Strategic Timing and Dynamic Pricing for Online Resource Allocation

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This paper optimizes dynamic pricing and real-time resource allocation policies for a platform facing non-transferable capacity, stochastic demand-capacity imbalances, and strategic customers with heterogenous price- and time-sensitivities. We characterize the optimal mechanism, which specifies a dynamic menu of prices and allocations. Service *timing* and *pricing* are used strategically to: (i) dynamically manage demand-capacity imbalances, and (ii) provide discriminated service levels. The balance between these two objectives depends on customer heterogeneity and customers' time-sensitivities. The optimal policy may feature *strategic idleness* (deliberately rejecting incoming requests for discrimination), *late service prioritization* (clearing the queue of delayed customers) and *deliberate late service rejection* (focusing on incoming demand by rationing capacity for delayed customers). Surprisingly, the price charged to time-sensitive customers is not increasing with demand—high demand may trigger lower prices.

By dynamically adjusting a menu of prices and service levels, the optimal mechanism increases profits significantly, as compared to dynamic pricing and static screening benchmarks. We also suggest a less information-intensive mechanism that is history-independent but fine-tunes the menu with incoming demand; this easier-to-implement mechanism yields close-to-optimal outcomes.

Key words: Dynamic Mechanism Design, Dynamic Pricing, Strategic Timing, Online Platforms.

1. Introduction

Consider an on-demand platform (e.g., Uber, Lyft, TaskRabbit), a delivery company (e.g., UPS, FedEx, Rivigo, Amazon), and a cloud computing provider (e.g., Microsoft, Google, Amazon). Clearly, they operate in very different environments; yet, they share common features. These platforms leverage a limited supply base (service providers for the on-demand platform; vehicles and staff for delivery; and servers for cloud computing) which does not accumulate over time—leaving capacity idle does not increase capacity thereafter. They all face stochasticities leading to excess demand in peak periods and excess capacity in off-peak periods. And they all deal with customers who have heterogeneous price-sensitivity and time-sensitivity—some are willing to pay a premium for expedited service while others are willing to wait more in exchange for a discount.

To manage these systems effectively, companies need to get access to real-time information and to react dynamically. While these abilities used to be out of reach in traditional markets, they are now common features in online markets—enabling online platforms to implement novel solutions to long-standing economic and operational problems. Motivated by these opportunities, this paper formalizes a problem of dynamic pricing and real-time resource allocation for a platform facing three core features: non-transferable capacity, stochastic demand-capacity imbalances, and strategic customers with heterogeneous price- and time-sensitivities.

This environment gives rise to two objectives: demand-capacity management and discrimination. To balance these objectives, the platform can use service *timing* and *pricing* as strategic devices. For demand-capacity management, timing and pricing can be leveraged to delay customer requests arising under high demand (potentially, at a price discount). And for discrimination, timing and (personalized) pricing can be leveraged to delay service to customers with high willingness to wait, and charge higher prices to customers with high willingness to pay.

To optimize timing and pricing, typical approaches make use of *dynamic pricing* and *screening mechanisms*. Dynamic pricing is commonly employed to manage demand stochasticity and capacity limitations—with significant revenue upsides for airlines, hotels, online advertisers, retailers, etc. (Talluri and Van Ryzin 2006). At the same time, dynamic pricing policies often rely on posted prices, which may lead to lost revenue opportunities and mismatches between service offers and customers' (private) preferences. Screening mechanisms, in contrast, elicit customers' preferences and tailor service offerings accordingly—for instance, by differentiating quality (Mussa and Rosen 1978) or by differentiating wait times (Afèche 2013). Such approaches often rely on static menus, thus potentially failing to adjust service offers based on real-time information.

Instead, this paper proposes a *dynamic screening menu*—the most general mechanism in our environment—combining principles of dynamic pricing and screening mechanisms. In our earlier examples, this menu would translate into a set of options with different prices and service levels (e.g., a menu of prices and wait times for the on-demand platform; of prices and lead times for delivery; and of prices and completion times for cloud computing). Ultimately, this paper provides guidelines to enhance the economic and operational performance of online markets, where companies can implement such dynamic menus by personalizing service offers for different customer segments and updating them based on real-time information.

We formalize this approach by designing a dynamic pricing and allocation mechanism in an environment featuring non-transferable capacity, stochastic demand-capacity imbalances, and customer heterogeneity. Non-transferrable capacity is captured by a perishable mass of suppliers arising in each period. Demand-capacity imbalances are captured by stochastic realizations of over-capacity and under-capacity demand. Customer heterogeneity is captured by a mix of time-sensitive customers (with high willingness to pay and low willingness to wait) and price-sensitive customers (with low willingness to pay and high willingness to wait). By specifying the timing of services, the mechanism leads to a partially-endogenous demand structure, creating inter-temporal dependencies. In each period, the platform elicits customers' (privately-observed) preferences, and optimizes the price and the probabilistic allocation of capacity to (i) incoming time-sensitive customers, (ii) incoming price-sensitive customers, and (iii) delayed price-sensitive customers. The problem is formulated as an infinite-horizon dynamic program that maximizes the platform's expected profit, subject to incentive compatibility, individual rationality and capacity constraints. We find that the platform prioritizes service to time-sensitive customers and extracts all the surplus from price-sensitive customers. The critical trade-off then lies in dynamically allocating capacity to price-sensitive customers and setting the price charged to time-sensitive customers.

We obtain the optimal policy, and show that it is history-dependent—highlighting the benefits of the dynamic menu of prices and allocations. Specifically, the optimal mechanism varies over three regions, depending on customer heterogeneity and the time preferences of price-sensitive customers:

- When heterogeneity is strong, the platform implements extreme discrimination by delaying all price-sensitive customers to maximize the price charged to time-sensitive customers. The policy features *strategic idleness*: customers can be delayed even when some suppliers are idle.
- When heterogeneity is weak and price-sensitive customers are mildly sensitive to wait times, the optimal policy involves *late service prioritization*: the platform prioritizes delayed price-sensitive customers over incoming ones. The policy may still feature strategic idleness if the platform chooses not to serve all incoming requests after fulfilling late demand.
- When heterogeneity is weak and price-sensitive customers are more sensitive to wait times, no capacity is left strategically idle. The platform may even forego late demand in order to prioritize timely services, resulting in a *deliberate late service rejection* policy.

Surprisingly, in a dynamic mechanism, the optimal price is not increasing with incoming demand: higher demand may trigger lower prices. Consider an instance where, under low demand, the platform delays service to price-sensitive customers to: (i) charge a higher price to time-sensitive customers, (ii) serve delayed customers, and (iii) create a demand backlog for the next period. Under high demand, the same strategy would result in a larger demand backlog, which the platform may not be able to fulfill in the next period—especially if demand is high again in the next period. The platform may instead provide more timely services to price-sensitive customers under high demand, and hence charge a lower price to time-sensitive customers in order to maintain incentive compatibility. The main driving force is that, as demand increases, discrimination becomes relatively less prominent and demand-capacity management becomes relatively more prominent. Using a numerical analysis, we show that this result generalizes: the price is non-monotonic with the *non-expiring portion of the demand* (i.e., the customers who will not renege if left unserved).

As compared to first-best allocation, the optimal mechanism induces a trade-off between the surplus generated and information rents. Under weak heterogeneity, the mechanism maximizes social welfare but leaves information rent to time-sensitive customers; under strong heterogeneity, it induces a surplus loss but the platform may capture all the surplus. Moreover, the proposed mechanism increases platform profits significantly vis-a-vis baseline pricing schemes based on dynamic pricing and static screening menus. We also propose an intermediate mechanism that adjusts the menu with incoming demand but not with history. It is less information-intensive but achieves close-to-optimal outcomes—thus providing an easily implementable approximation of the optimal mechanism. Yet, the optimal mechanism achieves further gains—up to 3–4% in additional profits.

Ultimately, this paper makes the following contributions to the literature and practice. First, it studies a novel problem: designing a dynamic screening mechanism under non-transferable capacity, stochastic demand-capacity imbalances and strategic customers with heterogeneous price- and time-sensitivities. Second, it derives the most general mechanism in this environment, which translates into a dynamic menu of prices and allocations—and cannot be implemented by means of posted prices or static menus. Third, the paper provides novel insights for managing online platforms by balancing rent seeking and system efficiency. The optimal mechanism gives rise to endogenous dynamics, alternating between queue-building and queue-clearing periods based on customer heterogeneity and customers' time preferences. These dynamics leads to the counterintuitive result that, in a dynamic mechanism, optimal prices do not necessarily increase with incoming demand. Finally, it demonstrates that the proposed mechanism achieves higher expected profits than baseline approaches based on dynamic posted prices and static screening mechanisms; moreover, it proposes a less information-intensive approximation of the optimal mechanism, which can be more easily implementable in practice and achieves close-to-optimal outcomes.

2. Related Literature

This paper relates mostly to the dynamic pricing literature. Extensive work has considered myopic customers (see Lazear (1986), Gallego and Van Ryzin (1994) for seminal work and Talluri and Van Ryzin (2006), Özer and Phillips (2012) for reviews). We instead focus on strategic customers.

Dynamic pricing with strategic customers started with Coase (1972), who conjectured that, when a monopolist does not have commitment power, the optimal price path converges to the marginal cost of production. Gul et al. (1986) formalized this result. Conlisk et al. (1984) and Sobel (1991) introduced dynamic customer arrivals into this problem, leading to cyclical price paths. Stokey (1979) showed that, when the monopolist has commitment power, the price remains constant—and positive—over time. Golrezaei et al. (2018) showed that the price path may decrease over time when customers are heterogeneous in price- and time-sensitivities. Board (2008) integrated stochastic customer arrivals in this literature. Besbes and Lobel (2015) extended this literature by introducing heterogeneous time preferences with stochastic customer arrivals. In this line of work, and in contrast to our setting, the firm does not face capacity restrictions.

Next, revenue management involves dynamic pricing for a firm with limited capacity to sell over a finite horizon. Su (2007) derived markup and markdown policies when customers with heterogeneous price- and time-sensitivities strategically time their purchase. Aviv and Pazgal (2008) showed that price commitments can benefit the firm. Liu and Van Ryzin (2008) showed that the firm may strategically ration capacity to drive early purchases at a higher price. Other studies have proposed dynamic screening mechanisms for revenue management. Vulcano et al. (2002) modeled a new group of buyers arriving and leaving in each period, with inter-temporal dependencies driven by the transfer of capacity over time—as opposed to the transfer of customer requests in our setting. Pai and Vohra (2013), Gallien (2006) and Board and Skrzypacz (2016) identified conditions under which the optimal mechanism can be implemented with an index rule or with posted prices.

As opposed to the revenue management literature, our setting features perishable capacity that cannot be transferred over time. This connects our paper to the problem of *dynamic resource allocation*. Extensive research has studied interventions such as load balancing, scheduling and staffing in queuing systems (e.g., Keskinocak and Tayur (2004) in manufacturing, Harchol-Balter (2013) in computer farms, Green (2006) in healthcare). We focus instead on pricing interventions.

Naor (1969) and Mendelson (1985) proposed static congestion pricing schemes to maximize welfare, and Maglaras and Zeevi (2003) designed static prices to maximize revenue in a heavy-traffic queuing system. Low (1974) proposed the first dynamic pricing scheme in a queuing system. Maglaras (2006) jointly optimized dynamic pricing and sequencing policies in a multi-class queue. Chen and Frank (2001), Besbes and Maglaras (2009) and Kim and Randhawa (2018) introduced customers that are both price-sensitive and delay-sensitive, using exact analysis, an asymptotic fluid approximation and a large-scale diffusion-based approximation, respectively. In these papers, customers make strategic purchasing decisions immediately upon arrival but are not forward-looking—leading to inter-temporal queuing effects but no inter-temporal effects on the demand structure. In contrast, Ahn et al. (2007) optimized production and pricing decisions when demand comprises incoming customers and already-delayed customers. Customers are modeled by a demand function, as opposed to an explicit model of strategic behaviors. Borgs et al. (2014) optimized a price path that guarantees service to each paying customer in a setting related to ours, featuring non-transferrable capacity and customers with heterogenous willingness to pay and time preferences. In practice, successful applications of pricing schemes for resource allocation include time-of-use pricing in electricity markets (e.g., Borenstein et al. 2002), time-dependent pricing in data markets (e.g., Ha et al. 2012), and pay-as-you-go solutions in cloud computing (e.g., Xu and Li 2013).

Our paper differs from that branch of the literature in that the firm proposes a menu of options, as opposed to a single posted price to all customers. Specifically, we design a screening mechanism, under which the platform elicits customer preferences to provide differentiated prices and service levels (Mussa and Rosen 1978, Rochet and Choné 1998, Deneckere and McAfee 1996). Several studies have designed price/lead-time menus in related operational settings with demand-capacity imbalances and strategic customers (Mendelson and Whang 1990, Lederer and Li 1997, Van Mieghem 2000, Afèche and Mendelson 2004, Maglaras and Zeevi 2005). In particular, Afèche (2013) proposed, in a setting with time-sensitive and price-sensitive customers, a menu with *strategic delay*, which involves delaying service to some customers for discriminatory purposes (also a feature of our menu). Afèche and Pavlin (2016) and Maglaras et al. (2017) extended this approach to multiple customer delay costs and multi-server systems, respectively. These studies propose a static, state-independent policy in a continuous-time queuing environment, whereas our paper designs a dynamic, state-dependent menu in a discrete-time setting.

As such, our paper relates to dynamic mechanism design (see Bergemann and Said 2011). Several papers have proposed dynamic menus prices and lead-time quotations (i.e., lead-time guarantees made upon customer arrivals) for resource allocation in service and manufacturing systems. Çelik and Maglaras (2008) proposed a menu with dynamic prices and lead times chosen within a set of predetermined options. Customers are represented by means of a demand function, which does not give rise to incentive compatibility issues—the menu is not a screening menu in that the items are not tailored to separate customer segments. Plambeck (2004) designed a screening menu with dynamic lead-time quotations but static prices. Akan et al. (2012) proposed a fully dynamic screening menu in a welfare-maximization setting—which differs from our profit-maximization problem. Ata and Olsen (2013) considered a single-server queuing model with two customer classes that have heterogeneous delay costs (using a convex-concave function) but homogeneous valuations. They proposed a dynamic screening menu and showed, using a heavy-traffic approximation, that the menu is asymptotically optimal—although not necessarily optimal otherwise.

Ultimately, our paper contributes to the dynamic resource allocation literature (i) by considering an environment featuring non-transferrable capacity, stochastic demand-capacity imbalances, and strategic customers with heterogeneous price- and time-sensitivities, and (ii) by developing a fully dynamic menu of prices and probabilistic allocations—the most general mechanism in this setting.

3. Model

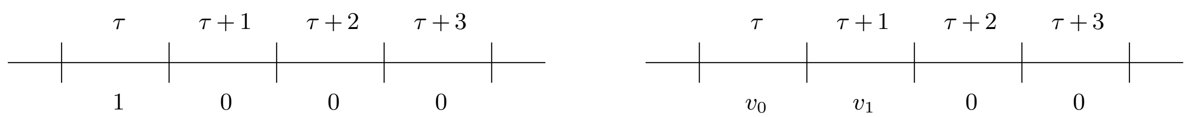
3.1. Setting and Assumptions

A platform operates continuously over a discretized infinite time horizon, and matches capacity with a demand of customers ("agents"). The components of our model are described below.

Non-transferrable capacity: We consider a constant continuum of suppliers, normalized to unit mass without loss of generality. Each service completion takes one period.

Stochastic demand-supply imbalances: In each period, a mass D of agents request a service. Demand can be high ($D = H$) or low ($D = L$).[1] We assume that $0 < L < 1 < H$, so that the platform faces an excess of supply under low demand and a shortage of supply under high demand. Demand realizations are independent and identically distributed over time, with $k = \mathbb{P}(D = H)$.

Agent heterogeneity: The demand comprises: (i) *time-sensitive* agents (r-type for "rush"), and (ii) *price-sensitive* agents (n-type for "non-rush"). An r-type agent derives a positive utility (normalized to 1) only from timely services. An n-type agent has a lower willingness to pay but higher willingness to wait, deriving a utility $v_0 < 1$ from a timely service, $v_1 < v_0$ from a service with a one-period delay, and zero from a service with a higher delay.[2] This is summarized in Figure 1.



(a) r-type agents (b) n-type agents

Figure 1 **Time-dependent utility of r-type and n-type agents joining the platform in period τ.**

Agent types are identically and independently distributed over time. We denote by σ the (time-invariant) probability that an incoming agent is of r-type.[3] The value of σ is commonly known but each agent's type is private information—leading to information asymmetries.

Assumption 1 states that serving all r-type agents is always feasible, even under high demand.[4]

ASSUMPTION 1. $\sigma H < 1$.

Platform problem: The platform optimizes service prices and the allocation of capacity to service agent requests. It maximizes expected discounted profits—with a discount rate of $\delta < 1$.

We abstract away from supply-side incentives by assuming homogeneous suppliers with a constant outside option, normalized to 0.[5] The demand side thus comprises the only information asymmetry, and the platform accrues all the revenue without leaving surplus to suppliers.

From the revelation principle, we focus without loss of generality on direct mechanisms in which agents report their types upon entering the platform (Myerson 1981). The mechanism specifies, for each type, an *allocation rule*—that is, a probability of service provision—and a per-service

[1] We relax this restriction numerically in Section 7 by considering a general demand distribution.

[2] We relax this restriction numerically in Section 7 by considering multi-period patience horizons.

[3] We relax this restriction in a simplified setting in Section 7 by introducing stochasticity in the customer mix.

[4] Otherwise, the solution would simply consist of serving r-type agents only and charging them a price of 1.

[5] We relax this restriction in a simplified setting in Section 7 by introducing endogenous supply-side participation.

payment rule.[6] Specifically, the platform makes six decisions. For an agent reporting an r-type, the mechanism specifies the price p_r and probability q_r of a timely service. For an agent reporting an n-type, the mechanism specifies (i) the price q_t and probability q_t of a timely service, and (ii) the price q_l and probability q_l of a late service *conditionally* on not getting a timely service. The pricing and allocation rules are determined prior to the agents' reports; after the reports, the platform commits to the pricing and allocation rule specified at the beginning of the period. The mechanism in each period is then defined in a sequentially rational manner.

3.2. Dynamic Programming Formulation

The platform's profit-maximization problem is formulated as an infinite-horizon dynamic program. The pricing and allocation policy is state-dependent but time-invariant, which is without loss of generality since the system follows stationary dynamics. The state variable includes (i) incoming demand D, and (ii) late demand (i.e., the number of n-type agents waiting for a late service, denoted by Γ). We assume that (D,Γ) is publicly observed by the agents upon reporting their types—as we shall see, this assumption is without loss of generality.

Let $(D',\Gamma'(D,\Gamma))$ be the state in the next period. Incoming demand D' is exogenous, equal to H or L with probabilities k and $1-k$. In contrast, $\Gamma'(D,\Gamma)$ is history dependent, given by $\Gamma'(D,\Gamma) = (1-q_t(D,\Gamma))\,(1-\sigma)D$: a mass $(1-\sigma)D$ of n-type agents are incoming, a fraction $q_t(D,\Gamma)$ of them receives a timely service, and the rest are transferred to the next period.

The expected payoffs of r-type and n-type agents are denoted by $U_r(D,\Gamma)$ and $U_n(D,\Gamma)$, respectively. For an r-type agent, the expected payoff depends solely on timely service provision:

$$U_r(D,\Gamma) = q_r(D,\Gamma)\,(1-p_r(D,\Gamma))\,. \tag{1}$$

The payoff of n-type agents includes the expected utility from a timely service, assigned with probability $q_t(D,\Gamma)$ at price $p_t(D,\Gamma)$, and the expected utility from a late service, assigned with probability $(1-q_t(D,\Gamma))\,q_l(D',\Gamma'(D,\Gamma))$ at price $p_l(D',\Gamma'(D,\Gamma))$. We obtain:

$$U_n(D,\Gamma) = q_t(D,\Gamma)\,(v_0 - p_t(D,\Gamma)) + (1-q_t(D,\Gamma))\begin{pmatrix} kq_l(H,\Gamma'(D,\Gamma))\,(v_1 - p_l(H,\Gamma'(D,\Gamma))) \\ +(1-k)q_l(L,\Gamma'(D,\Gamma))\,(v_1 - p_l(L,\Gamma'(D,\Gamma))) \end{pmatrix}. \tag{2}$$

Let $V(D,\Gamma)$ be the value function. We refer to the platform's problem as $(\mathcal{P})$. It is given by:

$$V(D,\Gamma) = \max_{\substack{q_r,q_t,q_l \\ p_r,p_t,p_l}} \quad p_r q_r \sigma D + p_t q_t (1-\sigma)D + p_l q_l \Gamma + \delta[kV(H,\Gamma') + (1-k)V(L,\Gamma')] \tag{3}$$

$$\text{s.t.} \quad U_r(D,\Gamma) \geq q_t(D,\Gamma)[1-p_t(D,\Gamma)], \quad (\mathcal{IC}_r) \tag{4}$$

$$U_n(D,\Gamma) \geq q_r(D,\Gamma)[v_0 - p_r(D,\Gamma)], \quad (\mathcal{IC}_n) \tag{5}$$

[6] We consider per-service payment rules because, in most applications, agents can opt out from the transaction after placing a request. This opt-out option translates into an ex post individual rationality constraint, i.e., each agent must receive a non-negative payoff after each realization of the stochastic allocation rule. But any mechanism satisfying ex post individual rationality can be implemented with per-service prices that agents pay only if services are provided.

$$p_r \leq 1, \; p_t \leq v_0, \; p_l \leq v_1, \tag{6}$$

$$1 \geq q_r \sigma D + q_t(1-\sigma)D + q_l \Gamma, \tag{7}$$

$$\Gamma' = (1-q_t)(1-\sigma)D. \tag{8}$$

Equation (3) maximizes the platform's expected discounted profit. The term $p_r q_r \sigma D$ is the expected profit from timely services provided to r-type agents; similarly, $p_t q_t(1-\sigma)D$ and $p_l q_l \Gamma$ are the expected profits derived from timely and late services provided to n-type agents. Equations (4) and (5) list the incentive compatibility constraints for r-type and n-type agents (referred to as $\mathcal{IC}_r$ and $\mathcal{IC}_n$, respectively). If an r-type agent misreports his type, he derives a value of $1 - p_t(D,\Gamma)$ with probability $q_t(D,\Gamma)$ (no value is derived from late services). If an n-type agent misreports his type, he will not be provided a late service by the platform, so he receives a value $v_0 - p_r(D,\Gamma)$ with probability $q_r(D,\Gamma)$. The individual rationality constraints (Equation (6)) ensure that service prices lie below agent valuations. The resource constraint (Equation (7)) limits the number of services provided to the unit capacity. Equation (8) defines the system's transitions.

PROPOSITION 1. *There exists a solution to problem* $(\mathcal{P})$.

3.3. Initial Results and Problem Transformation

We outline in Proposition 2 important properties of the optimal mechanism.

PROPOSITION 2. *The optimal solution to problem* $(\mathcal{P})$ *satisfies, in each state* (D,Γ)*:*

(i) $p_t(D,\Gamma) = v_0$, *and* $p_l(D,\Gamma) = v_1$.

(ii) $q_r(D,\Gamma) = 1$, *and* $p_r(D,\Gamma) \geq v_0$.

(iii) If $\Gamma > 0$, *then* $q_l(D,\Gamma) = \min\left\{1, \frac{1-\sigma D - q_t(D,\Gamma)(1-\sigma)D}{\Gamma}\right\}$.

(iv) Constraint $\mathcal{IC}_r$ *is binding, and hence* $p_r(D,\Gamma) = 1 - q_t(D,\Gamma)(1-v_0)$.

The first result asserts that the platform extracts all the surplus from n-type agents, who are always charged a price equal to their valuations (i.e., $p_t = v_0$ and $p_l = v_1$). Any smaller price would induce a profit loss for the platform without altering agent incentives.

The second result states that the price p_r is at least v_0—any lower price would violate incentive compatibility for n-type agents. Constraint $\mathcal{IC}_n$ is thus automatically satisfied. Moreover, the platform always serves r-type agents, since they generate the highest profits. In each period, the platform thus provides timely services to σD r-type agents and to $q_t(D,\Gamma)(1-\sigma)D$ n-type agents.

The third result asserts that any remaining capacity is used to serve late demand. Keeping capacity idle with extra demand for late services would induce a profit loss without affecting incentives. Nonetheless, late requests may still be rejected due to insufficient capacity.

The last part highlights the key trade-off faced by the platform between the quantity of timely services and the price. From the incentive compatibility constraint $\mathcal{IC}_r$, as $q_t(D,\Gamma)$ increases, n-type agents become more likely to receive a timely service, so misreporting becomes more attractive for r-type agents. The platform thus needs to charge a lower price p_r.[7]

Proposition 2 yields five relationships between six decision variables. Hence, we reformulate the platform's problem with a single decision variable Γ', defined as the number of n-type agents transferred to the next period.[8] Note that Γ' is bounded above by $(1-\sigma)D$: no more n-type agents can be transferred than the number of incoming n-type agents. Moreover, Γ' is bounded below by $H-1$ under high demand (from Equations (7) and (8)), which reflects that all n-type agents cannot receive a timely service under high demand. This is summarized as follows:

$$\text{If } D = H, \text{ then } \Gamma' \in [\underline{\Gamma}_H, \bar{\Gamma}_H], \text{ where } \bar{\Gamma}_H = (1-\sigma)H \text{ and } \underline{\Gamma}_H = H-1.$$
$$\text{If } D = L, \text{ then } \Gamma' \in [\underline{\Gamma}_L, \bar{\Gamma}_L], \text{ where } \bar{\Gamma}_L = (1-\sigma)L, \text{ and } \underline{\Gamma}_L = 0.$$

Let $\Gamma^*(D,\Gamma) \in [\underline{\Gamma}_D, \bar{\Gamma}_D]$ be the optimal policy function. We also denote by $\mathbf{\Gamma} = [\underline{\Gamma}_H, \bar{\Gamma}_H] \cup [\underline{\Gamma}_L, \bar{\Gamma}_L]$. In any state (D,Γ) and for an arbitrary choice of $\Gamma' \in [\underline{\Gamma}_D, \bar{\Gamma}_D]$, let $\tilde{V}(D,\Gamma,\Gamma')$ be the value function, given that the optimal policy will be applied thereafter. From Proposition 2, we have:

$$\tilde{V}(D,\Gamma,\Gamma') = \underbrace{\sigma D\left(1 - \tfrac{(1-\sigma)D-\Gamma'}{(1-\sigma)D}(1-v_0)\right)}_{r\text{-type agents}} + \underbrace{((1-\sigma)D-\Gamma')v_0}_{\text{timely services, } n\text{-type agents}} + \underbrace{\min\{\Gamma, 1-D+\Gamma'\}v_1}_{\text{late services, } n\text{-type agents}} + \underbrace{\delta\left(kV(H,\Gamma') + (1-k)V(L,\Gamma')\right)}_{\text{future value}}. \tag{9}$$

All constraints are embedded into Equation (9). Problem $(\mathcal{P})$ is therefore equivalent to:

$$V(D,\Gamma) = \max_{\Gamma' \in [\underline{\Gamma}_D, \bar{\Gamma}_D]} \tilde{V}(D,\Gamma,\Gamma'). \tag{10}$$

Proposition 3 establishes important properties of the value function V.

Proposition 3. *The value $V(D,\Gamma)$ is a non-decreasing and concave function of Γ, and is differentiable almost everywhere with respect to Γ.*

3.4. Discussion: Platform Commitment, Information and Strategic Behaviors

Before eliciting the optimal policy, we discuss a few elements of our model to (i) clarify the issue of commitment, (ii) show that the mechanism does not rely on the agents observing the state variable (D,Γ), and (iii) show that our mechanism does not permit additional deviations from the agents.

[7] Since p_r is the only pricing decision, we refer to it simply as "price" in the remainder of this paper.

[8] There is a one-to-one mapping between $q_t(D,\Gamma)$ and Γ' per Equation (8).

Platform Commitment and Sequential Rationality. Our mechanism assumes within-period commitment—and no future commitment. It is therefore sequentially rational. Under this formulation, the platform optimizes, in each period, the prices and allocations of *the services provided in that period* (that is, p_r and q_r for incoming r-type customers, p_t and q_t for incoming n-type customers, and p_l and q_l for already-delayed n-type customers).

An alternative (but ultimately equivalent) mechanism would optimize *the services provided to the agents who arrive in that period* (that is, p_r and q_r for incoming r-type customers, and p_t, q_t, p_l and q_l for incoming n-type customers). That mechanism would require inter-period commitment. But the two mechanisms are in fact equivalent—this alternative mechanism would thus also be sequentially rational, despite requiring inter-period commitment. Indeed, in our mechanism, the platform extracts all the surplus from late services (Proposition 2) so it has no incentive to promise prices and allocations for late services that it will deviate from later on.

Information Our incentive compatibility and individual rationality constraints implicitly assumed that the state (D,Γ) is publicly observable. This turns out to be without loss of generality.

Indeed, our mechanism specifies immediate services and an "uncertain component" governing late services for n-type agents. Immediate services (p_r, q_r, p_t and q_t) will be directly available to the customers, so the observability of the state does not play any role there. In theory, it could impact customer beliefs about the "uncertain component" (i.e., the price and allocation in the next period). However, the platform always leaves zero utility to n-type customers (Proposition 2). Customers thus do not benefit from observing the state. Stated differently, if the state (D,Γ) was privately observed, the platform would employ the same optimal mechanism—it would not benefit from hiding/signaling information regarding the state of the system. Technically, the main force driving this result is that all customers are effectively short lived: (i) r-type customers are short-lived by definition; and (ii) n-type customers do not derive any surplus from late services.

Strategic Behaviors In principle, two additional deviations need to be prevented by the incentive compatibility constraints. First, agents may strategically postpone their arrival time—similar to the strategic timing of purchase considered by Borgs et al. (2014) and Besbes and Lobel (2015). In our setting, this would not be a relevant option for r-type agents (who only derive a positive utility from timely services). This would not be beneficial to n-type agents either as they would get charged a price of at least v_0 (Proposition 2) while their valuation v_1 is strictly less than v_0.

Second, an n-type agent may re-enter the platform—that is, arrive in period τ, exit upon not receiving or rejecting a timely service, and re-enter in period $\tau+1$. This leaves three possible outcomes: (i) receiving an "immediate" service in $\tau+1$ at price $p_r \geq v_0$ or $p_t = v_0$ when he is only willing to pay $v_1 < v_0$; (ii) receiving a "late" service in $\tau+2$ at price v_1 when he is willing to pay zero; or (iii) receiving no service. Either way, the deviation is not profitable.

Thus, the optimal mechanism, obtained without explicitly considering these deviations, satisfies the stricter incentive compatibility constraints that would be obtained by considering them.

4. Characterizing the Optimal Policy

We define three regions based on customer heterogeneity and the time preferences of price-sensitive agents. We derive a closed-form expression of the optimal policy in each region.

4.1. Policy Structure

Recall that three groups are present on the platform: (i) a mass σD of incoming r-type agents, (ii) a mass $(1-\sigma)D$ of incoming n-type agents, and (iii) a mass Γ of delayed n-type agents. All r-type agents are always served (Proposition 2). The structure of the optimal policy depends on how the remaining capacity mass of $1-\sigma D$ is allocated across incoming and delayed n-type agents.

To facilitate the interpretation, we denote by $\zeta(D,\Gamma)$ the capacity left for late service provision:

$$\zeta(D,\Gamma) = 1 - D + \Gamma^*(D,\Gamma). \tag{11}$$

From Proposition 2, we know that the number of late services provided is equal to $\min\{\Gamma, \zeta(D,\Gamma)\}$; in other words, the platform can serve all late demand if and only if $\zeta(D,\Gamma) \geq \Gamma$.

Moreover, let $\chi(D,\Gamma)$ be the minimal number of incoming n-type agents that need to be transferred to the next period to keep sufficient capacity to serve all late demand in the current period:

$$\chi(D,\Gamma) = \max\left\{\min\left\{\bar{\Gamma}_D, \Gamma + D - 1\right\}, \underline{\Gamma}_D\right\}. \tag{12}$$

At one extreme, if $D+\Gamma \leq 1$, the queue is short enough so all late demand will be served regardless of the policy. Vice versa, if $\Gamma \geq 1-\sigma D$, the queue is too long to serve all late demand regardless of the policy. In-between, the platform's ability to serve late demand depends on its policy.

We identify the following structures of the optimal policy:

(i) late service prioritization : $\Gamma^*(D,\Gamma) \geq \chi(D,\Gamma)$. Upon serving r-type agents, the platform serves late demand up to capacity. If capacity remains available (for low values of Γ), the platform determines whether to also serve incoming n-type agents, yielding two possible outcomes:

- *strategic idleness*: $\Gamma^*(D,\Gamma) > \chi(D,\Gamma)$. The platform transfers strictly more n-type agents than required to serve all late demand. We then have $\zeta(D,\Gamma) > \Gamma$: the platform intentionally keeps capacity idle for discriminatory purposes.
- *no strategic idleness*: $\Gamma^*(D,\Gamma) = \chi(D,\Gamma)$. The platform transfers exactly as many n-type agents as necessary to serve all late demand. As long as effective demand exceeds capacity (i.e., $D+\Gamma \geq 1$), the platform utilizes its entire capacity (i.e, $\zeta(D,\Gamma) = \Gamma$). When $D+\Gamma < 1$, some capacity is left idle (i.e., $\zeta(D,\Gamma) > \Gamma$), but this does not arise from discriminatory purposes.

(ii) deliberate late service rejection : $\Gamma^*(D,\Gamma) < \chi(D,\Gamma)$. The platform rejects requests from some or all delayed n-type agents, in order to provide more timely services to incoming n-type agents.

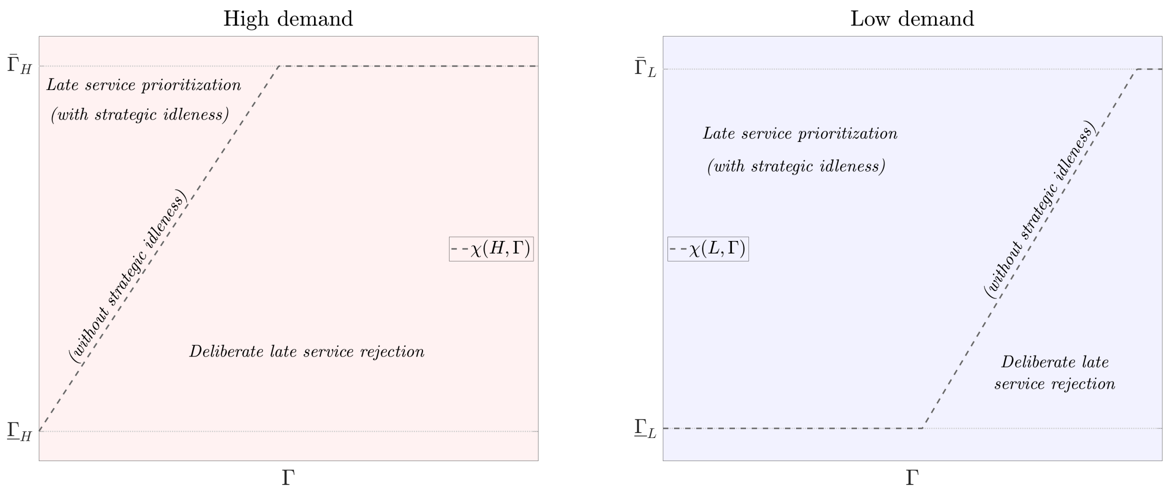


Figure 2 **Policy structures based on the comparison between $\Gamma^*(D,\Gamma)$ and $\chi(D,\Gamma)$, when $\sigma < 0.5$.**

Figure 2 depicts $\chi(D,\Gamma)$ (dashed line) and these policy structures as a function of D and Γ, when n-type agents comprise the majority of the demand ($\sigma < 0.5$).[9] The optimal policy will be represented as a function that associates $\Gamma^*(D,\Gamma)$ (on the vertical axis) to each value of D and Γ. As we shall see, the optimal policy never "crosses" the dashed line. In other words, the optimal policy either *(i)* features *late service prioritization* (i.e., lies above or on the dashed line); or *(ii)* features *deliberate late service rejection* (i.e., lies below or on the dashed line). Moreover, in case *(i)*, if the policy lies strictly above the dashed line for some values of Γ, then it exhibits *strategic idleness*.

4.2. Preliminary Steps

We denote by $V'(D,\Gamma)$ the partial derivative of $V(D,\Gamma)$ with respect to Γ (which exists almost everywhere from Proposition 3). From Equation (9), we obtain for each $(D,\Gamma) \in \{H,L\} \times \mathbf{\Gamma}$:

$$\frac{\partial \tilde{V}(D,\Gamma,\Gamma')}{\partial \Gamma'} = \underbrace{\frac{\sigma(1-v_0)}{1-\sigma}}_{r\text{-type}} - \underbrace{v_0}_{\substack{\text{timely}\\ n\text{-type}}} + \underbrace{\frac{\partial \min\{\Gamma, 1-D+\Gamma'\} v_1}{\partial \Gamma'}}_{\text{late, } n\text{-type}} + \underbrace{\delta\left(kV'(H,\Gamma') + (1-k)V'(L,\Gamma')\right)}_{\text{inter-temporal effects}}. \tag{13}$$

This equation highlights the core trade-off. Increasing $\Gamma' \in [\underline{\Gamma}_D, \bar{\Gamma}_D]$ has a negative effect: reducing the number of timely services provided to n-type agents, inducing a profit loss at rate v_0. But it also has one positive effect: increasing the price charged to r-type agents due to service differentiation. And it has two non-negative effects: providing more late services if $1-D+\Gamma' < \Gamma$ (i.e., if some late demand remains unserved), inducing a profit gain at rate v_1; and increasing future late demand.

[9] The problem's structure depends on whether $\sigma < 0.5$ or $\sigma \geq 0.5$. We identify the optimal policy in both cases but, for expositional ease, we focus on $\sigma < 0.5$ throughout this paper. This does not result in any loss of insight.

We now identify general properties of the optimal policy $\Gamma^*(D,\Gamma)$. Lemma 1 shows that $\Gamma^*(D,\Gamma)$ weakly increases with incoming and late demand. Lemma 2 asserts that, if some capacity is left idle, then the optimal policy does not depend on marginal deviations in late demand.

LEMMA 1. *The optimal policy function $\Gamma^*(D,\Gamma)$ is weakly increasing in D and Γ.*

LEMMA 2. *If the optimal policy is such that some capacity is left idle in $(D,\Gamma_0)\in\{H,L\}\times\mathbf{\Gamma}$, that is $\zeta(D,\Gamma_0)>\Gamma_0$, then we have $\Gamma^*(D,\Gamma)=\Gamma^*(D,\Gamma_0)$ for all $\Gamma\le\Gamma_0$.*

Given the model's rich set of parameters, we impose two restrictions on the demand structure to ensure analytical tractability (Assumption 2). First, demand is symmetric, i.e., excess demand under high demand is equal to excess supply under low demand. Second, demand fluctuations are large enough so the platform effectively faces the two objectives of managing demand-capacity imbalances and of discrimination. Under this assumption, the minimum number of n-type agents transferred to the next period under high demand (i.e., $\underline{\Gamma}_H=H-1$) is too large to be served entirely if realized demand is also high in the next period ($\sigma H+H-1>1$). The inter-temporal effect is thus zero when demand is high in consecutive periods, i.e., $V'(H,\Gamma)=0$ for each $\Gamma\in[\underline{\Gamma}_H,\bar{\Gamma}_H]$.

ASSUMPTION 2. *The parameters governing the structure of incoming demand satisfy:*

i) There exists $d\in(0,1)$ such that $H=1+d$ and $L=1-d$.

ii) The following inequalities are satisfied: $H-1>1-\sigma H\Leftrightarrow d>\frac{1-\sigma}{1+\sigma}$.

4.3. Optimal Policy

We define three regions in Figure 3 as a function of (i) *inter-type heterogeneity*, measured by the dispersion in valuations for timely services (i.e., 1 vs. v_0) and (ii) *time preferences*, measured by the dispersion in the valuations of n-type agents for timely vs. late services (i.e., v_0 vs. v_1). Region 1 is characterized by strong inter-type heterogeneity ($v_0\le\sigma$). Regions 2 and 3 are both characterized by weak inter-type heterogeneity but Region 2 ($\sigma<v_0\le\sigma+(1-\sigma)v_1$) features comparatively weaker time preferences than Region 3 ($v_0>\sigma+(1-\sigma)v_1$).

The optimal mechanism is shaped by the platform's two objectives—discrimination and demand-capacity management. As we shall see, discrimination becomes relatively less prominent and demand-capacity management becomes relatively more prominent as we move from Region 1 to Region 3 (i.e., as customer heterogeneity becomes weaker and as the time preferences of n-type agents become stronger) and as incoming demand increases.

