first tier analysis. Therefore, the LD theory covers the large majority of decisions and specifies the transition probabilities between EOQ cycle lengths. LD theory is a qualitative theory about adjustments within repetitive decision tasks, which calls for increasing probability for the choices that yield payoff improvements. We use a Markov model to quantify these adjustments. We specify the transitional probabilities as functions of whether a choice improves the current payoff and its absolute change in EOQ cycle length. We include both factors as payoffs are not monotonic in EOQ cycle length, and wish to accommodate potential viscosity in the magnitude of EOQ cycle length adjustment.¹¹

4.1 Branch Decision 1

As the experimental design prunes Branch 1 for the EOQ treatment, for the most part, the primary question is whether High cognitive load leads to larger probabilities of Non-EOQ actions. We formulate the probabilities of choosing Non-EOQ actions as simple Logit functions of time, habit formation and whether it is a High cognitive load treatment.

The Logit regression results are presented in Table 7: Panel A for the case $I_{t-1} < 10$ and Panel B for the case $I_{t-1} \ge 10$. For the prior case, deviations from an EOQ action occur due to the possibilities of stockouts. For the latter case, the only possible deviation from an EOQ

¹¹Other frequently adopted models of learning such as the experienced weighted attraction (EWA) learning models (Camerer and Ho, 1999) are unfit for our tasks. They are unfit due to two key features. Firstly, the payoffs for inventory orders are not monotonic in the number of months violating the "higher payoff implies higher probability" of choice property of EWA-like learning models. Secondly, there are a low number of positive orders decisions in many potential policies. For example, when an order of EOQ cycle length 12 is placed at the start of the year, only one observation is recorded that year.

action is to order a strictly positive amount, $q_t > 0$, leading to excess inventory. Furthermore, $NonEOQACC_{i,r-1}$ is the total rounds participant i has deviated from EOQ up through round r-1, which is intended to capture habit formation.

Table 7: Logit regression on the probability of deviating from an EOQ action

Panel A: $I_{t-1} < 10^{-1}$	0			Panel B:	$I_{t-1} \ge 10$	
$\overline{NonEOQ_{i,r}}$	(1)	(2)	(3)	(4)	(5)	(6)
$\overline{Year_r}$	-0.498***	-0.500***	-0.690***	-0.308***	-0.309***	-0.608***
	(0.126)	(0.128)	(0.160)	(0.096)	(0.097)	(0.134)
$Month_r$	0.194***	0.194***	0.153***	-0.115***	-0.114***	-0.141***
	(0.047)	(0.047)	(0.054)	(0.027)	(0.027)	(0.029)
High		0.255	0.216		-0.404	-0.193
		(0.425)	(0.311)		(0.376)	(0.296)
$NonEOQACC_{i,r-1}$			0.288***			0.307^{***}
			(0.039)			(0.040)
Constant	-3.379***	-3.507***	-3.175***	-1.754***	-1.573***	-1.311***
	(0.583)	(0.630)	(0.690)	(0.338)	(0.397)	(0.430)
N	3032	3032	2875	3286	3286	3286
χ^2	34.10***	36.06***	97.89***	22.66***	22.90***	75.76***
$Pr(NonEOQ_{i,r}) =$	$1 \ 0.034$	0.034	0.037	0.035	0.035	0.030

Standard errors in parentheses adjusted for clusters in individuals

Similar with the results reported in Pan et al. (2020), the estimated probability of a NonEOQ action at the average level of factors appear to be small. The number of years and the accumulation of experience of choosing Non-EOQ actions are significant both statistically and economically, which indicates there is significant learning to choose EOQ actions across the five years. The positive estimated value of the coefficient of $NonEOQACC_{i,r-1}$ captures the individual differences in the epiphany of the EOQ logic. The estimated coefficients for Months are statistically significant but lack economic significance. The marginal effects on the choice probabilities are extremely low and the effects have opposite signs in the two cases. This suggests that later in a year, stockouts are more and ordering while holding excess inventory is less likely. Surprisingly, there is no significant effect of having a high cognitive load on taking Non-EOQ actions. Thus the performance differences must come from the types of EOQ actions one takes under high cognitive load. Overall, these results suggest providing the more complicated choice set does lead to more Non-EOQ actions, rather these choices diminish with experience.

