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Authors	Gao, Pin; Fan, Xiaoshuai; Huang, Yangguang; Chen, Ying Ju
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Resource Allocation Among Competing Innovators*

PIN GAO† XIAOSHUAI FAN‡ YANGGUANG HUANG§ YING-JU CHEN¶

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Abstract

Many innovative products are designed to satisfy the demand of specific target consumers; thus, the innovators will inevitably compete with each other in the product market. We investigate how a profit-maximizing principal should properly allocate her limited resources to support the innovations of multiple potentially competing innovators. We find that, as the available resources increase, the optimal diversification of investment may first increase and then decrease. This interesting non-monotone pattern is driven by a trade-off between the risk of innovation failure and rent dissipation due to competition. Using this framework, we also analyze a nonprofit principal seeking to maximize the total number of successful innovations, the probability of at least one innovator succeeding, consumer surplus, and total social welfare. A nonprofit principal tends to invest more diversely compared to a for-profit counterpart.

Key words: innovation, competition, R&D, venture capital, resource allocation

1 Introduction

Innovation plays an essential role in economic growth by creating new products, developing new business models, and improving production processes. Venture capitals (VCs),

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†School of Data Science, Chinese University of Hong Kong, Shenzhen. Email: gaopin@cuhk.edu.cn.

‡Department of Information Systems and Management Engineering, College of Business, Southern University of Science and Technology, Shenzhen 518055, Guangdong, China. Email: fanxs@sustech.edu.cn.

§Department of Economics, Hong Kong University of Science and Technology. Email: huangyg@ust.hk.

¶School of Business and Management & School of Engineering, Hong Kong University of Science and Technology. Email: imchen@ust.hk.

governments, and private enterprises invest billions of dollars and other resources every year in R&D to foster innovation. Often, many innovators are targeting the demand of the same group of consumers. For example, as smartphones become popular, many startups develop similar ride-sharing apps targeting the same transportation market; to meet the global demand of COVID-19 vaccines, numerous pharmaceutical companies started over 100 vaccine development projects.[1] In practice, we observe that innovation investors may sometimes invest in multiple companies that are competitors in the product market. For example, several VCs invested in both Uber and Lyft before their IPOs (Eldar et al., 2020). Hong Kong government startup supporting funds invested in both Lalamove and GoGoVan with the same business model of matching van drivers with consumers.[2]

In this paper, we consider a central question faced by innovation investors: how to invest in potentially competing innovators. If the investor invests in only a small number of startups, there is a risk of missing a unicorn company. However, investing in many similar R&D projects or startups with similar business ideas not only requires more resources but also can intensify the competition in the product market. Igami (2017) characterizes an example in the hard disk industry. When more firms decide to innovate and develop the next generation of hard disks, each firm has a smaller market share and earns less profit. In practice, a VC investing in direct product market competitors can lead to a dismal outcome. For example, Grishin Robotics, a California-based VC, invest in two bike-sharing service companies, OBike and Gobee, in Hong Kong in early 2017.[3] The neck-and-neck competition of bike-sharing services soon drove both companies out of business in late 2018 (Du, 2018).

Both private investors and public funding authorities must carefully consider the implication of potential product market competition to the resource allocation decisions. Kaplan and Strömberg (2004) analyze how VCs evaluate startup companies by studying 67 investment memoranda. The results indicate that VCs not only consider companies' internal factors such as quality of management and performance to date but also external factors such as market size and competition. Many memos have statements like "company is targeting a significant market segment", "no competitors", "there is more than enough room for several competitors", "price competition could drive down margins". Based on a survey of VCs in California, Tyebjee and Bruno (1984) also find that the barrier to competitive entry is one major factor considered by VCs in the evaluation of investment

[1] See Le et al. (2020) and www.nytimes.com/interactive/2020/science/coronavirus-vaccine-tracker.html.

[2] See www.jumpstartmag.com/a-race-to-the-top-gogox-vs-lalamove.

[3] See www.dealstreetasia.com/stories/singapore-obike-45m-series-b-grishin-robotics-80447 and www.dealstreetasia.com/stories/79496-79496.

opportunities. In a study of U.S. Small Business Innovation Research (SBIR) programs, Lerner (2000) writes that "awards in many cases led to substantial spillovers. At the same time, the gains in employment and sales of the typical awardee could have come at the expense of small losses for many competitors."

In reality, we observe that investors adopt different investment strategies in different industries. Take Sequoia Capital's portfolio in China as an example.[4] In 2011, Sequoia invested $1.5 billion in JD.com, which soon became a leading e-commerce platform in China. Sequoia exited in 2014 after the successful IPO of JD.com (JD:Nasdaq). In 2017 and 2018, Sequoia invested $1.58 billion in another e-commerce platform, Pinduoduo (PDD:Nasdaq). Pinduoduo was a direct competitor to JD.com and later became China's largest e-commerce platform by the number of users in 2020.[5] From this anecdotal evidence, it seems that Sequoia intentionally avoided investing in multiple e-commerce startups at the same time.[6]

However, Sequoia sometimes invested in several startups that were direct competitors in the product market within the same period of time. Donald Valentine, founder of Sequoia Capital, said in an interview, "my position has always been, you find a great market and you build multiple companies in that market."[7] For example, in 2015, Sequoia invested in The ONE Smart Piano (`www.1tai.com`) and itan8 Pianist (`www.itan8.com`). Both companies offered online piano education via mobile apps with almost the same functions. In 2019, Sequoia invested in Mathplane (`www.mathplanet.com`) and Tongxing School (`www.tongxingschool.com`), who were competitors in online early childhood education. In late 2020 and 2021, Sequoia invested in WeDoctor (`www.guahao.com`), Just Health (`www.justhealth.cn`), Shanhu Health (`www.shanhuhealth.com`), Miaoshou (`www.miaoshou.net`), and Shukun Technology (`www.shukun.net`). All these companies aimed to apply information technology to health care and develop products for online health services. Why Sequoia concentrated its investment in one company in some industry but diversified in multiple startups in another industry? What are the factors affecting investors' adoption of different investment strategies?

These phenomena and questions motivate us to investigate the optimal resource allo-

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[4]Sequoia Capital's portfolio can be found at `www.itjuzi.com/investfirm/1`. ITJUZI (`www.itjuzi.com`) is a database of VC investments and startups in China.

[5]See `www.scmp.com/tech/big-tech/article/3125849/pinduoduo-founder-colin-huang-steps-down-chairman`.

[6]Softbank (9984:Tokyo) had a similar investment practice (`www.itjuzi.com/investfirm/441`). Softbank invested in Alibaba with $20 million in 2000 and $82 million in 2004. After the IPO of Alibaba (BABA:NYSE) in 2014, Softbank exited Alibaba and made several investments in e-commerce startups including Goodbaby, Snapdeal, MUSTIT, and YiKuYi. These e-commerce companies all had a competitive relationship with Alibaba.

[7]See `digitalassets.lib.berkeley.edu/roho/ucb/text/valentine_donald.pdf`.

cation strategy among competing innovators. We explore the investor's (principal, she) optimal resource allocation strategy among multiple innovators (agents, he), who are potential competitors in the post-innovation product market. The principal, who owns the essential resources for innovation, intends to maximize the investment returns from sharing the profit with the agents. We consider a sequential game. The principal first decides how to allocate her resources to competing agents. Then, given the resource allocation, each agent decides how much effort to exert on innovative activity. The success rate of innovation depends on both the amount of resources and effort levels. Agents who successfully innovate will launch products in the product market. If more than one product is launched, these products will compete with one another and affect the profit.