Region 1: Strong inter-type heterogeneity We have from Equation (13):

$$\frac{\partial\tilde{V}(D,\Gamma,\Gamma')}{\partial\Gamma'}\ge\frac{\sigma(1-v_0)}{1-\sigma}-v_0=\frac{\sigma-v_0}{1-\sigma}\ge 0,\ \forall(D,\Gamma)\in\{H,L\}\times\mathbf{\Gamma},\ \forall\Gamma'\in[\underline{\Gamma}_D,\bar{\Gamma}_D]. \tag{14}$$

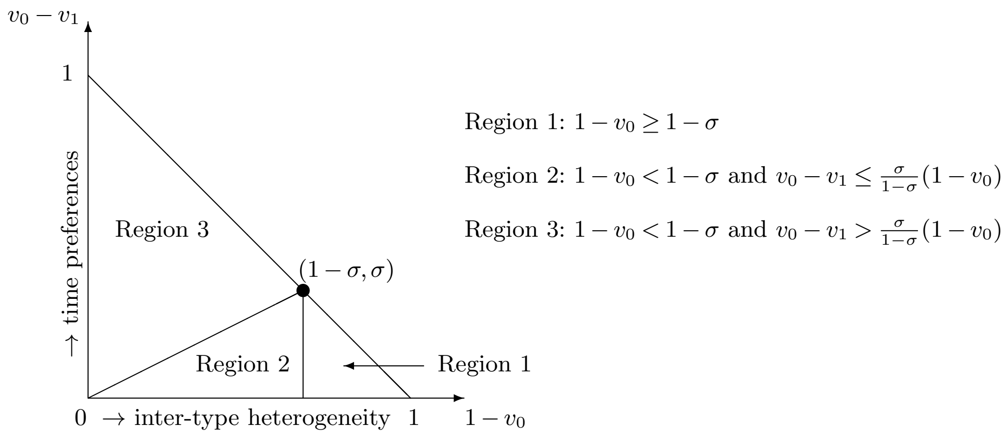


Figure 3 **Definition of Region 1, Region 2 and Region 3.**

Any loss from transferring incoming n-type agents is offset by the price increase alone. The optimal policy is thus to provide no timely service to incoming n-type agents and to charge a price $p_r = 1$ to r-type agents (thus leaving no information rent). Therefore, due to strong customer differentiation, the platform focuses on discrimination only with no regard for demand-capacity management.

PROPOSITION 4. *In Region 1, the optimal policy is:* $\Gamma^*(D,\Gamma) = \bar{\Gamma}_D, \quad \forall (D,\Gamma) \in \{H,L\} \times \mathbf{\Gamma}$.

Extreme discrimination results in idle capacity for low values of Γ, when $\Gamma < \zeta(D,\Gamma) = 1 - \sigma D$. When $\Gamma \geq \zeta(D,\Gamma) = 1 - \sigma D$, all n-type agents are still transferred to the next period, but no supplier is idle due to the high demand for late services.

KEY FINDING 1. In Region 1, the platform implements an extreme form of discrimination: no timely service is provided to n-type agents. The optimal policy exhibits *strategic idleness*.

Region 2: Weak inter-type heterogeneity, weak time preferences The price increase resulting from an increase in Γ' no longer offsets, by itself, the loss in timely services provided to n-type agents. Nonetheless, transferring n-type agents remains profitable as long as it enables the platform to serve more late demand in the current period. Specifically, we have, from Equation (13):

$$\frac{\partial \tilde{V}(D,\Gamma,\Gamma')}{\partial \Gamma'} \geq \frac{\sigma(1-v_0)}{1-\sigma} - v_0 + v_1 = \frac{\sigma - v_0 + v_1(1-\sigma)}{1-\sigma} \geq 0, \quad \text{if } 1 - D + \Gamma' \leq \Gamma. \tag{15}$$

The platform thus transfers to n-type agents as long as $1 - D + \Gamma' \leq \Gamma$, so that $\Gamma^*(D,\Gamma) \geq \chi(D,\Gamma)$.

LEMMA 3. *In Region 2, the optimal policy satisfies* $\Gamma^*(D,\Gamma) \geq \chi(D,\Gamma)$*,* $\forall (D,\Gamma) \in \{H,L\} \times \mathbf{\Gamma}$.

The policy thus features *late service prioritization*: the platform first serves all r-type customers, then fulfills late demand as much as possible. If capacity is insufficient (i.e., if $\Gamma > 1 - \sigma D$), some late requests will be rejected. Otherwise, the platform clears the entiere queue and then determines whether or not to provide timely services to incoming n-type agents. Unlike in Region 1, transferring more n-type agents than necessary to serve late demand induces a loss in the current period;

nonetheless, it may remain optimal if the resulting demand backlog provides sufficient future benefits. In this case, the optimal policy features *strategic idleness*, that is $\Gamma^*(D,\Gamma) > \chi(D,\Gamma)$. Formally, this occurs when, for $\varepsilon \to 0$:

$$\frac{\partial \tilde{V}(D,\Gamma,\Gamma')}{\partial \Gamma'} = \underbrace{\frac{\sigma(1-v_0)}{1-\sigma} - v_0}_{<0} + \underbrace{\delta\left(kV'(H,\Gamma') + (1-k)V'(L,\Gamma')\right)}_{\geq 0} \geq 0, \text{ for } \Gamma' = \Gamma^*(D,\Gamma) - \varepsilon. \quad (16)$$

From these observations, we elicit the optimal policy in Proposition 9 (Appendix A). Unlike in Region 1, the optimal policy is now state-dependent. In general terms, the optimal policy satisfies:

$$\Gamma^*(D,\Gamma) = \max\{\Gamma^*(D,0), \chi(D,\Gamma)\}, \ \forall (D,\Gamma) \in \{L,H\} \times \mathbf{\Gamma}.$$

The optimal policy function $\Gamma^*(D,\Gamma)$ thus stays constant at $\Gamma^*(D,0)$ as long as it does not fall below $\chi(D,\Gamma)$. If $\Gamma^*(D,0) > \chi(D,0)$, the policy features *strategic idleness* for low values of Γ. Once Γ grows to a point where $\chi(D,\Gamma) = \Gamma^*(D,0)$, $\Gamma^*(D,\Gamma)$ increases with slope 1 as long as feasible.

Key Finding 2. In Region 2, the optimal policy features *late service prioritization*. Some n-type agents may receive timely services but the optimal policy may still feature *strategic idleness*.

Region 2 is divided into three sub-regions (Sub-regions 2a, 2b and 2c, sorted by increasing order of v_0). Figure 4 shows the optimal policy $\Gamma^*(D,\Gamma)$ (rows 1 and 2), the allocation of capacity across r-type agents, incoming n-type agents, and delayed n-type agents (rows 3 and 4, with colored areas representing agents who receive a service), and the price $p_r(D,\Gamma)$ (row 5).

Note, first, that $\Gamma^*(D,\Gamma)$ lies consistently above $\chi(D,\Gamma)$, underscoring *late service prioritization* (Lemma 3). This property is also reflected in capacity allocation: if some late demand remains unserved, all incoming requests from n-type agents get transferred. Moreover, the policy may exhibit *strategic idleness*: for low values of D and Γ, some incoming n-type agents may be transferred while fewer services are provided than capacity. In Sub-region 2a (small v_0), discrimination is still a strong motive for the platform so the optimal mechanism involves *strategic idleness*; yet, some incoming n-type agents are served under high demand and low Γ. In Sub-region 2b, discrimination becomes less desirable and no capacity is left strategically idle under high demand. In Sub-region 2c, the optimal policy features no *strategic idleness* regardless of the demand, i.e., the platform exactly transfers the minimum number of n-type agents to serve late demand.

Region 3: Weak inter-type heterogeneity, strong time preferences Any revenue loss from transferring an n-type agent cannot be offset in the current period, even if it generates more late services (the first three terms of Equation (13) always sum up to a negative value). Therefore, $\Gamma^*(D,\Gamma) > \underline{\Gamma}_D$ only if late demand generate higher future benefits than the current-period loss.

The optimal policy, given in Proposition 10 (Appendix A), is again state-dependent. Transferring n-type agents can only be beneficial if it generates more late services in the current period. In

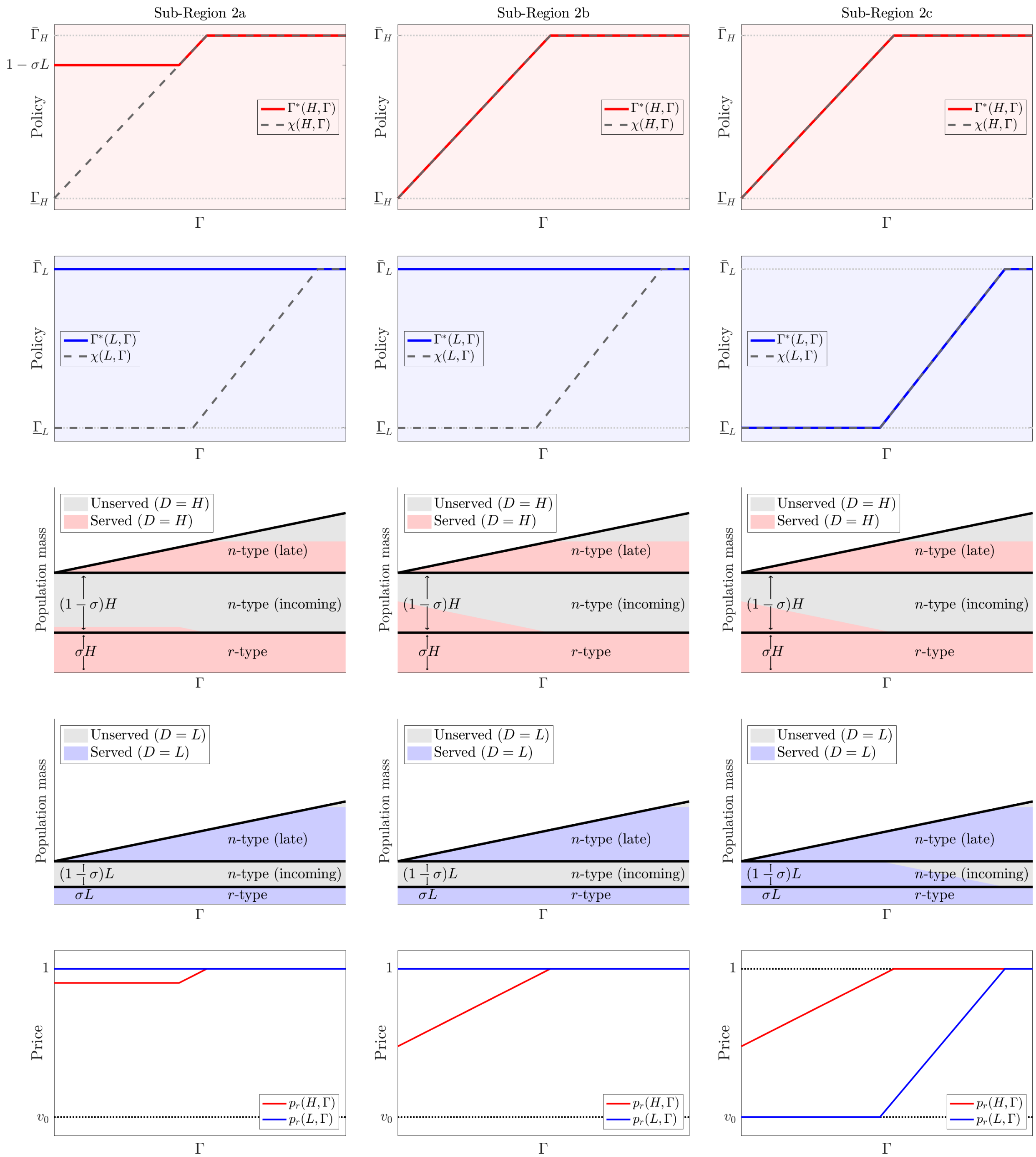


Figure 4 **Optimal policy $\Gamma^*(D,\Gamma)$, capacity allocation, and price $p_r(D,\Gamma)$ in Region 2, when $\sigma < 0.5$.**

other words, the inter-temporal benefits are not strong enough to justify transferring n-type agents on their own. Therefore, $\Gamma^*(D,0) = \underline{\Gamma}_D$. The optimal policy then increases along $\chi(D,\Gamma)$ until a threshold, after which is remains constant:

$$\text{There exists } \widehat{\Gamma}_D \in \left[\underline{\Gamma}_D, \bar{\Gamma}_D\right] \text{ such that } \Gamma^*(D,\Gamma) = \min\left\{\chi(D,\Gamma), \widehat{\Gamma}_D\right\}, \ \forall (D,\Gamma) \in \{L,H\} \times \mathbf{\Gamma}.$$

First, this equation shows that $\Gamma^*(D,\Gamma) \le \chi(D,\Gamma)$ in all states (D,Γ); therefore, the optimal policy exhibits *no strategic idleness*. As long as $\chi(D,\Gamma) \le \widehat{\Gamma}_D$, all late demand is served. In contrast, when $\chi(D,\Gamma) > \widehat{\Gamma}_D$, the optimal policy involves *deliberate late service rejection*.

Key Finding 3. In Region 3, the optimal policy may exhibit *deliberate late service rejection*.

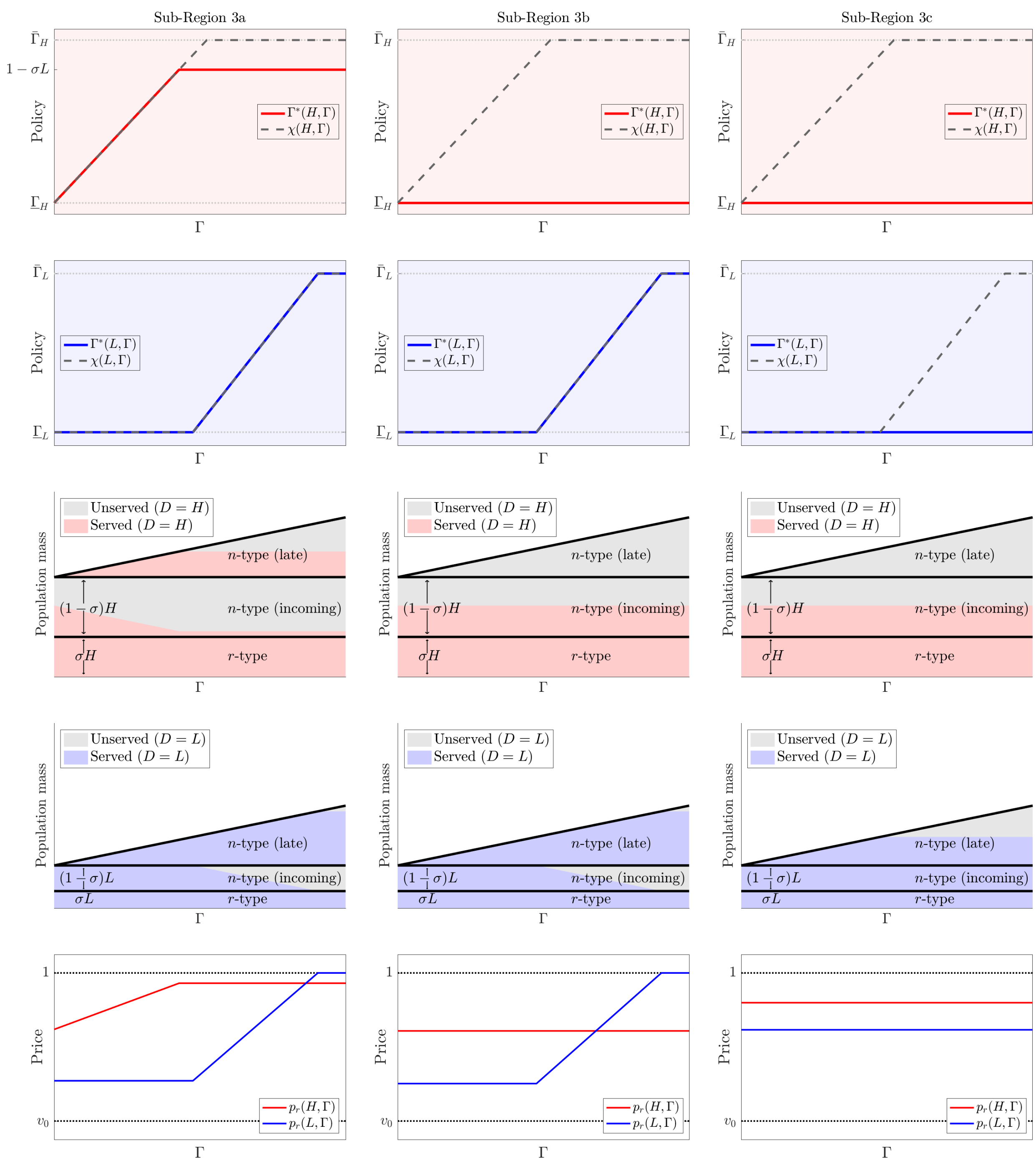


Figure 5 **Optimal policy $\Gamma^*(D,\Gamma)$, capacity allocation, and price $p_r(D,\Gamma)$ in Region 3, when $\sigma < 0.5$.**

Region 3 is also divided into three sub-regions (Sub-regions 3a, 3b and 3c, sorted by increasing order of v_0). The optimal policy is shown in Figure 5. Note that the optimal policy may exhibit *deliberate late service rejection*: in some cases, the platform serves some incoming n-type agents while leaving part of the late demand unserved. Also, the mechanism never results in *strategic idleness*: the number of services provided is either equal to the effective demand $(D+\Gamma)$ or to capacity. In Sub-region 3a, the platform serves all late demand as long as $\chi(D,\Gamma) \leq 1-\sigma L$ (i.e., as long as transferring n-type agents increases the number of late services if demand is low in the next period). Under high demand, when $\chi(D,\Gamma) > 1-\sigma L$, transferring additional n-type agents does not increase the future value; as a result, the policy features *deliberate late service rejection.* In Sub-region 3b, the platform only provides timely services under high demand, foregoing all late demand; under low demand, it keeps transferring n-type agents under the expectation that it will (at least partially) serve them in the subsequent period. In Sub-region 3c, the platform fully prioritizes timely services regardless of the incoming demand.

4.4. Non-Monotonicity of the Prices

In Region 1, the platform extracts the entire surplus ($p_r = 1$). But, in Regions 2 and 3, the platform leaves some information rent to r-type agents by trading off price vs. quantity of timely services.

Note, first, that the optimal price p_r (weakly) increases with late demand. Indeed, higher values of Γ result in more n-type agents transferred to the next period (Lemma 1), hence in higher prices.

One could expect a similar monotonic relationship with respect to the incoming demand D. However, this is not the case: the price p_r can be strictly higher under low demand than under high demand in Sub-regions 2a, 2b (Figure 4) and in Sub-regions 3a, 3b (Figure 5).

Key Finding 4. The optimal price does not increase monotonically with incoming demand.

To see the intuition behind this result, consider an instance where the platform transfers a large *proportion* of n-type agents to the next period under low demand. This is motivated by three factors: (i) charging a higher price to r-type agents, (ii) reserving capacity to serve late demand, and (iii) creating a demand backlog in the next period. Under high demand, transferring the same proportion of n-type agents would result in a larger backlog, which the platform may not be able to take full advantage of—especially if demand is also high in the next period. Instead, the platform may serve a larger proportion of incoming n-type agents under high demand, resulting in a higher price charged to r-type agents to maintain incentive compatibility. The main driving force behind these dynamics is that, among the two objectives of the platform, discrimination becomes relatively less prominent and demand-capacity management becomes relatively more prominent as incoming demand increases—so that the price may be higher under low demand than under high demand.

This phenomenon occurs for low values of Γ in Region 2 and for high values of Γ in Region 3. In Regions 2a and 2b, the platform adopts extreme discrimination under low demand—by transferring all incoming n-type agents (Figure 4). Under high demand, the demand-capacity management objective becomes stronger and the platform serves more incoming n-type agents (as long as late demand is small enough), resulting in a lower price charged to r-type agents. In Regions 3a and 3b, the demand-capacity management objective is stronger altogether. Under high demand, it is so strong that the platform may adopt *deliberate late service rejection*—especially when the late demand is large. Under low demand, the platform still prioritize late requests over timely requests from n-type agents in order to clear the queue and charge a higher price to r-type agents.

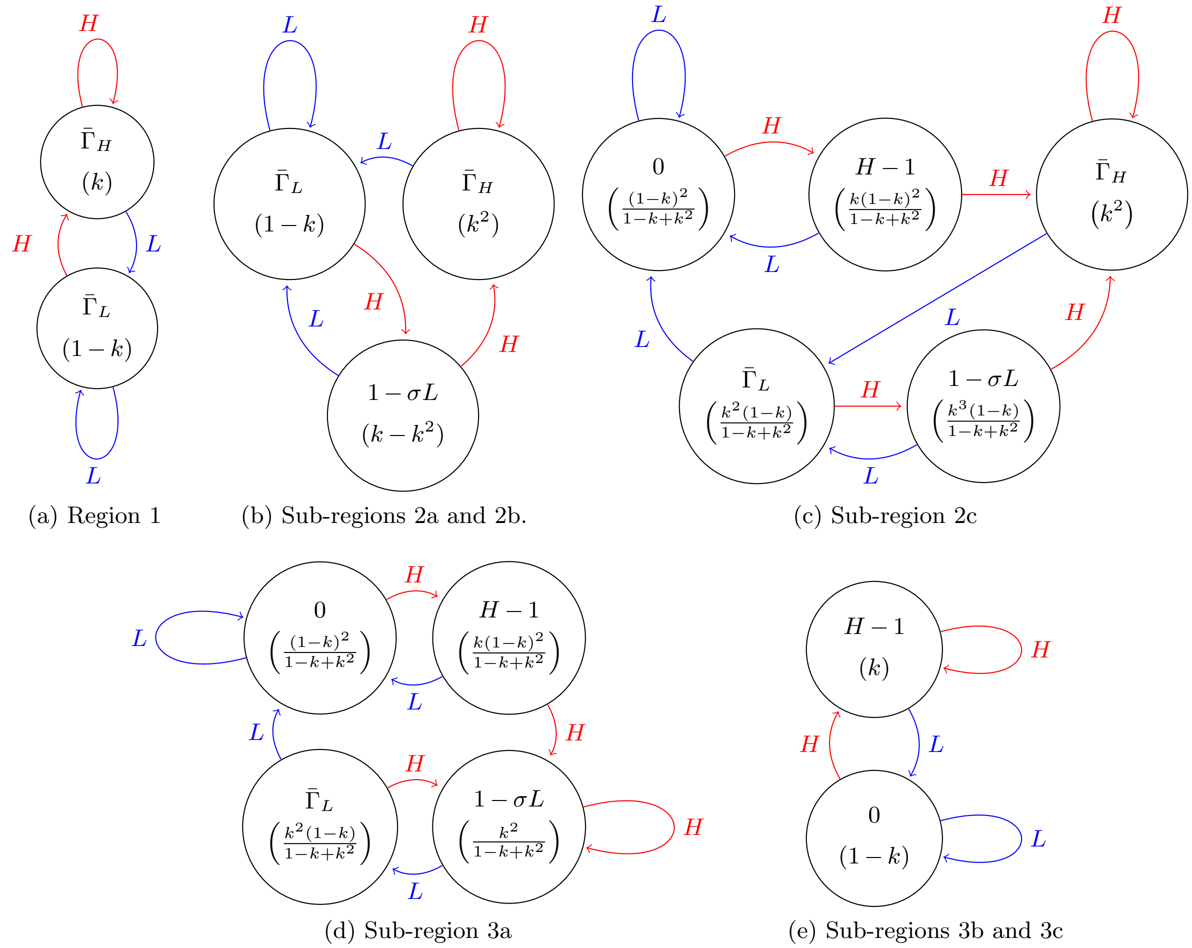
This result stems from the combination of the three features of our setting: (i) information asymmetries, (ii) capacity restrictions, and (iii) non-transferable capacity. Without information asymmetry, the platform would charge customer valuations regardless of incoming demand. With unrestricted capacity, the platform would focus on discrimination by serving either all customers or only r-type customers, regardless of incoming demand. And ig the platform was able to transfer idle capacity from one period to the next, it could leverage this extra degree of freedom to focus on discrimination more effectively, thus transferring a (weakly) higher proportion of n-type customers and charging a (weakly) higher price under high demand than under low demand.

4.5. Steady State and Transition Dynamics

To conclude, Figure 6 shows the steady-state distribution of the optimal mechanism. Each node denotes a value of Γ and each edge denotes a demand realization. The steady-state probability of each value of Γ is depicted within the corresponding node. We find that Γ takes at most five values in the steady state: $\underline{\Gamma}_L = 0$, $\bar{\Gamma}_L = (1-\sigma)L$, $\underline{\Gamma}_H = H - 1$, $1 - \sigma L$, and $\bar{\Gamma}_H = (1-\sigma)H$.

The main takeaway is that the optimal policy is history-dependent in the steady state—and thus cannot be implemented with posted prices and static mechanisms. In two extreme situations, the policy is state-independent: under very strong heterogeneity (Region 1) where the platform always ignores all requests from incoming n-type agents and under very weak heterogeneity (Regions 3b and 3c) where the platform always prioritizes timely requests. In-between (arguably, the most relevant instances in practice), the optimal policy depends on incoming and late demand. For instance, in Region 2c, the platform provides timely service to n-type agents only when Γ is low. These results indicate that the optimal mechanism gives rise to endogenous late demand dynamics: it alternates between queue-building periods—by creating a demand backlog for discrimination—and queue-clearing periods—by taking advantage of the queues for demand-capacity management.

Finally, Table 1 reports the steady-state probability of timely services for n-type agents (q_t), and the price charged to r-type agents (p_r). As v_0 increases, more n-type agents receive timely



(a) Region 1 (b) Sub-regions 2a and 2b. (c) Sub-region 2c

(d) Sub-region 3a (e) Sub-regions 3b and 3c

Figure 6 Steady state transition dynamics, with $\sigma < 0.5$.

services and r-type agents are thus charged a lower price. Importantly, our finding that the price does not monotonically increase with incoming demand is observed in steady state, and is thus not an artifact of the transient dynamics of the system (colored cells in Sub-regions 2a, 2b and 3a).

Table 1 Steady state allocations and prices, shown as $(q_t(D,\Gamma), p_r(D,\Gamma))$, when $\sigma < 0.5$.

State	Reg. 1	Reg. 2a&2b	Reg. 2c	Reg. 3a	Reg. 3b&3c
$(H,0)$			(q_t^α, p_r^α)	(q_t^α, p_r^α)	(q_t^α, p_r^α)
$(L,0)$			$(1, v_0)$	$(1, v_0)$	$(1, v_0)$
$(H,\bar{\Gamma}_L)$	(0,1)	(q_t^β, p_r^β)	(q_t^β, p_r^β)	(q_t^β, p_r^β)	
$(L,\bar{\Gamma}_L)$	(0,1)	(0,1)	$(1, v_0)$	$(1, v_0)$	
$(H,H-1)$			(0,1)	(q_t^β, p_r^β)	(q_t^α, p_r^α)
$(L,H-1)$			$(1, v_0)$	$(1, v_0)$	$(1, v_0)$
$(H,1-\sigma L)$		(0,1)	(0,1)	(q_t^β, p_r^β)	
$(L,1-\sigma L)$		(0,1)	(0,1)	(0,1)	
$(H,\bar{\Gamma}_H)$	(0,1)	(0,1)	(0,1)		
$(L,\bar{\Gamma}_H)$	(0,1)	(0,1)	(0,1)		

$$q_t^\alpha = \frac{1-\sigma H}{(1-\sigma)H}$$
$$q_t^\beta = \frac{(1-2\sigma)d}{(1-\sigma)H}$$
$$p_r^\alpha = 1 - q_t^\alpha(1-v_0)$$
$$p_r^\beta = 1 - q_t^\beta(1-v_0)$$

$$0 < q_t^\beta < q_t^\alpha < 1$$
$$v_0 < p_r^\alpha < p_r^\beta < 1$$

5. Welfare Implications

This section compares the steady-state surplus generated by the optimal mechanism against two benchmarks: (i) the first-best allocation rule in the absence of information asymmetry and (ii) a dynamic pricing mechanism that does not elicit agents' preferences (aimed to replicate, at a high level, prevalent practices in online markets such as surge pricing and congestion pricing).

5.1. First-best Allocation

First-best allocation can be interpreted in two ways. It captures the decisions of a surplus-maximizing social planner, who optimizes the allocation rule without specifying any payment rule. Equivalently, it captures the platform's profit-maximizing decisions in the absence of information asymmetry, in which case each agent is charged a price equal to his valuation (i.e., 1, v_0 and v_1).

The first-best allocation rule is defined by the variables $q_r^{FB}(D,\Gamma), q_t^{FB}(D,\Gamma), q_l^{FB}(D,\Gamma)$—analogous to $q_r(D,\Gamma), q_t(D,\Gamma), q_l(D,\Gamma)$. This problem, referred to as Problem $(\mathcal{P}_{FB})$, does not include incentive compatibility and individual rationality constraints. It is subject to the same resource constraint (Equation (18)) and transition function (Equation (19)) as Problem $(\mathcal{P})$.

$$V_{FB}(D,\Gamma) = \max_{q_r^{FB}, q_t^{FB}, q_l^{FB}} \quad q_r^{FB}\sigma D \times 1 + q_t^{FB}(1-\sigma)Dv_0 + q_l^{FB}\Gamma v_1 + \delta[kV_{FB}(H,\Gamma') + (1-k)V_{FB}(L,\Gamma')] \tag{17}$$

$$\text{s.t.} \quad 1 \geq q_r^{FB} sD + q_t^{FB}(1-\sigma)D + q_l^{FB}\Gamma, \tag{18}$$

$$\Gamma' = (1-q_t^{FB})(1-\sigma)D. \tag{19}$$

As in Section 3, all r-type agents are served, and late demand is served as long as capacity allows (Lemma EC.1). We can again simplify the problem with the single decision variable Γ', as follows:

$$\begin{aligned} V_{FB}(D,\Gamma) = \max_{\Gamma' \in [\underline{\Gamma}_D, \bar{\Gamma}_D]} \quad & \sigma D + ((1-\sigma)D - \Gamma')\, v_0 + \min\{\Gamma, 1 - D + \Gamma'\} v_1 \\ & + \delta\left(kV_{FB}(H,\Gamma') + (1-k)V_{FB}(L,\Gamma')\right). \end{aligned} \tag{20}$$

The only difference with Problem $(\mathcal{P})$ lies in the first term. In Problem $(\mathcal{P})$, this term included the price derived from the incentive compatibility constraint for r-type agents. In $(\mathcal{P}_{FB})$, it includes their valuation (equal to 1). Proposition 5 shows that Problem $(\mathcal{P}_{FB})$ is equivalent to Problem $(\mathcal{P})$ by transforming the parameter v_0 into $\tilde{v}_0 = \sigma + (1-\sigma)v_0$. Note that $\tilde{v}_0 \in (\sigma + (1-\sigma)v_1, 1)$. Therefore, Problem $(\mathcal{P}_{FB})$ is isomorphic to Problem $(\mathcal{P})$ in Region 3 (see Figure 5). This reflects that the social planner focuses on demand-capacity management as opposed to discrimination—by determining whether to serve late demand and delay service to incoming n-type agents vs. provide more timely services at the risk of leaving some late requests unserved.

Proposition 5. *Problem $(\mathcal{P}_{FB})$ with parameter v_0 is equivalent to Problem $(\mathcal{P})$ with parameter $\tilde{v}_0 = \sigma + (1-\sigma)v_0$. The optimal policy is identical to the optimal policy of Problem $(\mathcal{P})$ in Sub-Region 3a when $v_0 \in \left(v_1, \frac{1+\delta(1-k)+\delta^2(1-k)^2}{1+\delta(1-k)}v_1\right]$, in Sub-Region 3b when $v_0 \in \left(\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1+\delta(1-k)}v_1, (1+\delta(1-k))v_1\right]$, and in Sub-Region 3c when $v_0 \in ((1+\delta(1-k))v_1, 1)$.*

5.2. Dynamic Posted Prices

The dynamic pricing mechanism does not elicit agent preferences and thus charges a uniform price in each period. The system evolves as follows: (i) the platform posts a price in each period, (ii) each agent decides to accept the service, or not, (iii) the platform allocates capacity uniformly across all agents who accept the service (regardless of their types), and (iv) agents who decline the service may leave the platform or wait for a late service, depending on their type and history. We refer to the dynamic pricing problem as $(\mathcal{P}_{DP})$. The posted price, denoted by p_0, is optimized as follows:

$$V_{DP}(D,\Gamma) = \max_{p_0} \quad p_0 \times x(D,\Gamma,p_0) + \delta\left(kV_{DP}(H,\Gamma') + (1-k)V_{DP}(L,\Gamma')\right) \tag{21}$$

$$\text{s.t.} \quad x(D,\Gamma,p_0) = \begin{cases} \sigma D & p_0 > v_0, \\ \min\{D,1\} & p_0 \in (v_1,v_0], \\ \min\{D+\Gamma,1\} & p_0 \leq v_1. \end{cases} \tag{22}$$

$$\Gamma' = \begin{cases} (1-\sigma)D & p_0 > v_0, \\ (1-\sigma)D\left[1 - \frac{x(D,\Gamma,p_0)}{D}\right] & p_0 \in (v_1,v_0], \\ (1-\sigma)D\left[1 - \frac{x(D,\Gamma,p_0)}{D+\Gamma}\right] & p_0 \leq v_1. \end{cases} \tag{23}$$

Equation (21) maximizes the platform's expected discounted profit. The per-period profit is equal to the posted price p_0 times the number of services provided, denoted by $x(D,\Gamma,p_0)$. Constraints (22) and (23) specify the relationship between p_0, $x(D,\Gamma,p_0)$ and Γ'. When $p_0 > v_0$, only r-type agents accept the service; the platform thus provides σD services and the full mass $(1-\sigma)D$ of n-type agents is transferred. When $v_1 < p_0 \leq v_0$, all incoming agents are willing to accept the service.[10] But given the platform's finite capacity, only $\min\{D,1\}$ services can be provided, and a mass $(1-\sigma)D\left[1 - \frac{x(D,\Gamma,p_0)}{D}\right]$ is transferred. Last, if $p_0 \leq v_1$, all agents are willing to accept the service, so the number of services is then equal to $\min\{D+\Gamma,1\}$, and $\Gamma' = (1-\sigma)D\left[1 - \frac{x(D,\Gamma,p_0)}{D+\Gamma}\right]$.

PROPOSITION 6. *Problem $(\mathcal{P}_{DP})$ admits an optimal solution $p_0^*(D,\Gamma)$. It satisfies $p_0^*(D,\Gamma) \in \{1,v_0,v_1\}$. Moreover, $p_0^*(D,\Gamma) \in \{1,v_0\}$ for each (D,Γ) if and only if $v_1 \leq \max\{\sigma L, v_0 L\}$. In this case, for each $\Gamma \in \mathbf{\Gamma}$, we have under high demand: $p_0^*(H,\Gamma) = 1$ if $v_0 \leq \sigma H$ and $p_0^*(H,\Gamma) = v_0$ if $v_0 > \sigma H$. Under low demand, we have: $p_0^*(L,\Gamma) = 1$ if $v_0 \leq \sigma$ and $p_0^*(L,\Gamma) = v_0$ if $v_0 > \sigma$.*

If $v_1 \leq \max\{\sigma L, v_0 L\}$, the value of a late service for n-type agents is sufficiently low for the platform to never charge v_1—the corresponding price reduction would offset the increase in the quantity of services provided. In this case, the platform provides only timely services and the pricing policy is myopically optimal. Problem $(\mathcal{P}_{DP})$ is then equivalent to maximizing current-period profits based on incoming demand.

(i) When $v_0 \leq \sigma$, the platform only serves the r-type agents by setting a price of 1. The platform's expected profit is then $\sigma(kH + (1-k)L)$.

[10] In theory, n-type agents could reject the service and wait for a late service if they anticipate a lower price in the next period. This, does not occurs as the platform never charges a price strictly lower than v_1 at the optimum.

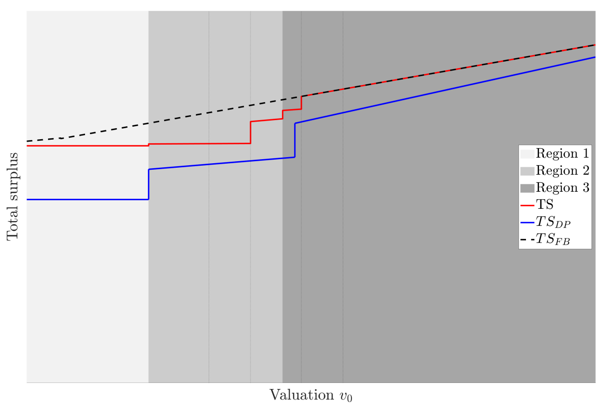
(ii) When $\sigma < v_0 \leq \sigma H$, the platform serves all the incoming demand in low-demand periods by setting a price of v_0, but only the r-type agents in high-demand periods by charging a price of 1. The platform's expected profit is then equal to $k\sigma H + (1-k)v_0 L$.

(iii) When $v_0 > \sigma H$, the platform always sets the price of v_0—but cannot serve all incoming requests under high demand. The platform's expected profit is $v_0\,(k + (1-k)L)$.

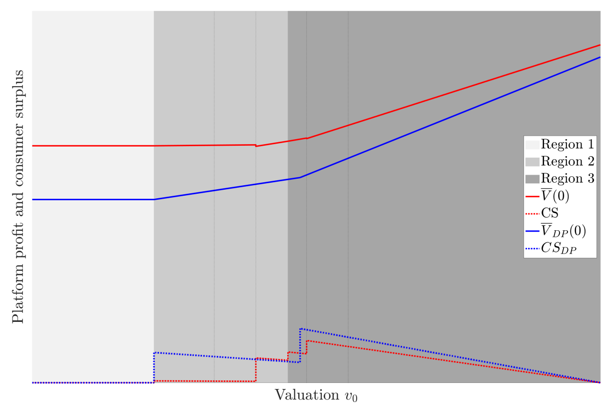
5.3. Performance Assessment

We now compare our mechanism to the two benchmarks based on three steady-state metrics: (i) the platform's expected discounted profit before the first-period demand realizes, i.e., $\overline{V}(0) = (1-k)V(L,0) + kV(H,0)$, (ii) the expected consumer surplus, denoted by CS, and (iii) the total expected surplus generated within the platform, denoted by TS. For tractability, we focus on the case where $v_1 \leq \max\{\sigma L, v_0 L\}$, and apply the result from Proposition 6.

Figure 7 shows the surplus generated by each policy (Figure 7a) and its distribution under the optimal mechanism and the dynamic pricing policy (Figure 7b), as a function of v_0.



(a) Total surplus



(b) Platform profit and consumer surplus

Figure 7 Surplus from the optimal mechanism, dynamic pricing ("DP") and the first-best mechanism ("FB").

As compared to the first-best outcome, the optimal menu induces a trade-off between the surplus generated and information rents. For the highest values of v_0 (Regions 3b and 3c), the optimal mechanism maximizes social welfare—by focusing on demand-capacity management rather than discrimination. However, the platform leaves some surplus to time-sensitive agents due to information asymmetries. For lower values of v_0, discrimination is an important concern for the platform, which induces a surplus loss. In Region 1, the platform extracts all the surplus without leaving any rent. In-between, the optimal mechanism induces both a surplus loss and information rents.

Key Finding 5. Under weak inter-type heterogeneity and strong time preferences, the optimal mechanism maximizes social welfare; yet, the platform leaves information rent to the agents. Otherwise, it induces a surplus loss; under strong heterogeneity, the platform captures all the surplus.

Next, eliciting agents' preferences through a dynamic menu increases the platform's expected profit, as compared to the dynamic pricing policy. The relative improvement can be significant, and increases with inter-type heterogeneity. The main benefits of our mechanism stem from personalization and targeting. Indeed, the dynamic pricing mechanism ends up rejecting some requests from r-type customers when the posted price is v_0 and demand is high, and losing all time-sensitive customers when the posted price is 1. Instead, by proposing a menu of prices and service levels, our mechanism can select which customers to serve or transfer, and can manage the queue of delayed customers more effectively. In some cases, the optimal mechanism even provides a Pareto improvement—higher platform profits and larger consumer surplus.

6. Benefits of Dynamic Menus

We compare our dynamic mechanism to a static benchmark. We also propose a "demand-dependent mechanism" that (i) adjusts the menu as a function of incoming demand, and (ii) allocates late services freely based on realized contingencies. Results shows that the static benchmark yields suboptimal outcomes. The demand-dependent mechanism achieves close-to-optimal outcomes, providing an easily-implementable approximation of the optimal mechanism. Yet, the optimal mechanism increases the platform's profits by up to 3–4% as compared to the demand-dependent mechanism.

6.1. Static Menu

The static mechanism proposes a state-independent menu. Its components (still denoted by q_t, q_l, p_r, p_t, p_l) remain constant regardless of the state variable (D,Γ). Each menu defines a steady-state distribution with four values: $\{\{H,L\}\times\{\Gamma_S^H,\Gamma_S^L\}\}$, where $\Gamma_S^H=(1-q_t)(1-\sigma)H$ and $\Gamma_S^L=(1-q_t)(1-\sigma)L$. For each $(D,\Gamma)\in\{\{H,L\}\times\{\Gamma_S^H,\Gamma_S^L\}\}$, the value function $V_S(D,\Gamma)$ satisfies:

$$V_S(D,\Gamma)=q_r\sigma Dp_r+q_t(1-\sigma)Dp_t+q_l\Gamma p_l+\delta\left(kV_S(H,\Gamma_S^D)+(1-k)V_S(L,\Gamma_S^D)\right).$$

The platform's problem, referred to as $(\mathcal{P}_S)$, maximizes the expected discounted profit before the first-period demand realizes, when $\Gamma=0$. The constraints are similar to Equations (4)–(8); for conciseness, we defer their full formulation to Appendix B.1. Problem $(\mathcal{P}_S)$ is defined as follows:

$$\overline{V}_S(0)=\max_{q_r,q_t,q_l,p_r,p_t,p_l}\left\{\begin{array}{l}k\left(q_r\sigma Hp_r+q_t(1-\sigma)Hp_t+\delta\mathbb{E}_D\left[V_S(D,\Gamma_S^H)\right]\right)\\+(1-k)\left(q_r\sigma Lp_r+q_t(1-\sigma)Lp_t+\delta\mathbb{E}_D\left[V_S(D,\Gamma_S^L)\right]\right)\end{array}\right\}$$

$$\text{s.t.}\quad \mathcal{IC}_r,\ \mathcal{IC}_n,\ \mathcal{IR},\ \mathcal{CC},\ \mathcal{TC}\ \text{(Appendix B.1)}.$$

Proposition 7. *The solution to problem $\mathcal{P}_S$ satisfies $q_r=1$, $p_t=v_0$ and $p_l=v_1$. Moreover:*

- *If $v_0\le\sigma+\delta(1-\sigma)v_1$ (Region 1, part of Region 2): $q_t=0$, $q_l=\frac{1-\sigma H}{(1-\sigma)H}$, $p_r=1$.*
- *If $v_0>\sigma+\delta(1-\sigma)v_1$ (part of Region 2, Region 3): $q_t=\frac{1-\sigma H}{(1-\sigma)H}$, $q_l=0$, $p_r=1-\frac{1-\sigma H}{(1-\sigma)H}(1-v_0)$.*

The static menu exhibits a "bang-bang" structure. The platform impleents extreme discrimination when $v_0 \leq \sigma + \delta(1-\sigma)v_1$ by providing no timely service to n-type agents; and it implements extreme late service rejection when $v_0 < \sigma + \delta(1-\sigma)v_1$ by only serving incoming requests. This contrasts with the dynamic mechanism, which adjusts the menu based on the system's state in Regions 2, 3a and 3b. Moreover, when the platform prioritizes timely services, it cannot serve all incoming requests under low demand, due to capacity constraints that would otherwise be violated under high demand. Last, the static mechanism results simultaneously in idle capacity and late service rejection under low demand ($q_l = 0$)—unlike the dynamic mechanism, which uses remaining capacity to serve late demand after providing timely services (Proposition 2).