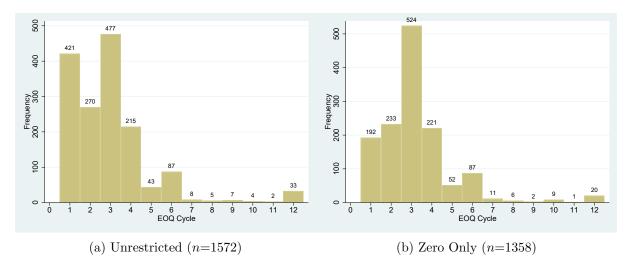
4.2 Branch Decision 2: A Markov model of EOQ cycle choice

At the Branch Decision 2, we consider how participants switch from one EOQ cycle length to another. Let $\tilde{s}_{i,k}$ denotes the largest integer less than or equal to $\frac{I_{t-1}+q_t}{10}$. Figure 4 shows histograms of EOQ cycle choices using this new definition in both Unrestricted and Zero Only treatments. This figure illustrates there are more optimal EOQ cycles of length three in the Zero Only treatment, and more extreme EOQ cycles of lengths one and twelve in the Unrestricted treatment.

^{*} p < 0.10, ** p < 0.05, *** p < 0.01

¹²For example, If a participant has a closing inventory of 2 units from previous period and orders 8 units, then $\tilde{s}_{i,k} = 1$.

Figure 4: EOQ cycle choice histograms for Zero Only and Unrestricted treatments



We use the Markov model presented in Pan et al. (2020) to examine the probability of switching to an at least as profitable EOQ action and the viscosity to making large changes to EOQ cycle length. The model has two parameters, α and λ . The parameter α is the probability of moving to the subset of EOQ cycles no worse than $\tilde{s}_{i,k-1}$. While λ measures the strength of the bias for small changes within the subsets. A decrease in λ corresponds to a growing bias in favor of smaller cycle length adjustments. Table 8 presents the maximum likelihood estimates of these parameters for each treatment cell.

Table 8: Maximum Likelihood Estimates for the Markov EOQ cycle choice model, standard errors in parentheses

Parameter	Unrestricted-Low	Unrestricted-High	Zero Only-Low	Zero Only-High
α	$0.708 \\ (0.041)$	$0.676 \\ (0.040)$	$0.760 \\ (0.031)$	$ \begin{array}{c} 0.712 \\ (0.032) \end{array} $
λ	-1.104 (0.217)	-1.320 (0.209)	$-0.709 \\ (0.178)$	-0.782 (0.137)

Table 9: Differences in parameter estimates and corresponding likelihood ratio test p-values for the Markov EOQ cycle choice model

Parameter	α		λ	
Treatment Comparison	Difference	p-value	Difference	p-value
Unrestricted-Low vs Unrestricted-High	0.032	0.575	0.216	0.475
Zero Only-Low vs Zero Only-High	0.048	0.284	0.074	0.744
Unrestricted-Low vs Zero Only-Low	-0.052	0.310	-0.396	0.159
Unrestricted-High vs Zero Only-High	-0.036	0.487	-0.538	0.032

In all treatments, α is approximately 70% indicating that participants are more likely to move into their current subset of EOQ cycles that is no worse than $\tilde{s}_{i,k-1}$. However, the absence

¹³The subset of EOQ cycles no worse than $\tilde{s}_{i,k-1}$ (NW), and the subset of EOQ cycles no better than $\tilde{s}_{i,k-1}$ (NB), are defined by the monthly average profit of the EOQ cycles conditional upon month. The values of the monthly average profit and a detailed listing on NW and NB subsets for different months can be found in Appendix C.

of the intervention and introducing cognitive load reduce the probability of switching to more profitable actions. The estimate of λ is larger in magnitude for the Unrestricted treatments, indicating a larger bias for small changes within the sets. The ability to order in any month leads to a greater degree of action lock-in. However, the estimated differences in the parameters are not statistically significant.