In equilibrium, the agent's effort level first increases and then decreases in the amount of resources allocated. This non-monotone relationship is rooted in the pattern of complementarity between the resources and the agent's effort in determining the success rate. When there is only a very small amount of resources, they must be complemented by a substantial amount of effort to achieve a reasonably high success rate. As a result, the agent is reluctant to exert effort or even chooses not to participate if resources are insufficient. However, when resources become excessively abundant, because of the substitutability between effort and resources, the agent tends to reduce his effort. Moreover, there is a strong effort-reduction effect if the other agent also receives many resources and actively innovates because the potential competition and rent dissipation in the product market discourage the agent from exerting effort *ex ante*.

In terms of the principal's resource allocation, we identify a key trade-off between the risk of innovation failure and rent dissipation due to competition among agents. When facing uncertainty in innovation, a profit-oriented principal can lower the risk of having no successful product development by diversifying her investment to more agents. However, if multiple similar innovative products are successfully developed, the profit from the product market will shrink. This trade-off, together with the agents' strategic choices of effort levels, leads to an interesting pattern in the optimal resource allocation strategy: the diversification of investment may not be monotone in the amount of resources. When the rent dissipation effect is at a moderate level, as the available resources increase, the principal will first invest in one agent, then switch to investing in multiple agents and finally switch back to investing in one agent.

When there is a shortage of resources, concentrating all of the resources to invest in one agent will guarantee the agent's active participation in his innovative project. Dividing the resources could cause each agent to receive too little investment to be incentivized. As the resources become more abundant, diverse investment can increase the overall success rate of innovation because each agent conducts innovation independently, and the probability of having at least one successful agent increases. However, diverse investment also increases the probability of more than one agent launching products on the market, causing a reduction in the total profit due to the business-stealing effect. As a result, the principal tends to concentrate her investment to avoid *ex post* competition when resources are abundant and the competition is intense.

We derive the main results of optimal resource allocation strategy in a baseline model with two agents. These results are robust to the cases of an endogenous profit-sharing rule, costly resources, two general classes of innovation success functions, and multiple agents. However, the optimal resource allocation strategy does not exhibit the non-monotone pattern if the principal is not profit oriented and aims at maximizing the number of successful innovations, the probability of at least one successful innovation, consumer surplus, or total social welfare. Intuitively, diversifying investment is likely to lead to more successful innovations, and having more innovative products always improves consumer and social welfare. We also explore the case of competing investors. Because of the rent dissipation problem (Choi, 1996) in the product market, introducing a new investor could make both investors and agents worse off.

2 Literature Review

Since the groundbreaking work of Schumpeter (1942), a large body of literature has explored the relationship between innovation and competition. One central question is how competition affects innovators' incentives. The classical Schumpeterian view emphasizes that the essential driving force of innovation is the monopoly rent. Aghion and Howitt (1992) and Caballero and Jaffe (1993), among many others, have shown that excessive competition discourages innovation. However, empirical studies, such as Nickell (1996) and Blundell et al. (1999), find positive effects of competition on innovation, followed by several theoretical works, such as Aghion et al. (2001). Aghion et al. (2005) combines both theoretical and empirical analyses and shows that the relationship between competition and innovation exhibits an inverted U-shape. Marshall and Parra (2019) argue that competition can stimulate innovation if the profit gap between the leader and the followers weakly increases with the number of firms. Most previous papers have focused on how firms choose their innovation strategies under various market structures. This paper emphasizes that *ex ante* innovation activities are affected by potential *ex post* market competition, and the principal must anticipate this effect when making investment decisions.[8]

Our paper contributes to three strands of the literature. First, our study sheds light on how financial resources affect operational outcomes.[9] Nelson (1961) notes the importance of parallel R&D efforts by multiple agents in uncertain innovation tasks. He suggests that resource allocation should be gradually concentrated as more information becomes available. Schlapp et al. (2015) further investigate the interaction among agents in effort exerting and information sharing. They show that the resource allocation decisions should align the incentives of agents to pursue a common goal and reveal their findings. Chao et al. (2009) show that resource allocation affects the incentives in pursuing incremental versus radical innovation projects. Ning and Babich (2017) find that firms obtaining resources by debt financing leads to risk shifting, resulting in more investment in high-risk R&D projects (lower free-riding incentives). These examples demonstrate that the allocation of financial resources has strong impacts on various aspects of innovation outcomes. Most previous studies in the literature have emphasized interaction among innovative agents during the process of R&D, such as information sharing (Schlapp et al., 2015) and free riding (Ning and Babich, 2017). In contrast, we consider the scenario in which agents conduct independent R&D projects but their profits from the product market are correlated. We provide a clear message to investors that the innovation outcomes depend not only on the competition during R&D but also on the competition after R&D.

Second, our study is related to the patent race literature. In many industries, the latest innovative product is protected by a patent. Agents compete by developing the next generation of products (Reinganum, 1985). In markets with patent races, competition affects innovation and welfare in a complicated way. For example, Marshall and Parra (2019) find that the pace of innovation critically depends on whether the profit advantage of the market leader increases or decreases in the competitive intensity. Shapiro (2000) point out that innovation carried out by multiple agents can lead to complementary technology, which can cause patent thicket problems and excessively high royalty rates. Kitch (1977) points out that the patent system allows the upstream patent owner to coordinate investment and information exchange in downstream patent races for follow-up innovation. Branstetter and Sakakibara (2002) study a Japan innovation policy supporting research consortia among innovative firms and conclude that "the design of consortium seems to be more important than the level of resources expended." In this paper, we emphasize

[8] This question is a popular operational problem faced by investors in reality. For example, Alibaba provides financial support to help retailers manage demand uncertainty. When making operational decisions, the platform must consider the implications for competition intensity among retailers (Dong et al., 2018).

[9] Babich and Kouvelis (2018) provide a survey of studies that combines financial decisions with operations choices and risk management.

the investor's role in influencing the innovation success rate, the product market profit, and welfare by her resource allocation strategy among competing agents.

Third, our model provides new insight into the literature on the design of innovation contests. Contests are a widely used mechanism to stimulate research or procure innovative products (Che and Gale, 2003). In a contest, agents exert costly effort to compete for one or several prizes granted based on the ranking of their performance. Most previous studies have focused on the design of the prize scheme (Terwiesch and Xu, 2008), information disclosure policy (Zhang and Zhou, 2015; Bimpikis et al., 2019), or other contest rules to induce agents to actively participate and exert effort. Typical objectives of the contest designer (principal) include maximizing total effort (Moldovanu and Sela, 2001), the highest or average performance outcomes (Hu and Wang, 2017; Ales et al., 2017; Körpeoğlu and Cho, 2018) and K-best outputs (Ales et al., 2014). In a contest, competition occurs during the process of innovation, and agents receive monetary transfers after exerting innovative effort. Our model is based on a scenario in which resources are first allocated to agents, and then the agents conduct innovation and compete for the "prize" from the product market. This setting suggests a new tool of contest design: the principal can divide her budget for post-innovation prizes and pre-innovation resources. The pre-innovation resource allocation among agents can be used to handicap some advantageous agents (Pérez-Castrillo and Wettstein, 2016), encourage participation, and maintain a proper competitive intensity.

3 The Model

3.1 Model Setup

We consider a one-principal-two-agent model with a resource allocation stage followed by an innovation stage. A principal has a finite amount of resources with capacity $\overline{B} > 0$ that can be allocated to two agents indexed by $i = 1, 2$. The resources are key inputs for innovation. In the resource allocation stage, the principal chooses a resource allocation, (b_1, b_2), subject to the budget constraint $b_1 + b_2 \leq \overline{B}$ with $b_i \geq 0$.

