On the Effects of Consumer Search and Firm Entry in a Multiproduct Competitive Market*

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Abstract

If searching for a better price becomes easier for consumers, conventional wisdom suggests that pricing pressure will increase on firms, thereby lowering prices, reducing firm profits and narrowing assortments. There is indeed evidence that recent search facilitating technologies, such as the Internet, have in some markets reduced prices. But there is also evidence that firms have been able to maintain some pricing power. Furthermore, there is evidence that the Internet has expanded variety in some markets and this broadening of the available assortment has increased consumer welfare significantly. This paper studies a model in which firms compete on price and assortment and consumers search for products that match their preferences at a reasonable price. Easier search does exhibit a competition intensifying effect in our model, and this effect puts pressure on firms to lower their prices and reduce assortments. But we also show that easier search exhibits a market expansion effect that encourages firms to expand their assortment. Due to broader assortments, consumers are more likely to find products that match their ideal preferences, improving the efficiency of the market. In fact, we demonstrate that the market expansion effect can even dominate the competition intensifying effect potentially leading to higher prices, broader assortments, more profits and expanded welfare.

Keywords: search, Internet, price competition, assortment, product variety, game theory, differentiated competition.

1 Introduction

It was widely believed that the emergence of the Internet would lead to brutal price competition and an erosion of retail profit. The argument is simple and compelling: the Internet lowers the costs consumers incur to search for goods and services, so consumers search more, resulting in more opportunities to compare prices and ultimately more severe price competition among sellers. There is indeed some evidence that the Internet has reduced prices in several markets (e.g., Brynjolfsson and Smith 2000, Brown and Goolsbee 2002, Sorensen 2000). Nevertheless, there is also evidence that firms are able to support prices above marginal cost (e.g., Brynjolfsson, Dick and Smith 2004, Clemons, Hann and Hitt 2002, Hortacsu and Syverson 2004).

This paper provides a simple explanation for why firms have been able to maintain at least some pricing power and hence profits. Suppose consumers care about the price they pay but also want to find a product or service that best matches their preferences. For example, a shopper may know she wishes to purchase a pair of shoes but is not exactly sure what pair she wants. Different retailers offer different prices and different selections of shoes, so our shopper searches among a sample of retailers to find a suitable pair at an acceptable price. All else being equal, the wider a store's assortment, the more likely the shopper will find a pair of shoes that closely matches her ideal. Although a wider assortment increases a retailer's demand, a wider assortment is costly (e.g., additional inventory costs, space requirements, etc.). Lower search costs then have two effects in this market. With a lower cost to search, a consumer is likely to search more stores, so each retailer is put into greater competition with other retailers. This *competition intensifying effect* is precisely the intuition behind the conventional wisdom that easier search reduces market prices. However, if consumers search more, then each retailer effectively gains access to a broader pool of potential customers. A larger pool of potential customers allows the retailer to justify a broader assortment. Because a larger assortment provides value to consumers, the retailer can then support a *higher* price and potentially even make *more* profit. In other words, when retailers choose their assortments in addition to their prices, lower search costs have a *market expansion effect* that dampens competition. In fact, we demonstrate that the market expansion effect can even dominate, i.e., lower search costs in a competitive market can lead to higher prices, more industry profits, and increased consumer and social welfare.

At the very least, the reduction in market prices and profits is not as severe as would be predicted by considering only the competition intensifying effect.

We present a model of multiple firms competing on price and assortment in which consumers search for a product that best fits their needs. This novel model setup lets us study the impact of search costs on prices, assortments, profits, and welfare. To further highlight the distinctiveness of search's market expansion effect, we compare the influence of search with the influence of firm entry.

2 Literature Review

Bakos (1997) and Anderson and Renault (1999) study search among differentiated single-product firms with exogenous differentiation. They find that easier search leads to lower equilibrium prices, corresponding, again, to the intuition that cheaper search intensifies price competition. Anderson and Renault (1999) find that equilibrium prices increase with the diversity of consumers' tastes. Despite this relationship, we note that easier search in their model continues to lower prices. In our model firms have a second strategic decision (their assortment breadth) that yields our qualitatively different conclusions. Klemperer (1992) studies a duopoly model where each firm offers a line of products, but the number of products carried by each firm is exogenous, whereas assortment breadth is endogenous in our model.

There is an extensive empirical literature that measures the impact of search on market dynamics. (See Baye, Morgan and Scholten 2005 for a review.) For example, Brynjolfsson, Hu, and Smith (2003) estimate the economic impact of increased product variety in online bookstores and show that consumer welfare has increased significantly more by broader assortments than by increased competition or lower prices. Our theoretical conclusions are consistent with their findings: easier search in a competitive equilibrium can lead to broader assortments and reasonably stable prices.

Several recent papers develop theoretical models to explain why easier search may not lead to excessive price competition. Lal and Sarvary (1999) argue that the Internet reduces consumer search costs for digital attributes of a product (such as its price and dimension) but not for non-digital attributes (such as how well it fits, in the case of clothing). This effectively increases a consumer's cost to search, so prices can rise. In an experimental study, Lynch and Ariely (2000) consider separately the ease of searching for price or quality information. They find that as quality information becomes easier to search, then consumers are less price sensitive. We assume consumers incur a single cost to learn each firm's price and product attributes. Most closely related to this paper is Kuksov (2004). Similar to our paper Kuksov establishes the counter-intuitive result that average prices can increase if search costs decrease. Despite this similar result, because our model set-up is quite different, we obtain different results and empirical predications concerning profits, assortments and welfare. We summarize these differences at the end of Section 5.

3 Model

There are n risk neutral multi-product firms who each offers a set of differentiated products within the same category to a market of risk neutral consumers. Firms simultaneously choose their prices and assortment. (Our qualitative results continue to hold if firms choose assortments first and then prices.)[1] Consumers then shop among firms and either choose to purchase one unit of a product from one firm or to abstain from purchasing (the no-purchase option). We normalize the consumer population to one (w.l.o.g.).

We study two variants of consumer search in our model, parallel and sequential search. (These are the common two variants in the search literature, see Baye, Morgan and Scholten 2005.) Both variants reflect the presence of search costs, yet they differ in the process with which consumers search. (Search can be costly even in electronic markets: e.g., Hann and Terwiesch 2003 show that consumers value the dis-utility of an online search activity at about $5 per search.) In the case of parallel search, consumers sample a fixed number of firms and then purchase their most preferred product among the firms searched, which might be the no-purchase option. With sequential search, consumers visit firms sequentially and decide after each visit whether to purchase an item, continue their search at another firm or to purchase nothing. We suspect other search models are qualitatively a mixture of these two extremes, and they are analytically more cumbersome.