We evaluate the predictive ability of our Markov learning model by comparing the log-likelihood of the experimental data for all four treatments under our estimated versions of the learning model against the log-likelihood evaluated under a model of simple multinomial Bernoulli choice. The outcome probabilities of the multinomial Bernoulli model are the empirical choice frequencies of a Non-EOQ action, and the EOQ cycle length choices 0-12. The increase in the log-likelihood value of our estimated Markov learning model measures improvement of prediction accuracy over unconditional choice frequencies. The value of the log-likelihood function for the multinomial Bernoulli choice model is -12297.54. This is demonstrably lower than the log-likelihood value of -3377.07 for our Markov learning models.

Overall we find the Unrestricted treatment leads to a small percentage of Non-EOQ actions, generating performance diminishing outcomes of excess inventories and stockouts. However, we find the likelihood of these events diminish over time and is surprisingly unaffected by high cognitive loads. The more complex choice sets of the Unrestricted treatment also leads to more inertia in EOQ cycle length choices inducing choice lock-in. This is a likely cause of participants choosing near rather than absolute optimal policies in the last two years. This is a similar phenomenon found in Caplin et al. (2011); as they increase choice set complexity participants tend to switch within a smaller range of values. The effect of the High cognitive load is for participants to exhibit a lower level of rationality once they choose EOQ actions; their probability of choosing EOQ cycles that generate at least the same level of average monthly profit is lower.

5 General Discussion

Decision models to optimise inventory management are a core element of operations management. Recent empirical research has challenged the assumptions that inventory managers are fully rational and expected profit maximising. Behavioural inventory management research has turned to experimental methods and behavioural modelling to examine systematic and predictable deviations from optimal decision-making. Extensive research has been done in environments with probabilistic outcomes and uncertainty, such as the newsvendor problem. Schweitzer and Cachon (2000) were among the first to analyse behaviour in the newsvendor setting and identify the pull-to-centre effect, which refers to the observation that people place order quantities that are between the profit maximising quantities and mean demand. Uppari and Hasija (2019) also analysed this effect by comparing different Prospect Theory models. The challenges seem to be persistent suboptimal choices, and difficulties to identify the process by which individuals learn to adjust in repetition. The bulk of behavioural modelling has focused on the preferences of inventory managers. However, the nature of the newsvendor problem

¹⁴See Caplin et al. (2011, page 2909) - Figure 4 for a comparison on the range of values observed with varying choice set complexity.

leads to a difficulty of disentangling aspects of risk preferences and the influence of deterministic attributes such as loss aversion, stockout and excess inventory disutilities.

We turn to a deterministic EOQ environment, one of the fundamental inventory models of supply chain management in practice, allowing isolation of the influence of deterministic attributes and identification of the learning process. We find a much more rapid convergence process from suboptimal to optimal choices. Moreover, we use the psychological EBA choice model as the baseline learning model and the LD theory to capture order adjustment. We incorporate policy attributes such as stockout and excess inventories into a hierarchical refinement of the choice set through an EBA approach which allows for a greater set of substitution patterns by the decision maker than is allowed by typical behavioural newsvendor models.

Even though both the EBA and LD models incorporate quite low rationalities, they predict the observed rapid improvement in order choices. In the EBA choice model, individuals learn to identify top attributes such as avoiding stockouts or excess inventory, namely the EOQ or non-EOQ actions in our setup. Once this step is taken, they proceed to learn the EOQ cycle length that maximises payoff. This process does not occur in a proportional way, rather individuals exhibit low rationality when probabilistically moving up and down the payoff gradient. Our low rationality approach establishes a baseline for future research to explore interventions embedded in more complicated environments, such as those including uncertainty, stochastic demand or time-varying costs.

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A Experiment Instructions and Interface

Instructions for different treatments are presented as the texts/sentences in italics and square brackets below.

A.1 Instruction Page

Welcome

Welcome to today's experiment. Please read the following instructions carefully as they are directly relevant to how much money you will earn today. Please do not communicate with other people during the experiment. Please note that **you are not permitted to use pen and paper or a mobile phone**. Please kindly switch your mobile phone off or put it on silent mode. Students causing a disturbance will be asked to leave the room. You will enter all of your decisions in today's experiment using only the computer mouse. Please do not attempt to use the keyboard or remove the keyboard cover. The information displayed on your computer monitor is private and specific to you. All monetary amounts in today's experiment are expressed as experimental currency units (ECU). The conversion rate for ECU and GBP is $300 \text{ ECU} = \pounds 1$ cash payment. Your payment will be rounded up to the nearest ten pence.