In the innovation stage, given (b_1, b_2), two agents simultaneously choose their effort levels (x_1, x_2). Agent i incurs a cost x_i by exerting effort $x_i \geq 0$. For each agent, the outcome of innovation is binary: either success or failure. The success rate of agent i follows the innovation success function:

$$p(b_i, x_i) = 1 - e^{-b_i x_i}, \tag{1}$$

which is increasing and concave in both b_i and x_i. This functional form implies that having more resources or exerting more effort will lead to a higher probability of successful innovation, but their marginal effects are decreasing. The resource is essential for innovation. If an agent does not receive any resources, his probability of success is zero regardless of how much effort he exerted.[10] If the agent fails to innovate, his profit is zero.

Once an agent successfully innovates, he launches a new product in the market. The profit from the product market is shared between the agent and the principal. The agent obtains a $\gamma \in (0, 1)$ fraction of the profit. The profit-sharing rate γ is exogenously determined.[11] The agent's payoff is the expected profit from the innovative product market less his effort cost. The principal receives the remaining $1 - \gamma$ fraction of the profit from both agents.

We normalize the profit from one innovative product in the market without competitors to 1. If both agents succeed, two competing products are launched. Let the profit of each product be $\alpha \in [0, 1]$. When $\alpha = 1$, the two innovative products target two separate groups of consumers and are not competing. However, when $\alpha < 1$, the two products are substitutes and compete for the same group of consumers. The parameter α measures the intensity of competition and the strength of the business-stealing effect. Appendix A demonstrates how values of α can be directly mapped to the institutional details regarding product market competition.

Discussion of model assumptions

Innovation success function. The exponential functional form of (1) has been widely used in the literature, for example, Reinganum (1982, 1983). It can be interpreted in a dynamic way. Suppose that the innovation is conducted over a period of duration 1, and the event of successful innovation follows a Poisson process with a rate of $b_i x_i$. Then, at the end of the period, the probability of success will be $1 - e^{-b_i x_i}$. This exponential innovation success function can also be treated as a static version of the Poisson bandit model (Bergemann and Valimaki, 2006). Bergemann and Hege (2005) use the Poisson bandit model to study the optimal flow of investment in financing an innovative entrepreneur. The static simplification enables us to study the problem of investing in multiple agents.

[10]For example, many startups need financial investment from VCs to implement their innovative ideas. Researchers conduct research projects conditional on receiving sufficient grants from funding authorities. In the absence of sufficient resources, it is almost impossible for these projects to succeed.

[11]In reality, the profit-sharing rate is usually determined by the funding authority's technology transfer policy or the VC market. We endogenize the principal's choice of γ in Section 4.1.

In Appendix C.2, we expand our analysis under two large classes of innovation success functions.

We restrict our attention to the case in which agents perform innovation activities independently. The interaction of two agents is purely driven by the possible competition in the product market. In reality, agents also interact in the innovation stage in the form of patent race (Marshall and Parra, 2019), research collaboration (Branstetter and Sakakibara, 2002), or knowledge spillover (Bloom et al., 2013). Interaction during innovation can be modeled by correlated success rates, but having two layers of interaction will substantially complicate the optimal resource allocation problem. In this paper, we focus on the post-innovation interaction, which helps us simplify the analysis and clearly delineate the key trade-off.

Principal's objective function. It is reasonable to assume that the principal maximizes returns from investment given a fixed budget and does not incur a marginal cost from investing more resources. This setting resembles most VCs and public funding authorities in reality. VCs typically collect funds in waves from institutional investors and high net worth individuals. Given a fixed amount of funds, VC managers make decisions on which startups to invest in and how much for the purpose of maximizing investment returns.[12] Public funding authorities typically obtain funds from government budgets and donations, so allocating more funds does not directly incur costs. In Appendix C.1, we show that having a positive marginal cost of investment does not qualitatively change our results. In Section 4.3, we consider nonprofit principals with alternative objectives such as number of successful innovations and consumer surplus.

Contract form. In this paper, we restrict the principal to adopting a simple profit-sharing contract form. This setting resembles the practice of equity investment by VCs and many technology transfer agreements by universities in reality. The investment contracts between the funding agencies and startups typically assign certain equity shares to the investors. We do not analyze more complicated contract forms for two reasons. First, it can be extremely challenging to adopt the mechanism design framework to analyze an environment with competing agents. For this reason, we do not consider a model in which agents have private information. Second, if we grant the principal more authority or control rights, she can simply shut down one agent's product when both agents succeed in

[12] VCs make most of their money based on the percentage of investment profits plus fund management fees. VC managers care about returns from investment and do not face a marginal cost from investing more money under their management (Gompers and Lerner, 1998).

innovation. Then, the key trade-off between the risk of innovation failure and rent dissipation would not exist.

3.2 Equilibrium Analysis of the Innovation Stage

The equilibrium analysis is conducted via backward induction. We divide the analysis into two cases: the principal investing in only one agent and investing positive amounts in both agents. In the latter case, the resources allocated to the two agents can be asymmetric, although the two agents are *ex ante* symmetric.

One agent receiving resources

Suppose that the principal only invests in agent i with $b_i > 0$. By (1), the other agent lacking the essential resources has zero probability of success. If agent i successfully innovates, he receives a profit γ from the product market. Given b_i, agent i chooses his effort x_i by maximizing his expected payoff $\gamma p(b_i, x_i) - x_i$. It is easy to show that the equilibrium effort level is

$$x_i^*(b_i) = \begin{cases} 0, & \gamma b_i < 1 \\ \frac{\ln \gamma b_i}{b_i}, & \gamma b_i \geq 1. \end{cases} \tag{2}$$

The red solid line in Figure 1 illustrates $x_i^*(b_i)$. The participation of the agent requires that the expected payoff is positive, which implies that $\gamma b_i > 1$. When $\gamma b_i \leq 1$, the profit share and the allocated resources are too small to encourage the agent to exert costly effort on innovation. When $\gamma b_i > 1$, the equilibrium effort level x^* follows a hump shape in b_i.[13] Intuitively, when a small amount of resources is allocated, the agent does not have a strong incentive to exert effort because the success rate is low. However, when abundant resources are allocated, the agent can rely on these resources and need not exert much effort. $x_i^*(\cdot)$ reaches its maximum at $b_i = \frac{e}{\gamma}$, which means that, if the agent's profit share γ is higher, the maximal effort can be induced by investing less resources.

Although the effort level is non-monotone in b_i, the equilibrium success rate,

$$p_i(b_i, x_i^*(b_i)) = 1 - e^{-b_i x_i^*} = \begin{cases} 0 & \gamma b_i \leq 1, \\ 1 - \frac{1}{\gamma b_i} & \gamma b_i > 1, \end{cases} \tag{3}$$

is increasing in b_i. Therefore, investing more resources stimulates innovation.

[13] The hump shape is not essential to our main results. See Appendix C.2.

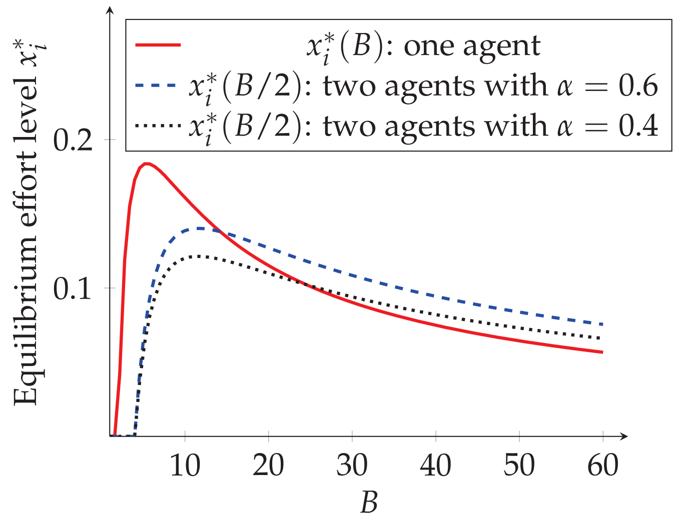


Figure 1: Agent's Equilibrium Effort Level x_i^* with $\gamma = 0.5$.