In both models firms set their prices and assortments to maximize their expected profit given consumer shopping behavior and consumers choose a search strategy to maximize

[1] See online technical appendix for details.

their utility given the price and assortment decisions of the firms. We focus on symmetric equilibria throughout the paper.

Our consumer choice process is based on the multinomial logit (MNL) model (see Anderson, de Palma and Thisse 1992). The basic form of the model works as follows. A consumer considers whether to purchase one unit from a set of products, S, or to purchase nothing, product 0, which we also call the no-purchase option. Let U_j be the consumer's utility from product j, $j \in S \cup \{0\}$: $U_j = (u_j - p_j) + \zeta_j$, where u_j is a constant, p_j is the price of product j ($p_0 = 0$), and ζ_j is a random variable with a zero-mean Gumbel distribution and variance $\pi^2\mu^2/6$, where μ is a scale parameter. Let $F(x)$ and $f(x)$ be the distribution and density functions of ζ_j and γ is Euler's constant,

$$F(x) = \exp[-\exp(-(x/\mu + \gamma))].$$

A consumer chooses the product with the highest realized utility. Let $q_j(S \cup \{0\})$ be the probability a consumer chooses product j given the choice set. It is well known that

$$q_j(S \cup \{0\}) = \Pr(U_j = \max\{U_i, i \in S \cup \{0\}\}) = \frac{\exp((u_j - p_j)/\mu)}{\sum_{j \in S} \exp((u_i - p_i)/\mu) + \exp(u_0/\mu)}. \quad (1)$$

It follows from (1) that demand for product j decreases as more products are added to the consumer's choice set: $q_j(S) > q_j(S^+)$, $\forall j \in S \subset S^+$.

Let U_{ik} be a consumer's utility associated with firm k's i^{th} product, $i \geq 0$: $U_{ik} = (u_{ik} - p_{ik}) + \zeta_{ik}$. We assume that all actual products are equally likely to be preferred, adjusting for any price differences: for all $i \geq 1$ and k, $\zeta_{ik} = \zeta$ and $u_{ik} = 0$ (normalized to zero w.l.o.g.). For the no-purchase product we choose $u_{0k} = u_0$ and $p_{0k} = p_0 = 0$ (normalized to zero w.l.o.g.). MNL based models have been widely used in empirical research on product variety/assortment related issues in both economics (e.g., Watson 2004) and marketing (e.g., Draganska and Jain 2005, 2006).

In our model, each firm offers a unique set of products (i.e., each firm is the exclusive provider of its assortment). This is appropriate in a market with substantial variety and diffuse preferences, such as women's shoes, eyeglasses, apparel, ceramic tiles, antiques, paintings, bicycles or paperback romance novels, among others. We feel our qualitative results should continue to hold even if firms have overlapping assortments. But overlapping assortments require modeling consumer expectations for how assortments overlap and how those expectations influence consumer search. Intuitively, the greater the overlap in assortments across firms, the lower the value of consumer search to find a suitable product. Therefore, in our model the value of search to consumers is maximized.

Because all products have the same probability of appealing to a consumer, it can be shown that it is optimal for each firm k to choose the same price for all of its products[2], p_k, (e.g., Shugan (1989) indicates that the majority of flavors within a product line are sold at the same price in the ice cream industry.) and the firm's assortment decision amounts to choosing the number of products to offer, x_k. For analytical simplicity, throughout this paper we treat x_k as a continuous variable. We assume each firm purchases each product for the same marginal cost (normalized to zero) but variety is costly from an operational perspective. In particular, let $c(x)$ be a firm's operational costs when it carries x products, $c(0) = 0$, $c'(x) \geq 0$ and $c''(x) \geq 0$.

In summary, we present an oligopoly model with n firms that provide an assortment of differentiated product variants to a set of consumers that may or may not purchase a unit (i.e. consumers have a no-purchase option). This model setup is distinct from the duopoly model presented by Kuksov (2004). Kuksov studies two firms that sell a single product variant to a set of consumers that purchase one unit with certainty (i.e., they do not have a no-purchase option).

4 Competition with Parallel Search

In this section consumers sample firms in parallel: each consumer *a priori* commits to search m firms. The consumer observes the product offerings of m randomly selected firms and then purchases the product that maximizes utility net price, which may be the no-purchase option. The consumer incurs a search cost of τ per firm included in the search, for a total search cost of $m\tau$.[3] Consumer choose m to maximize their expected utility conditional on the expectation that firms choose a symmetric equilibrium, i.e., each consumer expects each

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[2] See the online technical appendix for details on the optimality of identical pricing in an assortment.

[3] This model can be extended to the case that the cost of searching m firms is a convex and increasing function of m.

of the n firms to choose price p^* and assortment x^*. Naturally, each consumer makes an optimal purchase decision given the actual prices and assortments observed in the m chosen firms.

A parallel search strategy is appropriate if there is some lag to receive information from firms, such as requesting catalogs from several mail-order firms, or if consumers use a simple search heuristic that commits them to a fixed number of searches (e.g., "obtain price quotes from three insurance companies before choosing one"). Our parallel search model is analogous to the consideration set theory of consumer choice in the consumer behavior literature (e.g., Howard and Sheth 1969, Roberts and Lattin 1991). Consideration set theory posits that consumers first decide which brands/options to include in a consideration set and then they choose the best alternative among their pre-specified set.

Firms choose their prices and assortments optimally given their expectation of m. A symmetric equilibrium in this setting is a 3-tuple, $\{p^*, x^*, m^*\}$, such that consumers have no incentive to deviate from their search strategy and no firm has an incentive to deviate from its price and assortment strategy.

A consumer's search strategy maximizes $U(m)$, a consumer's expected net utility of searching m firms,

$$\max_{1 \leq m \leq n} U(m) = -p^* + \mu \ln m + \mu \ln x^* - m\tau,$$

where $U(m)$ follows from the properties of the Gumbel distribution and the MNL model (p. 60, Anderson, de Palma and Thisse 1992). The solution to the consumer's problem is

$$m^* = \min\left[n, \max\left(1, \frac{\mu}{\tau}\right)\right], \tag{2}$$

where we treat m as a continuous variable. The consumer's optimal search sample size, m^*, is decreasing in the search cost τ and is independent of p^* and x^*.[4]

Now consider the firms' pricing and assortment decisions. Let $\{p_i, x_i\}$ be firm i's choices and $\{p, x\}$ be firm i's expectation for the choices of the other firms, where recall firm i expects the other firms to follow a symmetric equilibrium. Conditional that firm i is included in a

[4] This convenient result is obtained because the consumer's maximum utility is the log of a multiplicative term that includes the number of products and price. Hence, m separates from both x^* and p^*.

consumer's search, that consumer purchases from firm i with probability

$$q_i(p_i, x_i|p, x, m^*) = \frac{x_i \exp(-p_i/\mu)}{x_i \exp(-p_i/\mu) + (m^* - 1)x \exp(-p/\mu) + \exp(u_0/\mu)}. \tag{3}$$

Although a consumer expects all firms to choose p^* and x^*, (3) implies that a consumer makes an optimal choice given the actual prices and assortments observed.