If you have any questions at any point during today's session, please raise your hand and one of the monitors will come to help.

Task

In today's experiment, you will be making **inventory management decisions** for an enterprise called 'S-Store'. S-Store sells coffee makers. You will perform this role for a sequence of 6 years. Every month you will decide how many coffee makers to order from the coffee maker supplier. Your earnings in this experiment will be proportional to the total profitability of S-Store. S-store will sell a new coffee maker model every year. Thus in the first month of a year your inventory always starts from zero. Further, any coffee makers remaining in inventory at the end of month 12 will be disposed of. To summarise, you will be making 12 monthly decisions for a year, and you will do this for 6 years in total.

You will have up to 4 minutes to complete your task for each year. Year 1 is a practice round, and you will have up to 7 minutes to complete the task for this year. You should use this as an opportunity to familiarize yourself with the software and decision tasks. If you don't finish within the time allowed, the computer will automatically execute the remaining month(s) sales with the existing inventory. You will not be able to add inventory. A 'wait page' displays automatically if you spend less than the allowed time in a year. You will only be able to proceed to the next year when the remaining time runs out.

Before the decision making portion of the experiment begins, there will be a **Quiz** consisting of 7 simple questions to check your understanding of the task. Please answer the questions carefully. If you missed 3 or more questions, you would be asked review the correct answers before you can proceed to the task.

[The following italic texts are additional for treatments with High Cognitive Loads]

PIN

In addition to the task, you will be given a 6-digit PIN at the beginning of each year. The PIN is case sensitive, and consisting of numbers, uppercase and lowercase letters. You will have 15 seconds to remember the PIN. This is your KEY to unlock an account which contains an extra reward of 300 ECU. You can open the account at the end of each year by correctly entering the PIN. You will only have one attempt to correctly enter the pin to claim this extra reward.

Payment

Year 0 is a practice round, and you will receive no earnings from your decisions in this year. For Years 1 through 5, your earnings will accumulate across years. At the end of the experiment you will be paid £5 show-up fee and your accumulated earnings, converted to Pounds. Note, negative profit may occur if poor coffee maker ordering decisions are made. To ensure that no one will leave the experiment with a payment less than £5, a negative total profit made in Year 1 to Year 5 will be treated as 0 earnings.

A.2 Background Information

[The following Background Information section shows up on every decision page.]

Your Role:

S-Store is open 360 days per year. You are the inventory manager for S-Store. In your role, you will control S-Store's inventory level which determines the store's total profits.

We now explain how S-Stores, and correspondingly you, earns profit. While we are explaining how the calculations are made, during the decision tasks the computer will carry out these calculations and report the results to you.

S-Store sells coffee makers at a price of **7 ECU** per unit. S-Store can sell up to **10 coffee** makers per month. A coffee maker can only be sold if there is a unit held in inventory. If you hold 10 or more units in inventory at the start of the month, S-Store will sell 10 coffee makers that month. However, if there are less than 10 units held in inventory at the start of the month then S-Store will only sell that amount. For example, if there are 2 units held in inventory at the beginning of a month then S-Store only sells 2 units that month. [(For Zero Only treatment only) You can only place an order when the current month's opening inventory is 0. For example, if the current month's opening inventory is 3 units, you cannot place an order this month, S-Store only sells 3 units this month.] S-Store's sales revenue for a month is calculated as follows:

Sales revenue = 7 ECU * Number of units sold.

Your job is to manage the store's inventory levels by each month choosing an inventory order. Prior to the start of each month you can order coffee makers from the supplier to add to the inventory. Your inventory management determines the S-Store's total costs. S-Store pays two types of costs. One is the **ordering cost**. Every time you order a positive amount you have to pay an order cost. This ordering cost is **45** ECU, and does not depend upon the size of the order. If you order zero coffee makers then you do not pay the 45 ECU ordering cost. Holding coffee makers in inventory is costly so S-Store pays a monthly **inventory holding cost**. S-Store pay's monthly inventory holding cost is based on the average number of coffee makers held in inventory multiplied by the per unit monthly inventory holding cost of **1** ECU. This is calculated as follows:

Inventory holding costs = 1 ECU * (Opening inventory + Order Quantity + Closing inventory)/2.