Both agents receiving resources

Given $b_1 > 0$ and $b_2 > 0$, the equilibrium effort levels, $(x_1^*(b_1, b_2), x_2^*(b_1, b_2))$, are determined by a static game between two agents. Let $y_i \equiv 1 - p(b_i, x_i)$ denote the failure rate; then, $x_i = -\frac{\ln y_i}{b_i}$. Because there is a one-to-one correspondence between x_i and y_i, it is equivalent to considering the agent choosing y_i instead of x_i. This transformation simplifies the algebra in the equilibrium analysis. We restrict $y_i > 0$ because achieving a zero failure rate ($y_i = 0$) requires an infinitely large effort. The boundary condition, $y_i = 1$, indicates that an agent exerts zero effort and thus has zero probability of success. Without loss of generality, let $b_1 \geq b_2$.

Given resources b_i and the other agent's failure rate y_{-i}, agent i's best response is determined by solving the following problem:

$$
\begin{aligned}
(4) \qquad Y(b_i, y_{-i}) &:= \underset{0<y_i\leq 1}{\text{argmax}} \{\gamma(1-y_i)(\alpha(1-y_{-i}) + y_{-i}) + \frac{\ln y_i}{b_i}\} \\
&= \underset{0<y_i\leq 1}{\text{argmax}} \left\{ 1 - y_i + \frac{\ln y_i}{\gamma[\alpha(1-y_{-i}) + y_{-i}]b_i} \right\}.
\end{aligned}
$$

We first consider the case of an interior solution in which both agents exert positive effort. Lemma 1 characterizes the condition under which both agents exert positive effort.

Lemma 1. *The sufficient and necessary condition for* $0 < y_1^* \leq y_2^* < 1$ *is*

$$
(5) \qquad \gamma b_2 > 1 \quad \textit{and} \quad 1 \leq \frac{b_1}{b_2} < 1 + \alpha(\gamma b_1 - 1).
$$

Under this condition, y_1^* and y_2^* are unique.

To encourage both agents to actively participate in innovation, the amount of resources allocated to each agent cannot be too small ($\gamma b_2 > 1$). Moreover, the allocation cannot be too imbalanced ($\frac{b_1}{b_2} < 1 + \alpha(\gamma b_1 - 1)$). Otherwise, the business-stealing effect will discourage the less resourceful agent from exerting any effort because the expected profit generated from the product market cannot compensate for his effort cost.

Under condition (5), the equilibrium failure probabilities, y_1^* and y_2^*, are determined by the first-order conditions (FOC) of (4), which can be rearranged as

$$\frac{1}{\gamma b_1} = \alpha(1 - y_2^*)y_1^* + y_1^* y_2^* \quad \text{and} \quad \frac{1}{\gamma b_2} = \alpha(1 - y_1^*)y_2^* + y_1^* y_2^*. \tag{6}$$

The solution, $y_1^*(b_1, b_2)$ and $y_2^*(b_1, b_2)$, exhibits the following comparative statics:

Lemma 2. *Given condition (5), y_i^* decreases in b_i and increases in b_{-i}.*

There is a qualitative difference in the effort exertion pattern between the cases of investing in one agent and investing in two agents. When investing in only one agent, a larger amount of resources always leads to a higher probability of successful innovation. However, in the latter case, increasing b_{-i} will discourage agent i from exerting more effort (lower y_i^*) because he is more likely to face competition in the product market. It is possible that reducing the total allocated resources can weaken competition and better incentivize the agents. Therefore, the principal must carefully consider the impact of potential product market competition facing multiple agents.

3.3 The Optimal Resource Allocation Strategy

Given a resource capacity, $\overline{B}$, how should the principal allocate her resources to the two agents? Based on the equilibrium failure rate in (4), the principal's optimization problem is

$$\begin{aligned} \Pi(\overline{B}) := \max_{b_1 \geq 0, b_2 \geq 0} \quad & (1-\gamma)[2\alpha(1-y_1^*)(1-y_2^*) + (1-y_1^*)y_2^* + (1-y_2^*)y_1^*] \\ \text{s.t.} \quad & b_1 + b_2 \leq \overline{B},\ y_1^* = Y(b_1, y_2^*),\ y_2^* = Y(b_2, y_1^*). \end{aligned} \tag{7}$$

The term $2\alpha(1 - y_1^*)(1 - y_2^*)$ is the total expected profit when both agents successfully innovate. This total profit decreases in α.

We start the analysis by first fixing a total amount of investment $B = b_1 + b_2 \leq \overline{B}$ with $b_1 \geq b_2$. Given B, the principal's investment decision degenerates to being one dimensional. Without loss of generality, let b_2 be the principal's choice variable. The principal's problem can be rewritten as

$$\max_{b_2 \in [0, \frac{B}{2}]} \tilde{\Pi}(B - b_2, b_2) := (1-\gamma)[2\alpha(1-y_1^*)(1-y_2^*) + (1-y_1^*)y_2^* + (1-y_2^*)y_1^*].$$

We can show that the solution satisfies the following proposition.

Proposition 1. *The optimal resource allocation scheme must exhibit one of the following properties: either $b_2^* = 0$ or $b_1^* = b_2^*$.*

Figure 2 plots the expected profit of each agent and the total profit of two agents as functions of b_2/B. From Panels (a) and (c), we see that $\tilde{\Pi}(B - b_2, b_2)$ is quasiconvex on the domain $b_2 \in [0, \frac{B}{2}]$. The total profit reaches its maximum at either the left boundary as in Panel (a) or at the right boundary as in Panel (c). Therefore, the principal will invest an equal amount in two agents or invest in only one agent.

This prediction is consistent with the allocation of R&D grants in practice. For example, the SBIR program assigns the same amount, \$150,000, to successful applicants for Phase-I R&D grants (Howell, 2017). Hong Kong Cyberport Creative Micro Fund allocates HKD100,000 to each funded startup.[14] In both examples, the funding authorities fund approximately 10% to 20% applicants among hundreds of them every year. Hence, an agent either obtains a fixed amount of funding or does not receive any funding at all. Intuitively, given a fixed amount of resources, the principal may not support all agents because she must guarantee that funded agents have sufficient resources and actively participate.

Investing in one agent

Based on Proposition 1, we compute and compare the principal's profits under the two investment strategies. Given a total investment $B > 0$, let $\Pi_m(B)$ denote the profit from investing in one agent. By (3), the principal's profit is

$$\Pi_m(B) = \begin{cases} 0 & \gamma B \leq 1, \\ (1-\gamma)(1 - \frac{1}{\gamma B}) & \gamma B > 1, \end{cases} \tag{8}$$

where the subscript m indicates that the resource allocation strategy induces a monopoly product in the product market. Because $\Pi_m(B)$ increases in B, the principal will exhaust her resources. We obtain the following result.

[14]See `www.cyberport.hk/en/about_cyberport/cyberport_youth/cyberport_creative_micro_fund`.