6.2. Demand-dependent Menu

The demand-dependent mechanism enhances the static one in two ways. First, the menu varies with realized demand $D \in \{H, L\}$. The prices and probabilities of timely services are thus denoted by $p_r(D)$, $p_t(D)$, $q_r(D)$, and $q_t(D)$. Second, the platform allocates late services depending on realized contingencies, independently from the menu—motivated by the fact that late services can enhance capacity utilization without altering the incentives of r-type agents. Let $p_l(D,\Gamma)$ and $q_l(D,\Gamma)$ thus denote the price and allocation variables. Each menu defines a steady-state distribution with four values: $\{\{H,L\} \times \{\Gamma^H_{DD}, \Gamma^L_{DD}\}\}$, where $\Gamma^H_{DD} = (1-q_t(H))(1-\sigma)H$ and $\Gamma^L_{DD} = (1-q_t(L))(1-\sigma)L$. For each $(D,\Gamma) \in \{\{H,L\} \times \{\Gamma^H_{DD}, \Gamma^L_{DD}\}\}$, the value function $V_{DD}(D,\Gamma)$ satisfies:

$$V_{DD}(D,\Gamma) = q_r(D)\sigma D p_r(D) + q_t(D)(1-\sigma)Hp_t(D) + q_l(D,\Gamma)\Gamma p_l(D,\Gamma) \\ + \delta\left(kV_{DD}(H,\Gamma^D_{DD}) + (1-k)V_{DD}(L,\Gamma^D_{DD})\right).$$

The platform's problem, referred to as $(\mathcal{P}_{DD})$, is given by:

$$\overline{V}_{DD}(0) = \max_{\substack{q_r(D),q_t(D),q_l(D,\Gamma) \\ p_r(D),p_t(D),p_l(D,\Gamma)}} \left\{ \begin{array}{l} k\left(q_r(H)\sigma Hp_r(H) + q_t(H)(1-\sigma)Hp_t(H) + \delta\mathbb{E}_D\left[V_{DD}(D,\Gamma^H_{DD})\right]\right) \\ +(1-k)\left(q_r(L)\sigma Lp_r(L) + q_t(L)(1-\sigma)Lp_t(L) + \delta\mathbb{E}_D\left[V_{DD}(D,\Gamma^L_{DD})\right]\right) \end{array} \right\}$$

$$[6pt] \quad \text{s.t.} \quad \mathcal{IC}_r,\ \mathcal{IC}_n,\ \mathcal{IR},\ \mathcal{CC},\ \mathcal{TC} \text{ (Appendix B.1).}$$

Proposition 11 (Appendix B) elicits the optimal demand-dependent mechanism. It identifies three regions: Region A (Region 1, part of Region 2), Region B (part of Region 2), and Region C (part of Region 2, Region 3). Figure 8 shows the optimal policy and the price charged to r-type agents.

The demand-dependent mechanism exhibits three main differences with the static benchmark. First, it does not involve extreme discrimination in Region B; instead, the platform only transfers n-type agents as long as the resulting late demand can be served in the next period if demand is low—by delaying $1 - \sigma L$ agents under high demand. Second, by differentiating service levels as a function of demand realizations, the demand-dependent mechanism can serve all incoming

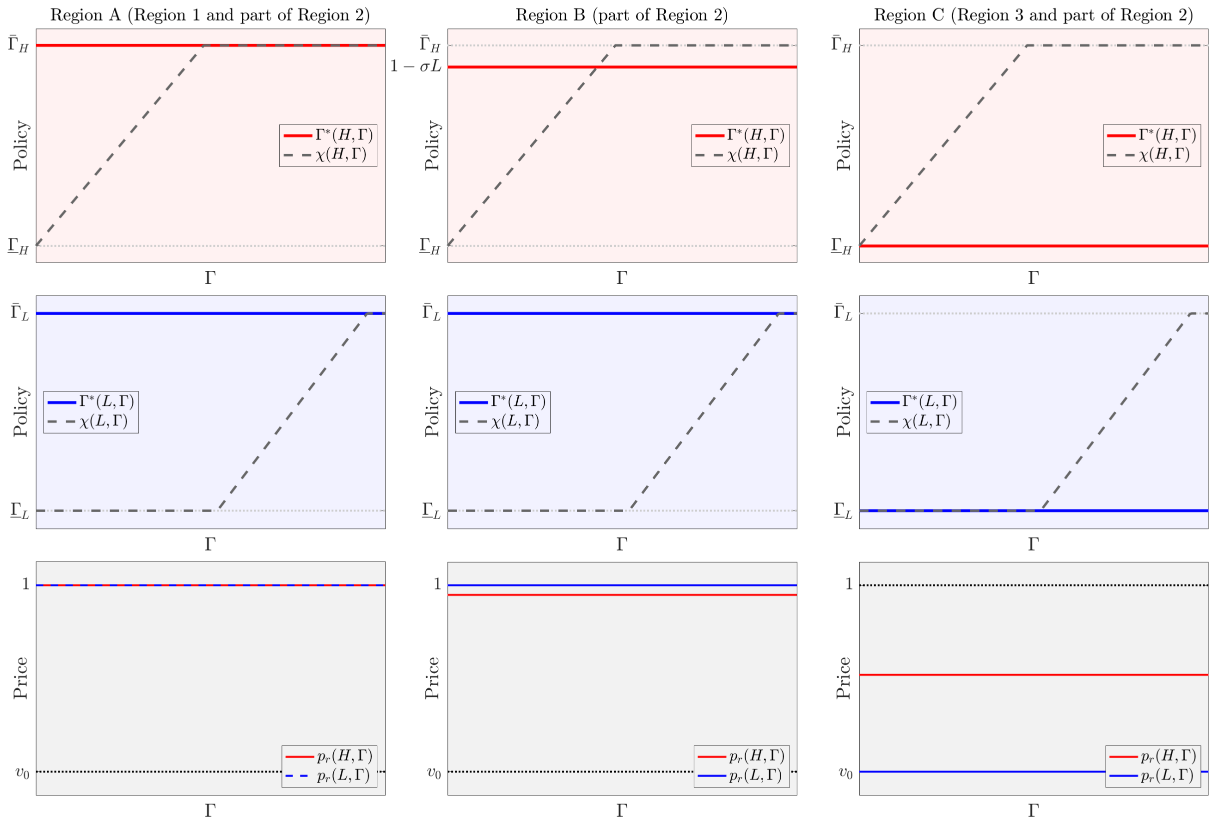


Figure 8 **Optimal policy and corresponding price under the demand-dependent mechanism, when $\sigma < 0.5$.**

requests under high and low demand in Region 3—by setting $q_t(H) = \frac{1-\sigma H}{(1-\sigma)H}$ and $q_t(L) = 1$. Third, adjusting the provision of late services as a function of realized contingencies ensures that no late request is rejected when capacity remains available upon provision of timely services.

Note, importantly, that the optimal price charged to r-type agents is lower under high demand than under low demand in Region B. This stems from a similar intuition as in the optimal mechanism. The platform provides no timely services to n-type agents under low demand—in part, for discriminatory purposes and, in part, under the expectation that these requests will be served in the next period. Under high demand, however, delaying all incoming requests from n-type agents would result in a long backlog that could not be served entirely, even under low demand. Therefore, only $1-\sigma L$ n-type agents are transferred, resulting in a lower price to maintain incentive compatibility. This underscores that the non-monotonicity of prices is *not* an idiosyncratic feature of the mechanism proposed in this paper but, instead, holds for broader classes of dynamic mechanisms.

Key Finding 6. Under the demand-dependent mechanism, the price does not increase monotonically with demand.

6.3. Performance Assessment and Implications

Figure 9 compares the platform's expected discounted profit under the optimal mechanism $\overline{V}(0)$, the static benchmark $\overline{V}_S(0)$, and the demand-dependent mechanism $\overline{V}_{DD}(0)$, as a function of v_0.

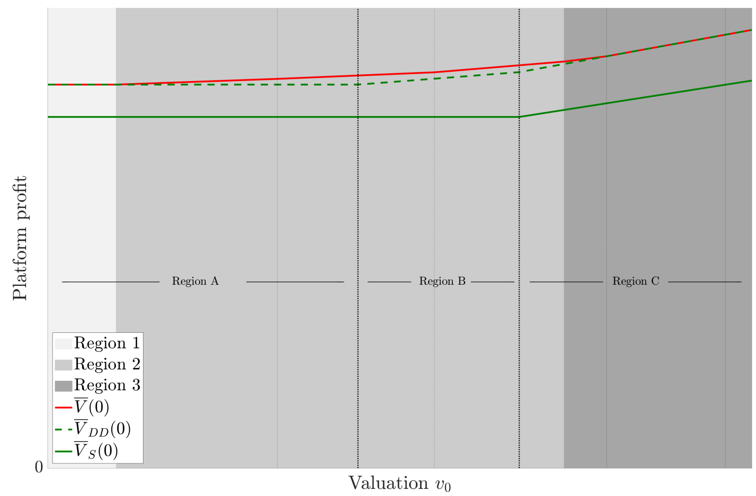


Figure 9 Comparison of dynamic mechanism with the static and demand-dependent mechanisms.

Recall that the optimal menu does not depend on Γ under strong heterogeneity (the platform disregards timely requests from n-type agents) and under weak heterogeneity (the platform maximizes timely services). In these cases (Regions 1, 3b and 3c), the static benchmark is sub-optimal anyway by failing to adjust late services as a function of Γ, hence failing to maximize capacity utilization. In contrast, the demand-dependent mechanism is optimal. In-between, the optimal mechanism tailors prices and allocations as a function of history—incoming and late demand. In these cases, the demand-dependent mechanism still performs much better than the static benchmark; yet, the optimal mechanism induces higher profits than the demand-dependent mechanism.

To quantify these points, let R_{DD}^{max} denote the maximum relative gain from the optimal mechanism, as compared to the demand-dependent mechanism. It is defined as:

$$R_{DD}^{max} = \max_{v_0 \in (v_1,1)} R_{DD}(v_0), \quad \text{where } R_{DD}(v_0) = \frac{\overline{V}(0) - \overline{V}_{DD}(0)}{\overline{V}_{DD}(0)}.$$

Proposition 8 provides a lower bound of R_{DD}^{max} (from the boundary between Regions B and C) and a tighter lower bound (from the boundary between Regions A and B) when $k = 0.5$ (i.e., when the expected demand is equal to the capacity and the system is in balance).

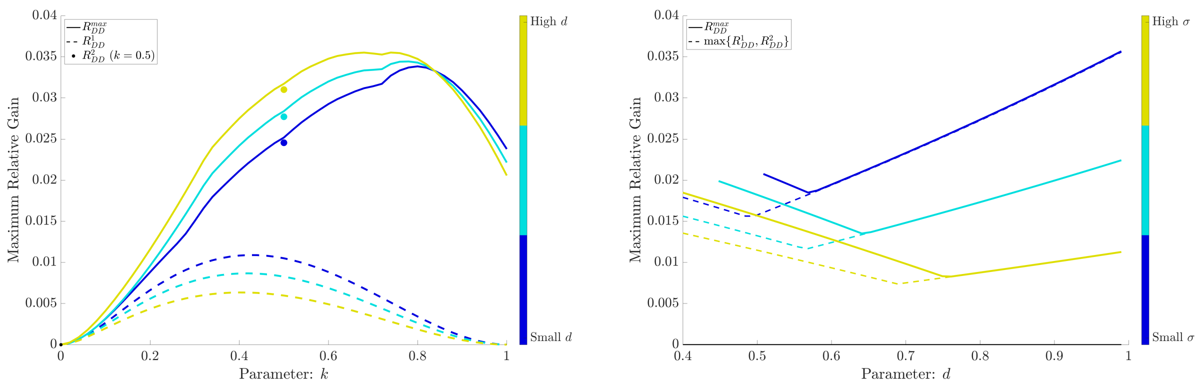
PROPOSITION 8. *When $\sigma < 0.5$ and $\delta \to 1$, we have $R_{DD}^{max} \geq R_{DD}^1(k,d,v_1,\sigma)$, where*

$$R_{DD}^1(k,d,v_1,\sigma) = \frac{(1-\sigma)Lv_1k^2(1-k)^2}{\sigma L(1-k) + \sigma Hk + (1-2\sigma)dv_1k + (1-\sigma)Lv_1(1-k+k^2) + (1-\sigma L)v_1(1-k)k}.$$

In a balanced system ($k = 0.5$), we have $R_{DD}^{max} \geq \max\{R_{DD}^1(0.5, d, v_1, \sigma),\ R_{DD}^2(d, v_1, \sigma)\}$, where:

$$R_{DD}^2(d, v_1, \sigma) = \begin{cases} \frac{(1-2\sigma)dv_1}{8\sigma+2(1-2\sigma)dv_1+6(1-\sigma)Lv_1+2(1-\sigma L)v_1} & \text{if } \sigma+\delta(1-\sigma)kv_1 \geq v_1, \\ \frac{(1-2\sigma)d\frac{(1-v_1)\sigma}{1-\sigma}}{4\sigma+2(1-2\sigma)d\frac{v_1-\sigma}{1-\sigma}+3(1-\sigma)Lv_1+(1-\sigma L)v_1} & \text{otherwise.} \end{cases}$$

Figure 10 plots R_{DD}^{max} and its lower bounds from Proposition 8. The lower bound R_{DD}^1 is relatively tight for low values of k but the optimal mechanism achieves stronger benefits otherwise. In a balanced system ($k = 0.5$), R_{DD}^2 achieves a much tighter lower bound, thus providing a good approximation of the maximum relative gain of the optimal mechanism.



(a) R_{DD}^{max} as a function of k and d

(b) R_{DD}^{max} as a function of d and σ (k=0.5)

Figure 10 Maximum relative gains (R_{DD}^{max}) of the optimal vs. demand-dependent mechanisms.

Overall, the optimal mechanism increases the platform's profits by up to 3–4%, as compared to the demand-dependent mechanism—by tailoring the items of the menu as a function of the system's history. By proceeding similarly, we find that the optimal mechanism increases the platform's profit by around 20%, as compared to the static mechanism (for a balanced system with $k = 0.5$).

These results also suggest that the optimal mechanism provides strongest improvements when the demand is volatile and the demand composition induces stronger trade-offs. Specifically:

- The optimal mechanism is most beneficial under demand uncertainty. Small k values lead to small demand backlogs hence limited benefits of tailoring the menu based on late demand. Larger k values create higher backlogs, hence stronger improvements of the optimal mechanism. In-between, the system exhibits strong volatility, resulting in high variability in Γ. As a result, adjusting the menu as a function of the system's history yields strongest benefits.
- The optimal mechanism is most beneficial under strong demand shocks (high d values), which lead to strong gains from the tailoring the menu with the system's history. The relative gains are also high with smallest d values, adjusting the menu based on incoming demand with the demand-dependent mechanism becomes marginally beneficial when H and L get closer.

- The benefits of the optimal mechanism are stronger when the demand composition induces stronger trade-offs. Indeed, as σ decreases, more capacity is available to serve n-type customers, strengthening the platform's degrees of freedom when allocating remaining capacity across timely and late services (vice versa, higher proportions of r-type customers increase the incidence of "obvious" decisions for all mechanisms).

In summary, the optimal mechanism comprises a menu of options, each catered to a specific customer segment and dynamically adjusted based on (exogenous and endogenous) contingencies. As compared to its static counterpart, this mechanism results in strong profit improvements. We also proposed a demand-dependent mechanism that adjusts the menu as a function of exogenous demand realizations. We showed that this mechanism—less information intensive and hence more easily implementable than the optimal mechanism—can yield close-to-optimal outcomes. Yet, the optimal mechanism still improves the platform's profits, by up to 3–4%, as compared to this approximation—by tailoring the menu as a function of the system's history.

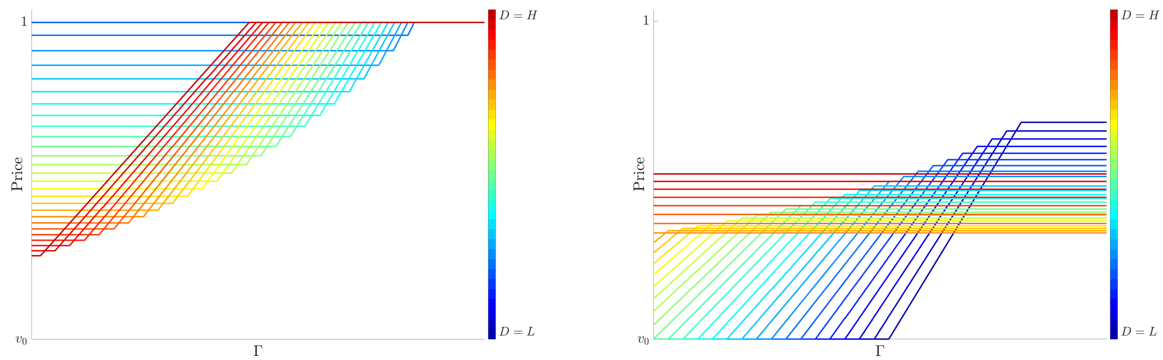
7. Robustness Checks

To this point, we have considered a simplified environment to analytically characterize the dynamics of pricing and allocation under non-transferable capacity, stochastic demand-capacity imbalances, and customer heterogeneity. We now relax key assumptions to capture general demand distributions, multi-period patience horizons, endogenous supply-side participation, and stochastic customer mix. As we shall see, the structure of the optimal policy and the non-monotonicity of the optimal price with incoming demand do not result from idiosyncrasies associated with our simplified setting but instead hold in each of these extensions. In fact, we generalize our main insight by showing that the optimal price is not monotonic with the non-expiring portion of the demand.

7.1. General Demand Distribution

Let D follow a general distribution. We derive the optimal policy computationally through value function iteration. All properties of the optimal policy remain valid. For instance, *strategic idleness* occurs under lower demand and *deliberate late service rejection* occurs under higher demand.

Figure 11 reports p_r as a function of Γ, for different values of D—obtained with a discrete uniform distribution between L and H. Note that the optimal price is not monotonic with demand. In Region 2 (Figure 11a), the platform focuses primarily on discrimination, leading to extreme discrimination and a price $p_r = 1$ under lower demand values. Under higher demand, the platform focuses more on demand-capacity management by providing more timely services and thus charging a lower price. For low values of Γ, p_r is actually weakly decreasing with demand. In Region 3 (Figure 11b), the platform focuses primarily on demand-capacity management. For higher demand values, it only provides timely services. Under lower demand, the platform instead prioritizes clearing the queue by transferring incoming n-type agents to serve them (at least partially) in the next period. This yields non-monotonic prices for high values of Γ.



(a) $\sigma = 0.45$, $v_0 = 0.6$, $v_1 = 0.4$ (akin to Region 2b) (b) $\sigma = 0.45$, $v_0 = 0.72$, $v_1 = 0.4$ (akin to Region 3b)

Figure 11 Optimal price p_r for multiple values of incoming demand D, as a function of late demand Γ.

7.2. Multi-period Patience Horizon

Let us assume that n-type agents can wait up to $M \geq 1$ periods for a late service. Let v_m be the utility derived from a service delayed by $m = 0, \cdots, M$ periods, with $v_0 > v_1 > \cdots > v_M$. Let "Group m" refer to n-type agents already delayed by m periods, for each $m \in \{1, 2, \cdots, M\}$. The platform optimizes prices and allocations for incoming agents and n-type agents in Groups 1 to M.

Let $(D, \overrightarrow{\Gamma})$ denote the state variable, where $\overrightarrow{\Gamma} = (\Gamma_1, \cdots, \Gamma_M)$ tracks the mass of agents in Group $m = 1, \cdots, M$. We still denote by $p_r(D, \overrightarrow{\Gamma})$ (resp. $p_t(D, \overrightarrow{\Gamma})$) and $q_r(D, \overrightarrow{\Gamma})$ (resp. $q_t(D, \overrightarrow{\Gamma})$) the price and allocation for incoming r-type (resp. n-type) agents. Let $p_m(D, \overrightarrow{\Gamma})$ and $q_m(D, \overrightarrow{\Gamma})$ be the price and allocation for agents in Group m. The generalized problem, $(\mathcal{P}_G)$, is given by:

$$V_G(D, \overrightarrow{\Gamma}) = \max_{\substack{p_r, p_t, p_1, \cdots, p_M \\ q_r, q_t, q_1, \cdots, q_M}} \quad p_r q_r \sigma D + p_t q_t (1-\sigma) D + \sum_{m=1}^{M} p_m q_m \Gamma_m + \delta \mathbb{E}_D \left[V_G(D, \overrightarrow{\Gamma'}) \right] \tag{24}$$

$$\text{s.t.} \quad \mathcal{IC}_r \text{ and } \mathcal{IC}_n, \tag{25}$$

$$p_r \leq 1, \; p_m \leq v_m, \; \forall m \in \{0, 1, \cdots, M\}, \tag{26}$$

$$1 \geq q_r \sigma D + q_t (1-\sigma) D + \sum_{m=1}^{M} q_m \Gamma_m, \tag{27}$$

$$\Gamma_1' = (1 - q_t)(1-\sigma) D, \tag{28}$$

$$\Gamma_m' = (1 - q_{m-1}) \Gamma_{m-1}, \; \forall m \in \{2, \cdots, M\}. \tag{29}$$

Equation (24) maximizes the platform's expected profit, subject to incentive compatibility constraints (Equation (25)), individual rationality constraints (Equation (26)), and capacity constraints (Equation (27)). The incentive compatibility constraint of r-type agents is identical to

Equation (4). The one of n-type agents involves additional terms reflecting the longer patience horizon; however, as in Problem $(\mathcal{P})$, it is not binding. The transition constraints define Γ_1' as the number of incoming n-type agents that are not served (Equation (28)) and, for each $m \in \{2, \cdots, M\}$, Γ_m' as the number of n-type agents in Group $m-1$ that remain unserved (Equation (29)).

In Appendix D, we provide an analogous version of Proposition 2, and reformulate Problem $(\mathcal{P}_G)$ with $\overrightarrow{\Gamma'}$ as the decision variable. We solve Problem $(\mathcal{P}_G)$ computationally using value iteration.

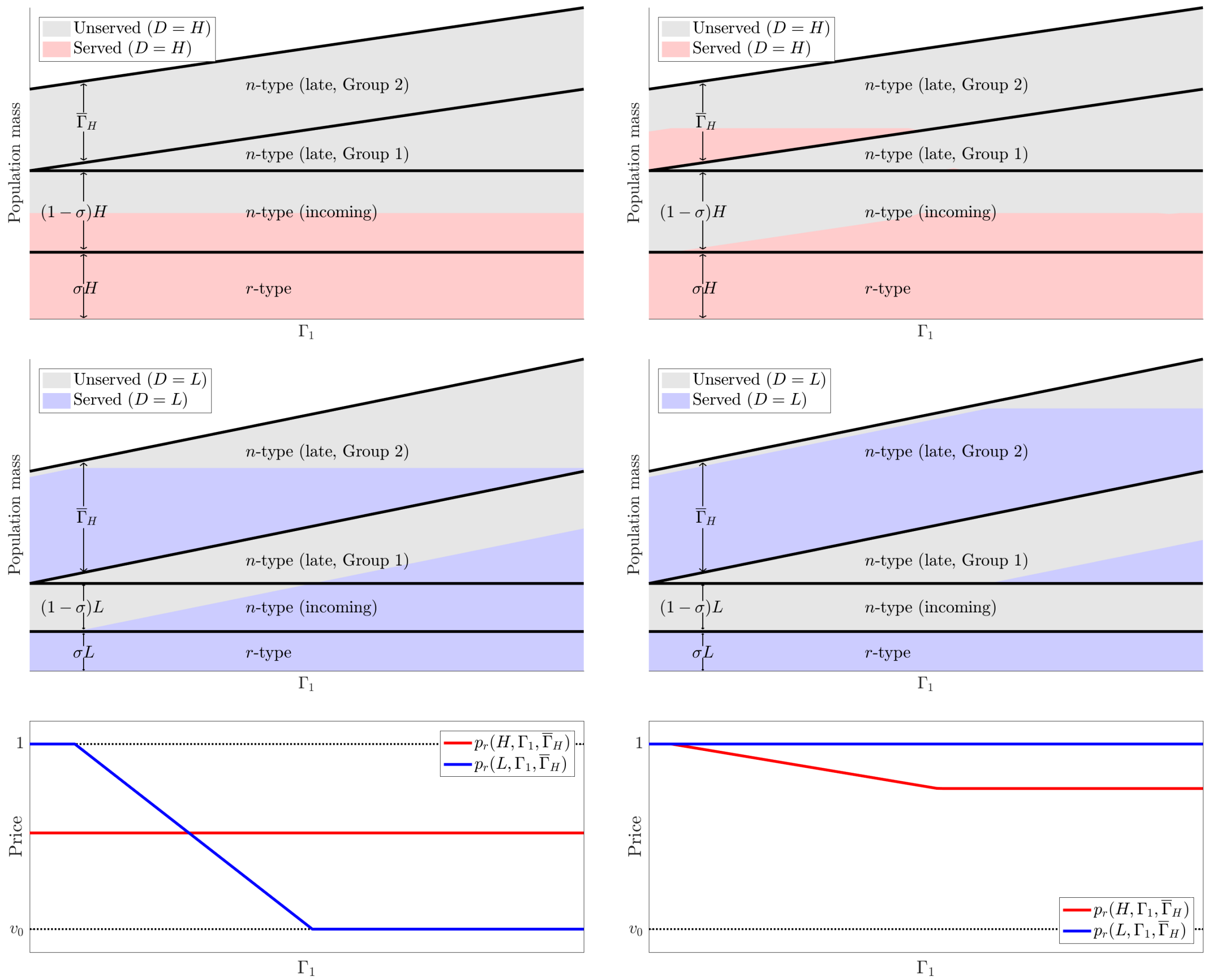
The platform still serves all r-type agents and extracts the entire surplus from n-type agents. It thus faces a similar trade-off between the quantity of incoming n-type agents served vs. the price charged to r-type agents—to balance discrimination and demand-capacity management. But the longer patience horizon introduces a new *queue clearing* dimension—that is, the allocation of remaining capacity (upon provision of timely services) across the M groups of delayed agents.

Two natural queue clearing strategies are first-come first-served (FCFS) allocations—from Group M to Group 1—and last-come first-served (LCFS) allocations—from Group 1 to Group M. The optimal policy features these strategies in some cases; for instance, FCFS (resp. LCFC) naturally arises when $\frac{v_{m+1}}{v_m}$ is large (resp. small). However, the optimal policy does not necessarily fall into these two categories: in some instances, the platform's queue clearing policy does not strictly prioritize one group over another. This is illustrated in Figure 12, when $M = 2$ and Γ_2 is large (i.e., $\Gamma_2 = \overline{\Gamma}_H$). For instance, in Figure 12a, under low demand, the queue clearing policy follows FCFS for lower values of Γ_1, but the platform shifts from serving Group 2 to serving Group 1 (thus deviating from FCFS) for higher values of Γ_1. The figure also shows the interdependencies between capacity allocations across groups: in some instances (e.g., in Figure 12b under high demand), increasing the population of Group 1 alters the allocation of capacity between Group 2 and incoming demand.

Turning to the price p_r, the observations are twofold. First, p_r may be lower under high demand than under low demand, generalizing the non-monotonicity of the price with incoming demand. Second, we extend this insight by noting that p_r is not monotonic with the *non-expiring portion of the queue* in a dynamic mechanism (i.e., with Γ_1 here, or with $\Gamma_1, \cdots, \Gamma_{M-1}$ with an M-period patience horizon).[11] This new insight can be explained as follows. When Γ_1 is sufficiently small, the platform delays timely requests from n-type agents in order to (i) charge a high price to r-type agents, (ii) serve agents in Group 2, and (iii) create a demand backlog for subsequent periods. But when Γ_1 gets larger, this would result in longer aggregate queues (across Groups 1 and 2), which the platform may not be able to satisfy. In this case, the platform foregoes more late requests from Group 2 and instead provides more timely services to n-type agents—resulting in a lower price p_r.

Key Finding 7. With a multi-period patience horizon, the optimal price is not monotonically increasing with incoming demand and with the non-expiring portion of the queue.

[11] The price p_r is non-decreasing with Γ_M (the expiring portion), as p_r was non-decreasing with Γ in our baseline.



(a) $v_0 = 0.9$, $\beta = 0.75$

(b) $v_0 = 0.95$, $\beta = 0.9$

Figure 12 Optimal capacity allocation and price p_r, as a function of Γ_1 (with $v_1 = \beta v_0$ and $v_2 = \beta v_1$).

7.3. Other Extensions: Endogenous Capacity and Stochastic Demand Composition

Finally, we relax the constant customer mix (by modeling σ as a stochastic parameter) and constant capacity (by introducing strategic suppliers). To retain tractability, we solve the model in a two-period horizon. Although less rich than our infinite-horizon model, the two-period problem captures (i) the tradeoff between the price charged to r-type customers and the timely services provided to n-type customers; (ii) the tradeoff between timely and late services provided to n-type customers; and (iii) intertemporal dynamics. The models and results are reported in Appendix C.

We find that the main results (that is, the structure of the policy and the non-monotonicity of the price) are robust to variations in the mix of customers and to the endogeneity of supply-side participation. At the same time, all additional results in these two extensions are expected: the price charged to r-type customers is monotonically increasing with the proportion of r-type customers; and the wage paid to suppliers is monotonically increasing with incoming demand. This suggests that the model presented in the paper captures the essential elements of our problem.

8. Conclusion

This paper studies a problem of dynamic pricing and real-time resource allocation for a platform facing non-transferable capacity, stochastic demand-capacity imbalances, and strategic customers with heterogeneous price- and time-sensitivities. In this environment, the platform uses service timing and pricing to balance demand-capacity management, and discrimination across heterogeneous customers. Discrimination becomes relatively more prominent and demand-capacity management becomes relatively less prominent as customer heterogeneity increases and as price-sensitive customers become more time-sensitive. We characterize the most general mechanism, which translates into a dynamic screening menu of prices and probabilistic allocations—catered to each customer segment and dynamically updated based on real-time information. This mechanism gives rise to endogenous demand dynamics—alternating between queue-building and queue-clearing periods.

The proposed dynamic menu can be implemented by leveraging online or mobile technologies, which enable platforms to fine tune prices and service levels over time and across customer segments. In the context of our motivating examples, such a menu could be implemented as follows:

– For an on-demand platform, the menu would propose different prices and wait times (e.g., \$10 service in 5 minutes, \$5 service in 30 minutes). Many platforms use dynamic pricing (e.g., surge pricing). Some also provide differentiated services (e.g., Uber X, Uber Pool, Uber Express Pool), leveraging *dynamic pricing* and *dynamic waiting* (Korolko et al. 2018). Our mechanism achieves similar objectives without resorting to separate service lines.

– In manufacturing and logistics, the menu would propose different prices and lead times (e.g., same-day delivery for \$30, 2-day delivery for \$10, 1-week delivery for free). Such differentiation is common in e-commerce. Our mechanism would dynamically update prices and lead times based on incoming demand, demand backlogs, delivery capacity, and customers' preferences.

– In cloud computing, the menu would propose different prices and computing resources—leading to different completion times. Currently, differentiation stems from guaranteed pay-as-you-go services vs. "best effort" services on a spot market. In practice, providers use dynamic pricing but perhaps insufficient service differentiation (Kilcioglu and Maglaras 2015).

One of our main findings is that the price charged to time-sensitive agents does not increase with demand—driven by information asymmetries, capacity restrictions and non-transferable capacity. To charge a high price, the platform needs to create service differentiation by delaying (or rejecting) requests from a high proportion of price-sensitive agents. Under higher demand, this strategy may result in a longer queue, which the platform might not be able to fulfill. As a result, under high demand, the platform may provide more timely services to price-sensitive customers, resulting in a lower price charged to time-sensitive customers to maintain incentive compatibility.

Our results show that this dynamic mechanism provides significant profit improvements, as compared to dynamic posted prices and static menus. We also propose a demand-dependent mechanism (less information-intensive and easier to implement) that is close to optimal; yet, the optimal mechanism achieves higher profits, by up to 3–4%. Ultimately, this paper combines principles of dynamic pricing and screening mechanisms by designing a dynamic menu of prices and service levels, each catered to a customer segment and dynamically adjusted based on real-time information.

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Appendix A: Optimal Policy in Regions 2 and 3

Region 2 We already know from Lemma 3 that the optimal policy satisfies $\Gamma^*(D,\Gamma) \geq \chi(D,\Gamma)$ for all $D\{H,L\}, \Gamma \in \mathbf{\Gamma}$. In other words, the optimal policy involves *late service prioritization.* But within that regime, the platform can adopt a *strategic idleness* policy (in which case $\Gamma^*(D,\Gamma) > \chi(D,\Gamma)$) or not (in which case $\Gamma^*(D,\Gamma) = \chi(D,\Gamma)$). Lemma 4 asserts that, if the platform policy does not feature *strategic idleness* for a certain Γ_0, then it does not feature *strategic idleness* too for any $\Gamma \geq \Gamma_0$ (for the same incoming demand D).

LEMMA 4. *In Region 2, if the optimal policy function satisfies $\zeta(D,\Gamma_0) = \Gamma_0$ in some state $(D,\Gamma_0) \in \{H,L\} \times \mathbf{\Gamma}$, then for each $\Gamma \geq \Gamma_0$, we have $\zeta(D,\Gamma) = \min\{\Gamma, 1-D+\bar{\Gamma}_D\}$.*

Lemmas 2, 3 and 4 imply that the entire optimal policy function $\Gamma^*(D,\Gamma)$ in Region 2 can be determined from $\Gamma^*(D,0)$. Indeed, $\Gamma^*(D,\Gamma)$ stays constant at $\Gamma^*(D,0)$ as long as $\Gamma^*(D,\Gamma) < \chi(D,\Gamma)$, that is as long as $\Gamma \in [0, 1-D+\Gamma^*(D,0))$.[12] In this range, the optimal policy involves *strategic idleness.* Then, $\Gamma^*(D,\Gamma) = \chi(D,\Gamma)$ for each $\Gamma \geq 1-D+\Gamma^*(D,0)]$—in which case the optimal policy no longer involves *strategic idleness.*

Proposition 9 elicits the optimal mechanism in Region 2, which is further divided into Sub-regions 2a, 2b and 2c—sorted by increasing order of v_0. Note that the optimal policy depends on whether the n-type or the r-type agents comprise the majority of the incoming demand (i.e., $\sigma < 0.5$ vs $\sigma \geq 0.5$).[13] Figure 4 shows the optimal policy when $\sigma < 0.5$ but the other case where $\sigma \geq 0.5$ is not qualitatively different.

PROPOSITION 9. *We denote by $\underline{v}_1 = \sigma + (1-\sigma)\delta(1-k)v_1$ and $\bar{v}_1 = \sigma + (1-\sigma)\frac{\delta+\delta^2(1-k)k}{1+\delta k}v_1$. The optimal policy in Region 2 is characterized as follows:*

Sub-region 2a: When $v_0 \in (\sigma, \underline{v}_1]$,

$$\Gamma^*(H,\Gamma) = \begin{cases} \min\{1-\sigma L, \bar{\Gamma}_H\} & \text{if } \Gamma \leq (1-\sigma)L, \\ \min\{\Gamma+H-1, \bar{\Gamma}_H\} & \text{if } \Gamma \geq (1-\sigma)L. \end{cases} \qquad \Gamma^*(L,\Gamma) = \bar{\Gamma}_L, \ \forall \Gamma \in \mathbf{\Gamma}.$$

Sub-region 2b: When $v_0 \in (\underline{v}_1, \bar{v}_1]$,

$$\Gamma^*(H,\Gamma) = \begin{cases} \Gamma+H-1 & \text{if } \Gamma \leq 1-\sigma H, \\ \bar{\Gamma}_H & \text{if } \Gamma \geq 1-\sigma H. \end{cases} \qquad \Gamma^*(L,\Gamma) = \min\{1-\sigma H, \bar{\Gamma}_L\} \ \forall \Gamma \in \mathbf{\Gamma}.$$

Sub-region 2c: When $v_0 \in (\bar{v}_1, \sigma + (1-\sigma)v_1]$,

$$\Gamma^*(H,\Gamma) = \begin{cases} \Gamma+H-1 & \text{if } \Gamma \leq 1-\sigma H, \\ \bar{\Gamma}_H & \text{if } \Gamma \geq 1-\sigma H. \end{cases} \qquad \Gamma^*(L,\Gamma) = \begin{cases} 0 & \text{if } \Gamma \leq 1-L, \\ \min\{\Gamma+L-1, \bar{\Gamma}_L\} & \text{if } \Gamma \geq 1-L. \end{cases}$$

Region 3 Lemma 5 shows that the optimal policy remains invariant as Γ increases as long as some late demand remains unserved, regardless of whether it comes from *deliberate late service rejection* ($\Gamma^*(D,\Gamma) < \chi(D,\Gamma)$) or from a large demand backlog ($\Gamma^*(D,\Gamma) = \chi(D,\Gamma) = \bar{\Gamma}_D$).

LEMMA 5. *In Region 3, if the optimal policy is such that some late demand is unserved in $(D,\Gamma_0) \in \{H,L\} \times \mathbf{\Gamma}$, that is $\zeta(D,\Gamma_0) < \Gamma_0$, then we have $\Gamma^*(D,\Gamma) = \Gamma^*(D,\Gamma_0)$ for all $\Gamma \geq \Gamma_0$.*

[12] This initial range is just a single point when $D = H$ and $\Gamma^*(H,0) = \underline{\Gamma}_H$.

[13] The two thresholds $1-\sigma H$ and $(1-\sigma)L$ satisfy $1-\sigma H \leq (1-\sigma)L \iff \sigma \geq 0.5$ (Assumption 2).

Next, Lemma 6 indicates that, if there is no late demand, the platform provides as many timely services as possible. In this case, transferring n-type agents to the next period would not generate more late services in the current period, and the inter-temporal effect is too weak to offset the associated current-period loss.

LEMMA 6. *In Region 3, the optimal policy satisfies* $\Gamma^*(D,0) = \underline{\Gamma}_D$, *for each* $D \in \{H, L\}$.