Calculation of S-Store's profits

Profits = Sales revenue - Ordering costs - Inventory holding costs

Your monthly earnings are equal to S-Store's monthly profits.

Examples:

1. Alice's closing inventory of last month is 20 units, she placed an order of 0 units in this month.

The demand for each month is 10 units.

She made sales of 10 units.

Her closing inventory of this month is 20 - 10 = 10 units.

Her profit in this month is equal to: 7 * 10 - 0 - 1 * (20 + 0 + 10)/2 = 55.

2. Alice's closing inventory of last month is 4 units, she placed an order of 5 units in this month

The demand for each month is 10 units.

She only made sales of 9 units. Her closing inventory of this month is 0 units. Her profit in this month is equal to: 7 * 9 - 45 - 1 * (4 + 5 + 0)/2 = 13.5.

A.3 Multiple Choice Questions prior to Decision Task

There are a couple of questions for you before the task, please use the information:

The demand for each month is 10 units.

Price of each coffee maker is 7.

Ordering cost is 45 per order.

Monthly inventory holding cost is 1 per unit.

Question 1 of 7

If the inventory level was 5 and you ordered 0 units. How many units will you SELL this month?

- A 0
- B 5
- C 10
- D 15

Question 2 of 7

If the inventory level was 0 and you ordered 15 units. How many units will you SELL this month?

- A 0
- B 5
- C 10
- D 15

Question 3 of 7

If you made sales of 10 units. What will be your SALES REVENUE this month?

- A 0
- B 10
- C 25
- D 70

Question 4 of 7

If you ordered 0 units. What will be your ORDERING COST this month?

- A 0
- B 1
- C45
- D 70

Question 5 of 7

If you ordered 1 unit. What will be your ORDERING COST this month?

- A 0
- B 1
- C45
- D 70

Question 6 of 7

If the inventory level was 0 and you ordered 10 units. You made sales of 10 units. What will be your HOLDING COST this month?

A 0

B 1

C 5

D 10

Question 7 of 7

If your sales revenue is 70. Your ordering cost is 0 and your holding cost is 10. What will be your PROFIT this month?

A 15

B 25

C 60

D 70

Figure 5 shows the result page of the multiple choice questions when participants had given more than 2 incorrect answers. Under such circumstances, they had to raise their hands to go through incorrectly answered questions with a monitor in order to obtain a passcode to proceed to the decision tasks.

Figure 5: Result Page of the Multiple Choice Questions

Results If the inventory level was 5 and you ordered 0 units. How many units will you SELL this month? B. 5 B. 5 True If the inventory level was 0 and you ordered 15 units. How many units will you SELL this month? D. 15 C. 10 False If you made sales of 10 units. What will be your SALES REVENUE this month? B 10 D 70 False If you ordered 0 units. What will be your ORDERING COST this month? False If you ordered 1 unit. What will be your ORDERING COST this month? ntory level was 0 and you ordered 10 units. You made sales of 10 units. What will be your C. 5 C. 5 True your sales revenue is 70. Your ordering cost is 0 and your holding cost is 10. What will be your D. 70 C. 60 False Explanation of answers: 1. There are 5 units in total held in inventory this month then S-Store sells 5 units that month 2. There are 15 units in total held in inventory this month, the demand is 10 units, then S-Store sells 10 units that month 3. S-Store's sales revenue for a month is calculated as follows: 7 ECU * Number of units sold = 7*10 = 70 4. You ordered 0 coffee makers then you do not pay the ordering cost 5. The ordering cost is 45 ECU, and does not depend upon the size of the order 6. S-Store's holding cost for a month is calculated as follows: 1 ECU inventory holding cost per unit * (Opening inventory + Order Quantity + Closing inventory) / 2 = 1 * (0+10+0)/2 = 5 7. S-Store's profit for a month is calculated as follows: Sales revenue - Ordering costs - Holding costs = 70 - 0 - 10 = 60 You answered 2 out of 7 questions correctly *Caution! You have missed a large number of questions. This suggests that you may struggle in this task. We suggest you raise your hand so that you can review the correct answers with the monitor lease ask the monitor for the passcode, when you are confidence about the questions, please enter your passcode and click 'Next' to continue Please enter your PASSCODE:

A.4 Decision Tasks

Prior to each year's decision tasks, a mini-instruction page appears. Figure 6 is an example with PIN task. For treatments with high cognitive loads, the pin page follows (Figure 7).