Our model of each firm's market share, $q_i(\cdot)$, is similar to the share equations in the literature on market-share models, but there are some important differences (Basuroy and Nguyen 1998; Bell, Keeney and Little 1975; Gruca, Kumar and Sudharshan 1992; Gruca and Sudharshan 1991; Karnani, 1985; Monahan 1987). Those models express firm i's sales as

$$SALES_i = \left(\frac{A_i}{A_i + \sum_{j \neq i} A_j}\right) \times SIZE\left(\sum A_i\right),$$

where A_i is the attractiveness of firm i and $SIZE(A)$ is the total sales in the market, which is possibly an increasing function of the firms' total attraction. Our model fixes the total potential sales of the market and each firm's share of that total potential depends on the sum of the firms' attractions plus the attraction of the no-purchase option (i.e., the adjustment for market size in our model is in the denominator rather than in the numerator). Existence of equilibrium can be an issue in market share models based on the MNL (Basuroy and Nguyen 1998; Gruca and Sudharshan 1991), but existence is not an issue in our model even though our model is also based on the MNL.[5]

Firm i's expected profit is

$$\Pi_i(p_i, x_i|p, x, m^*) = p_i \left(\frac{m^*}{n}\right) q_i(p_i, x_i|p, x, m^*) - c(x_i) \tag{4}$$

because m^*/n is the probability firm i is included in a consumer's random sample of m^*

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[5] Previous research derives each firm's attractiveness score to be of the form $a_i e^{\alpha_i p_i} e^{\beta_i x_i}$, where a_i, α_i and β_i are constants, p_i is the firm's price and m_i is its expenditure on a marketing variable (such as advertising or service). Price enters into our attractiveness in the same way, but our additional strategic variable, the number of products offered, is in linear form, x_i, rather than exponential form, $e^{\beta_i x_i}$, which is the key difference. Equilibrium existence does not appear to be an issue with multiplicative competitive interaction (MCI) models. Those models have attractiveness of the form $a_i p_i^{-\alpha} x_i^{\beta}$, which is similar to our form with respect to x_i, but different with respect to price.

firms. The next result establishes the existence and uniqueness of an equilibrium. All proofs are in the appendix.

Proposition 1 *In the parallel search model there exists a unique symmetric equilibrium,* $\{p^*, x^*, m^*\}$*. If* $c'(0) < \mu^2/(n\tau \exp(1 + u_0/\mu))$*, then*

$$m^* = \min\left[n, \max\left(1, \frac{\mu}{\tau}\right)\right]$$

and p^* *and* x^* *are implicitly defined by the following:*

$$p^* = \frac{\mu^2}{\mu - (n/m^*)x^* c'(x^*)}, \tag{5}$$

$$x^* = \frac{\mu}{nc'(x^*)} - \frac{\exp\left(\left(u_0 + p^*\right)/\mu\right)}{m^*}. \tag{6}$$

Otherwise, $p^* = \mu$*,* $m^* = 0$*,* $x^* = 0$*.*

Proposition 2 *In the parallel search model (assuming an equilibrium with positive assortments), while firm entry reduces both product variety,* x^**, and price,* p^**, easier search increases product variety,* x^**, and does not necessarily reduce price,* p^**. Specifically, if* $c'(x) = c$ *for some constant* c*, then there exists some threshold* $\bar{\tau}$ *such that easier search (decreasing* τ*) increases the equilibrium price when* $\tau > \bar{\tau}$*, otherwise easier search decreases the equilibrium price. Furthermore, while firm entry may not always increase the total market variety* nx^**, easier search always does.*

The effects of firm entry are consistent with the existing literature: firm entry lowers both equilibrium price and product variety and may not necessarily increase the total variety in the market. This is due to the competition intensifying effect of firm entry: a more crowded market results in more competition, which shrinks market-share (m^*/n), which reduces each firm's incentive to invest in variety. Furthermore, with less product variety and more competitors, firms must reduce prices.

The effects of search, on the other hand, are quite different than those of firm entry. For example, easier search *raises* the equilibrium product variety x^* and the total product variety in the market nx^*. As the search cost, τ, decreases, consumers are more likely to expand their search of firms before making a final purchase decision. Consequently, easier search pressures firms to compete for the patronage of a consumer with a greater number of competitors. This is similar to firm entry: easier search leads to more competition in the market, which induces firms to reduce their variety. This competition intensifying effect of easier search is identified in the choice probability (3): everything else equal, a higher value of m^* (which is caused by easier search) results in lower $q_i(p_i, x_i|p, x, m^*)$. But easier search has an additional effect on market dynamics: when consumers search more actively, a firm gains access to (in the sense of "is being visited by") more consumers, i.e., an increase in m^* also increases the m^*/n term in the objective function (4). The overall effect of easier search on the equilibrium product offering depends on the relative strength of these two effects. According to Proposition 2, in the case of parallel search, the market expansion effect of easier search always dominates the competition intensifying effect, i.e., $(m^*/n)q_i(p_i, x_i|p, x, m^*)$ is increasing in m^*. Hence, overall, easier search raises the equilibrium assortment of each firm and the total assortment in the market.

Proposition 2 also indicates that the market price may actually increase as search becomes easier (i.e., as τ decreases). This result is in sharp contrast to the conventional economic view that increased competition leads to a lower market price (e.g., Bakos 1997, Anderson and Renault 1999). Prices may rise because firms offer more variety, thereby generating greater consumer welfare that allows the firms to support higher prices. It can be shown that when the variety cost function is linear, there exists a key threshold, $\overline{m}$, that determines the influence of search on prices. When the search cost is sufficiently high that $m^* < \overline{m}$, then a decrease in the search cost increases equilibrium prices: with a high search cost the firms offer minimal variety, so a marginal expansion in variety creates a significant improvement in consumer welfare. When the search cost is already low, such that $m^* > \overline{m}$, then further expansion in variety does not increase consumer welfare enough to increase price. This result implies that poorly served markets (because of limited variety and high search costs) are likely to support price increases when technology is implemented to reduce search costs.