Proposition 10 elicits the optimal mechanism, which is further divided into Sub-regions 3a, 3b and 3c—sorted by increasing order of v_0. As in Region 2, the exact form of the optimal policy depends on whether the n-type or the r-type agents comprise the majority of the incoming demand (i.e., $\sigma < 0.5$ vs $\sigma \geq 0.5$).[14]

PROPOSITION 10. *We denote by* $\underline{\underline{v}}_1 = \sigma + (1-\sigma)\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1+\delta(1-k)} v_1$ *and* $\bar{\bar{v}}_1 = \sigma + (1-\sigma)(1+\delta(1-k))v_1$. *The optimal policy in Region 3 is characterized as follows:*

Sub-region 3a: When $v_0 \in (\sigma + (1-\sigma)v_1, \underline{\underline{v}}_1]$,

$$\Gamma^*(H,\Gamma) = \begin{cases} \min\{\Gamma + H - 1, \bar{\Gamma}_H\} & \text{if } \Gamma \leq (1-\sigma)L, \\ \min\{1-\sigma L, \bar{\Gamma}_H\} & \text{if } \Gamma \geq (1-\sigma)L. \end{cases} \qquad \Gamma^*(L,\Gamma) = \begin{cases} 0 & \text{if } \Gamma \leq 1-L, \\ \min\{\Gamma + L - 1, \bar{\Gamma}_L\} & \text{if } \Gamma \geq 1-L. \end{cases}$$

Sub-region 3b: When $v_0 \in (\underline{\underline{v}}_1, \bar{\bar{v}}_1]$,

$$\Gamma^*(H,\Gamma) = \underline{\Gamma}_H, \ \forall \Gamma. \qquad \Gamma^*(L,\Gamma) = \begin{cases} 0 & \text{if } \Gamma \leq 1-L, \\ \min\{\Gamma + L - 1, \bar{\Gamma}_L\} & \text{if } \Gamma \geq 1-L. \end{cases}$$

Sub-region 3c: When $v_0 \in (\bar{\bar{v}}_1, 1)$,

$$\Gamma^*(H,\Gamma) = \underline{\Gamma}_H, \ \forall \Gamma. \qquad \Gamma^*(L,\Gamma) = \underline{\Gamma}_L, \ \forall \Gamma.$$

Appendix B: Static and Demand-dependent Mechanism

B.1. Formulation of Static Mechanism

The full formulation of Problem $(\mathcal{P}_S)$ is given as follows:

$$\begin{aligned}
\overline{V}_S(0) = \max_{q_r,q_t,q_l,p_r,p_t,p_l} \quad & \left\{ \begin{array}{l} k\left(q_r\sigma H p_r + q_t(1-\sigma)Hp_t + \delta\mathbb{E}_D\left[V_S(D,\Gamma_S^H)\right]\right) \\ +(1-k)\left(q_r\sigma L p_r + q_t(1-\sigma)Lp_t + \delta\mathbb{E}_D\left[V_S(D,\Gamma_S^L)\right]\right) \end{array} \right\} \\
\text{s.t.} \quad & \mathcal{IC}_r : q_r(1-p_r) \geq q_t(1-p_t), \\
& \mathcal{IC}_n : q_t(v_0 - p_t) + q_l(v_1 - p_l) \geq q_t(v_0 - p_t), \\
& \mathcal{IR} : p_r \leq 1, p_t \leq v_0, p_l \leq v_1, \\
& \mathcal{CC} : q_r\sigma D + q_t(1-\sigma)D + q_l\Gamma \leq 1, \ \forall (D,\Gamma) \in \{H,L\} \times \{\Gamma_S^H, \Gamma_S^L\}, \\
& \mathcal{TC} : \Gamma_S^D = (1-q_t)(1-\sigma)D, \ \forall D \in \{H,L\}, \\
& \mathcal{SS} : V_S(D,\Gamma) = q_r\sigma D p_r + q_t(1-\sigma)Hp_t + q_l\Gamma p_l + \delta\mathbb{E}_{D'}\left(V_S(D',\Gamma_S^D)\right), \ \forall (D,\Gamma) \in \{H,L\} \times \{\Gamma_S^H, \Gamma_S^L\}.
\end{aligned}$$

B.2. Formulation of Demand-dependent Mechanism

The full formulation of Problem $(\mathcal{P}_{DD})$ is given as follows:

$$\overline{V}_{DD}(0) = \max_{\substack{q_r(D),q_t(D),q_l(D,\Gamma) \\ p_r(D),p_t(D),p_l(D,\Gamma)}} \left\{ \begin{array}{l} k\left(q_r(H)\sigma H p_r(H) + q_t(H)(1-\sigma)Hp_t(H) + \delta\mathbb{E}_D\left[V_{DD}(D,\Gamma_{DD}^H)\right]\right) \\ +(1-k)\left(q_r(L)\sigma L p_r(L) + q_t(L)(1-\sigma)Lp_t(L) + \delta\mathbb{E}_D\left[V_{DD}(D,\Gamma_{DD}^L)\right]\right) \end{array} \right\}$$

[14] The two thresholds $1-\sigma L$ and $(1-\sigma)H$ satisfy $1-\sigma L \leq (1-\sigma)H \iff \sigma \leq 0.5$ (Assumption 2).

$$\begin{aligned}
\text{s.t.}\quad & \mathcal{IC}_r : q_r(D)(1-p_r(D)) \geq q_t(D)(1-p_t(D)),\ \forall D \in \{H,L\} \\
& \mathcal{IC}_n : q_t(D)(v_0 - p_t(D)) + q_l(D,\Gamma)(v_1 - p_l(D,\Gamma)) \geq q_t(D)(v_0 - p_t(D)),\ \forall (D,\Gamma) \in \left\{\{H,L\} \times \{\Gamma_{DD}^H, \Gamma_{DD}^L\}\right\} \\
& \mathcal{IR} : p_r(D) \leq 1, p_t(D) \leq v_0, p_l(D,\Gamma) \leq v_1,\ \forall (D,\Gamma) \in \left\{\{H,L\} \times \{\Gamma_{DD}^H, \Gamma_{DD}^L\}\right\}, \\
& \mathcal{CC} : q_r(D)\sigma D + q_t(D)(1-\sigma)D + q_l(D,\Gamma)\Gamma \leq 1,\ \forall (D,\Gamma) \in \left\{\{H,L\} \times \{\Gamma_{DD}^H, \Gamma_{DD}^L\}\right\}. \\
& \mathcal{TC} : \Gamma_{DD}^D = (1-q_t(D))(1-\sigma)D,\ \forall D \in \{H,L\}, \\
& \mathcal{SS} : V_{DD}(D,\Gamma) = q_r(D)\sigma D p_r(D) + q_t(D)(1-\sigma)Hp_t(D) + q_l(D,\Gamma)\Gamma p_l(D,\Gamma) + \delta \mathbb{E}_{D'}\left(V_{DD}(D', \Gamma_{DD}^D)\right), \\
& \qquad\qquad\qquad\qquad\qquad\qquad \forall (D,\Gamma) \in \left\{\{H,L\} \times \{\Gamma_{DD}^H, \Gamma_{DD}^L\}\right\}.
\end{aligned}$$

B.3. Optimal Policy under Demand-dependent Mechanism

Proposition 11 elicits the optimal policy under the demand-dependent mechanism.

PROPOSITION 11. *The solution to problem $\mathcal{P}_{DD}$ satisfies $q_r(D) = 1$, $p_t(D) = v_0$, and $p_r(D) = 1 - q_t(D)(1 - v_0)$ for each $D \in \{H,L\}$. Moreover:*

- *When $\sigma < 0.5$.*
  - *— Region A: If $v_0 \leq \sigma + \delta(1-\sigma)v_1 k$, then $q_t(H) = 0, q_t(L) = 0$.*
  - *— Region B: If $\sigma + \delta(1-\sigma)v_1 k \leq v_0 \leq \sigma + \delta(1-\sigma)v_1$, then $q_t(H) = \frac{d(1-2\sigma)}{(1-\sigma)H}, q_t(L) = 0$.*
  - *— Region C: If $v_0 \geq \sigma + \delta(1-\sigma)v_1$, then $q_t(H) = \frac{1-\sigma H}{(1-\sigma)H}, q_t(L) = 1$.*
- *When $\sigma \geq 0.5$.*
  - *— Region A': If $v_0 \leq \sigma + \delta(1-\sigma)v_1(1-k)$, then $q_t(H) = 0, \quad q_t(L) = 0$.*
  - *— Region B': If $\sigma + \delta(1-\sigma)v_1(1-k) \leq v_0 \leq \sigma + \delta(1-\sigma)v_1$, then $q_t(H) = 0, q_t(L) = \frac{d(2\sigma-1)}{(1-\sigma)L}$.*
  - *— Region C: If $v_0 \geq \sigma + \delta(1-\sigma)v_1$, then $q_t(H) = \frac{1-\sigma H}{(1-\sigma)H}, q_t(L) = 1$.*

Appendix C: Two-period Model and Extensions

We first replicate our model in two-period setting (Section C.1). We then add stochasticity on the composition of demand across r-type and n-type customers (Section C.2) and endogenous supply-side participation (Section C.3). All proofs are reported in Appendix EC.4.

C.1. Baseline Two-period Model

We formulate the model presented in the paper in a finite horizon consisting of two periods, indexed by $\tau \in \{1,2\}$. We assume that there is no late demand in Period 1 (i.e., $\Gamma_1 = 0$) and that n-type agents cannot be served after Period 2. The platform's problem is thus given as follows:

$$\begin{aligned}
V_2(D_2,\Gamma_2) = \max_{\substack{q_{r,2},q_{t,2},q_{l,2} \\ p_{r,2},p_{t,2},p_{l,2}}} \quad & p_{r,2}q_{r,2}\sigma D_2 + p_{t,2}q_{t,2}(1-\sigma)D_2 + p_{l,2}q_{l,2}\Gamma_2, \\
\text{s.t.} \quad & \mathcal{IC}_{r,2} \text{ and } \mathcal{IC}_{n,2}, \\
& p_{r,2} \leq 1,\ p_{t,2} \leq v_0,\ p_{l,2} \leq v_1, \\
& 1 \geq q_{r,2}\sigma D_2 + q_{t,2}(1-\sigma)D_2 + q_{l,2}\Gamma_2. \\
V_1(D_1, 0) = \max_{\substack{q_{r,1},q_{t,1} \\ p_{r,1},p_{t,1}}} \quad & p_{r,1}q_{r,1}\sigma D_1 + p_{t,1}q_{t,1}(1-\sigma)D_1 + \delta\left(kV_2(H,\Gamma') + (1-k)V_2(L,\Gamma')\right), \\
\text{s.t.} \quad & \mathcal{IC}_{r,1} \text{ and } \mathcal{IC}_{n,1},
\end{aligned}$$

$$p_{r,1} \le 1,\ p_{t,1} \le v_0,$$
$$1 \ge q_{r,1}\sigma D_1 + q_{t,1}(1-\sigma)D_1,$$
$$\Gamma' = (1-q_{t,1})(1-\sigma)D_1.$$

The platform still extracts all the surplus from n-type customers, serves all r-type customers, serves late demand as long as capacity is sufficient, and faces a trade-off between the price charged to r-type customers and the timely services provided to n-type customers. We reformulate the problem by optimizing the number of n-type customers transferred to the next period denoted by $\Gamma'_\tau(D_\tau, \Gamma_\tau)$. The Bellman equations become:

$$V_1(D_1,0) = \max_{\Gamma'_1 \in [\underline{\Gamma}_{D_1}, \overline{\Gamma}_{D_1}]} \sigma D_1 \left(1 - \tfrac{(1-\sigma)D_1 - \Gamma'_1}{(1-\sigma)D_1}(1-v_0)\right) + \left((1-\sigma)D_1 - \Gamma'_1\right) v_0 + \delta \mathbb{E}_{D_2}\left(V_2(D_2, \Gamma'_1)\right),$$
$$V_2(D_2,\Gamma_2) = \max_{\Gamma'_2 \in [\underline{\Gamma}_{D_2}, \overline{\Gamma}_{D_2}]} \sigma D_2 \left(1 - \tfrac{(1-\sigma)D_2 - \Gamma'_2}{(1-\sigma)D_2}(1-v_0)\right) + \left((1-\sigma)D_2 - \Gamma'_2\right) v_0 + \min\left\{\Gamma_2, 1 - D_2 + \Gamma'_2\right\} v_1,$$
$$\text{where } \Gamma'_\tau \in [0, (1-\sigma)L] \text{ if } D_\tau = L \text{ and } \Gamma'_\tau \in [H-1, (1-\sigma)H] \text{ if } D_\tau = H.$$

Proposition 12. *The optimal solution is given in Table 2.*

Region	Definition	Period 1 policy	Period 2 policy
Region 1	$v_0 \in [v_1, \sigma)$	$\Gamma'_1(L) = \overline{\Gamma}_L$, $\Gamma'_1(H) = \overline{\Gamma}_H$	No timely service to n-type customers
Region 2-i	$v_0 \in [\sigma, \sigma + \frac{(1-\sigma)\delta(1-k)}{1+\delta(1-k)} v_1)$	$\Gamma'_1(L) = \overline{\Gamma}_L$, $\Gamma'_1(H) = 1 - \sigma L$	Prioritize late services
Region 2-ii	$v_0 \in [\sigma + \frac{(1-\sigma)\delta(1-k)v_1}{1+\delta(1-k)}, \sigma + \frac{(1-\sigma)\delta v_1}{1+\delta k})$	$\Gamma'_1(L) = \overline{\Gamma}_L$, $\Gamma'_1(H) = \underline{\Gamma}_H$	Prioritize late services
Region 2-iii	$v_0 \in [\sigma + \frac{(1-\sigma)\delta}{1+\delta k} v_1,\ \sigma + (1-\sigma)v_1)$	$\Gamma'_1(L) = \underline{\Gamma}_L$, $\Gamma'_1(H) = \underline{\Gamma}_H$	Prioritize late services
Region 3	$v_0 \in [\sigma + (1-\sigma)v_1, 1)$	$\Gamma'_1(L) = \underline{\Gamma}_L$, $\Gamma'_1(H) = \underline{\Gamma}_H$	Maximize timely services

Table 2 Optimal policy in the baseline setting

The optimal policy is similar—albeit less rich—than that in the infinite-horizon setting. Under strong inter-type heterogeneity (Region 1), the platform adopts extreme discrimination by providing timely services only to r-type customers. Under weaker inter-type heterogeneity (Regions 2 and 3), the platform prioritizes late services when time preferences are weak and timely services otherwise.

Moreover, the price charged to r-type customers is not monotonic with demand in Period 1. For instance, in Region 2-i, the platform provides timely services to n-type customers under high demand (so that $p_{r,1}(H) < 1$) but not under low demand (so that $p_{r,1}(L) = 1$). In Period 2, the price $p_{r,2}(D_2, \Gamma_2)$ is non-decreasing with D_2 due to the absence of inter-temporal dynamics—a crucial feature underlying our non-monotonicity result.

Corollary 1. *The price $p_{r,1}(D_1, \Gamma_1)$ is not an increasing function of D_1.*

C.2. Stochastic Demand Composition

In practice, many platforms face variations in the customer mix over time. For instance, demand composition may vary between peak periods (with perhaps a higher fraction of time-sensitive customers) and off-peak periods (with perhaps a higher fraction of price-sensitive customers). To address this situation, let σ_τ denote the fraction of incoming r-type customers in period $\tau = 1, 2$. We assume that σ_τ follows a binary distribution over $\{\underline{\sigma}, \overline{\sigma}\}$ with $\underline{\sigma} < \overline{\sigma}$ and $\mathbb{P}(\sigma = \overline{\sigma}) = \mu$.

We extend Assumptions 1 and 2 by assuming that $\overline{\sigma}H < 1$ (the platform can always serve all r-type customers) and that $H - 1 \geq 1 - \underline{\sigma}H$ (the platform cannot serve all the late demand in Period 2 if demand is high in both periods and if it provides no timely services in Period 1).

The state in Period τ becomes $(D_\tau, \Gamma_\tau, \sigma_\tau)$. The full formulation is given in Appendix EC.4.2. We reformulate the problem by optimizing the number of n-type customers transferred to the next period $\Gamma'_\tau(D_\tau, \Gamma_\tau, \sigma_\tau)$:

$$V_1(D_1, 0, \sigma_1) = \max_{\Gamma'_1} \ \sigma_1 D_1 \left(1 - \tfrac{(1-\sigma_1)D_1 - \Gamma'_1}{(1-\sigma_1)D_1}(1 - v_0)\right) + ((1-\sigma_1)D_1 - \Gamma'_1)\, v_0 + \delta \mathbb{E}\left(V_2(D_2, \Gamma'_1, \sigma_2)\right).$$

$$V_2(D_2, \Gamma_2, \sigma_2) = \max_{\Gamma'_2} \ \sigma_2 D_2 \left(1 - \tfrac{(1-\sigma_2)D_2 - \Gamma'_2}{(1-\sigma_2)D_2}(1 - v_0)\right) + ((1-\sigma_2)D_2 - \Gamma'_2)\, v_0 + \min\left\{\Gamma_2, 1 - D_2 + \Gamma'_2\right\} v_1,$$

$$\text{where } (1-\sigma_\tau)D_\tau - \min\left\{1 - \sigma D_\tau, (1-\sigma_\tau)D_\tau\right\} \leq \Gamma'_\tau(D_\tau, \Gamma_\tau, \sigma_\tau) \leq (1-\sigma_\tau)D_\tau.$$

Proposition 13 shows that the optimal policy follows the same structure as in the baseline model. The only distinction is that, as the r-type customers comprise a larger fraction of the demand, Region 1 becomes larger and Region 3 becomes smaller—so the platform becomes less likely to serve incoming n-type customers.

PROPOSITION 13. *The optimal solution in Period 2 is given in Table 3.*

Region	Definition	Period 2 policy
Region 1–σ_2	$v_0 \in [v_1, \sigma_2)$	No timely service to n-type customers
Region 2–σ_2	$v_0 \in [\sigma_2, \sigma_2 + (1-\sigma_2)v_1)$	Prioritize late services
Region 3–σ_2	$v_0 \in [\sigma_2 + (1-\sigma_2)v_1, 1)$	Maximize timely services

Table 3 Optimal policy in Period 2 with stochastic demand composition

Proposition 14 shows that, again, the price charged to r-type customers is not monotonic with demand. This stems from the same dynamics as in the baseline setting. It suggests that this result is not an artifact of the assumption that σ remains constant but holds in more general settings.

PROPOSITION 14. *The price $p_{r,1}(D_1, \Gamma_1, \sigma_1)$ is not an increasing function of D_1.*

Finally, Proposition 15 shows that the price charged to r-type customers $p_{r,\tau}(D_\tau, \Gamma_\tau, \sigma_\tau)$ is monotonic with the fraction of r-type customers σ_τ. This is an expected result, showing that the baseline model (with a constant σ) captures the main elements of our setting.

PROPOSITION 15. *The price $p_{r,\tau}(D_\tau, \Gamma_\tau, \sigma_\tau)$ is an increasing function of σ_τ.*

C.3. Endogenous Capacity

In practice, many platforms face endogenous supply-side dynamics—when suppliers are self-scheduled based on on-platform revenue opportunities. To address this situation, we still consider a unit mass of suppliers but each supplier now has an outside option u, uniformly drawn between 0 and $\bar{u}$. The distribution of outside options is publicly known but each supplier's outside option is private information. In each period $\tau \in \{1, 2\}$, the platform optimizes its pricing and allocation policy as well as a per-service wage w_τ. The choice of w_τ impacts the platform's capacity, denoted by $S(w_\tau) = \min\left\{\frac{w_\tau}{\bar{u}}, 1\right\}$.

The full problem formulation is given in Appendix EC.4.3. We reformulate the problem with two decision variables—the number of transferred n-type customers $\Gamma'_\tau(D_\tau,\Gamma_\tau)$ and the wage $w_\tau(D_\tau,\Gamma_\tau)$:

$$V_1(D_1,0)=\max_{w_1,\Gamma'_1}\left\{\begin{array}{l}\min\{\sigma D_1,S(w_1)\}\left(1-\frac{(1-\sigma)D_1-\Gamma'_1}{(1-\sigma)D_1}(1-v_0)-w_1\right)+\left((1-\sigma)D_1-\Gamma'_1\right)(v_0-w_1)\\+\delta\left((1-k)V_2(L,\Gamma'_1)+kV_2(H,\Gamma'_1)\right),\end{array}\right\}$$

$$V_2(D_2,\Gamma_2)=\max_{w_2,\Gamma'_2}\left\{\begin{array}{l}\min\{\sigma D_2,S(w_2)\}\left(1-\frac{(1-\sigma)D_2-\Gamma'_2}{(1-\sigma)D_2}(1-v_0)-w_2\right)+\left((1-\sigma)D_2-\Gamma'_2\right)(v_0-w_2)\\+\min\left\{\Gamma_2,S(w_2)-D_2+\Gamma'_2\right\}(v_1-w_2),\end{array}\right\}$$

$$\text{where } (1-\sigma)D_\tau-\max\left\{S(w_\tau)-\sigma D_\tau,0\right\}\le\Gamma'_\tau(D_\tau,\Gamma_\tau)\le(1-\sigma)D_\tau,\text{ and } 0\le w_\tau\le\bar u.$$

We assume that $\bar u$ is small enough so that it is always optimal for platform to increase wages in order to serve r-type customers or to provide late services to n-type customers—as shown in Lemma 7. Then, the platform determines whether to serve incoming n-type customers based on the discriminatory incentives under demand-side information asymmetries (as in earlier versions of the model) and the quadratic cost of service provision under supply-side information asymmetries (a new feature due to endogenous capacity).

Assumption 3. $\bar u\le\frac{v_1}{2}$.

Lemma 7. *The platform increases the wage until (i) all r-type customers are served; and (ii) if some late demand remains unserved, it is not possible to further increase supply (i.e., $S(w)=1$).*

Proposition 16. *The optimal solution in Period 2 is given in Table 4.*

Region	Definition	Period 2 policy
Region 1–$\bar u$	$v_0\in[v_1,\sigma+2(1-\sigma)\bar u\sigma L)$	Serve r-type customers and late demand No timely service to n-type customers $w_2(D_2,\Gamma_2)=\min\{\bar u,(\sigma D_2+\Gamma_2)\bar u\}$
Region 2.i–$\bar u$	$v_0\in[\sigma+2(1-\sigma)\bar u\sigma L,\sigma+2(1-\sigma)\bar u)$	Prioritize late services Provide timely services as long as $w_2(D_2,\Gamma_2)\le\frac{v_0-\sigma}{2(1-\sigma)}$ $w_2(D_2,\Gamma_2)=\max\left\{\min\left\{\bar u,(\sigma D_2+\Gamma_2)\bar u\right\},\min\left\{\frac{v_0-\sigma}{2(1-\sigma)},(D_2+\Gamma_2)\bar u\right\}\right\}$
Region 2.ii–$\bar u$	$v_0\in[\sigma+2(1-\sigma)\bar u,\sigma+(1-\sigma)v_1)$	Prioritize late services Provide as many timely service as possible $w_2(D_2,\Gamma_2)=\min\{\bar u,(D_2+\Gamma_2)\bar u\}$
Region 3–$\bar u$	$v_0\in[\sigma+(1-\sigma)v_1,1)$	Maximize timely services Provide as many late services as possible $w_2(D_2,\Gamma_2)=\min\{\bar u,(D_2+\Gamma_2)\bar u\}$

Table 4 Optimal policy in Period 2 with endogenous supply-side

Proposition 16 shows that the optimal policy has the same structure as the baseline. The platform still serves no incoming n-type customer in Region 1, prioritizes late services in Region 2 and prioritizes timely services in Region 3. Yet, there are two novelties. First, the platform may serve incoming n-type customers without providing as many timely services as possible. Indeed, in Region 2.ii–$\bar u$, the platform increases the wage up to a certain point—thus leaving some incoming n-type customers unserved (unlike in Region 2.i–$\bar u$). This does not stem from discrimination (unlike in Region 1) but from the added cost of service provision due to supply-side information asymmetries. Second, extreme discrimination is more prevalent than in the baseline model. Indeed, when $v_0 \in (\sigma, \sigma + (1-\sigma)\bar{u}\sigma L)$, the platform would serve incoming n-type customers with exogenous capacity, but does not with endogenous capacity. This is also due to the added cost of service provision—so the capacity is just sufficient to serve r-type customers and late demand.

Next, Proposition 17 shows that the price charged to r-type customers is not monotonic with demand. Again, this result is not an artifact of our setting with exogenous capacity but holds more generally.

Proposition 17. *The price $p_{r,1}(D_1, \Gamma_1)$ is not an increasing function of D_1.*

Finally, Proposition 18 shows that the wage is monotonic with incoming and late demand. Again, this suggests that the baseline model (with exogenous capacity) captures the main elements of our setting.

Proposition 18. *The wage $w_\tau(D_\tau, \Gamma_\tau)$ is a non-decreasing function of D_τ and Γ_τ.*

Appendix D: Reformulation of Problem $(\mathcal{P}_G)$

Proposition 19 extends Proposition 2 to Problem $(\mathcal{P}_G)$. The proof is identical, thus omitted for conciseness.

Proposition 19. *The optimal solution to problem $(\mathcal{P}_G)$ satisfies, in each state $(D, \overrightarrow{\Gamma})$:*

(i) $p_m(D, \overrightarrow{\Gamma}) = v_m$, *for each* $m \in \{0, 1, \cdots, M\}$.

(ii) $q_r(D, \overrightarrow{\Gamma}) = 1$, *and* $p_r(D, \overrightarrow{\Gamma}) \geq v_0$.

(iii) If $\Gamma_M > 0$, *then* $q_M(D, \overrightarrow{\Gamma}) = \min\left\{1, \frac{1-\sigma D - q_t(D,\overrightarrow{\Gamma})(1-\sigma)D - \sum_{m=1}^{M-1} q_m(D,\overrightarrow{\Gamma})\Gamma_m}{\Gamma_M}\right\}$.

(iv) Constraint $\mathcal{IC}_r$ *is binding, and hence* $p_r(D, \overrightarrow{\Gamma}) = 1 - q_t(D, \overrightarrow{\Gamma})(1 - v_0)$.

We now reformulate Problem $(\mathcal{P}_G)$ with $\overrightarrow{\Gamma}' = (\Gamma'_1, \cdots, \Gamma'_M)$ as the unique choice variable:

$$\begin{aligned}
V_G(D, \overrightarrow{\Gamma}) = \max_{\Gamma'} \quad & \sigma D \left(1 - \tfrac{(1-\sigma)D - \Gamma'_1}{(1-\sigma)D}(1 - v_0)\right) + ((1-\sigma)D - \Gamma'_2)v_0 + \sum_{m=1}^{M-1} v_m(\Gamma_m - \Gamma'_{m+1}) \\
& + \min\left\{\Gamma_M, 1 - D - \sum_{m=1}^{M-1}\Gamma_m + \sum_{m=1}^{M}\Gamma'_m\right\} v_M + \delta \mathbb{E}_D\left[V_G(D, \overrightarrow{\Gamma}')\right], \\
\text{s.t.} \quad & \Gamma'_1 \in [\underline{\Gamma}_D, \bar{\Gamma}_D], \quad \text{with } \bar{\Gamma}_D = (1-\sigma)D \text{ and } \underline{\Gamma}_D = \max(D-1, 0), \\
& \Gamma'_{m+1} \in [0, \Gamma_m], \ \forall m \in \{1, \cdots, M-1\}, \\
& 1 \geq D + \sum_{m=1}^{M-1}\Gamma_m - \sum_{m=1}^{M}\Gamma'_m.
\end{aligned}$$

Proofs of Statements

Appendix EC.1: Proofs on the Pricing and Allocation Mechanism

In this appendix, we proceed to prove the statements related to the characterization of the optimal policy under the pricing and allocation mechanism defined in the paper. For notational ease, let $\bar{V}$ be the unconditional value function before the realization of the demand takes place, i.e., $\bar{V}(\Gamma) = kV(H,\Gamma) + (1-k)V(L,\Gamma)$ for each $\Gamma \in \mathbf{\Gamma}$. We also denote its partial derivative with respect to Γ by $\bar{V}'$, so $\bar{V}'(\Gamma) = kV'(H,\Gamma) + (1-k)V'(L,\Gamma), \forall \Gamma \in \mathbf{\Gamma}$.

EC.1.1. Proof of Proposition 1

The spate space of Problem $(\mathcal{P})$ is included in $X = \{H, L\} \times [0, (1-\sigma)H]$. Let $B(X)$ be the set of bounded functions defined over X. We know that $(B(X), |\cdot|)$ defines a complete metric space (where $|\cdot|$ denotes the supremum metric).

Moreover, let $T = B(X) \to B(X)$ be a mapping such that, for all $f \in B(X)$ and $(D,\Gamma) \in X$:

$$
\begin{aligned}
Tf(D,\Gamma) = \max_{\substack{q_r(D,\Gamma), q_t(D,\Gamma), q_l(D,\Gamma) \\ p_r(D,\Gamma), p_t(D,\Gamma), p_l(D,\Gamma)}} \quad & p_r(D,\Gamma)q_r(D,\Gamma)\sigma D + p_t(D,\Gamma)q_t(D,\Gamma)(1-\sigma)D + p_l(D,\Gamma)q_l(D,\Gamma) \\
& \qquad + \delta[kf(H,\Gamma') + (1-k)f(L,\Gamma')] \\
\text{s.t.} \quad & \mathcal{IC}_r \text{ and } \mathcal{IC}_n, \\
& p_r(D,\Gamma) \leq 1,\ p_t(D,\Gamma) \leq v_0,\ p_l(D,\Gamma) \leq v_1, \\
& 1 \geq q_r(D,\Gamma)\sigma D + q_t(D,\Gamma)(1-\sigma)D + q_l(D,\Gamma)\Gamma, \\
& \Gamma'(D,\Gamma) = (1 - q_t(D,\Gamma))(1-\sigma)D.
\end{aligned}
$$

We note that $T(\cdot)$ satisfies Blackwell sufficiency conditions (i.e., monotonicity and discounting) to define a contraction mapping (Blackwell 1965):

- *Monotonicity:* If for $f, g \in B(X)$, $f(D,\Gamma) \geq g(D,\Gamma)$ for all $(D,\Gamma) \in X$ then we have $T(f) \geq T(g)$. This immediately follows from the definition of T
- *Discounting:* There exists a $\beta \in (0,1)$ such that $T(f(D,\Gamma) + c) \leq T(f(D,\Gamma) + \beta c$. Again, this immediately follows from the definition of T, with $\beta = \delta$.

Therefore $T(\cdot)$ is a contraction mapping. It follows directly that the equation $T(\cdot)$ has a unique fixed point (from Banach's fixed point theorem), hence Problem $(\mathcal{P})$ admits a unique solution (Blackwell 1965, Bertsekas 2012).

EC.1.2. Proof of Proposition 2

We consider a given state (D,Γ) and consider the optimal policy of Problem $\mathcal{P}$. For notational ease, we denote the policy by $(p_r, p_t, p_l, q_r, q_t, q_l)$, and removing the dependency on (D,Γ). To prove each statement in sequence, we will reason by contradiction and construct an alternative solution $(\tilde{p}_r, \tilde{p}_t, \tilde{p}_l, \tilde{q}_r, \tilde{q}_t, \tilde{q}_l)$ that strictly increases the platform's objective function.

Proof that the incentive constraint $\mathcal{IC}_r$ is binding: Let us consider an optimal solution and assume by contradiction that the incentive constraint $\mathcal{IC}_r$ is not binding, i.e., $q_r(1-p_r) > q_t[1-p_t]$. Then we necessarily have $p_r < 1$, and we define an alternative solution by setting $\tilde{p}_r = p_r + \varepsilon$, with $\varepsilon > 0$ such that $\tilde{p}_r \leq 1$ and $q_r(1-\tilde{p}_r) \geq q_t[1-p_t]$. By construction, Constraints (4) and (6) are satisfied. Moreover, Constraints (7) and (8) remain unchanged. Finally, Constraint (5) is also satisfied as the right-hand side decreases with p_r. Therefore, the solution remains feasible. Moreover, we have $q_r > 0$, so the platform's value function strictly increases when p_r is increased to $\tilde{p}_r$. This contradicts the optimality of $(p_r, p_t, p_l, q_r, q_t, q_l)$.

Proof that $p_r \geq v_0$: Let us assume by contradiction that $p_r < v_0$.

If $q_r > q_t$, then we consider the deviation such that $\tilde{p}_r = v_0$, $\tilde{p}_t = v_0$, and $\tilde{p}_l = v_1$, and $\tilde{q}_r = q_r$, $\tilde{q}_t = q_t$ and $\tilde{q}_l = q_l$. By construction, Construction, Constraint (6) is satisfied. Since $\tilde{p}_r = v_0$, Constraint (5) is also satisfied. Constraint (4) is also satisfied since $\tilde{p}_r = \tilde{p}_t$ and $q_r \geq q_t$. As we did not change the allocation variables, Constraint (7) is also satisfied. Finally, the platform's objective function strictly increases, as $\tilde{p}_r\tilde{q}_r\sigma D > p_r q_r \sigma D$, $\tilde{q}_t\tilde{p}_t(1-\sigma)D \geq q_t p_t(1-\sigma)D$ and $\tilde{q}_l\tilde{p}_l(1-\sigma)D \geq q_l p_l(1-\sigma)D$, and the future value remains unchanged since the value of $\tilde{\Gamma}' = (1-\tilde{q}_t)(1-\sigma)D = (1-q_t)(1-\sigma)D = \Gamma'$. This contradicts with the optimality of $(p_r, p_t, p_l, q_r, q_t, q_l)$.

If $0 < q_r < q_t$, then we consider the deviation such that $\tilde{p}_r = v_0$, $\tilde{p}_t = v_0$, and $\tilde{p}_l = v_1$, $\tilde{q}_r = \tilde{q}_t = sq_r + (1-\sigma)q_t$ and $\tilde{q}_l = q_l$. By construction, Construction, Constraint (6) is satisfied. Since $\tilde{p}_r = \tilde{p}_t$ and $\tilde{q}_r = \tilde{q}_t$, Constraint (4) is satisfied and since $\tilde{p}_r = v_0$, Constraint (5) is satisfied. Moreover, by construction, we have $\tilde{p}_r\tilde{q}_r\sigma D + \tilde{q}_t p_t(1-\sigma)D + \tilde{q}_l\Gamma = p_r q_r \sigma D + q_t p_t(1-\sigma)D + q_l\Gamma \leq 1$, so Constraint (7) is satisfied. Finally, the platform's objective function strictly increases as follows:

$$
\begin{aligned}
&\tilde{p}_r\tilde{q}_r\sigma D + \tilde{p}_t\tilde{q}_t(1-\sigma)D + \tilde{p}_l\tilde{q}_l\Gamma + \delta\bar{V}(\tilde{\Gamma}') \\
&\quad = v_0(\sigma q_r + (1-\sigma)q_t)\sigma D + v_0(\sigma q_r + (1-\sigma)q_t)(1-\sigma)D + v_1 q_l\Gamma + \delta\bar{V}((1-\tilde{q}_t)(1-\sigma)D) \\
&\quad = \underbrace{q_r v_0 \sigma D}_{> q_r p_r \sigma D} + \underbrace{q_t v_0(1-\sigma)D}_{\geq q_t p_t (1-\sigma)D} + \underbrace{v_1 q_l \Gamma}_{\geq p_l q_l \Gamma} + \delta\underbrace{\bar{V}((1-\tilde{q}_t)(1-\sigma)D)}_{\geq \bar{V}((1-q_t)(1-\sigma)D)} \\
&\quad > p_r q_r \sigma D + p_t q_t(1-\sigma)D + p_l q_l \Gamma + \delta\bar{V}(\Gamma')
\end{aligned}
$$

If$q_r = 0$, then in order not to violate $\mathcal{IC}_r$ we must also have $p_t = 0$. In this case, the platform could simply deviate to an $\tilde{q}_r = 1$, $\tilde{p}_r = 1$, $\tilde{q}_t = q_t = 0$, $\tilde{q}_l = \frac{q_l * \Gamma - \sigma D}{\Gamma}$, $\tilde{p}_l = v_1$, and can get strictly better off by not violating any of the constraints.[15]

[15] In this case the value of p_t is irrelevant as there are no timely rides are allocated to price-sensitive agents.

Note that the last inequality stems from the fact that $\tilde{q}_t = sq_r + (1-\sigma)q_t > q_t$ and $V(D,\Gamma)$ is non-decreasing in Γ, which can be easily verified. This again contradicts with the optimality of $(p_r, p_t, p_l, q_r, q_t, q_l)$. Notice that this also proves that $q_r \geq q_t$.

Proof that $p_t = v_0$ *and* $p_l = v_1$: Let us assume by contradiction that $p_t < v_0$ (resp. $p_l < v_1$). From the results above, we restrict our attention to the case where the incentive constraint $\mathcal{IC}_r$ is binding and $p_r \geq v_0$. In the case where $q_t = 0$ (resp. $q_l = 0$), we can set $p_t = v_0$ (resp. $p_l = v_1$) without loss of generality. We now assume that $q_t > 0$ (resp. $q_l > 0$). Since $p_t < v_0$ (resp. $p_l < v_1$) and $p_r \geq v_0$, Constraint $\mathcal{IC}_n$ is not binding. We consider $\tilde{p}_t = p_t + \varepsilon$ (resp. $\tilde{p}_l = p_l + \varepsilon$), with $\varepsilon > 0$ such that $p_t + \varepsilon \leq v_0$ (resp. $p_l + \varepsilon \leq v_1$) and such that the incentive constraint $\mathcal{IC}_n$ remains satisfied. By construction, this new solution does not violate Constraints (5) and (6). Constraints (7) and (8) remain unchanged, while Constraint (4) is loosened. Moreover, the new solution strictly increases the value function (Equation (3)). This contradicts the optimality of $(p_r, p_t, p_l, q_r, q_t, q_l)$.

Proof that $q_r = 1$: Suppose $q_r < 1$ to get a contradiction. If the resource constraint (Constraint (7)) is not binding, then the platform could increase q_r by an arbitrarily small ε and can get strictly better of. This contradicts with the optimality.

If, on the other hand, the resource constraint (Constraint (7)) is binding, then $q_r\sigma D + q_t(1-\sigma)D + q_l\Gamma = 1$. In the earlier steps we have already shown that $p_t = v_0$, $p_l = v_1$, and (due to the binding $\mathcal{IC}_r$) $p_r = 1 - \frac{q_t}{q_r}(1-v_0) \geq v_0$.

- If $q_l > 0$, then consider the following deviation, $\tilde{q}_r = q_r + \varepsilon$, and $\tilde{q}_l = q_l - \varepsilon\frac{\sigma D}{\Gamma}$ without altering any other variable (we obviously have $\Gamma > 0$ since $q_l > 0$). Clearly, under this deviation all the constraints of problem $\mathcal{P}$ are satisfied. Moreover the platform's objective value strictly increases as follows:

$$\begin{aligned}
&\tilde{p}_r\tilde{q}_r\sigma D + \tilde{q}_t p_t(1-\sigma)D + \tilde{q}_l\tilde{p}_l\Gamma + \delta\bar{V}(\tilde{\Gamma}')\\
&\quad = p_r q_r\sigma D + q_t p_t(1-\sigma)D + q_l p_l\Gamma + \delta\bar{V}(\Gamma') + p_r\sigma D\varepsilon - p_l\sigma D\varepsilon\\
&\quad > p_r q_r\sigma D + q_t p_t(1-\sigma)D + q_l p_l\Gamma + \delta\bar{V}(\Gamma') \quad \text{because } p_r \geq v_0 > v_1 = p_l
\end{aligned}$$

  This contradicts with the optimality of $(p_r, p_t, p_l, q_r, q_t, q_l)$.

- If $q_l = 0$, then we must have $q_t > 0$ since the resource constraint (Constraints (7)) is binding, and we always have $\sigma D < 1$ from Assumption 1. Consider the deviation in which $\tilde{q}_r = q_r + \varepsilon$, $\tilde{q}_t = q_t - \frac{\sigma}{1-\sigma}\varepsilon$, and $\tilde{p}_r = 1 - \frac{\tilde{q}_t}{\tilde{q}_r}(1-v_0)$. By construction, $\tilde{q}_r(1-\tilde{p}_r) = \tilde{q}_t(1-v_0)$, so Constraint (4) is satisfied. Moreover, we have by construction $\tilde{q}_r\sigma D + \tilde{q}_t(1-\sigma)D + \tilde{q}_l\Gamma = q_r\sigma D + q_t(1-\sigma)D + q_l\Gamma$, so Constraint (7)) is also satisfied. Finally, Constraint (5) is also satisfied, since it was satisfied under the original 6-tuple $(p_r, p_t, p_l, q_r, q_t, q_l)$, and now we have $\tilde{p}_r > p_r$. Note that the platform's objective function strictly increases as follows:

$$\tilde{p}_r\tilde{q}_r\sigma D + \tilde{p}_t\tilde{q}_t(1-\sigma)D + \tilde{p}_l\tilde{q}_l\Gamma + \delta\bar{V}(\tilde{\Gamma}')$$

$$
\begin{aligned}
&= \left(1 - \frac{\tilde{q}_t}{\tilde{q}_r}(1-v_0)\right)\tilde{q}_r\sigma D + v_0\tilde{q}_t(1-\sigma)D + p_l q_l \Gamma + \delta\bar{V}(\tilde{\Gamma}') \\
&= (\tilde{q}_r - \tilde{q}_t)\sigma D + \tilde{q}_t v_0 D + p_l q_l \Gamma + \delta\bar{V}(\tilde{\Gamma}') \\
&= (q_r - q_t)\sigma D + \left(\varepsilon + \frac{\sigma}{1-\sigma}\varepsilon\right)\sigma D + q_t v_0 D - \frac{\sigma}{1-\sigma} v_0 D\varepsilon + p_l q_l \Gamma + \delta\bar{V}(\tilde{\Gamma}') \\
&> (q_r - q_t)\sigma D + q_t v_0 D + p_l q_l \Gamma + \delta\bar{V}(\Gamma') \qquad \text{because } v_0 < 1 \text{ and } \tilde{\Gamma}' > \Gamma' \\
&= p_r q_r \sigma D + p_t q_t (1-\sigma) D + p_l q_l \Gamma + \delta\bar{V}(\Gamma') \qquad \text{because Constraint (4) is binding}
\end{aligned}
$$

This again contradicts with the optimality of $(p_r, p_t, p_l, q_r, q_t, q_l)$.

Proof that If $\Gamma > 0$*, then* $q_l(D,\Gamma) = \min\left\{1, \frac{1-\sigma D - q_t(D,\Gamma)(1-\sigma)D}{\Gamma}\right\}$: Let us assume that $\Gamma > 0$ and, by contradiction, that $q_l < 1$ and $1 > q_r\sigma D + q_t(1-\sigma)D + q_l\Gamma$. Then we can define $\tilde{q}_l = q_l + \varepsilon$, with $\varepsilon > 0$ such that $\tilde{q}_l < 1$ and $1 > q_r\sigma D + q_t(1-\sigma)D + \tilde{q}_l\Gamma$. By construction, Constraint (7) is satisfied, and Constraints (4), (6) and (8) remain unchanged. Moreover, note that, since we have proved that $p_t = v_0$, $p_l = v_1$ and $p_r \geq v_0$, Constraint (5) is automatically satisfied. Finally, since $p_l = v_1 > 0$ and $\Gamma > 0$, the new solution strictly increases the value function. This contradicts the optimality of $(p_r, p_t, p_l, q_r, q_t, q_l)$.

Proof that $p_r = 1 - q_t(1-v_0)$: We have already shown that the incentive constraint $\mathcal{IC}_r$ is binding, i.e., $q_r(1-p_r) = q_t[1-p_t]$. Since $q_r = 1$ and $p_t = v_0$, it yields directly $p_r = 1 - q_t(1-v_0)$.

This completes the proof.