An example of the ordering decision page is shown in Figure 8. Participants move the slider to enter their decision of order quantity for each month. Order quantities, costs, and profits of

Figure 6: An example of Instruction Page with PIN task

Year 4 Instructions On the next page, you will be given a 6-digit PIN. This is your KEY to unlock an account at the end of the year, to claim an extra reward of 300 ECU You will have 15 seconds to remember the PIN. After the PIN Page, you will be making monthly orders for S-Store from the supplier for this year. . To help you with understanding the task, at the beginning of the Order Page, you can find the basic formulas we introduced to you in the instructions . Next, you will be given information regarding the current month to remind you of the key information you will need. There will be a Monthly Record Table displayed on the screen to calculate the Sales revenue, Costs, and Profits for you. The table headings will be look like the following, and the content generates as you proceed to the next months Monthly Record Month Opening Inventory (units) Order Q Closing Inventory Inventory Holding Ordering (units) (units) (units) Revenue Costs Costs will be look like the following, and the content generates as you proceed to the next months Annual Profit Table Total Earnings(£) Click "Next" to proceed to the next page. You will have up to 4 minutes to complete your task for the year

Figure 7: PIN Page prior to Ordering Page



previous months are also displayed on the page. If participants completed the year's decision task within 4 minutes, they had to wait until the end of 4 minutes.

They were then prompted to enter the PIN (Figure 9), followed by the end of the year result page (Figure 10).

B Post-Experimental Survey, Demographics and Summary Statistics of Participants

Participants were asked to fill a simple questionnaire at the end of the experiment for us to collect some demographic information.

The following are some summary statistics of the participants.

Table 10: Demographics in Participants

Age (mean)	25.6
Gender (% female)	65%
Education (%Undergraduate)	51%

One can observe that 37% of the participants are from Social Science & Management, among which they may have training in operations management or have been exposed to the EOQ model before.

Figure 8: Ordering Page

You are making inventory orders for Year 4

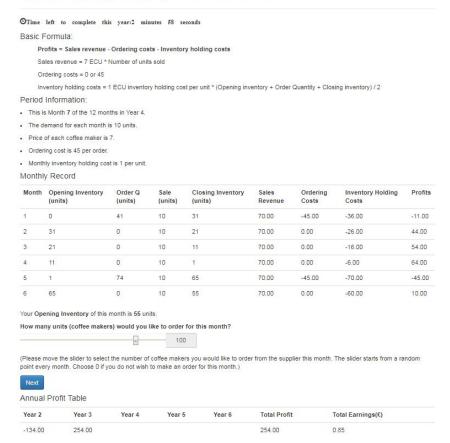


Figure 9: Enter the PIN Page

Year4 PIN - Reward

Figure 10: End of the Year Result Page

Year 4 Result

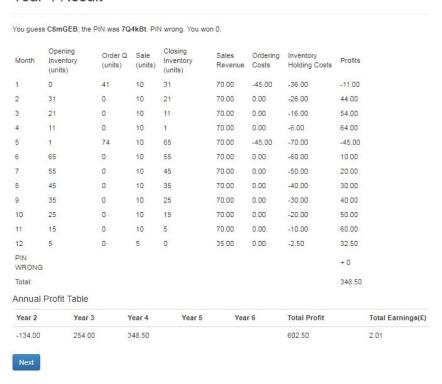


Figure 11: Post-Experimental Survey

Questionnaire

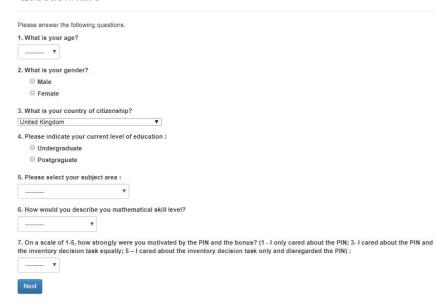
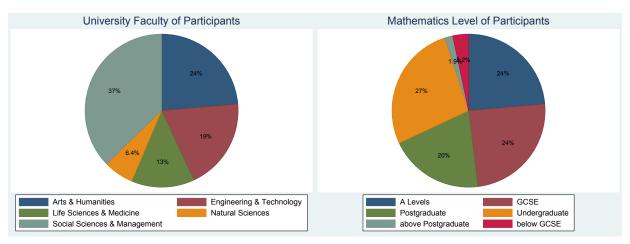