Proposition 3 *In the parallel search model, while firm entry reduces the equilibrium firm profit, easier search does not necessarily reduce the equilibrium firm profit. If the equilibrium price increases as the search cost decreases, then the firms' equilibrium profit increases.*

So firm entry indeed always reduces the firms' equilibrium profit, because the operating cost savings of less product variety are not enough to compensate for the revenue loss due to a lower price and market share. However, easier search may not necessarily lead to a profit decline: when easier search increases the equilibrium price, the revenue increase driven by a higher price offsets the cost increase associated with higher product variety, and equilibrium profit increases. (Note, this is a sufficient but not necessary condition: it is possible for equilibrium profits to increase even if equilibrium prices decrease.)

Proposition 4 *In the parallel search model, while firm entry does not necessarily increase consumer and social welfare, easier search always increases consumer welfare and can increase social welfare.*

Proposition 4 indicates that the social welfare (consumer welfare and industry profit) can increase as search becomes easier. More striking, Proposition 4 reveals that consumers always benefit from lower search costs even if prices rise because broader assortments generate sufficiently higher utilities to outweigh any welfare loss due to higher prices. Furthermore, we find that firm entry may neither benefit consumers nor society as a whole. These results are consistent with Brynjolfsson, Hu, and Smith (2003) who find that the consumer welfare gain from increased product variety available through electronic markets is significant and between 7 and 10 times as large as the gain from the increased number of competitors.

5 Competition with Sequential Search

With sequential search, upon observing a firm's offerings a consumer decides among three options: (1) purchase a product, (2) incur a search cost τ and engage in further search, or (3) discontinue the search process. As with parallel search, we assume consumers anticipate a symmetric equilibrium (each firm chooses price p^* and assortment x^*).

Suppose a consumer is visiting firm i with price p_i and her utility of the best product offered by firm i is y. Given her expectation of $\{p, x\}$, it is worthwhile for the consumer with search cost τ to search another firm who offers x products with price p instead of purchasing the current best product if the expected value of search is greater than the search cost τ, i.e.,

$$\int_{y-p_i+p}^{\infty} (\phi - (y - p_i + p))dH(\phi, x) \geq \tau, \tag{7}$$

where $H(\phi, x)$ is the CDF of the maximum utility of x products and

$$H(\phi, x) = \exp\left[-x \exp\left(-\left(\frac{\phi}{\mu} + \gamma\right)\right)\right].$$

As the left-hand side of (7) is decreasing in the term $y - p_i + p$ there exists a threshold $\bar{U}(x, \tau)$ for any given x and τ such that the consumer will choose to search another firm if $y - p_i + p \leq \bar{U}(x, \tau)$. Also, from (7), note that $\bar{U}(x, \tau)$ is increasing in x and decreasing in τ. Weitzman (1979) shows that this threshold search strategy is the optimal sequential search strategy.

The following Lemma establishes a useful property of the search threshold.

Lemma 5 *$H(\bar{U}(x,\tau),x)$ is independent of x, where $\bar{U}(x,\tau)$ is the smallest y such that the inequality (7) strictly holds.*

Based on Lemma 5, we write $H(\bar{U}(x,\tau),x)$ as $H(\tau)$ whenever it is convenient. Define $\rho(x,\tau) = \exp(-(\bar{U}(x,\tau)/\mu+\gamma))$. Because $H(\bar{U}(x,\tau),x) = \exp[-x\rho(x,\tau)]$, Lemma 5 implies that $x\rho(x,\tau)$ is also independent of x. Furthermore, $H(\tau)$ is decreasing in τ and $\rho(x,\tau)$ is increasing in τ.

We assume that consumers observe their no-purchase utility first. As a result, a consumer visits at least one firm only if her no-purchase utility is lower than $\bar{U}(x,\tau)-p$. Therefore, $H_0(\bar{U}(x,\tau)-p)$ is the fraction of consumers that actually enter the market, where H_0 is the CDF of the no-purchase utility and

$$H_0(\phi) = \exp\left[-\exp\left(-\left(\frac{\phi-u_0}{\mu}+\gamma\right)\right)\right].$$

A symmetric equilibrium in this setting is a 3-tuple, $\{p^*, x^*, \bar{U}(x^*,\tau)\}$, such that consumers have no incentive to deviate from their search strategy and no firm has an incentive to deviate from its price and assortment strategy given their expectations. Suppose firm i's choices are $\{p_i, x_i\}$ and all other firms' choices are $\{p, x\}$. If a consumer visits firm i, the probability she purchases at firm i is $1-H(\bar{U}(x,\tau)+(p_i-p), x_i)$, otherwise she is equally likely to choose among the firms that have not been visited. The probability that firm i is visited first is $1/n$ and, obviously, the probability it is not visited first is $(n-1)/n$. If firm i is not visited first, then with probability $H(\tau)$ the consumer does not purchase at the first firm and visits firm i in her next search with probability $1/(n-1)$. Thus, the probability that firm i is the second firm in the consumer's search is:

$$\left(\frac{n-1}{n}\right)H(\tau)\left(\frac{1}{n-1}\right) = \frac{H(\tau)}{n}.$$

In general, the probability a consumer eventually visits firm i is

$$\sum_{j=1}^{n}\frac{H(\tau)^{j-1}}{j} = \frac{1}{n}\left(\frac{1-H(\tau)^n}{1-H(\tau)}\right). \tag{8}$$

Hence, with probability $H(\bar{U}(x,\tau)+(p_i-p), x_i)H(\tau)^{n-1}$ a consumer will visit all n firms in the market and still cannot find a product (including the no-purchase option) that has a higher utility than her search threshold $\bar{U}(x,\tau)$. In our analysis we assume this probability is small, i.e., $H(\bar{U}(x,\tau)+(p_i-p),x_i)H(\tau)^{n-1}\approx 0$, which also implies that the probability (8) is approximately equal to

$$\frac{1}{n\left(1-H(\tau)\right)}.$$

This is a mild assumption as long as the total assortment in the market is reasonably large and the number of firms is not too small.