EC.1.3. Proof of Proposition 3

Let us first re-write the Bellman equation for Problem $(\mathcal{P})$:

$$
\begin{aligned}
V(D,\Gamma) &= \max_{\Gamma' \in [\underline{\Gamma}_D, \bar{\Gamma}_D]} \tilde{V}(D,\Gamma,\Gamma') \\
&= \max_{\Gamma' \in [\underline{\Gamma}_D, \bar{\Gamma}_D]} \left\{\sigma D\left(1 - \tfrac{(1-\sigma)D - \Gamma'}{(1-\sigma)D}(1-v_0)\right) + ((1-\sigma)D - \Gamma')v_0 + \min\{\Gamma, 1 - D + \Gamma'\}v_1 + \delta\bar{V}(\Gamma')\right\}
\end{aligned}
$$

To see that the value function $V(D,\Gamma)$ is non-decreasing in Γ, note that $\sigma D\left(1 - \tfrac{(1-\sigma)D - \Gamma'}{(1-\sigma)D}(1-v_0)\right) + ((1-\sigma)D - \Gamma')v_0 + \min\{\Gamma, 1 - D + \Gamma'\}v_1$ is non-decreasing in Γ, and that the term $\bar{V}(\Gamma')$ does not depend on Γ. Therefore, we directly obtain, for any $D \in \{L, H\}$ and any $\Gamma_a < \Gamma_b$:

$$
V(D,\Gamma_b) \geq \tilde{V}(D,\Gamma_b,\Gamma^*(D,\Gamma_a)) \geq \tilde{V}(D,\Gamma_a,\Gamma^*(D,\Gamma_a)) = V(D,\Gamma_a)
$$

Let us now turn to the proof of concavity. We define $\Gamma_\lambda = \lambda\Gamma_a + (1-\lambda)\Gamma_b$, and aim to show that $V(D,\Gamma_\lambda) \geq \lambda V(D,\Gamma_a) + (1-\lambda)V(D,\Gamma_b)$. We proceed by value iteration. Specifically, we initialize $V_0(D,\Gamma) = 0, \forall (D,\Gamma) \in \{H, L\} \times \mathbf{\Gamma}$, and consider the following recursive sequence of functions, for $n \geq 0$:

$$
\bar{V}_{n-1}(\Gamma) = kV_{n-1}(H,\Gamma) + (1-k)V_{n-1}(L,\Gamma), \forall \Gamma \in \mathbf{\Gamma},
$$

$$\tilde{V}_n(D,\Gamma,\Gamma') = \sigma D\left(1-\tfrac{(1-\sigma)D-\Gamma'}{(1-\sigma)D}(1-v_0)\right) + ((1-\sigma)D-\Gamma')v_0 + \min\{\Gamma, 1-D+\Gamma'\}v_1 + \delta\bar{V}_{n-1}(\Gamma'),$$
$$V_n(D,\Gamma) = \max_{\Gamma'\in[\underline{\Gamma}_D,\bar{\Gamma}_D]} \tilde{V}_{n-1}(D,\Gamma,\Gamma').$$

By value iteration, we know that the sequence V_n converges to the optimal value function V (Bertsekas 2012). We will show by induction over $n \geq 0$ that $V_n(D,\Gamma_\lambda) \geq \lambda V_n(D,\Gamma_a) + (1-\lambda)V_n(D,\Gamma_b)$. By taking the limit when $n \to \infty$, this will yield concavity of V. First, note that the property is clearly satisfied for $n = 0$. We now assume that it holds for some $n-1$ and show it for $n \geq 1$.

For each $\Gamma \in \mathbf{\Gamma}$, we introduce the following notations:

$$\Gamma_n^*(D,\Gamma) = \arg\max_{\Gamma'\in[\underline{\Gamma}_D,\bar{\Gamma}_D]} \tilde{V}_n(D,\Gamma,\Gamma').$$

We also define $y_\lambda = \lambda\Gamma_n^*(D,\Gamma_a) + (1-\lambda)\Gamma_n^*(D,\Gamma_b)$. We have: $V_n(D,\Gamma_\lambda) \geq \tilde{V}_n(D,\Gamma_\lambda,y_\lambda)$, which yields:

$$V_n(D,\Gamma_\lambda) \geq \sigma D\left(1-\tfrac{(1-\sigma)D-y_\lambda}{(1-\sigma)D}(1-v_0)\right) + ((1-\sigma)D-y_\lambda)v_0 + \min\{\Gamma_\lambda, 1-D+y_\lambda\}v_1 + \delta\bar{V}_{n-1}(y_\lambda)$$

Moreover, we have:

$$\begin{aligned}
&\lambda V(D,\Gamma_a) + (1-\lambda)V(D,\Gamma_b)\\
&= \lambda\left[\sigma D\left(1-\tfrac{(1-\sigma)D-\Gamma_n^*(D,\Gamma_a)}{(1-\sigma)D}(1-v_0)\right) + ((1-\sigma)D-\Gamma_n^*(D,\Gamma_a))\,v_0 + \min\{\Gamma_a, 1-D+\Gamma_n^*(D,\Gamma_a)\}\,v_1 + \delta\bar{V}_{n-1}\left(\Gamma_n^*(D,\Gamma_a)\right)\right]\\
&+ (1-\lambda)\left[\sigma D\left(1-\tfrac{(1-\sigma)D-\Gamma_n^*(D,\Gamma_b)}{(1-\sigma)D}(1-v_0)\right) + ((1-\sigma)D-\Gamma_n^*(D,\Gamma_b))\,v_0 + \min\{\Gamma_b, 1-D+\Gamma_n^*(D,\Gamma_b)\}\,v_1 + \delta\bar{V}_{n-1}\left(\Gamma_n^*(D,\Gamma_b)\right)\right]\\
&= \sigma D\left(1-\tfrac{(1-\sigma)D-y_\lambda}{(1-\sigma)D}(1-v_0)\right) + ((1-\sigma)D-y_\lambda)v_0\\
&+ \lambda\min\{\Gamma_a, 1-D+\Gamma_n^*(D,\Gamma_a)\}\,v_1 + (1-\lambda)\min\{\Gamma_b, 1-D+\Gamma_n^*(D,\Gamma_b)\}\,v_1\\
&+ \delta\left[\lambda\bar{V}_{n-1}\left(\Gamma_n^*(D,\Gamma_a)\right) + (1-\lambda)\bar{V}_{n-1}\left(\Gamma_n^*(D,\Gamma_b)\right)\right]
\end{aligned}$$

Note that the first line of the last equality stems from the fact that the first two terms of the function $\tilde{V}_n$ are linear in Γ'. Moreover, from the induction hypothesis, we know that $\bar{V}_{n-1}(y_\lambda) \geq \lambda\bar{V}_{n-1}\left(\Gamma_n^*(D,\Gamma_a)\right) + (1-\lambda)\bar{V}_{n-1}\left(\Gamma_n^*(D,\Gamma_b)\right)$. Therefore, a sufficient condition is to show that:

$$\min\{\Gamma_\lambda, 1-D+y_\lambda\} \geq \lambda\min\{\Gamma_a, 1-D+\Gamma_n^*(D,\Gamma_a)\} + (1-\lambda)\min\{\Gamma_b, 1-D+\Gamma_n^*(D,\Gamma_b)\}$$

We separate four cases:

1. If $\Gamma_a \leq 1-D+\Gamma_n^*(D,\Gamma_a)$ and $\Gamma_b \leq 1-D+\Gamma_n^*(D,\Gamma_b)$, then we have $\Gamma_\lambda \leq 1-D+y_\lambda$, and we directly obtain:

$$\underbrace{\min\{\Gamma_\lambda, 1-D+y_\lambda\}}_{=\Gamma_\lambda} = \lambda\underbrace{\min\{\Gamma_a, 1-D+\Gamma_n^*(D,\Gamma_a)\}}_{=\Gamma_a} + (1-\lambda)\underbrace{\min\{\Gamma_b, 1-D+\Gamma_n^*(D,\Gamma_b)\}}_{=\Gamma_b}.$$

2. If $\Gamma_a \leq 1-D+\Gamma_n^*(D,\Gamma_a)$ and $\Gamma_b > 1-D+\Gamma_n^*(D,\Gamma_b)$, then we distinguish two sub-cases:

– If $\Gamma_\lambda \le 1-D+y_\lambda$, we write $\lambda\Gamma_a+(1-\lambda)\left(1-D+\Gamma_n^*(D,\Gamma_b)\right) < \lambda\Gamma_a+(1-\lambda)\Gamma_b=\Gamma_\lambda$, which yields:

$$\underbrace{\min\{\Gamma_\lambda,1-D+y_\lambda\}}_{=\Gamma_\lambda} \ge \lambda\underbrace{\min\left\{\Gamma_a,1-D+\Gamma_n^*(D,\Gamma_a)\right\}}_{=\Gamma_a}+(1-\lambda)\underbrace{\min\left\{\Gamma_b,1-D+\Gamma_n^*(D,\Gamma_b)\right\}}_{=1-D+\Gamma_n^*(D,\Gamma_b)}.$$

– If $\Gamma_\lambda > 1-D+y_\lambda$, we write $\lambda\Gamma_a+(1-\lambda)\left(1-D+\Gamma_n^*(D,\Gamma_b)\right) \le \lambda\left(1-D+\Gamma_n^*(D,\Gamma_a)\right)+(1-\lambda)\left(1-D+\Gamma_n^*(D,\Gamma_b)\right)=1-D+y_\lambda$, which yields:

$$\underbrace{\min\{\Gamma_\lambda,1-D+y_\lambda\}}_{=1-D+y_\lambda} \ge \lambda\underbrace{\min\left\{\Gamma_a,1-D+\Gamma_n^*(D,\Gamma_a)\right\}}_{=\Gamma_a}+(1-\lambda)\underbrace{\min\left\{\Gamma_b,1-D+\Gamma_n^*(D,\Gamma_b)\right\}}_{=1-D+\Gamma_n^*(D,\Gamma_b)}.$$

3. If $\Gamma_a > 1-D+\Gamma_n^*(D,\Gamma_a)$ and $\Gamma_b \le 1-D+\Gamma_n^*(D,\Gamma_b)$, we proceed as in Case 2. by symmetry.
4. If $\Gamma_a > 1-D+\Gamma_n^*(D,\Gamma_a)$ and $\Gamma_b > 1-D+\Gamma_n^*(D,\Gamma_b)$, then we have $\Gamma_\lambda > 1-D+y_\lambda$, and we directly obtain:

$$\underbrace{\min\{\Gamma_\lambda,1-D+y_\lambda\}}_{=1-D+y_\lambda} = \lambda\underbrace{\min\left\{\Gamma_a,1-D+\Gamma_n^*(D,\Gamma_a)\right\}}_{=1-D+\Gamma_n^*(D,\Gamma_a)}+(1-\lambda)\underbrace{\min\left\{\Gamma_b,1-D+\Gamma_n^*(D,\Gamma_b)\right\}}_{=1-D+\Gamma_n^*(D,\Gamma_b)}.$$

This proves that $V_n(D,\Gamma_\lambda) \ge \lambda V_n(D,\Gamma_a)+(1-\lambda)V_n(D,\Gamma_b)$, and completes the proof that the value function V is concave in Γ. We conclude by invoking the fact that a concave function is differentiable almost everywhere.

EC.1.4. Some Remarks on the Partial Derivative of $V(D,\Gamma)$

In the remainder of the appendix, we extensively use the partial derivative of the value function $V(D,\Gamma)$ with respect to Γ, denoted by $V'(D,\Gamma)$. From Equation (9), we have:

$$\begin{aligned} V(D,\Gamma) = \sigma D\left(1-\frac{(1-\sigma)D-\Gamma^*(D,\Gamma)}{(1-\sigma)D}(1-v_0)\right)+\left((1-\sigma)D-\Gamma^*(D,\Gamma)\right)v_0 \\ +\min\{\Gamma,1-D+\Gamma^*(D,\Gamma)\}v_1+\bar{V}(\Gamma^*(D,\Gamma)). \end{aligned}$$

Therefore we can get the derivative of $V(D,\Gamma)$ as follows:

$$V'(D,\Gamma)=\frac{\partial\Gamma^*(D,\Gamma)}{\partial\Gamma}\left(\frac{\sigma-v_0}{1-\sigma}+\bar{V}'(\Gamma^*(D,\Gamma))\right)+\frac{\partial\min\{\Gamma,1-D+\Gamma^*(D,\Gamma)\}}{\partial\Gamma}v_1.$$

As we will see $\frac{\partial\Gamma^*(D,\Gamma)}{\partial\Gamma}$ is either equal to 1 or 0. Moreover, the second term on the RHS can have two different values: v_1 and 0. Therefore, $V'(D,\Gamma)$ can take four different expressions:

$$\text{If } \frac{\partial\Gamma^*(D,\Gamma)}{\partial\Gamma}=0 \text{ and } 1-D+\Gamma^*(D,\Gamma)\le\Gamma, \text{ then } V'(D,\Gamma)=0 \tag{EC.1}$$

$$\text{If } \frac{\partial\Gamma^*(D,\Gamma)}{\partial\Gamma}=0 \text{ and } 1-D+\Gamma^*(D,\Gamma)>\Gamma, \text{ then } V'(D,\Gamma)=v_1 \tag{EC.2}$$

$$\text{If } \frac{\partial\Gamma^*(D,\Gamma)}{\partial\Gamma}=1 \text{ and } 1-D+\Gamma^*(D,\Gamma)\le\Gamma, \text{ then } V'(D,\Gamma)=\frac{\sigma-v_0}{1-\sigma}+\bar{V}'(\Gamma^*(D,\Gamma)) \tag{EC.3}$$

$$\text{If } \frac{\partial\Gamma^*(D,\Gamma)}{\partial\Gamma}=1 \text{ and } 1-D+\Gamma^*(D,\Gamma)>\Gamma, \text{ then } V'(D,\Gamma)=\frac{\sigma-v_0}{1-\sigma}+v_1+\bar{V}'(\Gamma^*(D,\Gamma)) \tag{EC.4}$$

Before proceeding further, we highlight the intuition behind each of these four cases:

- We have Equation (EC.1), because in this case, increasing the value of Γ does not change the number of agents transferred to the next period, and the number of late rides.
- We have Equation (EC.2), because in this case, increasing the value of Γ increases the number of late rides at a 1:1 rate, without altering the number of agents transferred to the next period.
- We have Equation (EC.3), because in this case, increasing the value of Γ, increases the number of agents transferred to the next period at a 1:1 rate, without altering the number of late rides.
- We have Equation (EC.4), because in this case, increasing the value of Γ, increases both the number of late rides, and the number of agents transferred to the next period at a 1:1 rate.

EC.1.5. Proof of Lemma 1

It is sufficient to show that $\frac{\partial \tilde{V}(D,\Gamma,\Gamma')}{\partial \Gamma'}$ is a weakly increasing function of Γ and D. Notice that the first two terms of the RHS of equation (13) are constant. The fourth term (i.e., the intertemporal effect) does not depend on the value of Γ and D. Therefore, we just need to show that the third term (i.e., the effect on late rides) is weakly increasing function of Γ and D. Notice that this term is a step function of Γ', and satisfies:

$$\frac{\partial \min\{\Gamma, 1-D+\Gamma'\}v_1}{\partial \Gamma'} = \begin{cases} v_1 & \text{if } \Gamma' < \Gamma + D - 1, \\ 0 & \text{if } \Gamma' \geq \Gamma + D - 1. \end{cases}$$

This directly shows that $\frac{\partial \tilde{V}(D,\Gamma,\Gamma')}{\partial \Gamma'}$ is a weakly increasing function of Γ and D.

EC.1.6. Proof of Lemma 2

Let us consider Γ_0 such that $\zeta^*(D,\Gamma_0) > \Gamma_0$, i.e., $1 - D - \Gamma^*(D,\Gamma_0) > \Gamma_0$. We have $\frac{\partial \min\{\Gamma_0, 1-D-\Gamma^*(D,\Gamma_0)\}v_1}{\partial \Gamma'} = 0$. Since $\tilde{V}$ is differentiable almost everywhere, we have from Equation (13):

$$\frac{\partial \tilde{V}(D,\Gamma_0,\Gamma^*(D,\Gamma_0)-\varepsilon)}{\partial \Gamma'} = \frac{\sigma(1-v_0)}{1-\sigma} - v_0 + \delta \bar{V}'(\Gamma^*(D,\Gamma_0)-\varepsilon) \geq 0,$$
$$\frac{\partial \tilde{V}(D,\Gamma_0,\Gamma^*(D,\Gamma_0)+\varepsilon)}{\partial \Gamma'} = \frac{\sigma(1-v_0)}{1-\sigma} - v_0 + \delta \bar{V}'(\Gamma^*(D,\Gamma_0)+\varepsilon) \leq 0.$$

Let us now consider $\Gamma \leq \Gamma_0$. We have $\zeta(D,\Gamma,\Gamma^*(D,\Gamma_0) > \Gamma_0 \geq \Gamma$, so $\frac{\partial \min\{\Gamma, 1-D+\Gamma^*(D,\Gamma_0)\}v_1}{\partial \Gamma'} = 0$. We therefore obtain, as earlier:

$$\frac{\partial \tilde{V}(D,\Gamma,\Gamma^*(D,\Gamma_0)-\varepsilon)}{\partial \Gamma'} = \frac{\sigma(1-v_0)}{1-\sigma} - v_0 + \delta \bar{V}'(\Gamma^*(D,\Gamma_0)-\varepsilon) \geq 0,$$
$$\frac{\partial \tilde{V}(D,\Gamma,\Gamma^*(D,\Gamma_0)+\varepsilon)}{\partial \Gamma'} = \frac{\sigma(1-v_0)}{1-\sigma} - v_0 + \delta \bar{V}'(\Gamma^*(D,\Gamma_0)+\varepsilon) \leq 0.$$

In other words, we have proved that:

$$\frac{\partial \tilde{V}(D,\Gamma,\Gamma^*(D,\Gamma_0)-\varepsilon)}{\partial \Gamma'} = \frac{\partial \tilde{V}(D,\Gamma,\Gamma^*(D,\Gamma_0)-\varepsilon)}{\partial \Gamma'} \qquad \geq 0$$

$$\frac{\partial \tilde{V}(D,\Gamma,\Gamma^*(D,\Gamma_0)+\varepsilon)}{\partial \Gamma'} = \frac{\partial \tilde{V}(D,\Gamma,\Gamma^*(D,\Gamma_0)+\varepsilon)}{\partial \Gamma'} \qquad \leq 0$$

Therefore $\Gamma^*(D,\Gamma_0)$ is also an optimal choice in state (D,Γ) too.[16]

EC.1.7. Proof of Proposition 4

We have, from Equation (13):

$$\frac{\partial \tilde{V}(D,\Gamma,\Gamma')}{\partial \Gamma'} \geq \frac{\sigma(1-v_0)}{1-\sigma} - v_0 = \frac{\sigma - v_0}{1-\sigma} \geq 0,\ \forall (D,\Gamma) \in \{H,L\} \times \mathbf{\Gamma},\ \forall \Gamma' \in [\underline{\Gamma}_D, \bar{\Gamma}_D].$$

Therefore we always have: $\Gamma^*(D,\Gamma) = \bar{\Gamma}_D, \quad \forall (D,\Gamma) \in \{H,L\} \times \mathbf{\Gamma}$. Notice that at the boundary $v_0 = s$, $\Gamma^*(D,\Gamma) = \bar{\Gamma}_D$ is not necessarily unique, but is still an optimal solution.

EC.1.8. Proof of Lemma 3

By contradiction, let us consider Γ_0 such that $\zeta^*(D,\Gamma_0) < \Gamma_0$ and $\zeta^*(D,\Gamma_0) < 1 - D + \bar{\Gamma}_D$ (i.e., $1 - D + \Gamma^*(D,\Gamma_0) < \Gamma_0$ and $\Gamma^*(D,\Gamma) < \bar{\Gamma}_D$). This implies that:

$$\frac{\partial \min\{\Gamma_0, 1-D+\Gamma^*(D,\Gamma_0)\} v_1}{\partial \Gamma'} = v_1.$$

In consequence:

$$\frac{\partial \tilde{V}(D,\Gamma_0,\Gamma^*(D,\Gamma_0))}{\partial \Gamma'} \geq \frac{\sigma(1-v_0)}{1-\sigma} - v_0 + v_1 > 0$$

This contradicts the fact that $\Gamma^*(D,\Gamma_0)$ is optimal in state (D,Γ_0). Notice that at the boundary $v_0 = \sigma + (1-\sigma)v_1$, the claim is correct without loss of generality.

EC.1.9. Proof of Lemma 4

Let us consider Γ_0 such that $\zeta^*(D,\Gamma_0) = \min\{\Gamma_0, 1 - D + \bar{\Gamma}_D\}$. First, if $\zeta^*(D,\Gamma_0) = 1 - D + \bar{\Gamma}_D$, then $\Gamma^*(D,\Gamma_0) = \bar{\Gamma}_D$ and from Lemma 1, we know that for each $\Gamma \geq \Gamma_0$ we have $\Gamma^*(D,\Gamma) = \bar{\Gamma}_D$, so $\zeta^*(D,\Gamma) = 1 - D + \bar{\Gamma}_D$. We now assume that $\Gamma^*(D,\Gamma_0) < \bar{\Gamma}_D$. This implies that $\zeta^*(D,\Gamma_0) = \Gamma_0$, i.e., $\Gamma^*(D,\Gamma_0) = 1 - D + \Gamma_0$. Therefore, $\frac{\partial \min\{\Gamma_0, 1-D+\Gamma^*(D,\Gamma_0)\} v_1}{\partial \Gamma'} = 0$, and then, for an arbitrarily small $\varepsilon > 0$, we know that:

$$\frac{\partial \tilde{V}(D,\Gamma_0,\Gamma^*(D,\Gamma_0)+\varepsilon)}{\partial \Gamma'} = \frac{\sigma(1-v_0)}{1-\sigma} - v_0 + \delta \bar{V}'(\Gamma^*(D,\Gamma_0)+\varepsilon) \leq 0.$$

Let us consider $\Gamma > \Gamma_0$ and assume by contradiction that $\zeta^*(D,\Gamma) > \Gamma$.[17] This can be re-written as $1 - D + \Gamma^*(D,\Gamma) > \Gamma$, or $\Gamma^*(D,\Gamma) > D - 1 + \Gamma$. This implies that, for some $\varepsilon' > 0$ arbitrarily small, we have:

$$\frac{\partial \tilde{V}(D,\Gamma,D-1+\Gamma+\varepsilon')}{\partial \Gamma'} = \frac{\sigma(1-v_0)}{1-\sigma} - v_0 + \delta \bar{V}'(D-1+\Gamma+\varepsilon') > 0.$$

But for sufficiently small ε' and ε we must have $D - 1 + \Gamma + \varepsilon' > \Gamma^*(D,\Gamma_0) + \varepsilon$. Due to the concavity of the value function V, we therefore have $\bar{V}'(D-1+\Gamma+\varepsilon') \leq \bar{V}'(\Gamma^*(D,\Gamma_0)+\varepsilon)$. This is in contradiction with the two inequalities above.

[16] These arguments are based on the assumption that the value of $\Gamma^*(D,\Gamma_0)$ is in the interior of the interval $[\underline{\Gamma}_D, \bar{\Gamma}_D]$, so that it is possible to take the partial derivative at $\Gamma^*(D,\Gamma_0) \pm \varepsilon$. However, the result goes through for boundary values as well, we just need to consider $+\varepsilon$ or $-\varepsilon$ for lower and upper bounds respectively.

[17] Notice that, in Lemma 3, we already argued that $\zeta^*(D,\Gamma) \geq \min\{\Gamma, 1 - D + \bar{\Gamma}_D\}$.

EC.1.10. Proof of Proposition 9

We already showed that, in region 2, the optimal policy function will be automatically pinned down once we figure out the value of $\Gamma^*(D,0)$, as follows:

$$\Gamma^*(D,\Gamma)=\begin{cases}\Gamma^*(D,0) & \text{if } \Gamma\le 1-D+\Gamma^*(D,0),\\ \min\{\Gamma+D-1,\bar{\Gamma}_D\} & \text{if } \Gamma\ge 1-D+\Gamma^*(D,0).\end{cases}$$

We therefore aim to determine $\Gamma^*(D,0)$ for $D\in\{H,L\}$. Note that $\Gamma^*(D,0)$ can be (i equal to the lower bound $\underline{\Gamma}$ (ii an interior solution in the interval $(\underline{\Gamma}_D,\bar{\Gamma}_D)$, or (iii equal to the upper bound $\bar{\Gamma}_D$

In the following, we list all these possibilities for each demand realization $D\in\{H,L\}$ separately.

H1) $\Gamma^*(H,0)=\underline{\Gamma}_H$

H2) $\Gamma^*(H,0)=\Gamma_1\in(\underline{\Gamma}_H,\bar{\Gamma}_H)$

H3) $\Gamma^*(H,0)=\bar{\Gamma}_H$

L1) $\Gamma^*(L,0)=\underline{\Gamma}_L$

L2) $\Gamma^*(L,0)=\Gamma_2\in(\underline{\Gamma}_L,\bar{\Gamma}_L)$

L3) $\Gamma^*(L,0)=\bar{\Gamma}_L$

Now, we elaborate on all of these cases separately to find out whether and when these possibilities can be part of an optimal policy function $\Gamma^*(D,\Gamma)$. Before doing so, we first point out some initial observations regarding each of these cases. These observations will be useful in later stages. While describing underlying properties of these cases, the parameter value of σ becomes particularly important. In some cases, we further need to consider two subcases depending on whether $\sigma<0.5$ or $\sigma\ge 0.5$.

Case H1: If the optimal policy function satisfies H1, then we must have the following:

$$\Gamma^*(H,\Gamma)=\begin{cases}\Gamma+H-1 & \text{if } \Gamma\le 1-\sigma H,\\ \bar{\Gamma}_H & \text{if } \Gamma\ge 1-\sigma H.\end{cases}$$

$$\frac{\partial_+\tilde{V}(H,0,\underline{\Gamma}_H)}{\partial\Gamma'}=\frac{\sigma-v_0}{1-\sigma}+\delta\bar{V}'_+(\underline{\Gamma}_H)\le 0.$$

$$V'(H,\Gamma)=\begin{cases}\frac{\sigma-v_0}{1-\sigma}+v_1+\delta\bar{V}'(\Gamma+H-1) & \text{if } \Gamma\le 1-\sigma H, \qquad \text{(Equation (EC.4))}\\ 0 & \text{if } \Gamma\ge 1-\sigma H. \qquad \text{(Equation (EC.1))}\end{cases}$$

Notice that at $\Gamma=1-\sigma H$, the derivative $V'(D,\Gamma)$ does not exists. The two expressions provided above, the first one is when $\Gamma\le 1-\sigma H$, and the other is when $\Gamma\ge 1-\sigma H$, are respectively the left and the right derivatives of $V(D,\Gamma)$ at $\Gamma=1-\sigma H$. In what follows, we follow this convention, in that the expressions provided at a kink point of a piece-wise defined derivative are respectively the left and the right derivatives at the kink point.

Case H2: If the optimal policy function satisfies H2, then we must have the following:

$$\Gamma^*(H,\Gamma) = \begin{cases} \Gamma_1 & \text{if } \Gamma \le \Gamma_1 + 1 - H \\ \Gamma + H - 1 & \text{if } \Gamma \in [\Gamma_1 + 1 - H, 1 - \sigma H] \\ \bar{\Gamma}_H & \text{if } \Gamma \ge 1 - \sigma H \end{cases}$$

$$\frac{\partial_- \tilde{V}(H,0,\Gamma_1)}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + \delta \bar{V}'_-(\Gamma_1) \ge 0.$$
$$\frac{\partial_+ \tilde{V}(H,0,\Gamma_1\Gamma_1)}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + \delta \bar{V}'_+(\Gamma_1) \le 0.$$

$$V'(H,\Gamma) = \begin{cases} v_1 & \text{if } \Gamma \le \Gamma_1 + 1 - H \qquad \text{(Equation (EC.2))} \\ \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta \bar{V}'(\Gamma + H - 1) & \text{if } \Gamma \in [\Gamma_1 + 1 - H, 1 - \sigma H] \qquad \text{(Equation (EC.4))} \\ 0 & \text{if } \Gamma \ge 1 - \sigma H \qquad \text{(Equation (EC.1))} \end{cases}$$

Case H3: If the optimal policy function satisfies H3, then we must have the following:

$$\Gamma^*(H,\Gamma) = \bar{\Gamma}_H, \ \forall \Gamma \in \mathbf{\Gamma}.$$

$$\frac{\partial_- \tilde{V}(H,0,\bar{\Gamma}_H)}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + \delta \bar{V}'_-(\bar{\Gamma}_H) \ge 0$$

$$V'(H,\Gamma) = \begin{cases} v_1 & \text{if } \Gamma \le 1 - \sigma H \qquad \text{(Equation (EC.2))} \\ 0 & \text{if } \Gamma \ge 1 - \sigma H \qquad \text{(Equation (EC.1))} \end{cases}$$

Case L1: If the optimal policy function satisfies L1, then we must have the following:

$$\frac{\partial_+ \tilde{V}(L,0,0)}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + \delta \bar{V}'_+(\underline{\Gamma}_L) \le 0.$$

Subcase L1a: If $\sigma \ge 0.5$, then

$$\Gamma^*(L,\Gamma) = \begin{cases} 0 & \text{if } \Gamma \le 1 - L \\ \Gamma + L - 1 & \text{if } \Gamma \ge 1 - L \end{cases}$$

$$V'(L,\Gamma) = \begin{cases} v_1 & \text{if } \Gamma \le 1 - L \qquad \text{(Equation (EC.2))} \\ \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta V'(\Gamma + L - 1) & \text{if } \Gamma \ge 1 - L \qquad \text{(Equation (EC.4))} \end{cases}$$

Subcase L1b: If $\sigma < 0.5$, then

$$\Gamma^*(L,\Gamma) = \begin{cases} 0 & \text{if } \Gamma \le 1 - L \\ \Gamma + L - 1 & \text{if } 1 - L \le \Gamma \le 1 - \sigma L \\ \bar{\Gamma}_L & \text{if } \Gamma \ge 1 - \sigma L \end{cases}$$

$$V'(L,\Gamma) = \begin{cases} v_1 & \text{if } \Gamma \le 1 - L \qquad \text{(Equation (EC.2))} \\ \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta V'(\Gamma + L - 1) & \text{if } 1 - L \le \Gamma \le 1 - \sigma L \qquad \text{(Equation (EC.4))} \\ 0 & \text{if } \Gamma \ge 1 - \sigma L \qquad \text{(Equation (EC.1))} \end{cases}$$

Case L2: If the optimal policy function satisfies L2, then we must have the following:

$$\frac{\partial_- \tilde{V}(L,0,\Gamma_2)}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + \delta \bar{V}'_-(\Gamma_2) \geq 0.$$
$$\frac{\partial_+ \tilde{V}(L,0,\Gamma_2)}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + \delta \bar{V}'_+(\Gamma_2) \leq 0.$$

In this case we define subcases based on i) whether $\sigma < 0.5$, or $\sigma \geq 0.5$, and ii) whether $\Gamma_2 < 1 - \sigma H$ or $\Gamma_2 \geq 1 - \sigma H$. Notice that when $\Gamma_2 \geq 1 - \sigma H$, we automatically have $\sigma \geq 0.5$. Therefore we have three subcases in total.

Subcase L2a: If $\Gamma_2 < 1 - \sigma H$, and $\sigma \geq 0.5$, then

$$\Gamma^*(L,\Gamma) = \begin{cases} \Gamma_2 & \text{if } \Gamma \leq 1 - L + \Gamma_2, \\ \Gamma + L - 1 & \text{if } \Gamma \geq 1 - L + \Gamma_2. \end{cases}$$

$$V'(L,\Gamma) = \begin{cases} v_1 & \text{if } \Gamma \leq 1 - L + \Gamma_2, \qquad \text{(Equation (EC.2)} \\ \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta V'(\Gamma + L - 1) & \text{if } \Gamma \geq 1 - L + \Gamma_2. \qquad \text{(Equation (EC.4))} \end{cases}$$

Subcase L2a': If $\Gamma_2 < 1 - \sigma H$, and $\sigma < 0.5$, then

$$\Gamma^*(L,\Gamma) = \begin{cases} \Gamma_2 & \text{if } \Gamma \leq 1 - L + \Gamma_2, \\ \Gamma + L - 1 & \text{if } \Gamma \in [1 - L + \Gamma_2, 1 - \sigma L], \\ \bar{\Gamma}_L & \text{if } \Gamma \geq 1 - \sigma L. \end{cases}$$

$$V'(L,\Gamma) = \begin{cases} v_1 & \text{if } \Gamma \leq 1 - L + \Gamma_2, \qquad \text{(Equation (EC.2))} \\ \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta V'(\Gamma + L - 1) & \text{if } \Gamma \in [1 - L + \Gamma_2, 1 - \sigma L], \qquad \text{(Equation (EC.4))} \\ 0 & \text{if } \Gamma \geq 1 - \sigma L. \qquad \text{(Equation (EC.1))} \end{cases}$$

Subcase L2b: If $\Gamma_2 \geq 1 - \sigma H$, then

$$\Gamma^*(L,\Gamma) = \Gamma_2, \ \forall \Gamma \in \mathbf{\Gamma}.$$

$$V'(L,\Gamma) = v_1, \ \forall \Gamma \in \mathbf{\Gamma}.$$

Case L3: If the optimal policy function satisfies L3, then we must have the following:

$$\Gamma^*(L,\Gamma) = \bar{\Gamma}_L, \ \forall \Gamma \in \mathbf{\Gamma}.$$

$$\frac{\partial_- \tilde{V}(L,0,\bar{\Gamma}_L)}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + \delta \bar{V}'_-(\bar{\Gamma}_L) \geq 0$$

In this case, we have two subcases depending on the value of σ.

Subcase L3a: If $\sigma \geq 0.5$, then $\bar{\Gamma}_L > 1 - \sigma H$ and:

$$V'(L,\Gamma) = v_1, \ \forall \Gamma \in \mathbf{\Gamma}. \qquad \text{(Equation (EC.2))}$$

Subcase L3b: If $\sigma < 0.5$, then $\bar{\Gamma}_L \leq 1 - \sigma H$ and:

$$V'(L,\Gamma) = \begin{cases} v_1 & \text{if } \Gamma \leq 1 - \sigma L, \qquad \text{(Equation (EC.2))} \\ 0 & \text{if } \Gamma \geq 1 - \sigma L. \qquad \text{(Equation (EC.1))} \end{cases}$$

We carry out an analysis by separately examining the cases that are based on the value of $\Gamma^*(H,0)$, i.e., the cases H1 H2 and H3. Since the problem that we deal with is of a discrete nature, we are likely to end up with corner solutions. For the sake of a clarified exposition, as long as it does not cause any loss of generality, we focus on the corner solutions.

We first examine the instances in which we *must* have an interior solution for $\Gamma^*(H,0)$, i.e., the case H2. Suppose $\Gamma^*(H,0) = \Gamma_1 \in (\underline{\Gamma}_H, \bar{\Gamma}_H)$. The question is which one(s) of the cases L1, L2, and L3 may coexist with H2 in the optimal mechanism. The fact that we *must* have an interior solution for $\Gamma^*(H,0)$ implies that:

$$\frac{\sigma - v_0}{1-\sigma} + \delta \bar{V}'(\underline{\Gamma}_H) > 0.$$

Therefore, we can immediately preclude the case L1. We can also preclude L2, because it would require that $\Gamma(H,0) = \Gamma_1 = \Gamma_2 = \Gamma(L,0)$; and since $\Gamma_1 > \underline{\Gamma}_H > 1 - \sigma H$, this is possible only in subcase L2b. However in subcase L2b, we know that $V'(L,\Gamma_1) = v_1$. In addition, since $\Gamma_1 \in (\underline{\Gamma}_H, \bar{\Gamma}_H)$, we must have $V'(H,\Gamma_1) = 0$ (Case H2). Therefore, $\bar{V}'(\Gamma_1) = kV'(H,\Gamma_1) + (1-k)V'(L,\Gamma_1) = (1-k)v_1$. Then in order to comply with the optimality conditions of H2, i.e., $\frac{\partial_- \tilde{V}(H,0,\Gamma_1)}{\partial \Gamma'} \geq 0$ and $\frac{\partial_+ \tilde{V}(H,0,\Gamma_1)}{\partial \Gamma'} \leq 0$, we must have $v_0 = \sigma + \delta(1-k)(1-\sigma)v_1$. But in this case, the platform is indifferent between setting Γ_1 and $\bar{\Gamma}_H$ in state $(H,0)$, and simply can choose the former. Therefore we do not have to have an interior solution for $\Gamma^*(H,0)$. An analogous contradiction follows in subcase L3a.

Thus, only Case L3b can co-exist with an interior solution for $\Gamma(H,0)$. To comply with the optimality conditions of H2, we must have $\Gamma_1 = 1 - \sigma L$, since the value of $V'(L,\Gamma)$ changes only at $1 - \sigma L$. Finally, since $\bar{V}'_-(1-\sigma L) = v_1$, Condition H2 implies that $\frac{\sigma - v_0}{1-\sigma} + \delta(1-k)v_1 \geq 0$. All of these arguments regarding the case H2 is summarized in the following claim.

Claim EC.1. *In Region 2, if the optimal solution* $\Gamma(H,\Gamma)$ *is* unique *and satisfies Case H2, then we have:*

i) $\Gamma_1 = 1 - \sigma L$.

ii) $\sigma < 0.5$, *and* $\Gamma^*(L,\Gamma)$ *satisfies L3b.*

iii) $v_0 \leq \sigma + (1-\sigma)\delta(1-k)v_1$.

Next, we focus on Case H3, i.e., the case in which we have $\Gamma^*(H,0) = \bar{\Gamma}_H$. The underlying optimality condition in this case, $\frac{\sigma - v_0}{1-\sigma} + \delta \bar{V}'_-(\bar{\Gamma}_H) \geq 0$. By concavity of $\bar{V}$ (Proposition 3), this implies that $\frac{\sigma - v_0}{1-\sigma} + \delta \bar{V}'_-(\underline{\Gamma}_L) \geq \frac{\sigma - v_0}{1-\sigma} + \delta \bar{V}'_-(\Gamma_2) \geq 0$. This excludes Cases L1 and L2 and we only left with Case L3. Let us assume that Case H3 co-exists with Case L3b. However, we know that

$V'(H,\bar{\Gamma}_H)=0$ (Case H3), and also $V'(L,\bar{\Gamma}_H)=0$ (Case L3b). Therefore, $\frac{\sigma-v_0}{1-\sigma}+\delta\bar{V}'_-(\bar{\Gamma}_H)=\frac{\sigma-v_0}{1-\sigma}<0$, where the strict inequality follows from the fact that we are in Region 2 ($v_0>s$). This contradiction the above condition; hence, Case H3 can only co-exist with Case L3a. We then have $\bar{V}'_-(\bar{\Gamma}_H)=v_1$, so $\frac{\sigma-v_0}{1-\sigma}+\delta\bar{V}'_-(\bar{\Gamma}_H)\geq 0$ implies $v_0\leq\sigma+(1-\sigma)\delta(1-k)v_1$. This is summarized in the following claim.

Claim EC.2. *In region 2, if the optimal solution of* $\Gamma(H,\Gamma)$ *satisfies Case H3, then we have:*

i) $\sigma\geq 0.5$ *and* $\Gamma^*(L,\Gamma)$ *satisfies L3a.*

ii) $v_0\leq\sigma+(1-\sigma)\delta(1-k)v_1$.

Finally, we turn to Case H1, i.e, $\Gamma^*(H,0)=\underline{\Gamma}_H$. Following the same strategy as before, we consider all the cases L1, L2, and L3, and and see which one(s) can coexist with H1.

Let us first consider the coexistence of Case L3 with Case H1. Starting with Subcase L3a, we we know that $V'(L,\Gamma)=v_1$, for each $\Gamma\in\mathbf{\Gamma}$. Also, from Case H1, we know that $V'(H,\underline{\Gamma}_H)=0$. Therefore, $\frac{\partial\tilde{V}(H,0,\Gamma')}{\partial\Gamma'}=\frac{\sigma-v_0}{1-\sigma}+\delta(1-k)v_1\leq 0$ constant along $\Gamma'\in[\underline{\Gamma}_H,\bar{\Gamma}_H]$. Moreover, in Subcase L3a, we have $\bar{\Gamma}_L\geq 1-\sigma H$, therefore Assumption 2 implies that $V'(H,\bar{\Gamma}_L)=0$. Therefore, we must have: $\frac{\partial_-\tilde{V}(L,0,\bar{\Gamma}_L)}{\partial\Gamma'}=\frac{\sigma-v_0}{1-\sigma}+\delta(1-k)v_1\geq 0$. This together with the earlier expression requires that $\frac{\sigma-v_0}{1-\sigma}+\delta(1-k)v_1=0$. Hence, $\frac{\partial\tilde{V}(H,0,\Gamma')}{\partial\Gamma'}=0$, and the platform is indifferent between choosing anything in the interval $\underline{\Gamma}_H,\bar{\Gamma}_H]$ at $(H,0)$. This possibility, however, is already covered in Case H3 (Claim EC.2).

Now consider the coexistence of Subcase L3b together with Case H1, for which a necessary condition is $\bar{\Gamma}_L\leq\underline{\Gamma}_H$. This condition, however, is automatically satisfied since $\sigma<0.5$. Moreover, we have the following optimality conditions.

$$\frac{\partial_+\tilde{V}(H,0,\underline{\Gamma}_H)}{\partial\Gamma'}=\frac{\sigma-v_0}{1-\sigma}+\delta\bar{V}'_+(\underline{\Gamma}_H)\leq 0.$$
$$\frac{\partial_-\tilde{V}(L,0,\bar{\Gamma}_L)}{\partial\Gamma'}=\frac{\sigma-v_0}{1-\sigma}+\delta\bar{V}'_-(\bar{\Gamma}_L)\geq 0.$$

From Case H1, we know that $V'(H,\underline{\Gamma}_H)=0$. In addition, since $\bar{\Gamma}_L<\underline{\Gamma}_H<1-\sigma L$, we have $V'(L,\underline{\Gamma}_H)=V'(L,\bar{\Gamma}_L)=v_1$ (Case L3b). Finally, as we have $\bar{\Gamma}_L<1-\sigma H$ in Case L3b, the condition of Case H1 implies that $V'_-(H,\bar{\Gamma}_L)=\frac{\sigma-v_0}{1-\sigma}+v_1+\delta(1-k)v_1$. We therefore obtain:

$$\frac{\partial_+\tilde{V}(H,0,\underline{\Gamma}_H)}{\partial\Gamma'}=\frac{\sigma-v_0}{1-\sigma}+\delta(1-k)v_1\leq 0.$$
$$\frac{\partial_-\tilde{V}(L,0,\bar{\Gamma}_L)}{\partial\Gamma'}=\frac{\sigma-v_0}{1-\sigma}+\delta\left(k\left[\frac{\sigma-v_0}{1-\sigma}+v_1+\delta(1-k)v_1\right]+(1-k)v_1\right)\geq 0.$$

The following claim summarizes all of these arguments to characterize the conditions under which the optimal solution satisfies Cases H1 and L3b.