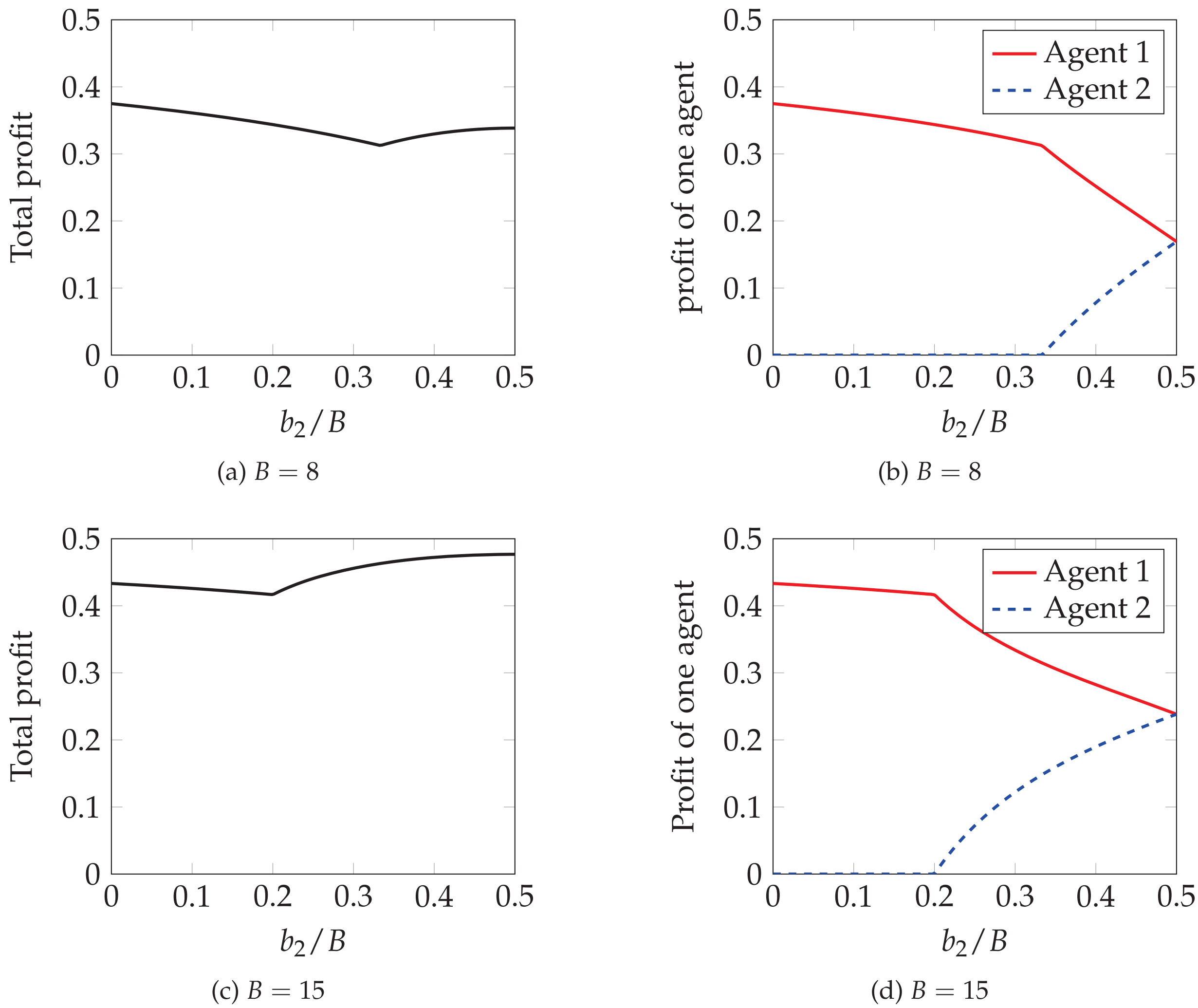


Figure 2: Illustration of Proposition 1, where $\gamma = 0.5$ and $\alpha = 0.6$.

Proposition 2. *When the principal only invests in one agent, the principal chooses* $(b_1, b_2) = (\overline{B}, 0)$.

Accordingly, the principal's expected payoff when investing in one agent is $\Pi_m(\overline{B})$ and the expected payoff of the agent who receives investment is

$$U_m(\overline{B}) = \begin{cases} 0 & \gamma\overline{B} \leq 1, \\ \gamma - \frac{1+\ln \gamma\overline{B}}{\overline{B}} & \gamma\overline{B} > 1. \end{cases} \tag{9}$$

U_m is also nondecreasing in $\overline{B}$. Thus, both the principal and the agent who receives investment are always better off under a larger resource endowment.

Investing in two agents

Given $B \leq \overline{B}$, by Proposition 1, $b_1 = b_2 = \frac{B}{2}$. By Lemma 1, when $\gamma B > 2$, each of the two agents will exert positive effort. By (6), the equilibrium failure rate of each agent is

$$y_1^* = y_2^* = y^* := \frac{4}{\alpha B\gamma + \sqrt{B\gamma}\sqrt{8 - 8\alpha + \alpha^2 B\gamma}}. \tag{10}$$

Obviously, y^* decreases in B, so investing more resources increases the failure rate of both agents. The equilibrium effort level, $x^*(\frac{B}{2}) = -\frac{lny^*}{B/2}$, exhibits a hump shape, which is depicted by the dash lines in Figure 1. Note that it require more resources to reach the peak of equilibrium effort when investing in two agents than that of one agent, and the maximum of $x_i^*(\frac{B}{2})$ is also less than that of $x_i^*(B)$. These phenomena indicate that the principal must devote more resources to incentivize competing agents under the potential business-stealing effect.

The principal's expected profit is

$$\begin{aligned}\Pi_d(B) =& (1-\gamma)[2\alpha(1-y^*)^2 + 2y^*(1-y^*)] \\ =& \begin{cases} 0, & \gamma B \leq 2, \\ (1-\gamma)(\alpha - \frac{4}{\gamma B} + \sqrt{\frac{\gamma\alpha^2 B - 8\alpha + 8}{\gamma B}}), & \gamma B > 2. \end{cases}\end{aligned} \tag{11}$$

where the subscript d indicates that the resource allocation strategy can induce a duopoly product market structure. The expected payoff of each agent is

$$\begin{aligned}U_d(B) =& \gamma(1-y^*)(\alpha(1-y^*) + y^*) + \frac{\ln y^*}{B/2} \\ =& \begin{cases} 0, & \gamma B \leq 2, \\ \frac{1}{2B}\left[\alpha B\gamma + \sqrt{B\gamma}\sqrt{\gamma\alpha^2 B - 8\alpha + 8} + 4\ln\frac{4}{\alpha B\gamma + \sqrt{B\gamma}\sqrt{8-8\alpha+\alpha^2 B\gamma}} - 4\right], & \gamma B > 2. \end{cases}\end{aligned} \tag{12}$$

The following lemma summarizes the properties of $\Pi_d(B)$, and $U_d(B)$.

Lemma 3. *If the principal adopts the equal investment strategy, $b_1 = b_2 = \frac{B}{2}$, and $\gamma B > 2$, then*

(i) When $\alpha \geq \frac{1}{2}$, the principal's profit $\Pi_d(\cdot)$ always increases in B. When $\alpha < \frac{1}{2}$, the principal's expected profit first increases and then decreases in B. It reaches the maximum at $B = \frac{8(1-\alpha)}{\gamma(1-2\alpha)}$.

(ii) The agent's expected payoff, $U_d(B)$, is always increasing in $[\frac{2}{\gamma}, B_d^*]$, where $B_d^* = \arg\max_B \Pi_d(B)$.

Lemma 3-(i) considers two cases. When $\alpha \geq \frac{1}{2}$, the business-stealing effect is weak. The total profit from the product market with two successful agents is larger than that with only one successful agent ($2\alpha \geq 1$), so $\Pi_d(B)$ is always increasing in B. In contrast, when $\alpha < \frac{1}{2}$, rent dissipation occurs, so $\Pi_d(B)$ decreases in B when B becomes large. As a result, investing too many resources can backfire for the principal, although the resources are not costly *per se*.