Thus, the probability a consumer visits firm i and also makes a purchase from firm i is

$$q_i(p_i,x_i|p,x,\tau)=\frac{H_0(\bar{U}(x,\tau)-p)\left[1-H(\bar{U}(x,\tau)+(p_i-p),x_i)\right]}{n\left[1-H(\tau)\right]}. \tag{9}$$

Firm i's expected profit is

$$\Pi_i(p_i,x_i|p,x,\tau)=p_iq_i(p_i,x_i|p,x,\tau)-c(x_i). \tag{10}$$

The following proposition establishes, under a relatively mild condition (e.g., which holds for linear and quadratic $c(x)$), the existence and uniqueness of an equilibrium for our sequential search model.

Proposition 6 *Define*

$$g(x)=\frac{c'(x)+xc''(x)}{x^2}.$$

If $g(x)$ is decreasing in x, there exists a unique symmetric equilibrium in the sequential search model, $\{p^*,x^*,\bar{U}(x^*,\tau)\}$, where $\bar{U}(x^*,\tau)$, p^*, and x^* are implicitly defined by the following:

$$\int_{\bar{U}(x^*,\tau)}^{\infty}(\phi-\bar{U}(x^*,\tau))dH(\phi,x^*)=\tau, \tag{11}$$

$$p^*=\frac{\mu\left[1-H(\tau)\right]}{x^*\rho(x^*,\tau)H(\tau)}, \tag{12}$$

and

$$x^*=\frac{\mu H_0(\bar{U}(x^*,\tau)-p^*)}{nc'(x^*)}. \tag{13}$$

Proposition 7 *In the sequential search model, while firm entry reduces product variety, x^*, and leaves price, p^*, unchanged, easier search raises x^* and reduces p^*. Furthermore, firm entry may not always increase the total variety available in the market, nx^*, while easier search always does.*

The qualitative findings of the sequential search model are consistent with those of the parallel search model. For example, while firm entry reduces the equilibrium product variety, easier search increases the equilibrium product variety and the total market variety. Again, the difference between easier search and firm entry is driven by the market expansion effect of easier search. However, in the sequential search model, the market expansion effect of easier search is not strong enough to elicit an increase in the equilibrium price. (But price competition is not as severe as it would be without the market expansion effect.)

Proposition 8 *In the sequential search model, while firm entry reduces the equilibrium firm profit, easier search does not necessarily reduce the equilibrium firm profit. If the consumers' expected no-purchase utility u_0 is sufficiently high, easier search increases equilibrium profit.*

As Proposition 8 shows, firm entry still reduces the firms' profits in equilibrium. This occurs for two reasons. First, because firms respond to new entry by reducing product variety and maintaining prices, the market as a whole is less attractive to consumers. As a result, fewer consumers enter the market. Second, each firm's market share falls post-entry. Although prices stay unchanged, the firms' demand post-entry decreases enough to lead to lower revenues and profits.

In contrast, Proposition 8 indicates that although easier search cannot elicit an increase in the equilibrium price, it can still lead to a profit increase, especially when the consumers' expected no-purchase utility u_0 is large. When u_0 is large, consumers are less likely to enter the market to search firms at all. In this scenario, the market expansion effect of easier search on the whole market (the term $H_0(\bar{U}(x,\tau)-p)$ in (9)), becomes stronger and more important to the firms' profitability. If u_0 is sufficiently large, this expansion of the total market size caused by easier search outweighs the price decline and the increase in product variety costs.

Proposition 9 *In the sequential search model, while firm entry reduces consumer welfare and does not necessarily increase social welfare, easier search always increases both consumer and social welfare.*

The qualitative findings of the sequential search model concerning welfare are consistent with those of the parallel search model. Thus, the market expansion effect and its impact on assortments, profits, and welfare is robust to the consumer search model. Since the effect of easier search on price differs in the parallel and sequential models, we conclude that the effect of easier search on assortments, profits, and welfare is stronger than the effect on prices.

There are subtle differences between the parallel and sequential search models, but the complexity of the MNL functional forms make a detailed comparison of the two models difficult. However, for a given search cost, we can show that consumers search more in the sequential model than in parallel model conditional that they choose to search, i.e., $E[N|\ N > 0] > m^*$, where N is the number of firms searched in the sequential model. Furthermore, $\partial m^*/\partial\tau \leq \partial E[N|N > 0]/\partial\tau < 0$, i.e., the amount of search in the parallel model is more sensitive to a change in the search cost than in the sequential model. Thus, a reduction in the search cost τ generates a greater increase in search with parallel search than with sequential search, thereby leading to a greater expansion in assortment. If initially τ is sufficiently high, then the expansion of the assortment in the parallel model is sufficiently great to support a price increase, whereas in the sequential search model the expansion of the assortment is never enough to lead to an increase in the equilibrium price. Independent of search mode, as search costs decrease, consumers search more in equilibrium.

In summary, we find that lower search costs (a) lead to an increase in industry sales (the market size expansion effect, consumers who abstained from purchase when faced with high search costs enter the market when search costs decrease), (b) lead to broader assortments (c) can increase social welfare, (d) can make consumers better off, even if prices increase, (e) increase search activity in equilibrium, (f) increase industry profit even if prices decrease, and (g) can increase profits for all firms. Our findings complement the work by Kuksov (2004), who - in a different model setup, finds that lower search costs (a) do not change overall industry sales (every consumer buys one unit as there is no no-purchase option), (b) do not lead to a change in assortment (as there is only one product variant in the market), (c) do not increase social welfare, (d) make consumers worse off if prices are increased, (e) do not lead to any search activity in equilibrium (i.e. every consumer visits exactly one firm), (f) increase industry profit only if prices increase, and (g) may increase industry profits but one of the two firms is always worse off, i.e., easier search will benefit one firm at the expense of the other.

6 Conclusion

Our work explores multiproduct firms' decision making and profitability in the presence of changing search costs or firm entry. In our model firm entry intensifies competition, resulting in less variety, lower prices and lower profits. Easier search (modeled as a lower search cost) also intensifies competition, as suggested by conventional wisdom (e.g., Bakos 1997). But we demonstrate that easier search also has a market expansion effect: if consumers search more, then a firm has access to more consumers. Due to this market expansion effect, which is not present with firm entry, firms in equilibrium invest in broader assortments. With more products to choose from, consumers are more likely to find products that match their ideal preferences, increasing the efficiency of the market which manifests itself in higher prices, more profits and increased welfare.

The advent of the Internet has clearly motivated the recent interest on the influence of search on market dynamics. Our results, as we have already stated, are consistent with an expansion of variety due to (in part) easier search associated with Internet enabled markets. But our results are not specific to any one technology - our model suggests that any technology that reduces search costs should expand the available assortment in a market. For example, according to our model, the development of better roads (e.g., pavement), telecommunications (e.g., telephone) and transportations (e.g., automobiles) in the early part of the last century should have contributed to larger retail formats (broader assortments). The Internet represents only the latest technological advancement in this trend.