Claim EC.3. *In region 2, if the optimal solution* $\Gamma^*(H,\Gamma)$ *satisfies Cases H1 and L3b together, then we must have:*

i) $\sigma < 0.5$.

ii) $v_0 \in [\sigma + (1-\sigma)\delta(1-k)v_1, \sigma + (1-\sigma)\frac{\delta+\delta^2(1-k)k}{1+\delta k}v_1]$.

We now investigate the coexistence of Case H1 and Case L1. We then have:

$$\frac{\partial_+ \tilde{V}(L,0,0)}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + \delta \bar{V}'_+(\underline{\Gamma}_L) \leq 0.$$

We have $V'(L,0) = v_1$ (Case L1). Moreover, we obtain from the conditions of Case H1:

$$\begin{aligned} V'_+(H,0) &= \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta k V'_+(H,H-1) + \delta(1-k)V'_+(L,H-1) \\ &= \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta(1-k)\left(\frac{\sigma - v_0}{1-\sigma} + v_1 + \delta k V'_+(H,0) + \delta(1-k)v_1\right) \end{aligned}$$

We obtain:

$$V'_+(H,0) = \frac{\sigma - v_0}{1-\sigma}\left(\frac{1+\delta(1-k)}{1-\delta^2(1-k)k}\right) + v_1\left(\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1-\delta^2(1-k)k}\right)$$

Therefore the condition that is needed for the optimality of $\underline{\Gamma}_L$ in state $(L,0)$ translates into: $v_0 \geq \sigma + (1-\sigma)\frac{\delta+\delta^2(1-k)k}{1+\delta k}v_1$. Then the next claim summarizes these arguments.

Claim EC.4. *In region 2, if the optimal solution $\Gamma(H,\Gamma)$ satisfies Cases H1 and L1, then we must have: $v_0 \geq \sigma + (1-\sigma)\frac{\delta+\delta^2(1-k)k}{1+\delta k}v_1$.*

We examine whether it is possible to have H1 and L2 in an optimal solution. First, consider Case L2b, i.e., when $\Gamma_2 \geq 1 - \sigma H$, which also requires that $\sigma \geq 0.5$. We know that $1 - L + \Gamma_2 \geq 1 - L + 1 - \sigma H = (1-\sigma)H$ per Assumption 2. Therefore, $\Gamma \leq 1 - L + \Gamma_2$ for all $\Gamma \in \mathbf{\Gamma}$. Since we are in Region L2, this implies that $V'(L,\Gamma) = v_1$ for all $\Gamma \in \mathbf{\Gamma}$. Therefore,

$$\begin{aligned} \frac{\partial \tilde{V}(L,0,\Gamma')}{\partial \Gamma'} &= \frac{\sigma - v_0}{1-\sigma} + \delta\left(kV'(H,\Gamma') + (1-k)V'(L,\Gamma')\right) \\ &= \frac{\sigma - v_0}{1-\sigma} + \delta(1-k)v_1 \qquad \text{because } V'(H,\Gamma) = 0 \text{ (Case H1) and } V'(L,\Gamma) = v_1 \end{aligned}$$

If $\frac{\sigma - v_0}{1-\sigma} + \delta(1-k)v_1 > 0$, then Γ_2 would be suboptimal at $(L,0)$, since the platform could simply choose $\bar{\Gamma}_L$ and get strictly better off. Therefore, we must have $\frac{\sigma - v_0}{1-\sigma} + \delta(1-k)v_1 \leq 0$. If $\frac{\sigma - v_0}{1-\sigma} + \delta(1-k)v_1 = 0$, then all the values within the interval $[1-\sigma H, \bar{\Gamma}_L]$ are optimal in state $(L,0)$. If on the other hand, $\frac{\sigma - v_0}{1-\sigma} + \delta(1-k)v_1 < 0$, then $1 - \sigma H$ is strictly better than all the other values included in the interval $[1-\sigma H, \bar{\Gamma}_L]$ at state $(L,0)$. In consequence, $\Gamma_2 = 1 - \sigma H$ without loss of any generality. We further need to make sure that setting $1 - \sigma H - \varepsilon$ is not strictly better at $(L,0)$. In other words, we must have:

$$\frac{\partial_- \tilde{V}(L,0,\Gamma')}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + \delta\left(kV'_-(H,\Gamma') + (1-k)V'_-(L,\Gamma')\right) \geq 0.$$

But we know that:

$$V'_{-}(H, 1-\sigma H) = \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta(1-k)v_1.$$

Then by using this we can get the corresponding necessary condition. The following claim summarizes these findings.

CLAIM EC.5. *In region 2, if the optimal solution $\Gamma(H,\Gamma)$ satisfies Cases H1 and L2b, then $\Gamma_2 = 1-\sigma H$ without loss of any generality. Moreover, we have:*

i) $\sigma \geq 0.5$.

ii) $v_0 \in [\sigma + (1-\sigma)\delta(1-k)v_1, \sigma + (1-\sigma)\frac{\delta+\delta^2(1-k)k}{1+\delta k}v_1]$.

If we have Case H1 together with Case L2a, i.e, $\Gamma_2 < 1-\sigma H$, and $\sigma \geq 0.5$, then following some algebra one can show that:

$$V'(H,\Gamma) = \begin{cases} \frac{\sigma-v_0}{1-\sigma} + v_1(1+\delta(1-k)) & \text{if } \Gamma \leq \Gamma_2 \\ \frac{\sigma-v_0}{1-\sigma}\left(\frac{1+\delta(1-k)}{1-\delta^2 k(1-k)}\right) + v_1\left(\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1-\delta^2 k(1-k)}\right) & \text{if } \Gamma \in [\Gamma_2, 1-\sigma H] \\ 0 & \text{if } \Gamma \in [1-\sigma H, (1-\sigma)H] \end{cases}$$

$$V'(L,\Gamma) = \begin{cases} v_1 & \text{if } \Gamma \leq 1-L+\Gamma_2 \\ \frac{\sigma-v_0}{1-\sigma}\left(\frac{1+\delta k}{1-\delta^2 k(1-k)}\right) + v_1\left(\frac{1+\delta}{1-\delta^2 k(1-k)}\right) & \text{if } \Gamma \geq 1-L+\Gamma_2 \end{cases}$$

The optimality conditions can be written as follows:

$$\frac{\partial_{-}\tilde{V}(L,0,\Gamma_2)}{\partial \Gamma'} = \frac{\sigma-v_0}{1-\sigma} + \delta \bar{V}'_{-}(\Gamma_2) \geq 0.$$
$$\frac{\partial_{+}\tilde{V}(L,0,\Gamma_2)}{\partial \Gamma'} = \frac{\sigma-v_0}{1-\sigma} + \delta \bar{V}'_{+}(\Gamma_2) \leq 0.$$

It yields:

$$\frac{\sigma-v_0}{1-\sigma} + \delta k\left(\frac{\sigma-v_0}{1-\sigma} + v_1(1+\delta(1-k))\right) + \delta(1-k)v_1 \geq 0$$

$$\frac{\sigma-v_0}{1-\sigma} + \delta k\left(\frac{\sigma-v_0}{1-\sigma}\left(\frac{1+\delta(1-k)}{1-\delta^2 k(1-k)}\right) + v_1\left(\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1-\delta^2 k(1-k)}\right)\right) + \delta(1-k)v_1 \leq 0$$

Following some simple algebra, one can find that these inequalities respectively boil down to:

$$\frac{\sigma-v_0}{1-\sigma}(1+\delta k) + v_1(\delta+\delta^2(1-k)k) \geq 0.$$

$$\frac{\sigma-v_0}{1-\sigma}(1+\delta k) + v_1(\delta+\delta^2(1-k)k) \leq 0.$$

These two conditions are satisfied together if and only if

$$\frac{\sigma-v_0}{1-\sigma}(1+\delta k) + v_1(\delta+\delta^2(1-k)k) = 0.$$

We obtain, for each $\Gamma' \in (\Gamma_2, 1-\sigma H]$:

$$\begin{aligned}
\frac{\partial \tilde{V}(L,0,\Gamma')}{\partial \Gamma'} &= \frac{\sigma - v_0}{1-\sigma} + \delta\left(k \underbrace{V'(H,\Gamma')}_{=V'(H,\Gamma_2)} + (1-k)\underbrace{V'(L,\Gamma')}_{=V'(L,\Gamma_2)}\right) \\
&= \frac{\sigma - v_0}{1-\sigma} + \delta k\left[\frac{\sigma - v_0}{1-\sigma}\left(\frac{1+\delta(1-k)}{1-\delta^2 k(1-k)}\right) + v_1\left(\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1-\delta^2 k(1-k)}\right)\right] + \delta(1-k)v_1 \\
&= 0
\end{aligned}$$

The equality stems from the fact that $V'(H,\Gamma')$, and $V'(L,\Gamma')$ stays constant over $\Gamma' \in (\Gamma_2, 1-\sigma H]$. Therefore, the platform does not get hurt by increasing $\Gamma^*(L,0)$ up to $1-\sigma H$. Hence it is without loss of generality to assume that $\Gamma_2 = 1-\sigma H$, and this case is already covered in Case L2b (Claim EC.5).

If we have Case H1 together with Case L2a', i.e, $\Gamma_2 < 1-\sigma H$, and $\sigma < 0.5$, then following some algebra one can show that:

$$V'(H,\Gamma) = \begin{cases} \frac{\sigma - v_0}{1-\sigma} + v_1(1+\delta(1-k)) & \text{if } \Gamma \le \Gamma_2 \\ \frac{\sigma - v_0}{1-\sigma}\left(\frac{1+\delta(1-k)}{1-\delta^2 k(1-k)}\right) + v_1\left(\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1-\delta^2 k(1-k)}\right) & \text{if } \Gamma \in [\Gamma_2, (1-\sigma)L] \\ \frac{\sigma - v_0}{1-\sigma} + v_1 & \text{if } \Gamma \in [(1-\sigma)L, 1-\sigma H] \\ 0 & \text{if } \Gamma \in [1-\sigma H, (1-\sigma)H] \end{cases}$$

$$V'(L,\Gamma) = \begin{cases} v_1 & \text{if } \Gamma \le 1-L+\Gamma_2 \\ \frac{\sigma - v_0}{1-\sigma}\left(\frac{1+\delta k}{1-\delta^2 k(1-k)}\right) + v_1\left(\frac{1+\delta}{1-\delta^2 k(1-k)}\right) & \text{if } \Gamma \in [1-L+\Gamma_2, 1-\sigma L] \\ 0 & \text{if } \Gamma \in [1-\sigma L, (1-\sigma)H] \end{cases}$$

Then checking the conditions that guarantee the optimality of Γ_2 at $(L,0)$, we get the same expressions above when we analyzed coexistence of Case L2a and Case H1. More precisely, putting the optimality conditions together, we reach that, it is possible to sustain Case H1 together with Case L2a' in an optimal policy, only if the following condition holds.

$$\frac{\sigma - v_0}{1-\sigma}(1+\delta k) + v_1(\delta + \delta^2(1-k)k) = 0.$$

This in turn implies that:

$$\frac{\partial \tilde{V}(L,0,\Gamma')}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + \delta\left(kV'_-(H,\Gamma') + (1-k)V'_-(L,\Gamma')\right) = 0,\ \forall \Gamma' \in (\Gamma_2, (1-\sigma)L].$$

Then the platform is indifferent between setting anything in $(\Gamma_2, (1-\sigma)L]$ in state $(L,0)$. Then this possibility is already covered in the coexistence of Cases H1 and L3b (Claim EC.3).

We conclude by putting Claims EC.1, EC.2, EC.3, EC.5, and EC.4 together.

EC.1.11. Proof of Lemma 5

Let us consider a state (D,Γ_0) such that $\zeta^*(D,\Gamma_0) < \Gamma_0$, i.e., the number of drivers that the platform can assign a late ride in state (D,Γ_0) is lower than the number of agents waiting for a late ride. Thus transferring more people to the next period in this optimal outcome would also increase the number of late rides that the platform generates in this period. However, it is sub-optimal to do so. Therefore, for $\varepsilon > 0$ sufficiently small, we obtain from Equation (13), using the fact that $\frac{\partial \min\{\Gamma_0, 1-D-\Gamma^*(D,\Gamma_0)\}v_1}{\partial \Gamma'} = v_1$:

$$\frac{\partial V(D,\Gamma_0,\Gamma^*(D,\Gamma_0)+\varepsilon)}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta \bar{V}'(\Gamma^*(D,\Gamma_0)+\varepsilon) \leq 0$$

Moreover, for each $\Gamma \geq \Gamma_0$, we have $\frac{\partial \min\{\Gamma_0, 1-D-\Gamma^*(D,\Gamma_0)\}v_1}{\partial \Gamma'} = v_1$, so for $\varepsilon > 0$ sufficiently small:

$$\begin{aligned}\frac{\partial V(D,\Gamma,\Gamma^*(D,\Gamma_0)+\varepsilon)}{\partial \Gamma'} &= \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta \bar{V}'(\Gamma^*(D,\Gamma_0)+\varepsilon) \\ &= \frac{\partial V(D,\Gamma_0,\Gamma^*(D,\Gamma_0)+\varepsilon)}{\partial \Gamma'} \\ &\leq 0\end{aligned}$$

Therefore, at each state (D,Γ) satisfying $\Gamma \geq \Gamma_0$, we have $\Gamma^*(D,\Gamma) \leq \Gamma^*(D,\Gamma_0)$. At the same time, we know from Lemma 1 that $\Gamma^*(D,\Gamma) \geq \Gamma^*(D,\Gamma_0)$. This concludes the proof.

EC.1.12. Proof of Lemma 6

In order to prove Lemma 6 we first show that $V'(D,\Gamma) \leq v_1$ for each $D \in \{H,L\}$. And for this we consider the two cases $D = L$, and $D = H$ separately.

- Let us prove that $V'(L,\Gamma) \leq v_1$. Note, first, that $V'(L,0) = v_1$. Indeed, from Lemma 2, we already know that $\Gamma^*(L,\Gamma)$ is constant over $[0,1-L]$ because $\zeta^*(L,\Gamma) = 1-L+\Gamma^*(L,\Gamma) > \Gamma$ for each $\Gamma < 1-L$. Therefore $\frac{\partial \Gamma^*(L,\Gamma)}{\partial \Gamma} = 0$, and since $1-L+\Gamma*(L,\Gamma) > \Gamma$ when $\Gamma \in [0,1-L)$, we have $V'(L,0) = v_1$ (Equation (EC.2)). Given the concavity of $V(D,\Gamma)$ as a function of Γ (Proposition 3), we obtain $V'(L,\Gamma) \leq v_1$ for each $\Gamma \in \mathbf{\Gamma}$.
- We now prove that $V'(H,\Gamma) \leq v_1$. We consider two possibilities separately: 1) $\Gamma^*(H,0) > \underline{\Gamma}_H$, and 2) $\Gamma^*(H,0) = \underline{\Gamma}_H$.
  - —If $\Gamma^*(H,0) > \underline{\Gamma}_H$, then $\zeta^*(H,0) > 0$. This implies that $\Gamma^*(H,\Gamma) = \Gamma^*(H,0)$ for all $\Gamma \in [0,\zeta^*(H,0))$ (Lemma 2). In other words, we have $\frac{\partial \Gamma^*(H,\Gamma)}{\partial \Gamma} = 0$, and $1-H+\Gamma^*(H,\Gamma) > \Gamma$ for all $\Gamma \in [0,\zeta^*(H,0))$. Then from Equation (EC.2), we must have $V'(H,0) = v_1$. Then $V'(H,\Gamma) \leq v_1$ for all $\Gamma \in \mathbf{\Gamma}$ per concavity.
  - —We now treat the case where $\Gamma^*(H,0) = \underline{\Gamma}_H$.

    If $\Gamma^*(H,\Gamma) = \underline{\Gamma}_H = H-1$ for all $\Gamma \in \mathbf{\Gamma}$. We have $\frac{\partial \Gamma^*(H,\Gamma)}{\partial \Gamma} = 0$. Moreover, $1-H+\Gamma^*(H,\Gamma) \leq 1-H+(1-\sigma)H = 1-\sigma H$ and $\Gamma = H-1 \geq 1-\sigma H$ per Assumption 2. Therefore, $\frac{\partial \min\{\Gamma, 1-H+\Gamma^*(H,\Gamma)\}}{\partial \Gamma} = 0$ and $V'(H,\Gamma) = 0$ fo all $\Gamma \in \mathbf{\Gamma}$.

If $\Gamma^*(H,\Gamma)$ is not constant over $\mathbf{\Gamma}$, then we prove that there exists $\bar{\varepsilon} > 0$ such that $\Gamma^*(H,\varepsilon) = \underline{\Gamma}_H + \varepsilon$ for all $\varepsilon < \bar{\varepsilon}$. Indeed, let us assume that $\Gamma^*(H,\varepsilon) > \underline{\Gamma}_H + \varepsilon$. Then $\zeta^*(H,\varepsilon) > \varepsilon$ and, from Lemma 2, $\Gamma^*(H,0) = \Gamma^*(H,\varepsilon)$. This contradicts that $\Gamma^*(H,0) = \underline{\Gamma}_H$. Conversely, if $\Gamma^*(H,\varepsilon) < \underline{\Gamma}_H + \varepsilon$ for every $\varepsilon > 0$, then $\zeta^*(H,\varepsilon) < \varepsilon$ so from Lemma 5, we need to have $\Gamma^*(H,\Gamma) = \Gamma^*(H,0)$ for all Γ. This contradicts the fact that $\Gamma^*(H,\Gamma)$ is not constant over $\mathbf{\Gamma}$.

Then we have $\frac{\partial \Gamma^*(H,\Gamma)}{\partial \Gamma} = 1$ so per Equation (EC.4):

$$V'(H,0) = \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta \bar{V}'(H,H-1)$$

We now prove that $V'(H,H-1) = 0$. First, note that: $1 - H + \Gamma^*(H,\Gamma) > \Gamma$ for each $\Gamma \geq H-1$, since $1 - H - \bar{\Gamma}_H = 1 - \sigma H < H - 1$ by Assumption 2m so we have $\frac{\partial \min\{\Gamma, 1-H+\Gamma^*(H,\Gamma)\}}{\partial \Gamma} = 0$. We now show that $\Gamma^*(H,\Gamma)$ stays constant for $\Gamma \geq H-1$. By contradiction, suppose $\Gamma^*(H,\Gamma)$ does not stay constant for $\Gamma \geq H-1$. Then by monotonicity (Lemma 1), there exist Γ_0 and Γ_1 satisfying $H - 1 \leq \Gamma_0 < \Gamma_1$, such that $\Gamma^*(H,\Gamma_1) > \Gamma^*(H,\Gamma_0)$, and $\Gamma^*(H,\Gamma_0)$ is not optimal at (H,Γ_1). Therefore:

$$\begin{aligned}
V(H,\Gamma_1) = &\, \sigma H \left(1 - \tfrac{(1-\sigma)H - \Gamma^*(H,\Gamma_1)}{(1-\sigma)H}(1-v_0)\right) + ((1-\sigma)H - \Gamma^*(H,\Gamma_1))v_0 + \\
&(1 - H + \Gamma^*(H,\Gamma_1)) + \delta \bar{V}(\Gamma^*(H,\Gamma_1)) \\
> &\, \sigma H \left(1 - \tfrac{(1-\sigma)H - \Gamma^*(H,\Gamma_0)}{(1-\sigma)H}(1-v_0)\right) + ((1-\sigma)H - \Gamma^*(H,\Gamma_0))v_0 + \\
&(1 - H + \Gamma^*(H,\Gamma_0)) + \delta \bar{V}(\Gamma^*(H,\Gamma_0))
\end{aligned}$$

But from the optimality of $\Gamma^*(H,\Gamma_0)$ in state (H,Γ_0), we know that:

$$\begin{aligned}
V(H,\Gamma_0) = &\, \sigma H \left(1 - \tfrac{(1-\sigma)H - \Gamma^*(H,\Gamma_0)}{(1-\sigma)H}(1-v_0)\right) + \\
&((1-\sigma)H - \Gamma^*(H,\Gamma_0))v_0 + (1 - H + \Gamma^*(H,\Gamma_0)) + \delta \bar{V}(\Gamma^*(H,\Gamma_0)) \\
\geq &\, \sigma H \left(1 - \tfrac{(1-\sigma)H - \Gamma^*(H,\Gamma_1)}{(1-\sigma)H}(1-v_0)\right) + \\
&((1-\sigma)H - \Gamma^*(H,\Gamma_1))v_0 + (1 - H + \Gamma^*(H,\Gamma_1)) + \delta \bar{V}(\Gamma^*(H,\Gamma_1))
\end{aligned}$$

This gives us a contradiction. Therefore, $\Gamma^*(H,\Gamma)$ stays constant for $\Gamma \geq H-1$ and $\frac{\partial \Gamma^*(H,\Gamma)}{\partial \Gamma} = 0$. It comes $V'(H,H-1) = 0$. We also showed that $V'(L,H-1) \leq v_1$. Finally we have $\frac{\sigma - v_0}{1-\sigma} + v_1 < 0$ since we are in Region 3. Therefore , $V'(H,0) \leq 0 + \delta \times (k \times 0 + (1-k) \times v_1) = v_1$, and hence by concavity $V'(H,\Gamma) \leq v_1$, for each $\Gamma \in \mathbf{\Gamma}$.

In summary, we have shown that $V'(D,\Gamma) \leq v_1$ for each $(D,\Gamma) \in \{H,L\} \times \mathbf{\Gamma}$. We now use that result to show that $\Gamma^*(H,0) = \underline{\Gamma}_D$, for each $D \in \{H,L\}$. Suppose by contradiction that $\Gamma^*(D,0) > \underline{\Gamma}_D$ for some $D \in \{H,L\}$. If $\Gamma^*(H,0) > \underline{\Gamma}_H$, then there exists an arbitrarily small $\varepsilon > 0$ such that:

$$\frac{\partial \tilde{V}(H,0,\Gamma^*(H,0)-\varepsilon)}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + \delta \bar{V}'(\Gamma^*(H,0)-\varepsilon)$$

$$\leq \frac{\sigma - v_0}{1-\sigma} + \delta v_1 \qquad \text{from the claim above}$$
$$< 0 \qquad \text{because } v_0 > \sigma + (1-\sigma)v_1 \text{ (Region 3)}$$

This contradicts the fact that $\Gamma^*(H,0)$ is optimal is state $(D,0)$. An analogous logic follows to show that $\Gamma^*(L,0) = \underline{\Gamma}_L$.

EC.1.13. Proof of Proposition 10

Following the preliminary steps that we had in the main text, we can conclude that, in region 3, the policy function $\Gamma^*(H,\Gamma)$ can be in three different shapes, which we denote by A1, A2, and A3:

A1) It is constant and equal to $\underline{\Gamma}_H$ for each $\Gamma \in \mathbf{\Gamma}$.

A2) It increases with slope 1 up to the upper bound $\bar{\Gamma}_H$.

$$\Gamma^*(H,\Gamma) = \begin{cases} \Gamma + H - 1 & \text{if } \Gamma \leq 1 - \sigma H \\ \bar{\Gamma}_H & \text{if } \Gamma \geq 1 - \sigma H \end{cases}$$

A3) It increases with slope 1 up to $\Gamma_1 \in (\underline{\Gamma}_H, \bar{\Gamma}_H)$ and stays constant afterwards.

$$\Gamma^*(H,\Gamma) = \begin{cases} \Gamma + H - 1 & \text{if } \Gamma \leq \Gamma_1 - H + 1 \\ \Gamma_1 & \text{if } \Gamma \geq \Gamma_1 - H + 1 \end{cases}$$

We already know the optimal policy satisfies $\Gamma^*(L,\Gamma) = \underline{\Gamma}_L = 0$ for all $\Gamma \leq 1 - L$. For the same reason that we had three possible values for $\Gamma^*(H,\Gamma)$, the policy function $\Gamma^*(L,\Gamma)$ can be in three different shapes in this region, which we denote by B1, B2, and B3:

B1) It is constant and equal to $\underline{\Gamma}_L = 0$ for each $\Gamma \in \mathbf{\Gamma}$.

B2) It stays constant over $\Gamma \leq 1 - L$, and then it increases with slope 1 as long as it is feasible.

—Sub-case B2a: If $\sigma \geq 0.5$, then $\Gamma^*(L,\Gamma) = \begin{cases} 0 & \text{if } \Gamma \leq 1 - L \\ \Gamma - 1 + L & \text{if } \Gamma \geq 1 - L \end{cases}$

—Sub-case B2b: If $\sigma < 0.5$, then $\Gamma^*(L,\Gamma) = \begin{cases} 0 & \text{if } \Gamma \leq 1 - L \\ \Gamma - 1 + L & \text{if } 1 - L \leq \Gamma \leq 1 - \sigma L \\ \bar{\Gamma}_L & \text{if } \Gamma \geq 1 - \sigma L \end{cases}$

B3) It stays constant over $\Gamma \leq 1 - L$, and then it increases with slope 1 up to a certain value $\Gamma_2 \in (\underline{\Gamma}_L, \bar{\Gamma}_L)$ and stays constant afterwards.

$$\Gamma^*(L,\Gamma) = \begin{cases} 0 & \text{if } \Gamma \leq 1 - L \\ \Gamma + L - 1 & \text{if } \Gamma \in [1 - L, \Gamma_2 + 1 - L] \\ \Gamma_2 & \text{if } \Gamma \in [\Gamma_2 + 1 - L, (1-\sigma)H] \end{cases}$$

In what follows, we analyze the three cases A1, A2, and A3 separately, and try to see under what circumstances these cases may arise in the optimal solution. This step involves in analyzing the coexistence of these cases with the three possibilities that we addressed for the function $\Gamma^*(L,\Gamma)$, i.e., cases B1, B2, and B3.

Case A1: In this case $V(H,\Gamma)$ is constant, and hence $V'(H,\Gamma)=0$, $\forall\Gamma\in\mathbf{\Gamma}$. Moreover, since it is optimal to set $\Gamma^*(H,\Gamma)$ to its minimum value, i.e., $\underline{\Gamma}_H = H-1$, we have $\frac{\partial_+\tilde{V}(H,\Gamma,\underline{\Gamma}_H)}{\partial\Gamma'}\le 0$. It therefore yields, from Equation (EC.4) that:

$$\frac{\sigma-v_0}{1-\sigma}+v_1+\delta(1-k)V'_+(L,\underline{\Gamma}_H)\le 0 \tag{EC.5}$$

Let us first consider the case in which $\frac{\sigma-v_0}{1-\sigma}+v_1+\delta(1-k)v_1\le 0$. In this case we must have $\Gamma^*(L,\Gamma)=0$, $\forall\Gamma\in\mathbf{\Gamma}$, since $V(D,\Gamma)$ is concave and $V'(L,0)=v_1$. Then $\Gamma^*(D,\Gamma)$ remains constant over $\mathbf{\Gamma}$, so Cases A1 and B1 can coexist.

Let us now consider the case where $\frac{\sigma-v_0}{1-\sigma}+v_1+\delta(1-k)v_1>0$. We already know that $\Gamma^*(L,\Gamma)=0$, and $V'(L,\Gamma)=v_1$, for each $\Gamma\in[0,1-L]$. Now, since $\frac{\sigma-v_0}{1-\sigma}+v_1+\delta(1-k)v_1>0$, we must have $\Gamma^*(L,\Gamma)=\Gamma+L-1$ as long as $\Gamma+L-1\le 1-L$. This immediately rules out Case B1. We also know that $\frac{\partial_+\tilde{V}(H,\Gamma,\underline{\Gamma}_H)}{\partial\Gamma'}\le 0$, for each $\Gamma\in\mathbf{\Gamma}$. But we have:

$$\frac{\partial_+\tilde{V}(L,\Gamma,L-1)}{\partial\Gamma'}=\frac{\partial_+\tilde{V}(H,\Gamma,\underline{\Gamma}_H)}{\partial\Gamma'}=\frac{\sigma-v_0}{\sigma}+v_1+\delta\bar{V}'(1-L)\le 0,\quad \forall\Gamma\ge 1-L.$$

Therefore, $\Gamma^*(L,\Gamma)\le\underline{\Gamma}_H=H-1=1-L$ without loss of generality, for each $\Gamma\in\mathbf{\Gamma}$. This yields $\Gamma^*(L,\Gamma)=\min\{\bar{\Gamma}_L,1-L,\Gamma+L-1\}$ for all $\Gamma>1-L$. In addition, note that, from Assumption 2, we have $\bar{\Gamma}_H-1+L<1-L$, so $\Gamma-1+L<1-L$ for all Γ. Therefore, this is incompatible with Case B3. Hence, the only remaining possibility is B2. In order it to coexist with A1, we need to make sure that Equation (EC.5) is satisfied. Since $V'_+(L,1-L)=\frac{\sigma-v_0}{1-\sigma}+v_1+\delta(1-k)v_1$, this is equivalent to:

$$\frac{\sigma-v_0}{1-\sigma}(1+\delta(1-k))+v_1(1+\delta(1-k)+\delta^2(1-k)^2)\le 0.$$

The following claim summarizes these findings.

Claim EC.6. *In region 3, if the optimal solution $\Gamma(H,\Gamma)$ satisfies Case A1, then there are two possibilities:*

i) $v_0\ge\sigma+(1-\sigma)v_1(1+\delta(1-k))$, *and $\Gamma^*(L,\Gamma)$ satisfies Case B1.*

ii) $v_0\in\left[\sigma+(1-\sigma)v_1\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1+\delta(1-k)},\sigma+(1-\sigma)v_1(1+\delta(1-k))\right)$, *and $\Gamma^*(L,\Gamma)$ satisfies Case B2.*

Case A2: In this case, we have, for all $\Gamma\ge\bar{\Gamma}+1-H$, $\frac{\partial_-\tilde{V}(H,\Gamma,\bar{\Gamma}_H)}{\partial\Gamma'}\ge 0$. Note that we have, in Case A2, $\Gamma^*(H,\Gamma)=\Gamma_1$, so $\frac{\partial\Gamma^*(D,\Gamma)}{\partial\Gamma}=0$ for $\Gamma\ge\Gamma_1-H+1$. Moreover, $1-H+\Gamma^*(H,\Gamma)\le 1-H+(1-\sigma)H=1-\sigma H$ and $\bar{\Gamma}_H=H-1\ge 1-\sigma H$ per Assumption 2, which yields $\frac{\partial\min\{\Gamma,1-D+\Gamma^*(D,\Gamma)\}}{\partial\Gamma}v_1$. Per Equation (EC.1), we obtain $V'_-(H,\bar{\Gamma}_H)=0$. This implies that

$$\frac{\sigma-v_0}{1-\sigma}+v_1+\delta(1-k)V'_-(L,\bar{\Gamma}_H))\ge 0$$

Due to the concavity of V (Proposition 3), it implies:

$$\frac{\sigma - v_0}{1-\sigma} + v_1 + \delta(1-k)V'_-(L,\Gamma)) \geq 0, \ \forall \Gamma \in \mathbf{\Gamma}.$$

Therefore, the optimal solution of $\Gamma^*(L,\Gamma)$ must be of the form B2. Moreover, we must have $\sigma \geq 0.5$, because otherwise we would have $1-\sigma L < \bar{\Gamma}_H$ and $V'(L,\bar{\Gamma}_H)=0$, which contradicts the above inequality $v_0 > \sigma + (1-\sigma)v_1$ in Region 3. Then the derivative of the value functions $V(L,\Gamma)$, and $V(H,\Gamma)$ can be written as follows:

$$V'(H,\Gamma) = \begin{cases} \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta \bar{V}'(\Gamma + H - 1) & \text{if } \Gamma \leq 1-\sigma H, \qquad \text{Equation (EC.4)} \\ 0 & \text{if } \Gamma \geq 1-\sigma H. \qquad \text{Equation (EC.1)} \end{cases}$$

$$V'(L,\Gamma) = \begin{cases} v_1 & \text{if } \Gamma \leq 1-L, \qquad \text{Equation (EC.2)} \\ \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta \bar{V}'(\Gamma + L - 1) & \text{if } \Gamma \geq 1-L. \qquad \text{Equation (EC.4)} \end{cases}$$

Since $\bar{\Gamma}_H + L - 1 = 1-\sigma H$ and $\bar{\Gamma}_H + L - 1 \leq 1 - L$ due to Assumption 2, we have:

$$V'_-(L,\bar{\Gamma}_H) = \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta k V'_-(H, 1-\sigma H) + \delta(1-k)v_1.$$

Moreover, we have:

$$\begin{aligned} V'_-(H,1-\sigma H) &= \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta(1-k)V'_-(L,(1-\sigma)H) \quad \text{because } V'_-(H,(1-\sigma)H)=0 \\ &= \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta(1-k)\left(\frac{\sigma - v_0}{1-\sigma} + v_1 + \delta k V'_-(H,1-\sigma H) + \delta(1-k)v_1\right) \\ &\qquad \text{because } (1-\sigma)H \geq 1-L \text{ due to Assumption 1} \\ &= \frac{\sigma - v_0}{1-\sigma}\left(\frac{1+\delta(1-k)}{1-\delta^2(1-k)k}\right) + v_1\left(\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1-\delta^2(1-k)k}\right). \end{aligned}$$

Hence we can get:

$$V'_-(L,\bar{\Gamma}_H) = \frac{\sigma - v_0}{1-\sigma}\left(\frac{1+\delta k}{1-\delta^2(1-k)k}\right) + v_1\left(\frac{1+\delta}{1-\delta^2(1-k)k}\right).$$

Therefore the following comprises a necessary condition:

$$\frac{\sigma - v_0}{1-\sigma}\left(\frac{1+\delta(1-k)}{1-\delta^2(1-k)k}\right) + v_1\left(\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1-\delta^2(1-k)k}\right) \geq 0.$$

The following claim summarizes these findings regarding the case A2.

Claim EC.7. *In region 3, if the optimal solution $\Gamma(H,\Gamma)$ satisfies Case A2 only if:*

i) $\sigma \geq 0.5$*, and* $\Gamma^*(L,\Gamma)$ *satisfies B2a.*

ii) $v_0 < \sigma + (1-\sigma)v_1 \frac{1+\delta(1-k)+\delta^2(1-k)^2}{1+\delta(1-k)}$.

Case A3: As is in Case A2, for each $\Gamma \in \underline{\Gamma}_H, \bar{\Gamma}_H]$, $\Gamma^*(H,\Gamma)$ is constant, and $1-H+\Gamma^*(H,\Gamma) \leq \Gamma$; therefore we have $V'(H,\Gamma)=0$ from Equation (EC.1). Thus, if the optimal policy $\Gamma^*(H,\Gamma)$ is of the form A3, then for all $\Gamma > \Gamma_1 + 1 - H$ we must have:

$$\frac{\partial_- \tilde{V}(H,\Gamma,\Gamma_1)}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta(1-k)V'_-(L,\Gamma_1) \geq 0, \tag{EC.6}$$

$$\frac{\partial_+ \tilde{V}(H,\Gamma,\Gamma_1)}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta(1-k)V'_+(L,\Gamma_1) \leq 0, \tag{EC.7}$$

Moreover we have:

$$V'(H,\Gamma) = \begin{cases} \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta(1-k)V'_L(\Gamma + H - 1) & \text{if } \Gamma \leq \Gamma_1 + 1 - H \\ 0 & \text{if } \Gamma \geq \Gamma_1 + 1 - H \end{cases}$$

We distinguish three cases.

A3-1) If $\Gamma_1 \geq \bar{\Gamma}_L$, and $s \geq 0.5$, then $\Gamma^*(L,\Gamma)$ is of the form B2a. To see this, first note that, due to the optimality of Γ_1 at $\Gamma_1 - H + 1$, we know that:

$$\frac{\sigma - v_0}{1-\sigma} + v_1 + \delta \bar{V}_-(\Gamma_1) \geq 0.$$

But then, for each $\Gamma > 1 - L$, setting the value Γ' equal to $L - 1 + \Gamma$, which is feasible, is optimal. In other words:

$$\frac{\partial \tilde{V}(L,\Gamma,\Gamma + L - 1)}{\partial \Gamma'} = \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta \bar{V}'(\Gamma + L - 1) \geq 0, \quad \forall \Gamma \geq 1 - L.$$

This stems from the fact that $\bar{V}'(\Gamma + L - 1) \geq \bar{V}'(\Gamma_1)$ since $\Gamma + L - 1 < \Gamma_1$ and $V(D,\Gamma)$ is a concave function of Γ. In this case, we also know that:

$$V'_L(\Gamma) = \begin{cases} v_1 & \text{if } \Gamma \leq 1 - L \\ \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta \bar{V}'(\Gamma + L - 1) & \text{if } \Gamma \geq 1 - L \end{cases}$$

Suppose $\Gamma \leq \Gamma_1 + 1 - H$, then we have

$$V'(H,\Gamma) = \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta(1-k)V'(L,\Gamma + H - 1).$$

Moreover, since $H - 1 = 1 - L$ (Assumption 2), we have $\Gamma + H - 1 > 1 - L$, and:

$$V'(L,\Gamma + H - 1) = \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta k V'(H,\Gamma) + \delta(1-k)V'(L,\Gamma)$$

In addition, when $\Gamma \leq \Gamma_1 + 1 - H$, from Assumption 2 we must have $\Gamma \leq 1 - L$ since $\Gamma_1 + 1 - H < (1-\sigma)H + 1 - H = 1 - \sigma H < H - 1 = 1 - L$. Therefore, $V'(L,\Gamma) = v_1$. It comes:

$$V'(H,\Gamma) = \frac{\sigma - v_0}{1-\sigma} + v_1 + \delta(1-k)\left(\frac{\sigma - v_0}{1-\sigma} + v_1 + \delta k V'(H,\Gamma) + \delta(1-k)v_1\right).$$

Therefore,

$$V'(H,\Gamma)=\begin{cases}\frac{\sigma-v_0}{1-\sigma}\left(\frac{1+\delta(1-k)}{1-\delta^2(1-k)k}\right)+v_1\left(\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1-\delta^2(1-k)k}\right) & \text{if } \Gamma\leq\Gamma_1+1-H\\ 0 & \text{if } \Gamma\geq\Gamma_1+1-H\end{cases}$$

$$V'(L,\Gamma)=\begin{cases} v_1 & \text{if } \Gamma\leq 1-L\\ \frac{\sigma-v_0}{1-\sigma}\left(\frac{1+\delta k}{1-\delta^2(1-k)k}\right)+v_1\left(\frac{1+\delta}{1-\delta^2(1-k)k}\right) & \text{if } \Gamma\in[1-L,\Gamma_1]\\ \frac{\sigma-v_0}{1-\sigma}+v_1\left(1+\delta(1-k)\right) & \text{if } \Gamma\geq\Gamma_1\end{cases}$$

Then the inequalities (EC.6), and (EC.7) can be written as:

$$\frac{\sigma-v_0}{1-\sigma}+v_1+\delta(1-k)\left(\frac{\sigma-v_0}{1-\sigma}\left(\frac{1+\delta k}{1-\delta^2(1-k)k}\right)+v_1\left(\frac{1+\delta}{1-\delta^2(1-k)k}\right)\right)\geq 0,$$
$$\frac{\sigma-v_0}{1-\sigma}+v_1+\delta(1-k)\left(\frac{\sigma-v_0}{1-\sigma}+v_1\left(1+\delta(1-k)\right)\right)\leq 0.$$

After rewriting these conditions we get:

$$\frac{\sigma-v_0}{1-\sigma}\left(\frac{1+\delta(1-k)}{1-\delta^2(1-k)k}\right)+v_1\left(\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1-\delta^2(1-k)k}\right)\geq 0,$$
$$\frac{\sigma-v_0}{1-\sigma}\left(1+\delta(1-k)\right)+v_1\left(1+\delta(1-k)+\delta^2(1-k)^2\right)\leq 0.$$

These conditions can hold at the same time if and only if both holds with equality. But this means that the value function $V(H,\Gamma)$ is constant and hence $\Gamma^*(H,\Gamma)$ could be increased up to $\bar{\Gamma}_H$ (the value function would be unchanged). This, however, is already covered in case A2

A3-2) If $\Gamma_1\geq\bar{\Gamma}_L$, and $s<0.5$, then by proceeding as previously, we show that $\Gamma^*(L,\Gamma)$ is of the form B2. But since not all the values of $\Gamma+L-1$ are included in $[\underline{\Gamma}_L,\bar{\Gamma}_L]$ because $\bar{\Gamma}_H-L+1>\bar{\Gamma}_L$ (since $\sigma<0.5$), $\Gamma^*(L,\Gamma)$ is now of the form B2b. Note that $V'(L,\Gamma)=0$, for every $\Gamma\in[1-\sigma L,(1-\sigma)H]$. This implies that we must have $\Gamma_1\leq 1-\sigma L$; because otherwise $V'(H,\Gamma_1+1-H-\varepsilon)=\frac{\sigma-v_0}{1-\sigma}+v_1+\delta(1-k)V'(L,\Gamma+H-1)$ for $\varepsilon>0$ infinitesimally small. Then by following similar steps with the previous case, we reach:

$$V'(L,\Gamma)=\begin{cases} v_1 & \text{if } \Gamma\leq 1-L\\ \frac{\sigma-v_0}{1-\sigma}\left(\frac{1+\delta k}{1-\delta^2(1-k)k}\right)+v_1\left(\frac{1+\delta}{1-\delta^2(1-k)k}\right) & \text{if } \Gamma\in[1-L,\Gamma_1]\\ \frac{\sigma-v_0}{1-\sigma}+v_1\left(1+\delta(1-k)\right) & \text{if } \Gamma\in[\Gamma_1,1-\sigma L]\\ 0 & \text{if } \Gamma\in[1-\sigma L,(1-\sigma)H]\end{cases}$$

Following the arguments of the previous section one can see that $\Gamma_1=1-\sigma L$, and the following constitutes a necessary condition.

$$\frac{\sigma-v_0}{1-\sigma}+v_1+\delta(1-k)\left(\frac{\sigma-v_0}{1-\sigma}\left(\frac{1+\delta k}{1-\delta^2(1-k)k}\right)+v_1\left(\frac{1+\delta}{1-\delta^2(1-k)k}\right)\right)\geq 0,$$

which is equivalent to:

$$\frac{\sigma-v_0}{1-\sigma}\left(\frac{1+\delta(1-k)}{1-\delta^2(1-k)k}\right)+v_1\left(\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1-\delta^2(1-k)k}\right)\geq 0.$$

A3-3) If $\Gamma_1 \le \bar{\Gamma}_L$, then Γ_1 is amongst the possible realizations of $\Gamma^*(L,\Gamma)$. However, in this case, the value of $\Gamma + L - 1$, the maximal value of Γ' in state (L,Γ), is always less than Γ_1. To see this note that $\Gamma + L - 1 \le (1-\sigma)H + L - 1 = 1 - \sigma H < H - 1 < \Gamma_1$ from Assumption 2. This suggests that the optimal policy $\Gamma'(L,\Gamma)$ is linearly increasing with slope 1 when $\Gamma > 1 - L$, but it never reaches to the maximal value. The former point follows from the fact that $\frac{\sigma - v_0}{1-\sigma} + v_1 + \bar{V}'_-(\Gamma_1) \ge 0$, and due to the concavity of $\bar{V}$, $\frac{\sigma - v_0}{1-\sigma} + v_1 + \bar{V}'_-(\Gamma') \ge 0$ for each $\Gamma' \le \Gamma_1$. In consequence, the optimal policy function $\Gamma^*(L,\Gamma)$ is in form of B2a and $\sigma \ge 0.5$. We conclude as in the case where $\Gamma_1 > \underline{\bar{\Gamma}}$ and $\sigma \ge 0.5$ (Case A3-1).