Lemma 3-(ii) states that an agent's expected payoff increases in B before reaching the peak of $\Pi_d(B)$. Intuitively, the principal's interest is aligned with those of the agents under the proportional profit sharing rule, so it must be suboptimal for the principal to adopt an investment strategy that hurts the agents.

Comparison between two strategies

Now, we compare the principal's expected profit in (8) and (11) under the two investment strategies. Because $\Pi_d(B) = 0$ when $\gamma B < 2$, we focus on the case in which $\gamma B > 2$.

Proposition 3. *$\Pi_m(B)$ and $\Pi_d(B)$ exhibit the following properties:*

(i) $\max\{\Pi_m(B), \Pi_d(B)\}$ *is increasing in B.*

(ii) $\Pi_m(B) - \Pi_d(B)$ *is quasiconvex in* $B \in (\frac{2}{\gamma}, \infty)$.

Proposition 3-(i) implies that the principal will always exhaust her resources, i.e., $b_1^* + b_2^* = \overline{B}$. Note that as $B \to \frac{2}{\gamma}$, $\Pi_d(B) \to 0$, so when B is low, $\Pi_m(B) - \Pi_d(B) > 0$, and the principal invests in only one agent. Proposition 3-(ii) implies that when $B \geq \frac{2}{\gamma}$, $\Pi_m(B) - \Pi_d(B)$ may first decrease and become negative (depending on α and γ); thus, investing in two agents might become optimal. The quasiconvexity implies that there only exists at most one continuous region of $\overline{B}$ where investing in two agents is optimal.

Based on the results above, we obtain the optimal resource allocation strategy as follows.

Theorem 1. *Given α, γ, and $\overline{B}$, the optimal resource allocation strategy is as follows:*

(i) If $\gamma\overline{B} \leq 1$, the principal invests nothing and obtains zero profit, i.e., $(b_1^*, b_2^*) = (0, 0)$.

(ii) If $1 < \gamma\overline{B} \leq 2$, the principal invests all of her resources in one agent, i.e., $(b_1^*, b_2^*) = (\overline{B}, 0)$.

(iii) If $\gamma\overline{B} > 2$, the principal uses all of her resources. As shown in Figure 3,

a. When $\alpha \leq 6\sqrt{2} - 8$ (region a), the principal invests in one agent, i.e., $(b_1^*, b_2^*) = (\overline{B}, 0)$.

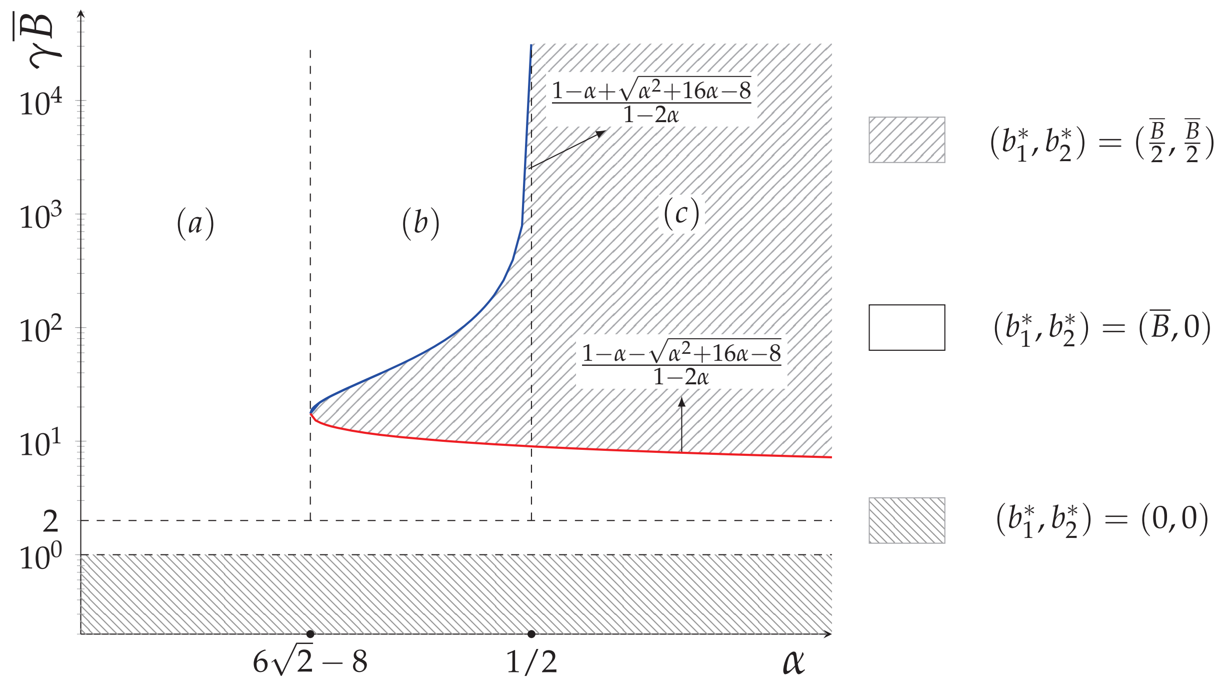


Figure 3: Principal's Optimal Resource Allocation Strategy

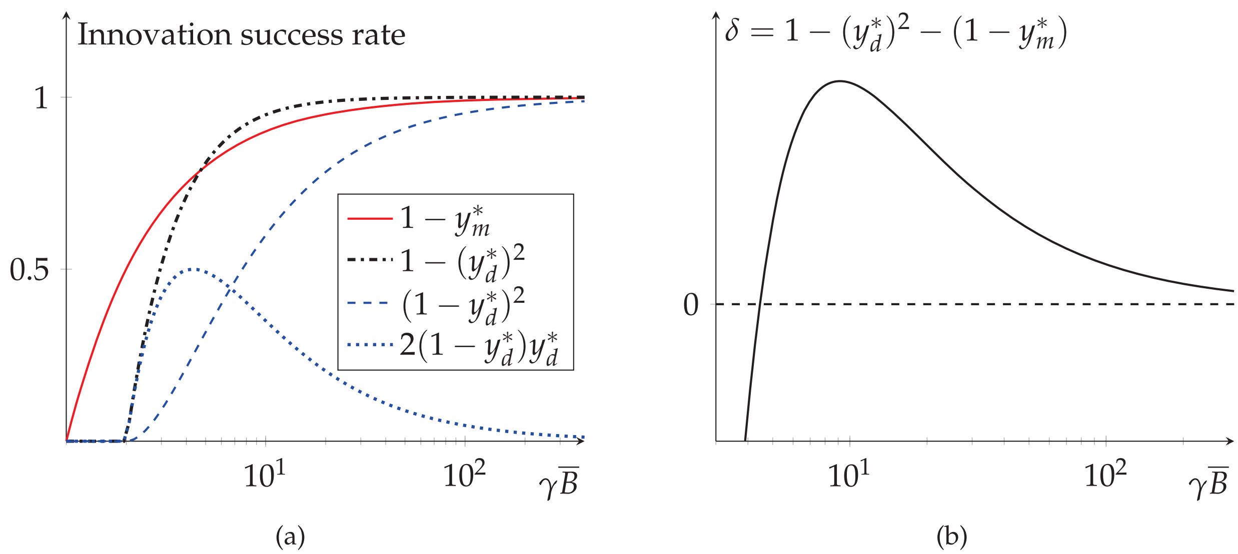
b. *When* $6\sqrt{2} - 8 < \alpha < \frac{1}{2}$ *(region b), if* $\gamma\overline{B} \in [\frac{1-\alpha-\sqrt{\alpha^2+16\alpha-8}}{1-2\alpha}, \frac{1-\alpha+\sqrt{\alpha^2+16\alpha-8}}{1-2\alpha}]$ *(shaded area), the principal invests equally in two agents, i.e.,* $(b_1^*, b_2^*) = (\frac{\overline{B}}{2}, \frac{\overline{B}}{2})$*; otherwise the principal invests in one agent, i.e.,* $(b_1^*, b_2^*) = (\overline{B}, 0)$.

c. *When* $\alpha \geq \frac{1}{2}$ *(region c), if* $\gamma\overline{B} \geq \frac{1-\alpha-\sqrt{\alpha^2+16\alpha-8}}{1-2\alpha}$ *(shaded area), the optimal allocation strategy is* $(b_1^*, b_2^*) = (\frac{\overline{B}}{2}, \frac{\overline{B}}{2})$*; otherwise, it should be* $(b_1^*, b_2^*) = (\overline{B}, 0)$.