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7 Appendix

Outlines of proofs are provided in this appendix. See the on-line technical appendix for more details.

Proof of Proposition 1: To simplify notation, we use the following abbreviations: $\Pi_i = \Pi_i(p_i, x_i|p, x, m^*)$ and $q_i = q_i(p_i, x_i|p, x, m^*)$ and

$$q_i = \frac{x_i}{x_i + a(p_i)}; \quad a(p_i) = \frac{(m^* - 1)x\exp(-p/\mu) + \exp(u_0/\mu)}{\exp(-p_i/\mu)}.$$

Differentiate (4) to obtain the following first order conditions:

$$\begin{aligned} (m^*/n)\, q_i - (m^*/n)\, p_i\, (1 - q_i)\, q_i/\mu &= 0 \\ \frac{(m^*/n)\, p_i\, (1 - q_i)\, q_i}{x_i} - c'(x_i) &= 0 \end{aligned}$$

Note, firm i optimizes p_i and x_i given an expectation that consumers will search m^* firms and the other firms will choose $\{p, x\}$. A symmetric equilibrium then requires that the firms and consumers have correct expectations regarding everyone's behavior and behavior is optimal given these expectations. Simplification of the first order conditions yield (5) and (6). Evaluation of the Hession determines that this solution is indeed a maximum. The symmetric equilibrium is unique because the first order conditions have a unique solution (and hence the only candidate equilibrium). The condition on $c'(0)$ ensures an interior equilibrium. ■

Proof of Proposition 2: The right hand side of (6) is decreasing in n, so x^* is decreasing in n. From (5), p^* is increasing in x^*, so p^* is decreasing in n as well. It is straightforward to show that nx^* can be decreasing in n when $c(x) = cx$. It is not difficult to verify that the right-hand side of (6) is increasing in m^*, so x^* is increasing in m^*. Because m^* is decreasing in the consumer search cost τ, easier search increases x^*.

To illustrate that easier search can increase p^*, assume $c(x) = cx$. The equilibrium price is then implicitly defined by

$$p^* - \frac{\mu^2 m^2}{\mu m^2 - \mu m + nce^{\left(\frac{u_o + p^*}{\mu}\right)}} = 0$$

Define $\hat{p}$ such that

$$2nce^{\left(\frac{u_o + \hat{p}}{\mu}\right)} = \mu m.$$

Note that $\partial\hat{p}/\partial m > 0$. From the implicit function theorem, the sign of $\partial p^*/\partial m$ is positive if $p^* > \hat{p}$, otherwise it is negative. It can be shown that there is a unique $\bar{m}$ such that

$p^* = \hat{p} = \mu\bar{m}(\bar{m} - 1/2)$. It follows that $\partial p^*(\bar{m})/\partial m = 0$. Hence, for all $m < \bar{m}$, $\hat{p} < p^*$, which implies $\partial p^*/\partial m > 0$. Furthermore, for all $m > \bar{m}$, $\hat{p} > p^*$, which implies $\partial p^*/\partial m < 0$. Thus, for sufficiently high search costs, $m^* < \overline{m}$, and the equilibrium price is increasing as search becomes easier (as m^* increases) and for sufficiently low search costs, $m^* > \overline{m}$ and the equilibrium price is decreasing as search becomes easier (as m^* continues to increases).■

Proof of Proposition 3: Differentiate the firm's equilibrium profit with respect to n:

$$\frac{\partial \Pi^*}{\partial n} = \frac{(\mu m^* - nx^*c'(x^*))\frac{\partial x^*}{\partial n}\left[\mu m^* x c''(x) + nx^*c'(x^*)^2\right] + \mu m^* x^* c'(x^*)\frac{\partial(nx^*c'(x^*))}{\partial n}}{(\mu m^* - nx^*c'(x^*))^2} < 0.$$

To ensure $p^* \geq 0$, we must have $\mu m^* - nx^*c'(x^*) \geq 0$. Because p^* is decreasing in n, $nx^*c'(x^*)$ must be decreasing in n. Together with the convexity of $c(x^*)$ and $\partial x^*/\partial n < 0$, the inequality follows.

Differentiating the firm's equilibrium profit with respect to m^*:

$$\frac{\partial \Pi^*}{\partial m^*} = \frac{\mu n x^* c'(x^*)\left[m^*\left(c'(x^*) + x^*c''(x^*)\right)\frac{\partial x^*}{\partial m^*} - x^*c'(x^*) + \frac{c'(x^*)(\mu m^* - nx^*c'(x^*))}{\mu}\frac{\partial x^*}{\partial m^*}\right]}{(\mu m^* - nx^*c'(x^*))^2}. \tag{14}$$

The above derivative cannot be signed unambiguously. Differentiating (5) with respect to m^* and collecting terms, we have

$$\frac{\partial p^*}{\partial m^*} = \frac{\mu^2 n\left[m^*\left(c'(x^*) + x^*c''(x^*)\right)\frac{\partial x^*}{\partial m^*} - x^*c'(x^*)\right]}{(\mu m^* - nx^*c'(x^*))^2}.$$

The sign of $\partial p^*/\partial m^*$ is determined by the sign of the term in the square brackets in the numerator, which is the same as the first term in the square brackets in the numerator of (14). As we discussed above, the second term in the square brackets in the numerator of (14) is always positive. Therefore, if $\partial p^*/\partial m^* \geq 0$, it is sufficient to ensure $\partial \Pi^*/\partial m^* \geq 0$. This implies that if easier search leads to a price increase in the market, it will surely increase the equilibrium firm profit as well. ■

Proof of Proposition 4: In the parallel search model, equilibrium consumer welfare is

$$CW = \mu \ln\left[m^* x^* \exp\left(-p^*/\mu\right) + \exp\left(u_0/\mu\right)\right] - m^*\tau,$$

where the first term is the expected utility of searching m^* firms, which is the expected value of the maximum of the utilities of the no-purchase option and m^*x^* products with price p^* offered by m^* firms, and the second term is the cost of searching m^* firms. Since $m^* = \min\left[n, \max\left(1, \frac{\mu}{\tau}\right)\right]$, $m^*\tau$ is non-increasing in τ. Thus, the sign of $\partial CW/\partial m^*$ is the same as the sign of $\partial\left(\mu \ln m^* x^* - p^*\right)/\partial m^*$. Differentiating, we find that to show