The following claim summarizes our analysis in Case A3 . It points out that, if the policy function $\Gamma^*(H,\Gamma)$ is of the form A3, then we can only have the second possibility (A3-2) out of the three.

CLAIM EC.8. *In region 3, if the optimal solution $\Gamma(H,\Gamma)$ satisfies Case A3 only if:*

i) $\Gamma_1 = 1 - \sigma L$.

ii) $\sigma < 0.5$ *and* $\Gamma^*(L,\Gamma)$ *is of the form B2b.*

iii) $v_0 \le \sigma + (1-\sigma)v_1 \frac{1+\delta(1-k)+\delta^2(1-k)^2}{1+\delta(1-k)}$,

We conclude by putting Claims EC.6, EC.8, and EC.7 together.

Appendix EC.2: Proofs on First-best and Dynamic Pricing Mechanisms

EC.2.1. Proof of Proposition 5

We first characterize properties of the optimal solution in Lemma EC.1, analogous to Proposition 2. The proof of these properties follows from the same arguments, and is thus omitted for conciseness.

LEMMA EC.1. *The optimal solution to Problem $\mathcal{P}_{FB}$ satisfies:*

(i) $q_r^{FB}(D,\Gamma) = 1$ *for each* $(D,\Gamma) \in \{H,L\} \times \mathbf{\Gamma}$.

(ii) If $\Gamma > 0$, *then* $q_l^{FB}(D,\Gamma) = \min\left\{1, \frac{1-\sigma D - q_t^{FB}(D,\Gamma)(1-\sigma)D}{\Gamma}\right\}$.

We now turn to the proof of Proposition 5. We have, by definition, $\tilde{v}_0 = \sigma + (1-\sigma)v_0$. By construction, the following equality is satisfied:

$$\sigma D\left(1 - \frac{(1-\sigma)D - \Gamma'}{(1-\sigma)D}(1-\tilde{v}_0)\right) + ((1-\sigma)D - \Gamma')\tilde{v}_0 = \sigma D + ((1-\sigma)D - \Gamma')\, v_0$$

From Equation (9), Problem $\mathcal{P}$ with valuation parameter $\tilde{v}_0$ is given by:

$$\begin{aligned} V(D,\Gamma) = &\max_{\Gamma' \in [\underline{\Gamma}_D, \bar{\Gamma}_D]} \left\{ \sigma D\left(1 - \frac{(1-\sigma)D - \Gamma'}{(1-\sigma)D}(1-\tilde{v}_0)\right) + ((1-\sigma)D - \Gamma')\tilde{v}_0 + \min\{\Gamma, 1 - D - \Gamma'\}v_1 \right. \\ &\left. \qquad + \delta\left(kV(H,\Gamma') + (1-k)V(L,\Gamma')\right) \right\} \\ = &\max_{\Gamma' \in [\underline{\Gamma}_D, \bar{\Gamma}_D]} \sigma D + ((1-\sigma)D - \Gamma')\, v_0 + \min\{\Gamma, 1 - D + \Gamma'\}v_1 + \delta\left(kV_f(H,\Gamma') + (1-k)V_f(L,\Gamma)\right) \end{aligned}$$

From Equation (20), we obtain that the problem is equivalent to Problem $\mathcal{P}_{FB}$. The optimal policy is then obtained by applying the result of Proposition 10 by noting that $\tilde{v}_0 \in (\sigma + (1-\sigma)v_1, 1)$ for any $v_0 \in (v_1, 1)$ and that the relationship between v_0 and $\tilde{v}_0$ is monotonic. Specifically, we have:

- $\tilde{v}_0 \in (\sigma + (1-\sigma)v_1, \underline{\underline{v}}_1]$, if and only if $v_0 \in \left(v_1, \frac{1+\delta(1-k)+\delta^2(1-k)^2}{1+\delta(1-k)} v_1\right]$.
- $\tilde{v}_0 \in (\underline{\underline{v}}_1, \bar{\bar{v}}_1]$, if and only if $v_0 \in \left(\frac{1+\delta(1-k)+\delta^2(1-k)^2}{1+\delta(1-k)} v_1, (1+\delta(1-k))v_1\right]$.
- $\tilde{v}_0 \in (\bar{\bar{v}}_1, 1)$, if and only if $v_0 \in ((1+\delta(1-k))v_1, 1)$.

EC.2.2. Proof of Proposition 6

From the formulation of $\mathcal{P}_{DP}$, the Blackwell sufficiency conditions (monotonicity, and discounting) are clearly satisfied, hence a solution to Problem $\mathcal{P}_{DP}$ always exists. Moreover, since the quantity of rides provided $x(D, \Gamma, p_0)$, as well as the number of people transferred to the next period Γ' both stay constant over $(0, v_1]$, $(v_1, v_0]$, and $(v_0, 1]$, the optimal price always satisfies $p_0(D, \Gamma) \in \{1, v_0, v_1\}$.

In a given state (D, Γ), we assume, without loss of generality, that in case the platform is indifferent between charging $p_0 \in \{1, v_0\}$ and v_1, it chooses price p_0. We first prove that, if $p_0^*(D, \Gamma) \in \{1, v_0\}$ for each (D, Γ), then the optimal policy follows the one given in the proposition. We then show that this policy is applied if and only if $v_1 \leq \max\{\sigma L, v_0 L\}$.

We assume that $p_0^*(D, \Gamma) \in \{1, v_0\}$ for each (D, Γ). In this case, $V_{DP}(D, \Gamma)$ does *not* depend on the value of Γ. Therefore, the platform determines the price p_0 by optimizing the current-period profit, i.e. $p_0 \times x(D, \Gamma, p_0)$. Specifically, for each Γ, $p_0^*(H, \Gamma) = 1$ if $\sigma H \geq v_0$, and $p_0^*(H, \Gamma) = v_0$ otherwise. Similarly, $p_0^*(L, \Gamma) = 1$ if $\sigma L \geq v_0 L$, and $p_0^*(L, \Gamma) = v_0$ otherwise. We therefore obtain the policy given in the proposition.

We now show that this policy is applied if and only if $v_1 \leq \max\{\sigma L, v_0 L\}$.

- Let us assume that $p_0^*(D, \Gamma) \in \{1, v_0\}$ for each (D, Γ) and, by contradiction, that $v_1 > \max\{\sigma L, v_0 L\}$. Note that, since $p_0^*(D, \Gamma) \in \{1, v_0\}$, $V_s(D, \Gamma)$ does not depend on Γ. Let us consider $\Gamma_0 > 1 - L$. We have:

  $$\text{If } p_0^*(L, \Gamma_0) = 1, \text{ then } V_{DP}(L, \Gamma_0) = \sigma L + \delta\left(kV_{DP}(H, (1-\sigma)L) + (1-k)V_{DP}(L, (1-\sigma)L)\right)$$
  $$\text{If } p_0^*(L, \Gamma_0) = v_0, \text{ then } V_{DP}(L, \Gamma_0) = v_0 L + \delta\left(kV_{DP}(H, 0) + (1-k)V_{DP}(L, 0)\right)$$

  However, if the platform deviates in the period under consideration by charging a price v_1 without changing the policy in any of the subsequent periods, then its total expected discounted profit, denoted by $\tilde{V}_s(L, \Gamma_0, v_1)$, becomes:

  $$\tilde{V}_{DP}(L, \Gamma_0, v_1) = v_1 + \delta\left(kV_{DP}(H, \Gamma') + (1-k)V_{DP}(L, \Gamma')\right) \quad \text{where } \Gamma' = (1-\sigma)L\left[1 - \frac{1}{L + \Gamma_0}\right]$$

  Since, by assumption, $v_1 > \max\{\sigma L, v_0 L\}$, and $V_{DP}(D, \Gamma)$ does not depend on Γ, we obtain that $\tilde{V}_{DP}(L, \Gamma_0, v_1) > V_s(L, \Gamma_0)$. This contradicts the optimality of $p_0^*(L, \Gamma_0)$.

- Let us assume that $v_1 \leq \max\{\sigma L, v_0 L\}$. Note that setting $p_0 = v_1$ can bring a per-period profit of at most v_1. However, we have identified above a policy that yields a per-period profit of $\max\{\sigma L, v_0 L\}$ in low-demand periods and of $\max\{\sigma H, v_0\}$ in high-demand periods. Therefore, this policy dominates any policy that involves charging v_1 in some states.

Appendix EC.3: Proofs on Static and Demand-dependent Mechanisms

EC.3.1. Proof of Proposition 7

For each $(D,\Gamma) \in \{H, L\} \times \{\Gamma_S^H, \Gamma_S^L\}$, we have:

$$V_S(D,\Gamma) = \pi(D,\Gamma) + \delta\left(kV_S(H,\Gamma_S^D) + (1-k)V_S(L,\Gamma_S^D)\right),$$

where $\pi(D,\Gamma) = q_r\sigma D p_r + q_t(1-\sigma)Dp_t + q_l\Gamma p_l$ is the within-period profit. Following some algebra:

$$\begin{aligned}\overline{V}_S(0) &= k\pi(H,0) + (1-k)\pi(L,0)\\ &+ \frac{\delta}{1-\delta}\left(k^2\pi(H,\Gamma_S^H) + k(1-k)\pi(H,\Gamma_S^L) + (1-k)k\pi(L,\Gamma_S^H) + (1-k)^2\pi(L,\Gamma_S^L)\right)\end{aligned}$$

Under the optimal static menu, the capacity constraint must bind in state $(D,\Gamma) = (H, \Gamma_S^H)$:

$$q_r\sigma H + q_t(1-\sigma)H + q_l\underbrace{(1-q_t)(1-\sigma)H}_{=\Gamma_S^H} = 1.$$

From this equation, we obtain:

$$\begin{aligned} q_t &\leq \frac{1-\sigma H}{(1-\sigma)H},\\ q_l &= \frac{1-\sigma H - q_t(1-\sigma)H}{(1-q_t)(1-\sigma)H}.\end{aligned}$$

Following the same logic as in Proposition 2, we can see that the optimal solution satisfies:

$$q_r = 1, \qquad p_t = v_0, \qquad p_l = v_1, \qquad p_r = 1 - q_t(1-v_0).$$

Therefore, the platform's profit-maximization problem becomes:

$$\max_{q_t \leq \frac{1-\sigma H}{(1-\sigma)H}} \left\{\begin{array}{l} k\left[\sigma H(1-q_t(1-v_0)) + q_t v_0(1-\sigma)H\right] + (1-k)\left[\sigma L(1-q_t(1-v_0)) + q_t v_0 (1-\sigma)L\right]\\ \frac{\delta k^2}{1-\delta}\left[\sigma H(1-q_t(1-v_0)) + q_t v_0(1-\sigma)H + \frac{1-\sigma H - q_t(1-\sigma)H}{(1-q_t)(1-\sigma)H}(1-q_t)(1-\sigma)Hv_1\right]\\ +\frac{\delta k(1-k)}{1-\delta}\left[\sigma H(1-q_t(1-v_0)) + q_t v_0(1-\sigma)H + \frac{1-\sigma H - q_t(1-\sigma)H}{(1-q_t)(1-\sigma)H}(1-q_t)(1-\sigma)Lv_1\right]\\ +\frac{\delta(1-k)k}{1-\delta}\left[\sigma L(1-q_t(1-v_0)) + q_t v_0(1-\sigma)L + \frac{1-\sigma H - q_t(1-\sigma)H}{(1-q_t)(1-\sigma)H}(1-q_t)(1-\sigma)Hv_1\right]\\ +\frac{\delta(1-k)^2}{1-\delta}\left[\sigma L(1-q_t(1-v_0)) + q_t v_0(1-\sigma)L + \frac{1-\sigma H - q_t(1-\sigma)H}{(1-q_t)(1-\sigma)H}(1-q_t)(1-\sigma)Lv_1\right]\end{array}\right\}$$

Following some algebra, the problem becomes:

$$\max_{q_t \leq \frac{1-\sigma H}{(1-\sigma)H}} \quad Constant + \frac{1}{1-\delta}\left[kH + (1-k)L\right]\left[q_t(v_0 - \sigma - \delta(1-\sigma)v_1)\right]$$

Therefore the optimal solution satisfies: $q_t = 0$ and $q_l = \frac{1-\sigma H}{(1-\sigma)H}$ if $v_0 \leq \sigma + \delta(1-\sigma)v_1$, and $q_t = \frac{1-\sigma H}{(1-\sigma)H}$ and $q_l = 0$ otherwise.

EC.3.2. Proof of Proposition 11

For each $(D,\Gamma) \in \{H,L\} \times \{\Gamma^H_{DD}, \Gamma^L_{DD}\}$, we have:

$$V_{DD}(D,\Gamma) = \pi(D,\Gamma) + \delta\left(kV_{DD}(H,\Gamma^D_{DD}) + (1-k)V_{DD}(L,\Gamma^D_{DD})\right),$$

where $\pi(D,\Gamma) = q_r(D)\sigma D p_r(D) + q_t(D)(1-\sigma)Dp_t(D) + q_l(D,\Gamma)\Gamma p_l(D,\Gamma)$ is the within-period profit. Following some algebra, we get:

$$\begin{aligned}\overline{V}_{DD}(0) &= k\pi(H,0) + (1-k)\pi(L,0) \\ &+ \frac{\delta}{1-\delta}\left(k^2\pi(H,\Gamma^H_{DD}) + k(1-k)\pi(H,\Gamma^L_{DD}) + (1-k)k\pi(L,\Gamma^H_{DD}) + (1-k)^2\pi(L,\Gamma^L_{DD})\right)\end{aligned}$$

Following the same logic as in Proposition 2, we can see that the optimal solution satisfies:

$$\begin{aligned} q_r(D) &= 1,\ \forall D \in \{H,L\}, \\ p_t(D) &= v_0,\ \forall D \in \{H,L\}, \\ p_l(D,\Gamma) &= v_1,\ \forall (D,\Gamma) \in \left\{\{H,L\} \times \{\Gamma^H_{DD}, \Gamma^L_{DD}\}\right\} \\ p_r(D) &= 1 - q_t(D)(1-v_0),\ \forall D \in \{H,L\}, \\ q_t(H) &\le \frac{1-\sigma H}{(1-\sigma)H}. \end{aligned}$$

For a given menu, the number of late services provided in state $(D,\Gamma) \in \{\{H,L\} \times \{\Gamma^H_{DD}, \Gamma^L_{DD}\}\}$ is given by $\min\{R_D, \Gamma\}$, where $R_D = 1 - \sigma D - q_t(D)(1-\sigma)D$ is the number of suppliers that remain available upon provision of timely services. The platform's problem becomes:

$$\max_{\substack{q_t(H) \le \frac{1-\sigma H}{(1-\sigma)H}, \\ q_t(L)}} \left\{ \begin{array}{l} k\left[\sigma H(1-q_t(H)(1-v_0)) + q_t(H)v_0(1-\sigma)H\right] \\ +(1-k)\left[\sigma L(1-q_t(L)(1-v_0)) + q_t(L)v_0(1-\sigma)L\right] \\ +\frac{\delta k^2}{1-\delta}\left[\sigma H(1-q_t(H)(1-v_0)) + q_t(H)v_0(1-\sigma)H + v_1\min\{R^H, \Gamma^H_{DD}\}\right] \\ +\frac{\delta k(1-k)}{1-\delta}\left[\sigma H(1-q_t(H)(1-v_0)) + q_t(H)v_0(1-\sigma)H + v_1\min\{R^H, \Gamma^L_{DD}\}\right] \\ +\frac{\delta(1-k)k}{1-\delta}\left[\sigma L(1-q_t(L)(1-v_0)) + q_t(L)v_0(1-\sigma)L + v_1\min\{R^L, \Gamma^H_{DD}\}\right] \\ +\frac{\delta(1-k)^2}{1-\delta}\left[\sigma L(1-q_t(L)(1-v_0)) + q_t(L)v_0(1-\sigma)L + v_1\min\{R^L, \Gamma^L_{DD}\}\right] \end{array} \right\}$$

Note that $1-\sigma H - q_t(H)(1-\sigma)H \le (1-q_t(H))(1-\sigma)H$, since $1-\sigma H \le (1-\sigma)H$; therefore $\min\{R^H, \Gamma^H_{DD}\} = R^H$. Similarly $1-\sigma L - q_t(L)(1-\sigma)L \ge (1-q_t(L))(1-\sigma)H$, since $1-\sigma L \le (1-\sigma)L$; therefore $\min\{R^L, \Gamma^L_{DD}\} = \Gamma^L_{DD}$. We then distinguish two cases, following simple algebra:

1) If $q_t(H)H - q_t(L)L \ge \frac{d(1-2\sigma)}{1-\sigma}$, then $\min\{R^H, \Gamma^L_{DD}\} = R^H$ and $\min\{R^L, \Gamma^H_{DD}\} = \Gamma^H_{DD}$.
2) If $q_t(H)H - q_t(L)L \le \frac{d(1-2\sigma)}{1-\sigma}$, then $\min\{R^H, \Gamma^L_{DD}\} = \Gamma^L_{DD}$ and $\min\{R^L, \Gamma^H_{DD}\} = R^L$.

In order to solve the above problem, we define two sub-problems, which we refer to as $(\mathcal{P}^1_{DD})$ and $(\mathcal{P}^2_{DD})$, based on whether $q_t(H)H - q_t(L)L \ge \frac{d(1-2\sigma)}{1-\sigma}$ or not. Then by comparing the solutions of Problems $(\mathcal{P}^1_{DD})$ and $(\mathcal{P}^2_{DD})$, we will derive the optimal solution of the overall Problem $(\mathcal{P}_{DD})$.

Problem $(\mathcal{P}^1_{DD})$ With $\min\{R^H, \Gamma^L_{DD}\} = R^H$, $\min\{R^L, \Gamma^H_{DD}\} = \Gamma^H_{DD}$, the problem becomes:

$$\max_{\substack{q_t(L) \\ q_t(H) \le \frac{1-\sigma H}{(1-\sigma)H}}} \quad Constant + \frac{1}{1-\delta}\begin{Bmatrix} q_t(H)Hk\left[v_0 - \sigma - \delta v_1(1-\sigma)(2-k)\right] \\ +q_t(L)L(1-k)\left[v_0 - \sigma - \delta v_1(1-\sigma)(1-k)\right] \end{Bmatrix}$$
$$\text{s.t.} \quad q_t(H)H - q_t(L)L \ge \frac{d(1-2\sigma)}{1-\sigma}.$$

- When $\sigma < 0.5$
  - —When $v_0 \le \sigma + \delta(1-\sigma)v_1$, the solution to Problem $(\mathcal{P}^1_{DD})$ satisfies:
  $$q_t(H) = \frac{d(1-2\sigma)}{(1-\sigma)H}, \quad q_t(L) = 0.$$
  - —When $v_0 \ge \sigma + \delta(1-\sigma)v_1$, the solution to Problem $(\mathcal{P}^1_{DD})$ satisfies:
  $$q_t(H) = \frac{1-\sigma H}{(1-\sigma)H}, \quad q_t(L) = 1$$
- When $\sigma \ge 0.5$.
  - —When $v_0 \le \sigma + \delta(1-\sigma)v_1(1-k)$, the solution to Problem $(\mathcal{P}^1_{DD})$ satisfies:
  $$q_t(H) = 0, \quad q_t(L) = 0.$$
  - —When $v_0 \in [\sigma + \delta(1-\sigma)v_1(1-k), \sigma + \delta(1-\sigma)v_1]$, the solution to Problem $(\mathcal{P}^1_{DD})$ satisfies:
  $$q_t(H) = 0, \quad q_t(L) = \frac{d(2\sigma-1)}{(1-\sigma)L}.$$
  - —When $v_0 \ge \sigma + \delta(1-\sigma)v_1$, the solution to Problem $(\mathcal{P}^1_{DD})$ satisfies:
  $$q_t(H) = \frac{1-\sigma H}{(1-\sigma)H}, \quad q_t(L) = 1$$

Problem $(\mathcal{P}^2_{DD})$ With $\min\{R^H, \Gamma^L_{DD}\} = \Gamma^L_{DD}$, $\min\{R^L, \Gamma^H_{DD}\} = R^L$, the problem becomes:

$$\max_{\substack{q_t(L) \\ q_t(H) \le \frac{1-\sigma H}{(1-\sigma)H}}} \quad Constant + \frac{1}{1-\delta}\begin{Bmatrix} q_t(H)Hk\left[v_0 - \sigma - \delta v_1(1-\sigma)k\right] \\ +q_t(L)L(1-k)\left[v_0 - \sigma - \delta v_1(1-\sigma)(1+k)\right] \end{Bmatrix}$$
$$\text{s.t.} \quad q_t(H)H - q_t(L)L \le \frac{d(1-2\sigma)}{1-\sigma}.$$

- When $\sigma < 0.5$
  - —When $v_0 \le \sigma + \delta(1-\sigma)v_1 k$, the solution to Problem $(\mathcal{P}^2_{DD})$ satisfies:
  $$q_t(H) = 0, \quad q_t(L) = 0.$$
  - —When $v_0 \in [\sigma + \delta(1-\sigma)v_1 k, \sigma + \delta(1-\sigma)v_1]$, the solution to Problem $(\mathcal{P}^2_{DD})$ satisfies:
  $$q_t(H) = \frac{d(1-2\sigma)}{(1-\sigma)H}, \quad q_t(L) = 0.$$

—When $v_0 \geq \sigma + \delta(1-\sigma)v_1$, the solution to Problem $(\mathcal{P}^2_{DD})$ satisfies:

$$q_t(H) = \frac{1-\sigma H}{(1-\sigma)H}, \quad q_t(L) = 1.$$

- When $\sigma \geq 0.5$.

  —When $v_0 \leq \sigma + \delta(1-\sigma)v_1$, the solution to Problem $(\mathcal{P}^1_{DD})$ satisfies:

$$q_t(H) = 0, \quad q_t(L) = \frac{d(2\sigma - 1)}{(1-\sigma)H}.$$

  —When $v_0 \geq \sigma + \delta(1-\sigma)v_1$, the solution to Problem $(\mathcal{P}^1_{DD})$ satisfies:

$$q_t(H) = \frac{1-\sigma H}{(1-\sigma)L}, \quad q_t(L) = 1$$

Now by using the solutions of $(\mathcal{P}^1_{DD})$ and $(\mathcal{P}^2_{DD})$, we characterize the solution to Problem $(\mathcal{P}_{DD})$.

- When $\sigma < 0.5$, the solution to problem $(\mathcal{P}^2_{DD})$ solves the global problem. Indeed, the solutions of $(\mathcal{P}^1_{DD})$ and $(\mathcal{P}^2_{DD})$ differ only when $v_0 \leq \sigma + \delta(1-\sigma)v_1 k$; in this region, the solution to Problem $(\mathcal{P}^2_{DD})$ $(p_t(H) = p_t(L) = 0)$ yields a higher value of the objective function.
- When $\sigma \geq 0.5$, the solution to problem $(\mathcal{P}^1_{DD})$ solves the global problem. Indeed, the solutions of $(\mathcal{P}^1_{DD})$ and $(\mathcal{P}^2_{DD})$ differ only when $v_0 \leq \sigma + \delta(1-\sigma)v_1(1-k)$; in this region, the solution to Problem $(\mathcal{P}^1_{DD})$ $(p_t(H) = p_t(L) = 0)$ yields a higher value of the objective function.

EC.3.3. Proof of Proposition 8

Recall that the maximum relative gain is defined as:

$$R_{max} = \max_{v_0 \in (v_1, 1)} R(v_0), \quad \text{where } R(v_0) = \frac{\overline{V}(0) - \overline{V}_{DD}(0)}{\overline{V}_{DD}(0)}.$$

Proof that $R_{max} \geq R_1(k, d, v_1, \sigma)$. Let us denote $v_0^+ = \sigma + \delta(1-\sigma)v_1$, which defines the boundary between Regions B and C for the demand-dependent mechanism. Moreover, as $\delta \to 1$, v_0^+ converges to the boundary between Region 2c and Region 3a for the optimal mechanism.

Demand-dependent Mechanism:

In Region B, $\Gamma^L_{DD} = (1-\sigma)L$ and $\Gamma^H_{DD} = 1 - \sigma L$. Let $\overline{\mathbf{V}}_{DD}$ be the vector of steady-state values:

$$\overline{\mathbf{V}}_{DD} = \begin{bmatrix} \overline{V}_{DD}(\Gamma^L_{DD}) \\ \overline{V}_{DD}(\Gamma^H_{DD}) \end{bmatrix}.$$

We have:

$$\overline{\mathbf{V}}_{DD} = \begin{bmatrix} (1-k)\left[\sigma L + (1-\sigma)Lv_1\right] + k[\sigma H p_r^\beta + (1-2\sigma)dv_0 + (1-\sigma)Lv_1] \\ (1-k)[\sigma L + (1-\sigma L)v_1] + k[\sigma H p_r^\beta + (1-2\sigma)dv_0 + (1-\sigma)Lv_1] \end{bmatrix} + \delta \begin{bmatrix} (1-k) & k \\ (1-k) & k \end{bmatrix} \times \overline{\mathbf{V}}_{DD},$$

where

$$p_r^\beta = 1 - \frac{(1-2\sigma)d}{(1-\sigma)H}(1-v_0).$$

Moreover, we have:

$$\overline{V}_{DD}(0) = (1-k)\left[\sigma L\right] + k\left[\sigma H\right] + \delta\left[(1-k)\ \ k\right] \times \overline{\mathbf{V}}_{DD}$$

After some algebra, we get:

$$\overline{V}_{DD}(0) = \tfrac{1}{1-\delta}\Big[\sigma L(1-k) + \underbrace{\sigma H p_r^{\beta} k + (1-2\sigma)dv_0 k}_{=\sigma Hk + (1-2\sigma)d\frac{v_0-\sigma}{1-\sigma}k} + \delta(1-\sigma)Lv_1(1-k+k^2) + \delta(1-\sigma L)v_1(1-k)k\Big]$$

General Mechanism:

In Region 2c, there are 5 steady-state values of Γ: $0 < (1-\sigma)L < H-1 < 1-\sigma L < (1-\sigma)H$. Moreover, we know that $\overline{V}(1-\sigma L) = \overline{V}((1-\sigma)H)$, since the first-period and continuation profits are identical with these two different values of Γ. Then, let $\overline{\mathbf{V}}$ be a vector defined as:

$$\overline{\mathbf{V}} = \begin{bmatrix} \overline{V}(0) \\ \overline{V}((1-\sigma)L) \\ \overline{V}(H-1) \\ \overline{V}(1-\sigma L) \end{bmatrix}.$$

We have:

$$\overline{\mathbf{V}} = \underbrace{\begin{bmatrix} (1-k)\left[Lv_0\right] + k[\sigma H p_r^{\alpha} + (1-\sigma H)v_0] \\ (1-k)\left[Lv_0 + (1-\sigma)Lv_1\right] + k[\sigma H p_r^{\beta} + (1-2\sigma)dv_0 + (1-\sigma)Lv_1] \\ (1-k)\left[Lv_0 + dv_1\right] + k[\sigma H + (1-\sigma H)v_1] \\ (1-k)\left[\sigma L + (1-\sigma L)v_1\right] + k[\sigma H + (1-\sigma H)v_1] \end{bmatrix}}_{\mathbf{F}} + \delta \underbrace{\begin{bmatrix} (1-k) & 0 & k & 0 \\ (1-k) & 0 & 0 & k \\ (1-k) & 0 & 0 & k \\ 0 & (1-k) & 0 & k \end{bmatrix}}_{\mathbf{A}} \times \overline{\mathbf{V}},$$

where

$$p_r^{\alpha} = 1 - \frac{1-\sigma H}{(1-\sigma)H}(1-v_0), \text{ and } \ p_r^{\beta} = 1 - \frac{(1-2\sigma)d}{(1-\sigma)H}(1-v_0).$$

Then $\overline{\mathbf{V}} = (\mathbf{I} - \delta\mathbf{A})^{-1}\mathbf{F}$ yields:

$$\overline{V}(0) = \frac{1}{1-\delta}\left(\sum_{i=1}^{4} \frac{\mu_i}{\mu_1+\mu_2+\mu_3+\mu_4}\ \mathbf{F}(i)\right),$$

where

$$\mu_1 = 1 + \delta^2k^2 - \delta k - \delta^2 k,$$
$$\mu_2 = \delta^3k^2 - \delta^3k^3,$$
$$\mu_3 = \delta k + \delta^3k^3 - \delta^2k^2 - \delta^3k^2,$$
$$\mu_4 = \delta^2k^2.$$

Then by denoting $\bar{\mu} = \mu_1+\mu_2+\mu_3+\mu_4 = 1 + \delta^2k^2 - \delta^2 k$, and after some algebra we get:

$$\overline{V}(0) = \tfrac{1}{(1-\delta)\bar{\mu}}\begin{pmatrix} Lv_0(1-k)(\mu_1+\mu_2+\mu_3) + \sigma L(1-k)\mu_4 + \sigma Hk\bar{\mu} + (\mu_1(1-\sigma H) + \mu_2(1-2\sigma)d)\,\frac{v_0-\sigma}{1-\sigma}k \\ +(1-\sigma)Lv_1\mu_2 + dv_1(1-k)\mu_3 + (1-\sigma L)v_1(1-k)\mu_4 + (1-\sigma H)v_1k(\mu_3+\mu_4) \end{pmatrix}$$

Relative Gain at $v_0 = v_0^+$:

We get:

$$R(v_0^+) = \frac{\begin{pmatrix} (\mu_1+\mu_2+\mu_3)L(v_0^+-\sigma)(1-k)+\mu_1(1-\sigma H)\frac{v_0^+-\sigma}{1-\sigma}k-(\mu_1+\mu_3+\mu_4)(1-2\sigma)d\frac{v_0^+-\sigma}{1-\sigma}k+dv_1(1-k)\mu_3 \\ +(1-\sigma)Lv_1\left(\mu_2-\delta(1-k+k^2)\bar{\mu}\right)+(1-\sigma L)v_1(1-k)\left(\mu_4-\delta k\bar{\mu}\right)+(1-\sigma H)v_1k(\mu_3+\mu_4) \end{pmatrix}}{\sigma L(1-k)+\sigma Hp_r^\beta k+(1-2\sigma)dv_0^+k+\delta(1-\sigma)Lv_1(1-k+k^2)+\delta(1-\sigma L)v_1(1-k)k}.$$

By using the fact that $v_0 = v_0^+$, and taking the limit when $\delta \to 1$, we get:

$$\lim_{\delta\to 1} R(v_0^+) = \frac{(1-\sigma)Lv_1k^2(1-k)^2}{\sigma L(1-k)+\sigma Hk+(1-2\sigma)dv_1k+(1-\sigma)Lv_1(1-k+k^2)+(1-\sigma L)v_1(1-k)k}.$$

Proof that $R_{max} \geq R_2(d, v_1, \sigma)$ when $k = 0.5$. Let us denote $v_0^- = \max\{v_1, \sigma + \delta(1-\sigma)kv_1\}$, which either defines the boundary between Regions A and B for the demand-dependent mechanism, or the case where $v_0 = v_1$ (if Region A does not exists). Moreover, when $k = 0.5$, we know that, when $v_0^- = \sigma + \delta(1-\sigma)kv_1$, v_0^- defines the boundary between Regions 2a and 2b for the optimal mechanism. When $v_0^- = v_1$, v_0^- may belong to Region 2b or to Region 2c.

Demand-dependent Mechanism:

In Region B, $\Gamma_{DD}^L = (1-\sigma)L$ and $\Gamma_{DD}^H = 1-\sigma L$. We already know that:

$$\overline{V}_{DD}(0) = \frac{1}{1-\delta}\left[\sigma L(1-k)+\sigma Hk+(1-2\sigma)d\frac{v_0-\sigma}{1-\sigma}k+\delta(1-\sigma)Lv_1(1-k+k^2)+\delta(1-\sigma L)v_1(1-k)k\right]$$

General Mechanism

There are 3 steady-state values of Γ: $(1-\sigma)L < 1-\sigma L < (1-\sigma)H$. We know that $\overline{V}(1-\sigma L) = \overline{V}((1-\sigma)H)$. Let $\overline{\mathbf{V}}$ be a vector defined as:

$$\overline{\mathbf{V}} = \begin{bmatrix} \overline{V}((1-\sigma)L) \\ \overline{V}(1-\sigma L) \end{bmatrix}.$$

We have:

$$\overline{\mathbf{V}} = \underbrace{\begin{bmatrix} (1-k)\left[\sigma L+(1-\sigma)Lv_1\right]+k[\sigma Hp_r^\beta+(1-2\sigma)dv_0+(1-\sigma)Lv_1] \\ (1-k)[\sigma L+(1-\sigma L)v_1]+k[\sigma H+(1-\sigma H)v_1] \end{bmatrix}}_{\mathbf{F}} + \delta\begin{bmatrix} (1-k) & k \\ (1-k) & k \end{bmatrix} \times \overline{\mathbf{V}},$$

where

$$p_r^\beta = 1 - \frac{(1-2\sigma)d}{(1-\sigma)H}(1-v_0).$$

From Region 2b's optimal policy ($\Gamma^*(H,0) = H-1$, and $\Gamma^*(H,H-1) = (1-\sigma)H$) we get:

$$\begin{aligned} \overline{V}(0) &= (1-k)\left[\sigma L\right]+k\left[\sigma Hp_r^\alpha+(1-\sigma H)v_0\right]+\delta k(1-k)\left[\sigma L+dv_1\right]+\delta k^2\left[\sigma H+dv_1\right] \\ &\quad +\delta(1-k)\overline{V}((1-\sigma)L)+\delta^2k(1-k)\overline{V}((1-\sigma)L)+\delta^2k^2\overline{V}((1-\sigma)H) \end{aligned}$$

This can be rewritten as follows:

$$\overline{V}(0) = (1-k)\sigma L(1+\delta k)+k\sigma H(1+\delta k)+k(1-\sigma H)\frac{v_0-\sigma}{1-\sigma}+\delta kdv_1+\frac{\delta}{1-\delta}\left[(1-k)\;\;\delta k\right]\mathbf{F}$$

After some algebra, we get:

$$\overline{V}(0) = \frac{1}{1-\delta}\begin{bmatrix} \sigma L(1-k) + \sigma H + (1-\delta)k(1-\sigma H)\frac{v_0-\sigma}{1-\sigma} + \delta(1-\delta)kdv_1 + \delta(1-2\sigma)d\frac{v_0-\sigma}{1-\sigma}k(1-k) \\ +\delta(1-\sigma)Lv_1(1-k) + \delta^2(1-\sigma L)v_1k(1-k) + \delta^2(1-\sigma H)v_1k^2 \end{bmatrix}$$

Relative Gain at $v_0 = v_0^-$:

We get:

$$R(v_0^-) = \frac{-((1-\delta)k+\delta^2 k)(1-2\sigma)d\frac{v_0-\sigma}{1-\sigma} - \delta(1-\sigma)Lv_1k^2 + \delta^2(1-\sigma H)v_1k^2 - (\delta-\delta^2)(1-\sigma L)v_1(1-k)k}{\sigma L(1-k) + \sigma Hk + (1-2\sigma)d\frac{v_0-\sigma}{1-\sigma}k + \delta(1-\sigma)Lv_1(1-k+k^2) + \delta(1-\sigma L)v_1(1-k)k}.$$

By using the facts that $\delta \to 1$, $k = 0.5$ and $v_0^- = \max\{\sigma + \delta(1-\sigma)kv_1, v_1\}$, we get the ratio:

$$R(v_0^-) \to \begin{cases} \frac{(1-2\sigma)dv_1}{8\sigma+2(1-2\sigma)dv_1+6(1-\sigma)Lv_1+2(1-\sigma L)v_1} & \text{if } v_0^- = \sigma + \delta(1-\sigma)kv_1, \\ \frac{(1-2\sigma)d\frac{(1-v_1)\sigma}{1-\sigma}}{4\sigma+2(1-2\sigma)d\frac{v_1-\sigma}{1-\sigma}+3(1-\sigma)Lv_1+(1-\sigma L)v_1} & \text{if } v_0^- = v_1. \end{cases}$$

Appendix EC.4: Proofs of Appendix C

EC.4.1. Baseline Two-period Model

The following lemma asserts that the initial findings provided in Proposition 2 carry over to the two-period setting. The proof uses identical arguments, and is thus omitted for conciseness.

Lemma EC.2. *The solution to above problem satisfies, at each $\tau \in \{1,2\}$, in each state (D_τ, Γ_τ):*

(i) $p_{t,\tau}(D_\tau,\Gamma_\tau) = v_0$, and $p_{l,\tau}(D_\tau,\Gamma_\tau) = v_1$.

(ii) $q_{r,\tau}(D_\tau,\Gamma_\tau) = 1$, and $p_{r,\tau}(D_\tau,\Gamma_\tau) \geq v_0$.

(iii) If $\Gamma_\tau > 0$, then $q_{l,\tau}(D_\tau,\Gamma_\tau) = \min\left\{1, \frac{1-q_{r,\tau}(D_\tau,\Gamma_\tau)\sigma D_\tau - q_{t,\tau}(D_\tau,\Gamma_\tau)(1-\sigma)D_\tau}{\Gamma_\tau}\right\}$.

(iv) Constraint $\mathcal{IC}_{r,\tau}$ is binding, and hence $p_{r,\tau}(D_\tau,\Gamma_\tau) = 1 - q_{t,\tau}(D_\tau,\Gamma_\tau)(1-v_0)$.

EC.4.1.1. Proof of Proposition 12

Period 2: The problem in Period 2 can be written as follows:

$$V_2(D_2,\Gamma_2) = \max_{\Gamma_2'} \widetilde{V}_2(D_2,\Gamma_2,\Gamma_2'), \quad \text{where:}$$
$$\widetilde{V}_2(D_2,\Gamma_2,\Gamma_2') = \sigma D_2\left(1 - \frac{(1-\sigma)D_2-\Gamma_2'}{(1-\sigma)D_2}(1-v_0)\right) + ((1-\sigma)D_2 - \Gamma_2')\,v_0 + \min\{\Gamma_2, 1-D_2+\Gamma_2'\}v_1.$$

Taking the partial derivative with respect to Γ_2' we get:

$$\frac{\partial\widetilde{V}_2(D_2,\Gamma_2,\Gamma_2')}{\partial\Gamma_2'} = \begin{cases} \frac{\sigma-v_0}{1-\sigma} & \text{If } \Gamma_2 < 1-D_2+\Gamma_2' \\ \frac{\sigma-v_0}{1-\sigma} + v_1 & \text{If } \Gamma_2 \geq 1-D_2+\Gamma_2' \end{cases} \tag{EC.8}$$

In Region 1, we have $v_0 \leq \sigma$, so $\frac{\partial\widetilde{V}_2(D_2,\Gamma_2,\Gamma_2')}{\partial\Gamma_2'} \geq 0$. Therefore, the platform provides no timely service to n-type customers.

In Region 2, we have $v_0 \in (\sigma, \sigma+(1-\sigma)v_1]$, so $\frac{\partial\widetilde{V}_2(D_2,\Gamma_2,\Gamma_2')}{\partial\Gamma_2'} \geq 0$ if and only if $\Gamma_2 \geq 1-D_2+\Gamma_2'$. Therefore, the platform rejects timely services to n-type customers as long as it increases the number of late services. In other words, the platform prioritizes late services.

In Region 3, we have $v_0 \in (\sigma + (1-\sigma)v_1, 1)$, so $\frac{\partial \widetilde{V}_2(D_2,\Gamma_2,\Gamma_2')}{\partial \Gamma_2'} < 0$. Therefore, the platform provides as many timely services as possible.