Figure 12: Distribution of University Programs Participants study and Mathematics levels of participants (self reported)



(a) Distribution of University Programs Participants study

(b) Mathematics levels of participants

Table 11: Regression on PIN and demographic information

	(1) Annual Profit	(2) Annual Profit
Year 1	-129.08***	-130.78***
	(19.30)	(18.84)
Year 2	-71.18***	-70.65***
	(16.84)	(16.90)
Year 3	-31.00**	-30.90**
	(13.76)	(13.23)
Year 4	-13.95	-13.74
	(12.28)	(11.89)
Unrestricted	-22.32**	-33.26***
	(11.22)	(12.56)
High-Correct PIN	27.26^{*}	19.09
	(15.96)	(16.13)
Age		-2.42***
		(0.78)
Male		16.14
		(11.88)
Postgrad		-7.44
		(12.40)
STEM		8.66
		(13.73)
Math level		-5.98
		(5.20)
Constant	417.18***	505.75***
	(15.61)	(33.00)
N	385	385
R^2	0.18	0.22

Standard errors in parentheses * p < 0.10, ** p < 0.05, *** p < 0.01

C Possible NW NB sets by month

The monthly average profit conditional upon month is reported in Table 12. Notice that the profit function depends upon t and will penalize relatively long EOQ cycles that generates excess inventory at the year's end.

Table 12: Average monthly profit for alternative EOQ cycle choice given the current month

s	1	2	3	4	5	6	12^{1}
Month 1-7	20	37.5	40	38.75	36	32.5	-
Month 8	20	37.5	40	38.75	36	26	-
Month 9	20	37.5	40	38.75	28.75	18.75	-
Month 10	20	37.5	40	30	20	10	-
Month 11	20	37.5	27.5	17.5	7.5	-2.5	-
Month 12	20	10	0	-10	-20	-30	-

 $[\]tilde{s}_{i,k} = 12$ always offers the lowest average monthly payoff

Based on the monthly average profit table, we can compute the No worse than and No better than sets for each EOQ cycle conditional upon month as follows:

Table 13: The No worse than and No better than sets for each EOQ cycle by month

Months 2-8	$\begin{split} \tilde{s}_{i,k-1} &= 1 \\ \tilde{s}_{i,k-1} &= 2 \\ \tilde{s}_{i,k-1} &= 3 \\ \tilde{s}_{i,k-1} &= 4 \\ \tilde{s}_{i,k-1} &= 5 \\ \tilde{s}_{i,k-1} &= 6 \\ \tilde{s}_{i,k-1} &= 12 \end{split}$	$NW = \{1, 2, 3, 4, 5, 6\}$ $NW = \{2, 3, 4\}$ $NW = \{3\}$ $NW = \{3, 4\}$ $NW = \{2, 3, 4, 5\}$ $NW = \{2, 3, 4, 5, 6\}$ $NW = \{1, 2, 3, 4, 5, 6, 12\}$	$NB = \{1, 12\}$ $NB = \{1, 2, 5, 6, 12\}$ $NB = \{1, 2, 3, 4, 5, 6, 12\}$ $NB = \{1, 2, 4, 5, 6, 12\}$ $NB = \{1, 5, 6, 12\}$ $NB = \{1, 6, 12\}$ $NB = \{12\}$
Month 9	$ \begin{aligned} \tilde{s}_{i,k-1} &= 1 \\ \tilde{s}_{i,k-1} &= 2 \\ \tilde{s}_{i,k-1} &= 3 \\ \tilde{s}_{i,k-1} &= 4 \\ \tilde{s}_{i,k-1} &= 5 \\ \tilde{s}_{i,k-1} &= 6 \\ \tilde{s}_{i,k-1} &= 12 \end{aligned} $	$NW = \{1, 2, 3, 4, 5\}$ $NW = \{2, 3, 4\}$ $NW = \{3\}$ $NW = \{3, 4\}$ $NW = \{2, 3, 4, 5\}$ $NW = \{1, 2, 3, 4, 5, 6\}$ $NW = \{1, 2, 3, 4, 5, 6, 12\}$	$NB = \{1, 6, 12\}$ $NB = \{1, 2, 5, 6, 12\}$ $NB = \{1, 2, 3, 4, 5, 6, 12\}$ $NB = \{1, 2, 4, 5, 6, 12\}$ $NB = \{1, 5, 6, 12\}$ $NB = \{6, 12\}$ $NB = \{12\}$
Month 10	$\begin{split} \tilde{s}_{i,k-1} &= 1 \\ \tilde{s}_{i,k-1} &= 2 \\ \tilde{s}_{i,k-1} &= 3 \\ \tilde{s}_{i,k-1} &= 4 \\ \tilde{s}_{i,k-1} &= 5 \\ \tilde{s}_{i,k-1} &= 6 \\ \tilde{s}_{i,k-1} &= 12 \end{split}$	$NW = \{1, 2, 3, 4, 5\}$ $NW = \{2, 3\}$ $NW = \{3\}$ $NW = \{2, 3, 4\}$ $NW = \{1, 2, 3, 4, 5\}$ $NW = \{1, 2, 3, 4, 5, 6\}$ $NW = \{1, 2, 3, 4, 5, 6, 12\}$	$NB = \{1, 5, 6, 12\}$ $NB = \{1, 2, 4, 5, 6, 12\}$ $NB = \{1, 2, 3, 4, 5, 6, 12\}$ $NB = \{1, 4, 5, 6, 12\}$ $NB = \{1, 5, 6, 12\}$ $NB = \{6, 12\}$ $NB = \{12\}$
Month 11	$\begin{split} \tilde{s}_{i,k-1} &= 1 \\ \tilde{s}_{i,k-1} &= 2 \\ \tilde{s}_{i,k-1} &= 3 \\ \tilde{s}_{i,k-1} &= 4 \\ \tilde{s}_{i,k-1} &= 5 \\ \tilde{s}_{i,k-1} &= 6 \\ \tilde{s}_{i,k-1} &= 12 \end{split}$	$NW = \{1, 2, 3\}$ $NW = \{2\}$ $NW = \{2, 3\}$ $NW = \{1, 2, 3, 4\}$ $NW = \{1, 2, 3, 4, 5\}$ $NW = \{1, 2, 3, 4, 5, 6\}$ $NW = \{1, 2, 3, 4, 5, 6, 12\}$	$NB = \{1, 4, 5, 6, 12\}$ $NB = \{1, 2, 3, 4, 5, 6, 12\}$ $NB = \{1, 3, 4, 5, 6, 12\}$ $NB = \{4, 5, 6, 12\}$ $NB = \{5, 6, 12\}$ $NB = \{6, 12\}$ $NB = \{12\}$
Month 12	$\begin{split} \tilde{s}_{i,k-1} &= 1 \\ \tilde{s}_{i,k-1} &= 2 \\ \tilde{s}_{i,k-1} &= 3 \\ \tilde{s}_{i,k-1} &= 4 \\ \tilde{s}_{i,k-1} &= 5 \\ \tilde{s}_{i,k-1} &= 6 \\ \tilde{s}_{i,k-1} &= 12 \end{split}$	$NW = \{1\}$ $NW = \{1, 2\}$ $NW = \{1, 2, 3\}$ $NW = \{1, 2, 3, 4\}$ $NW = \{1, 2, 3, 4, 5\}$ $NW = \{1, 2, 3, 4, 5, 6\}$ $NW = \{1, 2, 3, 4, 5, 6, 12\}$	$NB = \{1, 2, 3, 4, 5, 6, 12\}$ $NB = \{2, 3, 4, 5, 6, 12\}$ $NB = \{3, 4, 5, 6, 12\}$ $NB = \{4, 5, 6, 12\}$ $NB = \{5, 6, 12\}$ $NB = \{6, 12\}$ $NB = \{12\}$