$\partial\left(\mu \ln m^* x^* - p^*\right)/\partial m^* \geq 0$, it is sufficient to show $\frac{\partial p^*}{\partial m^*} \leq \mu/m^*$. Differentiating (6) and using $\frac{\partial x^*}{\partial m^*} \geq 0$, we find $\frac{\partial p^*}{\partial m^*} \leq \mu/m^*$ is true. Thus, consumer welfare always increases as search costs decrease. The social welfare in the parallel search model is the sum of consumer welfare and the total profit for all n firms. Since we have shown that equilibrium firm profit can increase as search costs decrease in Proporsition 3, the social welfare also can increase. Numerical studies show that both consumer and social welfare is not monotone in n, i.e., firm entry may reduce or increase consumer and social welfare.■

Proof of Lemma 5: To show $H(\bar{U}(x,\tau),x) = \exp\left[-x\exp\left(-\left(\bar{U}(x,\tau)/\mu+\gamma\right)\right)\right]$ is independent of x, it is sufficient to show the term in the square bracket is independent of x. Differentiating the term with respect to x, we have

$$\frac{\partial\left[x\exp\left(-\left(\bar{U}(x,\tau)/\mu+\gamma\right)\right)\right]}{\partial x} = \exp\left(-\left(\bar{U}(x,\tau)/\mu+\gamma\right)\right)\left[x - \frac{1}{\mu}\frac{\partial \bar{U}(x,\tau)}{\partial x}\right].$$

Thus, we just need to show that $\partial\bar{U}(x,\tau)/\partial x = \mu/x$. Note that $\bar{U}(x,\tau)$ is the solution of the following equation

$$\Psi(x) = \int_{\bar{U}(x,\tau)}^{\infty} (\phi - \bar{U}(x,\tau))dH(\phi,x) = \tau.$$

Differentiating by using Leibnitz's Rule and Implicit Function Theorem, we have

$$\frac{\partial\bar{U}(x,\tau)}{\partial x} = -\left(\frac{\mu}{x}\right)\frac{\int_{\bar{U}(x,\tau)}^{\infty}\frac{\partial\left[(\phi-\bar{U}(x,\tau))h(\phi,x)\right]}{\partial\phi}d\phi - \int_{\bar{U}(x,\tau)}^{\infty}h(\phi,x)d\phi}{\int_{\bar{U}(x,\tau)}^{\infty}h(\phi,x)d\phi} = \frac{\mu}{x},$$

since $\int_{\bar{U}(x,\tau)}^{\infty}\frac{\partial\left[\left[\phi-\bar{U}(x,\tau)\right]h(\phi,x)\right]}{\partial\phi}d\phi = \left[\phi-\bar{U}(x,\tau)\right]h(\phi,x)\Big|_{\bar{U}(x,\tau)}^{\infty} = 0.$■

Proof of Proposition 6: Let $\rho(p_i - p, x, \tau) = \exp(-((\bar{U}(x,\tau) + (p_i - p))/\mu + \gamma))$. To simplify notation, we use the following abbreviations: $\rho(p_i) = \rho(p_i - p, x, \tau)$, $H(x) = H(\bar{U}(x,\tau),x)$ and $H(p_i,x_i) = H(\bar{U}(x,\tau) + (p_i - p), x_i)$. Furthermore, let

$$A = \frac{H_0(\bar{U}(x,\tau)+p)}{n\left(1 - H(\bar{U}(x,\tau),x)\right)}, \text{ and } Q_i = \left[1 - H(p_i,x_i)\right].$$

Differentiating (10) to obtain the following first order conditions:

$$AQ_i + p_iA\frac{\partial Q_i}{\partial p_i} = 0,$$

$$p_iA\frac{\partial Q_i}{\partial x_i} - c'(x_i) = 0.$$

Because for any symmetric equilibrium $\{p^*, x^*\}$ there exists a unique $\bar{U}(x^*,\tau)$ satisfying (11), it is sufficient to show the existence and uniqueness of a $\{p^*, x^*\}$ equilibrium for a given $\bar{U}(\cdot)$. To prove existence of the equilibrium we show that firm i's expected profit function (10) is jointly quasi-concave in $\{p_i, x_i\}$. To establish uniqueness, we show if function $g(x) = (c'(x) + xc''(x))/x^2$ is decreasing in x, there exists a unique positive $\{p^*, x^*\}$ that solves the first-order conditions of all firms simultaneously.$\blacksquare$

We need the following lemma for the proof of Proposition 7.

Lemma 10 *For a given x, $\rho(x,\tau)H(\tau)/(1-H(\tau))$ is decreasing in τ.*

Proof. Define functions $K(\omega) = \exp(-x\exp(-\omega))$, where x is a constant, and $T(\omega) = \exp(-\omega) K(\omega)/(1-K(\omega))$. Thus, $K'(\omega) = x\exp(-\omega)K(\omega)$ and $T(\omega) = K'(\omega)/(x(1-K(\omega)))$. We then have

$$T'(\omega) = \frac{x\exp(-\omega)K(\omega)\left[x\exp(-\omega) + \exp(-x\exp(-\omega)) - 1\right]}{x(1-K(\omega))^2} \geq 0.$$

The inequality follows from the fact that function $f(\epsilon) = \epsilon + \exp(-\epsilon) - 1$ is convex and minimized at $\epsilon = 0$ with $f(0) = 0$. Therefore, $T(\omega)$ is an increasing function. The lemma follows from $\rho(x,\tau)H(\tau)/(1-H(\tau)) = T\left(\bar{U}(x,\tau)/\mu + \gamma\right)$ and $\bar{U}(x,\tau)$ is decreasing in τ.$\blacksquare$

Proof of Proposition 7: From (12), it is clearly that p^* is independent of n. Condition (13) can be written as

$$w(x^*) = x^*c'(x^*) - \frac{\mu}{n}H_0(\bar{U}(x^*,\tau) - p^*) = 0. \tag{15}$$

Because $w(x^*)$ is increasing in n and quasi-convex with $w(0) = 0$, x^*, which is the unique positive solution of (15), is decreasing in n. To illustrate the possibility that nx^* can be decreasing in n, consider a linear cost function $c(x) = cx$ with $c'(x) = c$. For this linear cost function, defining $\upsilon^* = nx^*$ and multiplying both sides of equation (15) by n, it yields $c\upsilon^* - \mu H_0(\bar{U}(\upsilon^*/n,\tau) - p^*) = 0$. Since the right-hand side of the equation is quasi-convex in υ^* and increasing in n, $\upsilon^* = nx^*$, which is the unique positive solution of the above equation, is decreasing in n. By Lemma 10, $\rho(x^*,\tau)H(\tau)/(1-H(\tau))$ is decreasing in the search cost τ. Thus, from (12), p^* is increasing in τ. Because $w(x^*)$ in condition (15) is increasing in τ and quasi-convex with $w(0) = 0$, x^* is decreasing in τ.$\blacksquare$