The term $\gamma\overline{B}$ measures the abundance of resources from the agents' perspective. In Theorem 1-(i), the resources are scarce ($\gamma\overline{B} \leq 1$). Even if an agent obtains all the available resources, the effort cost still outweighs the expected profit. Hence, the agent will exert zero effort; thus, the principal will hold her resources. When there are a slightly more resources ($1 < \gamma\overline{B} \leq 2$), Theorem 1-(ii) shows that the principal can earn a positive profit by granting the resources to a single agent. The resources are still not sufficient for diversification because investing in two agents will lead to insufficient resources to incentivize either of them to exert effort (Lemma 1).

When the amount of resources is sufficiently large to incentivize two agents ($\gamma\overline{B} > 2$), the key trade-off between the risk of innovation failure and rent dissipation emerges. The strength of rent dissipation effect is determined by α. If the product market competition is intense (small α), the principal is more inclined to invest in only one agent. Hence, Figure 3 is divided into the left region and right region based on α, corresponding to investing in one agent and two agents, respectively.

Conversely, the risk of innovation failure is mainly determined by $\gamma\overline{B}$ and investment strategy. Figure 4-(a) compares the innovation success rates under two investment strategies. y_m^* is the failure rate of agent 1 given $(b_1, b_2) = (\overline{B}, 0)$. y_d^* is the failure rate of one agent defined in (10) given $(b_1, b_2) = (\overline{B}/2, \overline{B}/2)$. Panel (b) depicts $\delta = 1 - (y_d^*)^2 - (1 - y_m^*)$, which is the difference between the probability of having at least one successful agent under the equal investment strategy $(1 - (y_d^*)^2)$ and the success rate of devoting all resources to one agent $(1 - y_m^*)$. δ is an important quantity that measures the benefit of diverse investment in reducing the risk of innovation failure.



Note: In Panel (a), $1 - y_m^*$ indicate the success rate of agent 1 given $(b_1, b_2) = (\overline{B}, 0)$. The other three lines are the probability that at least one agent succeeds $(1 - (y_d^*)^2)$, both agent succeed $((1 - y_d^*)^2)$, and only one agent succeeds $(2(1 - y_d^*)y_d^*)$, respectively, given $(b_1, b_2) = (\frac{\overline{B}}{2}, \frac{\overline{B}}{2})$. Panel (b) plots the probability that at least one agent succeeds $(b_1, b_2) = (\frac{\overline{B}}{2}, \frac{\overline{B}}{2})$ minus the probability thzt agent 1 succeeds given $(b_1, b_2) = (\overline{B}, 0)$.

Figure 4: Innovation Success Rates and δ under Two Investment Strategies

Theorem 1-(iii)-a states that when the rent dissipation effect is strong ($\alpha \leq 6\sqrt{2} - 8$), the principal wants to avoid product market competition which leads to very low profits, so she cares about the probability of inducing a monopoly product market. As shown in Figure 4-(a), the probability of having exactly one successful agent is always higher when investing in one agent than investing in two agents. Hence, the principal will devote all resources to one agent.

Theorem 1-(iii)-c characterizes the case in which the total profits from two competing agents are greater than the profit from one successful agent ($\alpha > \frac{1}{2}$). In this case, the most important thing for the principal is to avoid having no product in the market. Figure 4-(b) shows that when $\gamma\overline{B}$ is small, $\delta < 0$, so devoting all resources to one agent leads to a higher probability of having a successful agent; but when $\gamma\overline{B}$ becomes large, $\delta > 0$, and dividing the resources between two agents yields benefits from lowering the risk of innovation failure and also increases the total profit if both agents succeed. We find that the threshold condition to justify diverse investment is $\gamma\overline{B} \geq \frac{1-\alpha-\sqrt{\alpha^2+16\alpha-8}}{1-2\alpha}$ (red line in Figure 3).

Theorem 1-(iii)-b demonstrates the most interesting case with an intermediate level of α. Figure 4-(b) shows that, as $\gamma\overline{B}$ increases, δ is initially negative, and then becomes positive, but gradually converges to zero. Hence, when resources are limited, there is no benefit from diverse investment, so the principal will concentrate her resources. With more resources, δ becomes positive, and the principal invests in two agents as the benefit from diverse investment outweighs the rent dissipation ($\alpha < \frac{1}{2}$). However, when $\gamma\overline{B}$ becomes sufficiently large, the benefit from diverse investment shrinks. Moreover, the loss from rent dissipation becomes large, because the probability of having one successful agent decreases, and the probability of having a competitive product market increases, as illustrated by two blue dotted lines in Figure 4-(a). As a result, with abundant resources, the small benefit from diverse investment in reducing the risk of innovation failure is dominated by the loss from rent dissipation. In summary, as $\gamma\overline{B}$ increases, and the optimal resource allocation rule switches from investing in one agent to investing in two agents and then back to investing in one agent.

The optimal investment strategy above is somewhat consistent with the example of Sequoia's portfolio in the introduction. Establishing a new e-commerce platform requires a huge amount of investment (> \$1.5 billion), and the competition between e-commerce platforms can be very intense. Moreover, the uncertainty involved in developing the product of an online platform is relatively low. In contrast, there was a great deal of uncertainty in developing successful products in newly emerged markets of online education and online health care, but the required amount of investment for each startup was relatively small. That might be the reason that Sequoia only supported one e-commerce platform at a time, but diversified its investment in several startups in online education and health service.

4 Extensions

In this section, we show that the main features of the optimal resources allocation strategy are robust under an endogenous profit sharing rule and multiple agents. We also explore the optimal investment strategy of a nonprofit principal. Appendix C covers other extensions including costly resources, generalization of innovation success function, and the case of competing investors.

4.1 Endogenous Profit Sharing

In the baseline model, we assume that the principal will charge an exogenously fixed share $1-\gamma$ of the profit. In addition, our analysis shows that given γ, when $\gamma\overline{B} \geq 1$, the principal should exhaust her resources by investing all of them in one agent or investing an equal amount in each of two agents. Because the resources are essential for successful innovation, the principal might have the power to strategically determine the profit-sharing rate γ.

Suppose that the principal can endogenously choose a profit-sharing rate γ in addition to the resource allocation. Given any γ, the optimal resource allocation will still follow Theorem 1. Similar to the analysis of the baseline model, we first separately investigate the case of investing in one agent and the case of investing in two agents, and then we compare the profits between these two cases to derive the optimal resource allocation rule.