Period 1: The problem in Period 1 can be written as follows:

$$V_1(D_1,0) = \max_{\Gamma_1'} \ \widetilde{V}_1(D_1,0,\Gamma_1'), \quad \text{where:}$$
$$\widetilde{V}_1(D_1,0,\Gamma_1') = \sigma D_1 \left(1 - \tfrac{(1-\sigma)D_1 - \Gamma_1'}{(1-\sigma)D_1}(1-v_0)\right) + \left((1-\sigma)D_1 - \Gamma_1'\right) v_0 + \delta \mathbb{E}_{D_2}\left(V_2(D_2,\Gamma_1')\right).$$

Taking the partial derivative with respect to Γ_1' we get:

$$\frac{\partial \widetilde{V}_1(D_1,0,\Gamma_1')}{\partial \Gamma_1'} = \frac{\sigma - v_0}{1-\sigma} + \delta \mathbb{E}_{D_2}\left(V_2'(D_2,\Gamma_1')\right). \tag{EC.9}$$

From the optimal policy in Period 2, we know that, when $\sigma < 0.5$:

$$\mathbb{E}_{D_2}\left(V_2'(D_2,\Gamma_2)\right) = \begin{cases} k\frac{\sigma-v_0}{1-\sigma} + v_1 & \text{if } \Gamma_2 \in [0, 1-\sigma H), \\ (1-k)v_1 & \text{if } \Gamma_2 \in [1-\sigma H, H-1), \\ (1-k)\left(\frac{\sigma-v_0}{1-\sigma} + v_1\right) & \text{if } \Gamma_2 \in [H-1, 1-\sigma L), \\ 0 & \text{if } \Gamma_2 \geq 1-\sigma L. \end{cases}$$

In Region 1, $\frac{\partial \widetilde{V}_1(D_1,\Gamma_1,\Gamma_1')}{\partial \Gamma_1'} \geq 0$, so the platform provides no timely service to n-type customers.

In Region 2, the sign of $\frac{\partial \widetilde{V}_1(D_1,\Gamma_1,\Gamma_1')}{\partial \Gamma_1'}$ depends on the value of $\mathbb{E}_{D_2}\left(V_2'(D_2,\Gamma_1')\right)$.

- Under low demand, $\Gamma_1'(L,0) \leq (1-\sigma)L \leq 1-\sigma H$. Therefore, $\mathbb{E}_{D_2}\left(V_2'(D_2,\Gamma_1')\right) = k\frac{\sigma-v_0}{1-\sigma} + v_1$ and $\frac{\partial \widetilde{V}_1(L,0,\Gamma_1')}{\partial \Gamma_1'} = \frac{\sigma-v_0}{1-\sigma}(1+\delta k) + \delta v_1$. We obtain that $\frac{\partial \widetilde{V}_1(L,0,\Gamma_1')}{\partial \Gamma_1'} \geq 0$ if and only if $v_0 \leq \sigma + \frac{(1-\sigma)\delta v_1}{1+\delta k}$ (Regions 2-i and 2-ii). So $\Gamma_1'(L,0) = \overline{\Gamma}_L$ in Regions 2-i and 2-ii and $\Gamma_1'(L,0) = \underline{\Gamma}_L$ in Region 2-iii.
- Under high demand, $\Gamma_1'(H,0) \in [H-1, (1-\sigma)H]$, so:

$$\frac{\partial \widetilde{V}_1(H,0,\Gamma_1')}{\partial \Gamma_1'} = \begin{cases} \frac{\sigma-v_0}{1-\sigma}(1+\delta(1-k)) + \delta(1-k)v_1 & \text{if } \Gamma_1' \in [H-1, 1-\sigma L), \\ \frac{\sigma-v_0}{1-\sigma} < 0 & \text{if } \Gamma_1' \in [1-\sigma L, (1-\sigma)H]. \end{cases} \tag{EC.10}$$

  Therefore, $\Gamma_1'(H,0) \leq 1-\sigma L$ in Region 2. We obtain $\Gamma_1'(H,0) = 1-\sigma L$ if $v_0 \leq \sigma + \frac{(1-\sigma)\delta(1-k)}{1+\delta(1-k)}v_1$ (Region 2-i), and $\Gamma_1'(H,0) = \underline{\Gamma}_H$ otherwise (Regions 2-ii and 2-iii).

In Region 1, $\frac{\partial \widetilde{V}_1(D_1,\Gamma_1,\Gamma_1')}{\partial \Gamma_1'} < 0$, so the platform provides as many timely service as possible.

EC.4.2. Stochastic Demand Composition

The platform's problem is given by:

$$\begin{aligned} V_2(D_2,\Gamma_2,\sigma_2) = \max_{\substack{q_{r,2},q_{t,2},q_{l,2} \\ p_{r,2},p_{t,2},p_{l,2}}} \quad & p_{r,2}q_{r,2}\sigma_2 D_2 + p_{t,2}q_{t,2}(1-\sigma_2)D_2 + p_{l,2}q_{l,2}\Gamma_2 \\ \text{s.t.} \quad & \mathcal{IC}_{r,2} \text{ and } \mathcal{IC}_{n,2}, \\ & p_{r,2} \leq 1, \ p_{t,2} \leq v_0, \ p_{l,2} \leq v_1, \\ & 1 \geq q_{r,2}\sigma_2 D_2 + q_{t,2}(1-\sigma_2)D_2 + q_{l,2}\Gamma_2. \end{aligned}$$

$$
\begin{aligned}
V_1(D_1,0,\sigma_1) = \max_{\substack{q_{r,1},q_{t,1}\\ p_{r,1},p_{t,1}}} \quad & p_{r,1}q_{r,1}\sigma_1 D_1 + p_{t,1}q_{t,1}(1-\sigma_1)D_1 + \delta\mathbb{E}_{D_2,\sigma_2}\left(V_2(D_2,\Gamma',\sigma_2)\right)\\
\text{s.t.} \quad & \mathcal{IC}_{r,1} \text{ and } \mathcal{IC}_{n,1},\\
& p_{r,1}\leq 1,\ p_{t,1}\leq v_0,\\
& 1\geq q_{r,1}\sigma_1 D_1 + q_{t,1}(1-\sigma_1)D,\\
& \Gamma' = (1-q_{t,1})(1-\sigma_1)D_1.
\end{aligned}
$$

Lemma EC.3 provides analogous statements to Proposition 2. The proofs of Lemma EC.3 and Proposition 13 follow usual arguments and are omitted for conciseness.

LEMMA EC.3. *The optimal solution to above problem satisfies, at each τ, in each state (D_τ,Γ_τ):*

(i) $p_{t,\tau}(D_\tau,\Gamma_\tau,\sigma_\tau) = v_0$*, and* $p_{l,\tau}(D_\tau,\Gamma_\tau,\sigma_\tau) = v_1$*.*

(ii) $q_{r,\tau}(D_\tau,\Gamma_\tau,\sigma_\tau) = 1$*, and* $p_{r,\tau}(D_\tau,\Gamma_\tau,\sigma_\tau)\geq v_0$*.*

(iii) If $\Gamma_\tau > 0$*, then* $q_{l,\tau}(D_\tau,\Gamma_\tau,\sigma_\tau) = \min\left\{1, \frac{1-q_{r,\tau}(D_\tau,\Gamma_\tau)\sigma_\tau D_\tau - q_{t,\tau}(D_\tau,\Gamma_\tau,\sigma_\tau)(1-\sigma_\tau)D_\tau}{\Gamma_\tau}\right\}$*.*

(iv) Constraint $\mathcal{IC}_{r,\tau}$ *is binding, and hence* $p_{r,\tau}(D_\tau,\Gamma_\tau,\sigma_\tau) = 1 - q_{t,\tau}(D_\tau,\Gamma_\tau,\sigma_\tau)(1-v_0)$*.*

EC.4.2.1. Proof of Proposition 14

It suffices to show that there exists $\sigma_1\in\{\underline{\sigma},\overline{\sigma}\}$, and Γ_1 such that $p_{r,1}(H,\Gamma_1,\sigma_1) < p_{r,1}(L,\Gamma_1,\sigma_1)$. Recall that the platform's problem in Period 1 is given by:

$$
\begin{aligned}
&V_1(D_1,0,\sigma_1) = \max_{\Gamma_1'} \widetilde{V}_1(D_1,0,\sigma_1,\Gamma_1'), \quad \text{where:}\\
&\widetilde{V}_1(D_1,0,\sigma_1,\Gamma_1') = \sigma_1 D_1\left(1 - \tfrac{(1-\sigma_1)D_1-\Gamma_1'}{(1-\sigma_1)D_1}(1-v_0)\right) + \left((1-\sigma_1)D_1 - \Gamma_1'\right)v_0 + \delta\mathbb{E}_{D_2,\sigma_2}\left(V_2(D_2,\Gamma_1',\sigma_2)\right).
\end{aligned}
$$

We obtain:

$$
\frac{\partial\tilde{V}_1(D_1,0,\sigma_1,\Gamma_1')}{\partial\Gamma_1'} = \frac{\sigma_1 - v_0}{1-\sigma_1} + \delta\mathbb{E}_{D_2,\sigma_2}\left(V_2'(D_2,\Gamma_1',\sigma_2)\right).
$$

Let us assume that $\overline{\sigma} < \underline{\sigma} + (1-\underline{\sigma})v_1$ and that $v_0\in[\overline{\sigma},\underline{\sigma}+(1-\sigma)v_1]$. In this region, the optimal policy in Period 2 involves late service prioritization (Region 2–σ). Therefore, for any $\sigma_2\in\{\underline{\sigma},\overline{\sigma}\}$:

$$
V_2(L,\Gamma_2,\sigma_2) = \begin{cases}
Lv_0 + \Gamma_2 v_1 & \text{if } \Gamma_2\in[0,1-L),\\
\sigma_2 L\left((1 - \frac{(1-\sigma_2)L-\Gamma_2}{(1-\sigma_2)L}(1-v_0)\right) + ((1-\sigma_2)L - \Gamma_2)v_0 + \Gamma_2 v_1 & \text{if } \Gamma_2\in[1-L,1-\sigma_2 L),\\
\sigma_2 L + (1-\sigma_2 L)v_1 & \text{if } \Gamma_2\geq 1-\sigma_2 L.
\end{cases}
$$

$$
V_2(H,\Gamma_2,\sigma_2) = \begin{cases}
\sigma_2 H\left((1 - \frac{(1-\sigma_2)H-\Gamma_2}{(1-\sigma_2)H}(1-v_0)\right) + ((1-\sigma_2)H - \Gamma_2)v_0 + \Gamma_2 v_1 & \text{if } \Gamma_2\in[0,1-\sigma_2 H),\\
\sigma_2 H + (1-\sigma_2 H)v_1 & \text{if } \Gamma_2\geq 1-\sigma_2 H.
\end{cases}
$$

From our assumptions, we know that $1-\overline{\sigma}H < 1-\underline{\sigma}H < 1-L < 1-\underline{\sigma}H < 1-\overline{\sigma}L$. Therefore:

$$\mathbb{E}_{D_2,\sigma_2}\left(V_2'(D_2,\Gamma_2,\sigma_2)\right) = \begin{cases} (1-k)\left(\mu(\frac{\overline{\sigma}-v_0}{1-\overline{\sigma}}) + (1-\mu)(\frac{\underline{\sigma}-v_0}{1-\underline{\sigma}})\right) + v_1 & \text{if } \Gamma_2 < 1-\overline{\sigma}H, \\ (1-k)(1-\mu)(\frac{\underline{\sigma}-v_0}{1-\underline{\sigma}}) + (1-k\mu)v_1 & \text{if } \Gamma_2 \in [1-\overline{\sigma}H, 1-\underline{\sigma}H), \\ (1-k)v_1 & \text{if } \Gamma_2 \in [1-\underline{\sigma}H, 1-L), \\ (1-k)\left(\mu(\frac{\overline{\sigma}-v_0}{1-\overline{\sigma}}) + (1-\mu)(\frac{\underline{\sigma}-v_0}{1-\underline{\sigma}}) + v_1\right) & \text{if } \Gamma_2 \in [1-L, 1-\overline{\sigma}L), \\ (1-k)(1-\mu)\left((\frac{\underline{\sigma}-v_0}{1-\underline{\sigma}}) + v_1\right) & \text{if } \Gamma_2 \in [1-\overline{\sigma}L, 1-\underline{\sigma}L), \\ 0 & \text{if } \Gamma_2 \geq 1-\underline{\sigma}L. \end{cases}$$

Let us focus on the case with $\sigma_1 = \overline{\sigma}$. Let us then define $\underline{v}_0 < \bar{v}_0 \in [\overline{\sigma}, \underline{\sigma} + (1-\sigma)v_1]$ as:

$$\underline{v}_0 = \frac{\overline{\sigma}(1-\underline{\sigma})(1+\delta(1-k)\mu) + \underline{\sigma}(1-\overline{\sigma})\delta(1-k)(1-\mu) + (1-\underline{\sigma})(1-\overline{\sigma})\delta(1-k)}{(1-\underline{\sigma})(1+\delta(1-k)\mu) + (1-\overline{\sigma})\delta(1-k)(1-\mu)},$$
$$\bar{v}_0 = \frac{\overline{\sigma}(1-\underline{\sigma}) + \underline{\sigma}(1-\overline{\sigma})\delta(1-k)(1-\mu) + (1-\underline{\sigma})(1-\overline{\sigma})\delta(1-k)}{(1-\underline{\sigma}) + (1-\overline{\sigma})\delta(1-k)(1-\mu)}.$$

After some algebra, we can check that $v_0 \in [\underline{v}_0, \bar{v}_0] \subset [\overline{\sigma}, \underline{\sigma} + (1-\sigma)v_1]$, that $\Gamma_1'(L,0,\overline{\sigma}) = (1-\overline{\sigma})L$, and that $\Gamma_1'(H,0,\overline{\sigma}) = 1-\overline{\sigma}L < (1-\overline{\sigma})H$. Therefore, $p_{r,1}(L,0,\overline{\sigma}) = 1$, and $p_{r,1}(H,0,\overline{\sigma}) < 1$.

EC.4.2.2. Proof of Proposition 15

Let us first prove that $\Gamma_\tau'(D_\tau,\Gamma_\tau,\sigma_\tau)$ is non-decreasing with σ_τ. Starting with Period 2, we have:

$$\frac{\partial \tilde{V}_2(D_2,\Gamma_2,\sigma_2,\Gamma_2')}{\partial \Gamma_2'} = \begin{cases} \frac{\sigma_2-v_0}{1-\sigma_2} & \text{if } \Gamma_2 < 1-D_2-\Gamma_2', \\ \frac{\sigma_2-v_0}{1-\sigma_2} + v_1 & \text{if } \Gamma_2 \geq 1-D_2-\Gamma_2'. \end{cases}$$

Since $\frac{\partial \tilde{V}_2(D_2,\Gamma_2,\sigma_2,\Gamma_2')}{\partial \Gamma_2'}$ is non-decreasing with σ_2, we obtain $\Gamma_2'(D_2,\Gamma_2,\sigma_2) \geq \Gamma_2'(D_2,\Gamma_2,\sigma_2')$ if $\sigma_2 > \sigma_2'$.

Similarly, we have, in Period 1:

$$\frac{\partial \tilde{V}_1(D_1,0,\sigma_1,\Gamma_1')}{\partial \Gamma_1'} = \frac{\sigma_1 - v_0}{1-\sigma_1} + \delta \mathbb{E}_{D_2,\sigma_2}\left(V_2(D_2,\Gamma_1',\sigma_2)\right).$$

Again, $\frac{\partial \tilde{V}_1(D_1,\Gamma_1,\sigma_1,\Gamma_1')}{\partial \Gamma_1'}$ is non-decreasing with σ_1, so $\Gamma_1'(D_1,\Gamma_1,\sigma_1) \geq \Gamma_1'(D_1,\Gamma_1,\sigma_1')$ if $\sigma_1 > \sigma_1'$.

For any $\tau = 1,2$, let us consider $\sigma_\tau > \sigma_\tau'$. We have:

$$\begin{aligned} p_{r,\tau}(D_\tau,\Gamma_\tau,\sigma_\tau) &= 1 - \frac{(1-\sigma_\tau)D_\tau - \Gamma'(D_\tau,\Gamma_\tau,\sigma_\tau)}{(1-\sigma_\tau)D_\tau}(1-v_0) \\ &= v_0 + \frac{\Gamma'(D_\tau,\Gamma_\tau,\sigma_\tau)}{(1-\sigma_\tau)D_\tau}(1-v_0) \\ &> v_0 + \frac{\Gamma'(D_\tau,\Gamma_\tau,\sigma_\tau')}{(1-\sigma_\tau')D_\tau}(1-v_0) \\ &= p_{r,\tau}(D_\tau,\Gamma_\tau,\sigma_\tau') \end{aligned}$$

EC.4.3. Endogenous Capacity

The platform's problem is given by:

$$V_2(D_2,\Gamma_2) = \max_{\substack{w_2 \\ q_{r,2},q_{t,2},q_{l,2} \\ p_{r,2},p_{t,2},p_{l,2}}} \quad (p_{r,2}-w_2)\,q_{r,2}\sigma D_2 + (p_{t,2}-w_2)\,q_{t,2}(1-\sigma)D_2 + (p_{l,2}-w_2)\,q_{l,2}\Gamma_2$$

$$\begin{aligned}
&\text{s.t.} \quad \mathcal{IC}_{r,2} \text{ and } \mathcal{IC}_{n,2},\\
&\qquad p_{r,2} \leq 1,\ p_{t,2} \leq v_0,\ p_{l,2} \leq v_1,\\
&\qquad S(w_2) \geq q_{r,2}\sigma D_2 + q_{t,2}(1-\sigma)D_2 + q_{l,2}\Gamma_2.
\end{aligned}$$

$$\begin{aligned}
V_1(D_1,0) = \max_{\substack{w_1\\ q_{r,1},q_{t,1}\\ p_{r,1},p_{t,1}}} \quad & (p_{r,1}-w_1)\,q_{r,1}\sigma D_1 + (p_{t,1}-w_1)\,q_{t,1}(1-\sigma)D_1 + \delta\left(kV_2(H,\Gamma') + (1-k)V_2(L,\Gamma')\right)\\
\text{s.t.} \quad & \mathcal{IC}_{r,1} \text{ and } \mathcal{IC}_{n,1},\\
& p_{r,1} \leq 1,\ p_{t,1} \leq v_0,\\
& S(w_1) \geq q_{r,1}\sigma D_1 + q_{t,1}(1-\sigma)D,\\
& \Gamma' = (1-q_{t,1})(1-\sigma)D_1.
\end{aligned}$$

Lemma EC.4 is the analog of Lemma EC.2. Its proof is omitted for conciseness.

LEMMA EC.4. *The optimal solution to above problem satisfies, at each τ, in each state (D_τ,Γ_τ):*

(i) $p_{t,\tau}(D_\tau,\Gamma_\tau) = v_0$, *and* $p_{l,t}(D_\tau,\Gamma_\tau) = v_1$.

(ii) $q_{r,\tau}(D,\Gamma) = \min\left\{1, \frac{S(w_\tau)}{\sigma D_\tau}\right\}$, *and* $p_{r,\tau}(D_\tau,\Gamma_\tau) \geq v_0$.

(iii) If $\Gamma > 0$, *then* $q_{l,\tau}(D_\tau,\Gamma_\tau) = \min\left\{1, \frac{S(w_\tau) - q_{r,\tau}(D_\tau,\Gamma_\tau)\sigma D_\tau - q_{t,\tau}(D_\tau,\Gamma_\tau)(1-\sigma)D_\tau}{\Gamma_\tau}\right\}$.

(iv) Constraint $\mathcal{IC}_{r,\tau}$ *is binding, and hence* $p_{r,\tau}(D_\tau,\Gamma_\tau) = 1 - q_{t,\tau}(D_\tau,\Gamma_\tau)(1-v_0)$.

EC.4.3.1. Proof of Lemma 7

We first show that the platform always serves r-type customers. Suppose by contradiction that $q_{r,\tau}(D,\Gamma) < 1$. From Lemma EC.4, we know that the platform serves only r-type customers ($q_{l,\tau}(D,\Gamma) = q_{t,\tau}(D,\Gamma) = 0$) and charges a price $p_{r,\tau}(D_\tau,\Gamma_\tau) = 1$. We obtain that $S(w_\tau(D_\tau,\Gamma_\tau)) = \frac{w_\tau(D_\tau,\Gamma_\tau)}{\bar{u}} < 1$. The platform's profit is then:

$$\Pi_\tau = \frac{w_\tau}{\bar{u}}\left(1 - w_\tau\right).$$

Let us now increase the wage by $\varepsilon > 0$. The platform can serve more r-type customers without serving any n-type customers, so that $p_{r,\tau}(D_\tau,\Gamma_\tau) = 1$. The resulting profit is

$$\widetilde{\Pi}_\tau = \left(\frac{w_\tau + \varepsilon}{\bar{u}}\right)(1 - w_\tau - \varepsilon) = \frac{w_\tau}{\bar{u}}\left(1 - w_\tau\right) + \frac{\varepsilon}{\bar{u}}\left(1 - 2(w_\tau + \varepsilon)\right).$$

Note that $w_\tau < \bar{u} \leq \frac{v_1}{2} < \frac{1}{2}$, so $1 - 2w_\tau > 0$. For $\varepsilon > 0$ sufficiently small, we thus obtain $\widetilde{\Pi}_\tau > \Pi_\tau$.

We now show that, if some late demand is left unserved in Period 2, then $S(w_2(D_2,\Gamma_2)) = 1$. Suppose by contradiction that $q_{l,2}(D_2,\Gamma_2) < 1$ and $S(w_2(D_2,\Gamma_2)) < 1$ (i.e., $w_2(D_2,\Gamma_2) < \bar{u}$). Then, note that the cost of service provision is $S(w_2(D_2,\Gamma_2)) \times w_2(D_2,\Gamma_2) = \frac{w_2(D_2,\Gamma_2)^2}{\bar{u}}$. Let Rev_2 be the platform's revenue in Period 2. Its profit is then given by:

$$\Pi_2 = Rev_2 - \frac{w_2^2}{\bar{u}},$$

Now, consider increasing the wage by $\varepsilon > 0$. This increases the capacity by $\frac{\varepsilon}{\bar{u}}$. Let us allocate the additional capacity to provide more late services. This deviation does not change the price $p_{r,2}(D_2, \Gamma_2)$. The platform's revenue thus increases by $\frac{\varepsilon}{\bar{u}} v_1$. Its profit becomes:

$$\widetilde{\Pi}_2 = Rev_2 + \frac{\varepsilon}{\bar{u}} v_1 - \frac{(w_2 + \varepsilon)^2}{\bar{u}} = \Pi_2 + \frac{\varepsilon}{\bar{u}} (v_1 - 2w_2 - \varepsilon).$$

Note that $w_\tau < \bar{u} \leq \frac{v_1}{2}$, so $v_1 - 2w_\tau > 0$. For $\varepsilon > 0$ sufficiently small, we thus obtain $\widetilde{\Pi}_\tau > \Pi_\tau$.

EC.4.3.2. Proof of Proposition 16

First, we prove that in Region 3–$\bar{u}$ ($v_0 \geq \sigma + (1-\sigma) v_1$), the platform prioritizes timely services over late services. Suppose by contradiction that $q_{l,2}(D_2, \Gamma_2) > 0$ and $q_{t,2}(D_2, \Gamma_2) < (1-\sigma) D_2$. Denote the number of timely (resp., late) services by A_t (resp., A_l), and let $w_2(D_2, \Gamma_2)$ be the corresponding wage level. Then the platform's profit in Period 2 is given by:

$$\Pi_2 = \sigma D_2 \left(1 - \frac{A_t}{(1-\sigma) D_2} (1 - v_0)\right) + A_t v_0 + A_l v_1 - \frac{(w_2(D_2, \Gamma_2))^2}{\bar{u}}.$$

Let us increase A_t and decrease A_l by $\varepsilon > 0$. This does not change the total number of services provided nor the wage level. The platform's profit in Period 2 becomes:

$$\widetilde{\Pi}_2 = \sigma D_2 \left(1 - \frac{A_t + \varepsilon}{(1-\sigma) D_2} (1 - v_0)\right) + (A_t + \varepsilon) v_0 + (A_l - \varepsilon) v_1 - \frac{(w_2(D_2, \Gamma_2))^2}{\bar{u}}.$$

But then $\frac{\partial \widetilde{\Pi}_2}{\partial \varepsilon} = \frac{v_0 - \sigma - (1-\sigma) v_1}{1-\sigma} > 0$, so $\widetilde{\Pi}_2 > \Pi_2$. Thus, in Region 3–$\bar{u}$, the platform provides as many timely services as possible, and then serves late demand as much as possible.

Second, we show that in Region 1–$\bar{u}$ and in Region 2–$\bar{u}$, the platform prioritizes late services over timely services provided to n-type customers. Suppose by contradiction that $q_{t,2}(D_2, \Gamma_2) > 0$ and $q_{l,2}(D_2, \Gamma_2) < \Gamma_2$. Let us increase A_l and decrease A_t by $\varepsilon > 0$. This does not change the total number of services provided nor the wage level. The platform's profit in Period 2 becomes:

$$\widetilde{\Pi}_2 = \sigma D_2 \left(1 - \frac{A_t - \varepsilon}{(1-\sigma) D_2} (1 - v_0)\right) + (A_t - \varepsilon) v_0 + (A_l + \varepsilon) v_1 - \frac{(w_2(D_2, \Gamma_2))^2}{\bar{u}}.$$

Then $\frac{\partial \widetilde{\Pi}_2}{\partial \varepsilon} = -\frac{v_0 - \sigma - (1-\sigma) v_1}{1-\sigma} > 0$, so for $\varepsilon > 0$ we obtain $\widetilde{\Pi}_2 > \Pi_2$. Thus, in Region 1–$\bar{u}$ and in Region 2–$\bar{u}$, the platform first serves r-type customers, then the late demand, and then it *may* provide timely services to n-type customers. Let, again, rewrite the problem by changing the decision variable into the number of timely services provided to n-type customers, denoted by A_t. We know that $A_t = 0$ when $\sigma D_2 + \Gamma_2 \geq 1$. To distinguish between Region 1–$\bar{u}$ and in Region 2–$\bar{u}$, we now focus on the case where $\sigma D_2 + \Gamma_2 < 1$. Let $\widetilde{V}(D_2, \Gamma_2, A_t)$ be the platform's profit in Period 2 in state (D_2, Γ_2) for a given choice of $A_t \in [0, \min\{1 - \sigma D_2 + \Gamma_2, (1-\sigma) D_2\}]$. We have:

$$\tilde{V}_2(D_2, \Gamma_2, A_t) = \sigma D_2 \left(1 - \tfrac{A_t}{(1-\sigma) D_2} (1 - v_0)\right) + A_t v_0 + \Gamma_2 v_1 - (\sigma D_2 + \Gamma_2 + A_t)^2 \bar{u}.$$

We obtain:

$$\frac{\partial \tilde{V}_2(D_2,\Gamma_2,A_t)}{\partial A_t} = \frac{v_0-\sigma}{1-\sigma} - 2(\sigma D_2+\Gamma_2+A_t)\bar{u},$$

$$A_t = \max\left\{0, \min\left\{\frac{v_0-\sigma}{2(1-\sigma)\bar{u}} - \sigma D_2 - \Gamma_2, 1-\sigma D_2-\Gamma_2, (1-\sigma)D_2\right\}\right\}.$$

After serving r-type customers and the late demand, the platform can thus increase the wage up to $\frac{v_0-\sigma}{2(1-\sigma)}$ to provide timely services to n-type customers.

- In Region 1–$\bar{u}$, $v_0 \le \sigma + 2(1-\sigma)\bar{u}\sigma L$, so $\sigma D_2+\Gamma_2 \ge \sigma L \ge \frac{v_0-\sigma}{2(1-\sigma)\bar{u}}$. Therefore, $A_t=0$: no n-type customer receives a timely service. We obtain $w_2(D_2,\Gamma_2)=\min\{\bar{u},(\sigma D_2+\Gamma_2)\bar{u}\}$.
- In Region 2.i–$\bar{u}$, $\sigma L \le \frac{v_0-\sigma}{2(1-\sigma)\bar{u}} \le 1$. The platform increases the wage to provide timely services as long as it is feasible and $w_2(D_2,\Gamma_2) \le \frac{v_0-\sigma}{2(1-\sigma)}$. If the platform provides timely services, the wage is $\min\left\{\frac{v_0-\sigma}{2(1-\sigma)},(D_2+\Gamma_2)\bar{u}\right\}$. Otherwise, the wage is $\bar{u}$ if $\sigma D_2+\Gamma_2 \ge 1$ or $(\sigma D_2+\Gamma_2)\bar{u}$ when $\frac{v_0-\sigma}{2(1-\sigma)} \le \sigma D_2+\Gamma_2 < 1$. We obtain $w_2(D_2,\Gamma_2) = \max\left\{\min\{\bar{u},(\sigma D_2+\Gamma_2)\bar{u}\}, \min\left\{\frac{v_0-\sigma}{2(1-\sigma)},(D_2+\Gamma_2)\bar{u}\right\}\right\}$.
- In Region 2.ii–$\bar{u}$, $\frac{v_0-\sigma}{2(1-\sigma)\bar{u}} \ge 1$, so $A_t=\min\{1-\sigma D_2-\Gamma_2,(1-\sigma)D_2\}$: the platform increases the wage as long as it can provide timely services. Thus, $w_2(D_2,\Gamma_2)=\min\{\bar{u},(D_2+\Gamma_2)\bar{u}\}$.

EC.4.3.3. Proof of Proposition 17

Recall that the platform's problem is given by:

$$V_1(D_1,0) = \max_{\Gamma_1'} \tilde{V}_1(D_1,0,\Gamma_1'), \quad \text{where:}$$

$$\widetilde{V}_1(D_1,0,\Gamma_1') = \sigma D_1\left(1-\tfrac{(1-\sigma)D_1-\Gamma_1'}{(1-\sigma)D_1}(1-v_0)\right) + ((1-\sigma)D_1-\Gamma_1')\,v_0 - (D_1-\Gamma_1')^2\bar{u} + \delta\mathbb{E}_{D_2}\left(V_2(D_2,\Gamma_1')\right).$$

Therefore:

$$\frac{\partial \tilde{V}_1(D_1,0,\Gamma_1')}{\partial \Gamma_1'} = \frac{\sigma-v_0}{1-\sigma} + 2\bar{u}(D_1-\Gamma_1') + \delta\mathbb{E}_{D_2}\left(V_2'(D_2,\Gamma_1')\right).$$

Let us focus on Region 2.ii–$\bar{u}$. We have:

$$V_2(L,\Gamma_2) = \begin{cases} Lv_0+\Gamma_2 v_1 - (L+\Gamma_2)^2\bar{u} & \text{if } \Gamma_2 \in [0,1-L), \\ \sigma L\left((1-\tfrac{(1-\sigma)L-\Gamma_2}{(1-\sigma)L}(1-v_0)\right) + ((1-\sigma)L-\Gamma_2)v_0+\Gamma_2 v_1 - \bar{u} & \text{if } \Gamma_2 \in [1-L,1-\sigma L), \\ \sigma L + (1-\sigma L)v_1 - \bar{u} & \text{if } \Gamma_2 \ge 1-\sigma L. \end{cases}$$

$$V_2(H,\Gamma_2) = \begin{cases} \sigma H\left((1-\tfrac{(1-\sigma)H-\Gamma_2}{(1-\sigma)H}(1-v_0)\right) + ((1-\sigma)H-\Gamma_2)v_0+\Gamma_2 v_1 - \bar{u} & \text{if } \Gamma_2 \in [0,1-\sigma H), \\ \sigma H + (1-\sigma H)v_1 - \bar{u} & \text{if } \Gamma_2 \ge 1-\sigma H. \end{cases}$$

Indeed:

- If $D_2=L$ and $\Gamma_2 \in [0,1-L)$, the platform provides σL services to r-type customers at price v_0, Γ_2 late services, and $(1-\sigma)L$ timely services to n-type customers. Overall, the platform provides $L+\Gamma_2$ services by setting a wage equal to $(L+\Gamma_2)^2\bar{u}$.

- If $D_2 = L$ and $\Gamma_2 \in [1-L, 1-\sigma L)$, the platform provides σL services to r-type customers at price $1 - \frac{(1-\sigma)L-\Gamma_2}{(1-\sigma)L}(1-v_0)$, Γ_2 late services, and $1-\sigma L-\Gamma_2$ timely services to n-type customers. The platform sets its capacity to 1 by setting a wage of $\bar{u}$.
- If $D_2 = L$ and $\Gamma_2 \geq 1-\sigma L$, the platform provides σL services to r-type customers at price 1, and $1-\sigma L$ late services. The platform sets its capacity to 1 by setting a wage of $\bar{u}$.
- If $D_2 = H$ and $\Gamma_2 \in [0, 1-\sigma H)$, the platform provides σH services to r-type customers at price $1 - \frac{(1-\sigma)H-\Gamma_2}{(1-\sigma)H}(1-v_0)$, Γ_2 late services, and $1-\sigma H-\Gamma_2$ timely services to n-type customers. The platform sets its capacity to 1 by setting a wage of $\bar{u}$.
- If $D_2 = H$ and $\Gamma_2 \geq 1-\sigma H$, the platform provides σH services to r-type customers at price 1, and $1-\sigma H$ late services. The platform sets its capacity to 1 by setting a wage of $\bar{u}$.

With $\sigma < 0.5$, we have $(1-\sigma)L < 1-\sigma H < H-1 = d = 1-L < 1-\sigma L < (1-\sigma)H$. We obtain:

$$V_2'(L,\Gamma_2) = \begin{cases} v_1 - 2\bar{u}(L+\Gamma_2) & \text{if } \Gamma_2 \in [0, 1-L), \\ \frac{\sigma-v_0}{1-\sigma} + v_1 & \text{if } \Gamma_2 \in [1-L, 1-\sigma L), \\ 0 & \text{if } \Gamma_2 \geq 1-\sigma L. \end{cases}$$

$$V_2'(H,\Gamma_2) = \begin{cases} \frac{\sigma-v_0}{1-\sigma} + v_1 & \text{if } \Gamma_2 \in [0, 1-\sigma H), \\ 0 & \text{if } \Gamma_2 \geq 1-\sigma H. \end{cases}$$

$$\mathbb{E}_{D_2}\left(V_2'(D_2,\Gamma_2)\right) = \begin{cases} k\left(\frac{\sigma-v_0}{1-\sigma} + v_1\right) + (1-k)\left(v_1 - 2\bar{u}(L+\Gamma_2)\right) & \text{if } \Gamma_2 \in [0, 1-\sigma H), \\ (1-k)\left(v_1 - 2\bar{u}(L+\Gamma_2)\right) & \text{if } \Gamma_2 \in [1-\sigma H, H-1), \\ (1-k)\left(\frac{\sigma-v_0}{1-\sigma} + v_1\right) & \text{if } \Gamma_2 \in [H-1, 1-\sigma L), \\ 0 & \text{if } \Gamma_2 \geq 1-\sigma L. \end{cases}$$

We know that $\Gamma_1'(L,0) \leq (1-\sigma)L < 1-\sigma H$. Therefore, we obtain, from the first-order conditions:

$$\Gamma_1'(L) = \min\left\{\max\left\{0, \frac{1}{2\bar{u}(1+\delta(1-k))}\left(\frac{\sigma-v_0}{1-\sigma}(1+\delta k) + \delta v_1 + 2\bar{u}L(1-\delta(1-k))\right)\right\}, (1-\sigma)L\right\}.$$

Similarly, we know that $\Gamma_1'(H,0) \geq H-1$. Clearly, $\Gamma_1'(H,0) < 1-\sigma L$; otherwise, $\mathbb{E}_{D_2}\left(V_2'(D_2,\Gamma_2)\right) = 0$ and, since we are in Region 2.ii–$\bar{u}$, we get $\frac{\partial \tilde{V}_1(D_1,0,\Gamma_1')}{\partial \Gamma_1'} = \frac{\sigma-v_0}{1-\sigma} + 2\bar{u}(D_1 - \Gamma_1') \leq 0$. Therefore, $\mathbb{E}_{D_2}\left(V_2'(D_2,\Gamma_2)\right) = (1-k)\left(\frac{\sigma-v_0}{1-\sigma} + v_1\right)$. We obtain, from the first-order conditions:

$$\Gamma_1'(H) = \min\left\{\max\left\{H-1, H + \frac{1}{2\bar{u}}\left(\frac{\sigma-v_0}{1-\sigma}(1+\delta(1-k)) + \delta(1-k)v_1\right)\right\}, (1-\sigma)H\right\}$$

Suppose that $v_0 = \sigma + 2(1-\sigma)\bar{u}$ (which is admissible in Region 2.ii–$\bar{u}$). Then:

$$\Gamma_1'(L) = \min\left\{\max\left\{0, \frac{1}{2\bar{u}(1+\delta(1-k))}\left(-2\bar{u}(1+\delta k) + \delta v_1 + 2\bar{u}L(1-\delta(1-k))\right)\right\}, (1-\sigma)L\right\},$$

$$\Gamma_1'(H) = \min\left\{\max\left\{H-1, H-1-\delta(1-k) + \frac{\delta(1-k)v_1}{2\bar{u}}\right\}, (1-\sigma)H\right\}.$$

Then, let us set k is sufficiently large, and consider $\bar{u} \in \left[\frac{\delta(1-k)v_1}{1-\sigma H+\delta(1-k)}, \frac{\delta v_1}{2(1+\delta k)+L((2-\sigma)\delta(1-k)-\sigma)}\right]$. This is possible because, as $k \to 1$, this interval converges to $\left[0, \frac{\delta v_1}{2(1+\delta)-L\sigma}\right] \subset \left[0, \frac{v_1}{2}\right]$. For these values of $\bar{u}$, we obtain $\Gamma_1'(L,0) = (1-\sigma)L$, and $\Gamma_1'(H,0) = H-1$, and thus $p_{r,1}(H,0) < p_{r,1}(L,0) = 1$.

EC.4.3.4. Proof of Proposition 18

Recall that the platform adjusts the wage to match capacity with the number of services. From Lemma 7, the platform increases the supply as much as possible to provide more late services. The number of services, hence the wage, thus increases with late demand. We now show that they also increase with incoming demand. In period $\tau = 2$, this result follows from Proposition 13:

- In Region 1–$\bar{u}$, the platform never provides a timely service to n-type customers. The number of services is then $\min\{1, \sigma D_2 + \Gamma_2\}$, which is clearly non-decreasing with with D_2.
- In Region 2.i–$\bar{u}$, the platform provides σD_2 services to r-type customers and $\min(1-\sigma D_2, \Gamma_2)$ services to n-type customers. If the platform does not provide any timely service to n-type customers, it thus provides $\min(1, \sigma D_2 + \Gamma_2)$ services—which is non-decreasing with D_2. Otherwise, the number of timely services provided to n-type customers satisfies $A_t \in \left\{\frac{v_0-\sigma}{2(1-\sigma)\bar{u}} - \sigma D_2 - \Gamma_2, 1-\sigma D_2 - \Gamma_2, (1-\sigma)D_2\right\}$, so the total number of services falls within $\frac{v_0-\sigma}{2(1-\sigma)}\bar{u}$, 1, or $D_2 + \Gamma_1$, which is also non-decreasing with D_2.
- In Region 2.ii–$\bar{u}$ and Region 3–$\bar{u}$, the platform aims to satisfy all the demand, and thus provides $\min\{1, D_2 + \Gamma_2\}$ services—which is again non-decreasing with D_2.

Turning to Period 1, let us denote the number of timely services provided to n-type customers by $A_1(D_1) = (1-\sigma)D_1 - \Gamma_1'$. Moreover, the wage level satisfies $w_1(D_1, 0) = (\sigma D_1 + A_1(D_1))\bar{u}$ and the cost of service provision is $(\sigma D_1 + A_1(D_1))^2\bar{u}$. We can write the platform's problem as follows:

$$V_1(D_1, 0) = \max_{w_1, A} \tilde{V}_1(D_1, A), \quad \text{where:}$$

$$\tilde{V}_1(D_1, A) = \sigma D_1 \left(1 - \frac{A}{(1-\sigma)D_1}(1-v_0)\right) + Av_0 - (\sigma D_1 + A)^2\bar{u} + \delta \mathbb{E}_{D_2}\left(V_2(D_2, (1-\sigma)D_1 - A)\right).$$

Moreover, the value function V_2 is concave. This can be shown using the same arguments as in the main paper. The proof is omitted for conciseness.

It is sufficient to show that the number of services is larger under high demand than under low demand. Suppose by contradiction that $\sigma H + A_1(H) < \sigma L + A_1(L)$. From the optimality of $A_1(L)$ and $A_1(H)$, we have, for $\varepsilon > 0$ sufficiently small:

$$\frac{\partial \tilde{V}_1(L, A)}{\partial A}\Big|_{A=A_1(L)-\varepsilon} = \frac{\sigma}{1-\sigma}(1-v_0) + v_0 - \mathbb{E}_{D_2}\left(V_2'(D_2, (1-\sigma)L - A_1(L) + \varepsilon)\right) - 2\bar{u}(\sigma L + A_1(L) - \varepsilon) > 0,$$

$$\frac{\partial \tilde{V}_1(H, A)}{\partial A}\Big|_{A=A_1(H)+\varepsilon} = \frac{\sigma}{1-\sigma}(1-v_0) + v_0 - \mathbb{E}_{D_2}\left(V_2'(D_2, (1-\sigma)H - A_1(H) - \varepsilon)\right) - 2\bar{u}(\sigma H + A_1(H) + \varepsilon) < 0.$$

Therefore we have:

$$\begin{aligned}
\mathbb{E}_{D_2}\left(V_2'(D_2, (1-\sigma)H - A_1(H) - \varepsilon)\right) &\geq \frac{\sigma}{1-\sigma}(1-v_0) + v_0 - 2\bar{u}(\sigma H + A(H) + \varepsilon) \\
&> \frac{\sigma}{1-\sigma}(1-v_0) + v_0 - 2\bar{u}(\sigma L + A(L) + \varepsilon) \\
&\geq \mathbb{E}_{D_2}\left(V_2'(D_2, (1-\sigma)L - A_1(L) - \varepsilon)\right).
\end{aligned}$$

For small $\varepsilon > 0$, $(1-\sigma)H - A(H) - \varepsilon > (1-\sigma)L - A(L) - \varepsilon$. It contradicts the concavity of V_2.