Proof of Proposition 8: Differentiating the firm's equilibrium profit with respect to n and arranging terms, we have

$$\frac{\partial \Pi^*}{\partial n} = \left(\frac{1 - H(\tau) - x^*\rho(x^*,\tau)H(\tau)}{x^*\rho(x^*,\tau)H(\tau)}\right)c'(x^*)\frac{\partial x^*}{\partial n} + \frac{[1-H(\tau)]\,x^*c''(x^*)}{x^*\rho(x^*,\tau)H(\tau)}\frac{\partial x^*}{\partial n} \leq 0.$$

The inequality follows from $1-H(\tau)-x^*\rho(x^*,\tau)H(\tau)\geq 0$, $c(x^*)$ is convex, and $\partial x^*/\partial n<0$.

Define $\theta(\bar{U}(x^*,\tau))=(1-H(\tau))\,/\,(\rho(x^*,\tau)H(\tau))$. Differentiating the firm's equilibrium profit with respect to $\bar{U}(x^*,\tau)$ and abbreviating notation, we have

$$\frac{\partial\Pi^*}{\partial\bar{U}}=\left(\frac{1-x^*\rho(x^*,\tau)-H(\tau)+\rho(x^*,\tau)\left(1-H(\tau)\right)\frac{\partial x^*}{\partial\bar{U}}}{\rho(x^*,\tau)H(\tau)}\right)c'(x^*)+\theta c''(x^*)\frac{\partial x^*}{\partial\bar{U}}. \tag{16}$$

The above derivative cannot be signed unambiguously. Let consider the linear cost function $c(x)=cx$ with $c'(x)=c$. Then, (16) becomes

$$\frac{\partial\Pi^*}{\partial\bar{U}}=\left(\frac{1-x^*\rho(x^*,\tau)-H(\tau)+x^*\rho(x^*,\tau)\left(1-H(\tau)\right)\frac{\rho_0(x^*,\tau)}{\mu}}{\rho(x^*,\tau)H(\tau)}\right)c'(x^*)+\theta c''(x^*)\frac{\partial x^*}{\partial\bar{U}}.$$

For $\rho_0(x^*,\tau)/\mu=1$, the numerator of the term inside the parentheses becomes $1-H(\tau)-x^*\rho(x^*,\tau)H(\tau)\geq 0$, which makes $\partial\Pi^*/\partial\bar{U}(x^*,\tau)\geq 0$. All terms in the numerator are independent of x^* except the term $\rho_0(x^*,\tau)$. Because $w(x^*)$ in condition (31) is increasing in u_0 and quasi-convex with $w(0)=0$, x^* is decreasing in u_0. Hence, $\rho_0(x^*,\tau)=\exp\left(-\left(\left(\bar{U}(x^*,\tau)-p^*-u_0\right)/\mu+\gamma\right)\right)$ is increasing in u_0. Thus, the numerator inside the parentheses is more likely to be positive in which case $\partial\Pi^*/\partial\bar{U}\geq 0$ is more likely to hold, which implies easier search does not necessarily reduce the equilibrium firm profit.■

Proof of Proposition 9: Using the definition of $\bar{U}\left(x^*,\tau\right)$, the conditional expected net utility for a consumer enters the market and search is

$$\begin{aligned}E[\text{consumer utility}|\text{search}] &= \frac{\int_{\bar{U}(x^*,\tau)}^{\infty}\xi dH\left(\xi\right)-\tau}{\left(1-H\left(\tau\right)\right)}-p^*\\ &= \bar{U}\left(x^*,\tau\right)-p^*.\end{aligned}$$

Thus, the expected consumer welfare is

$$\begin{aligned}CW &= H_0\left(\bar{U}\left(x^*,\tau\right)-p^*\right)\left(\bar{U}\left(x^*,\tau\right)-p^*\right)+\int_{\bar{U}(x^*,\tau)-p^*}^{\infty}\xi h_0\left(\xi\right)d\xi\\ &= E\left[\max\left[\bar{U}\left(x^*,\tau\right)-p^*,U_0\right]\right],\end{aligned}$$

where U_0 is the utility of consumer's no-purchase option which is a random variable with CDF H_0 and pdf h_0. It is clearly that $E\left[\max\left[\bar{U}\left(x^*,\tau\right)-p^*,U_0\right]\right]$ is increasing in $\bar{U}\left(x^*,\tau\right)-p^*$. Therefore, consumer welfare CW increases as search cost τ decreases since $\bar{U}\left(x^*,\tau\right)$ increases and p^* decreases as τ decreases. As more firms enter the market, $\bar{U}\left(x^*,\tau\right)$ decreases since x^* decreases, and p^* is unchanged. As a result, $\bar{U}\left(x^*,\tau\right)-p^*$ decreases. Therefore, firm entry always reduces consumer welfare.

The social welfare is

$$SW = H_0\left(\bar{U}\left(x^*, \tau\right) - p^*\right)\bar{U}\left(x^*, \tau\right) + \int_{\bar{U}(x^*,\tau)-p^*}^{\infty} \xi h_0\left(\xi\right) d\xi - nc\left(x^*\right).$$

To simplify notation, we use the following abbreviations: $h_0 = h_0\left(\bar{U}\left(x^*, \tau\right) - p^*\right)$, $H_0 = H_0\left(\bar{U}\left(x^*, \tau\right) - p^*\right)$, and $\bar{U} = \bar{U}\left(x^*, \tau\right)$. Differentiating with respect to x^* and collecting terms, we have

$$\frac{\partial SW}{\partial x^*} = H_0\frac{\partial \bar{U}}{\partial x^*} + p^* h_0 \frac{\partial \bar{U}}{\partial x^*} - nc'\left(x^*\right).$$

From (13), we know $H_0 = nx^*c'\left(x^*\right)/\mu$, and in the proof of Lemma 5 we have shown that $\frac{\partial \bar{U}}{\partial x^*} = \mu/x^*$. Therefore, $\partial SW/\partial x^*$ simplifies to

$$\frac{\partial SW}{\partial x^*} = p^* h_0 \frac{\partial \bar{U}}{\partial x^*} \geq 0.$$

Since x^* increases as search costs decrease, it implies that social welfare also increases as search costs decrease. Numerical studies show that social welfare is not monotone in n, i.e., firm entry may reduce or increase social welfare.■