Based on Proposition 2, if the principal invests in only one agent, she should grant all resources $\overline{B}$ to this agent as long as $\overline{B} \geq \frac{1}{\gamma}$.[15] The agent's equilibrium failure rate y_i^* is determined by maximizing his expected payoff function with $b_i = \overline{B}$:

$$\max_{0<y_i\leq 1} \gamma(1-y_i) + \frac{\ln y_i}{\overline{B}}.$$

We can easily show that $y_i^* = \min\{\frac{1}{\gamma\overline{B}}, 1\}$. The principal's problem is

$$\max_{0<\gamma<1} (1-\gamma)(1-\min\{\frac{1}{\gamma\overline{B}}, 1\}).$$

Proposition 4. *Given $\overline{B} > 1$, when the principal only invests in one agent, the optimal profit-sharing rate is $\gamma^* = \frac{1}{\sqrt{\overline{B}}}$. Accordingly, the principal's profit is $(1-\frac{1}{\sqrt{\overline{B}}})^2$; the agent's equilibrium*

[15]If $\bar{B} \leq 1$, then $\gamma\bar{B} \leq 1$ for all γ. By (2) and Theorem 1, it is optimal to allocate zero resources.

effort is $x^*(\overline{B}) = \frac{\ln(\sqrt{\overline{B}})}{\overline{B}}$*; and the agent's payoff is* $\frac{1}{\sqrt{\overline{B}}} - \frac{1}{\overline{B}} + \frac{1}{\overline{B}} \ln \frac{1}{\sqrt{\overline{B}}}$.

Note that the optimal profit-sharing rate γ^* decreases in the total amount of available resources $\overline{B}$. Intuitively, the principal's choice of γ reflects a balance between value creation (larger γ) and value capture (smaller γ). The agent's effort, $x^*(\overline{B}) = \frac{\ln(\sqrt{\overline{B}})}{\overline{B}}$, reaches the global maximum at $\overline{B} = e$. With $\overline{B} > e$, as $\overline{B}$ increases, the agent exerts less effort and creates less value. As the agent relies more on the abundant resources to conduct innovation, and the principal will capture a larger share of the total profit. As a result, in Proposition 4, the agent's payoff first increases and then decreases in $\overline{B}$. This is different from the case of exogenous profit sharing in which the agent's payoff (9) always increases in $\overline{B}$.

Next, by Theorem 1, if the principal finds it optimal to invest in two agents, the resource allocation will be $(\frac{\overline{B}}{2}, \frac{\overline{B}}{2})$. Applying a similar analysis as in Section 3.2, the equilibrium failure probabilities are

$$y_1^* = y_2^* = y^* = \begin{cases} 1, & \gamma\overline{B} \le 2 \\ \frac{4}{\alpha\gamma\overline{B} + \sqrt{\gamma\overline{B}}\sqrt{8-8\alpha+\alpha^2\gamma\overline{B}}}, & \gamma\overline{B} > 2. \end{cases} \tag{13}$$

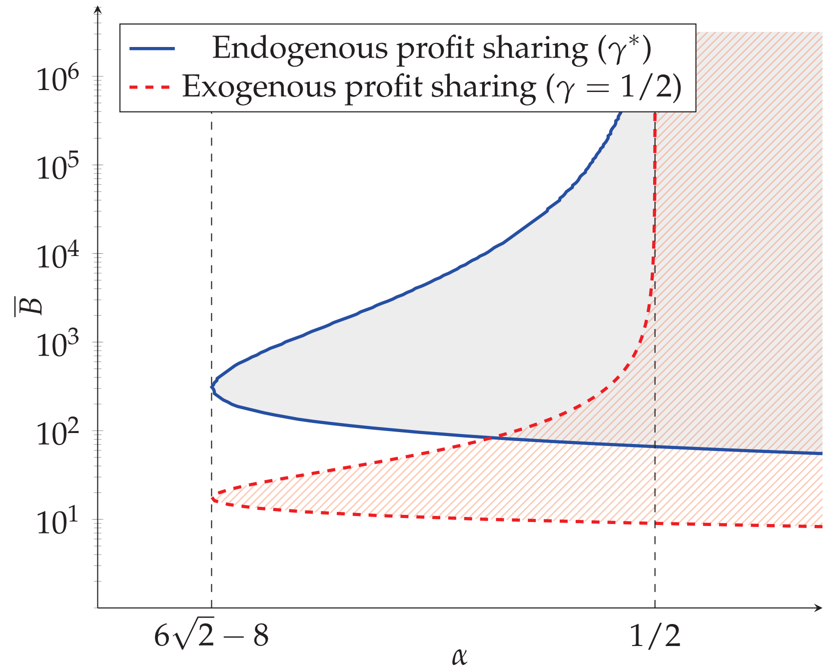
$$\max_{0<\gamma<1} (1-\gamma)[2\alpha(1-y^*)^2 + 2(1-y^*)y^*]. \tag{14}$$

When $\overline{B} \le 2$, an agent is unwilling to exert positive effort. The innovation activity fails for sure ($y^* = 1$), and the principal always obtains a zero payoff independent of the profit-sharing rule γ. When $\overline{B} > 2$, (14) yields the following result.

Proposition 5. *Given* $\overline{B} > 2$*, when the principal invests equally in two agents, the principal's profit is quasiconcave in* $\gamma \in (0,1)$*. Therefore, the optimal sharing rule* γ^* *uniquely exists, and* γ^* *decreases in* $\overline{B}$ *and increases in* α*.*

Note that because γ^* decreases in $\overline{B}$, the principal will always use up her resources under endogenous profit sharing when investing in two agents. This is different from Lemma 3 under exogenous profit sharing in which the principal can retain some resources to avoid fierce product competition. To understand this difference, note that the equilibrium failure rate y^* in (13) is a function of γB. As long as γB is fixed, the agent's equilibrium effort and the total profit from the product market will not change. When the available resources ($\overline{B}$) increase, the principal can always increase her profit by reducing

γ and increasing B to keep γB unchanged. Hence, it is optimal for the principal to exhaust her resources under endogenous profit sharing.



Note: The region of investing in none is not drawn. The shaded and non-shaded regions are the cases where the investor invests in two agents and a single agent, respectively.

Figure 5: Optimal Resource Allocation with Endogenous and Exogenous γ.

Based on Propositions 4 and 5, we reach the following result:

Theorem 2. *In the case of endogenous profit sharing, the optimal resource allocation rule is as follows:*

(i) When $\alpha \leq 6\sqrt{2} - 8$, the principal always invests in one agent, i.e., $(b_1^*, b_2^*) = (\overline{B}, 0)$, and the optimal sharing rule is $\gamma^* = \frac{1}{\sqrt{\overline{B}}}$.

(ii) When $6\sqrt{2} - 8 < \alpha < \frac{1}{2}$, as $\overline{B}$ increases, the investor will switch from investing in one agent to investing equally in two agents, and then back to investing in one.

(iii) When $\alpha \geq \frac{1}{2}$, as $\overline{B}$ increases, the investor will first invest in one agent and then invest equally in two agents.

Therefore, when the principal has the power to determine the "tax" rate, the optimal allocation rule is qualitatively similar to that under exogenous profit sharing.

4.2 Multiple Agents

We relax the restriction of having only two agents. Suppose that there are a large or infinite number of agents who can receive the principal's resources and engage in innovation. All of the agents are symmetric and share the same innovation success function (1). A principal with limited resources $\overline{B}$ can choose to invest in n agents with a desired total amount of B. If $m \leq n$ agents successfully innovate and launch their products on the market, each agent obtains α^{m-1} profit with $\alpha \in (0,1)$.[16]

We focus on the symmetric case in which the principal allocates a total amount of resources $B \leq \overline{B}$ evenly to n agents. Each agent receives resources $b = B/n$. Under the resource allocation rule $(b, b, ..., b)$, a generic agent chooses his failure rate by solving the following problem.

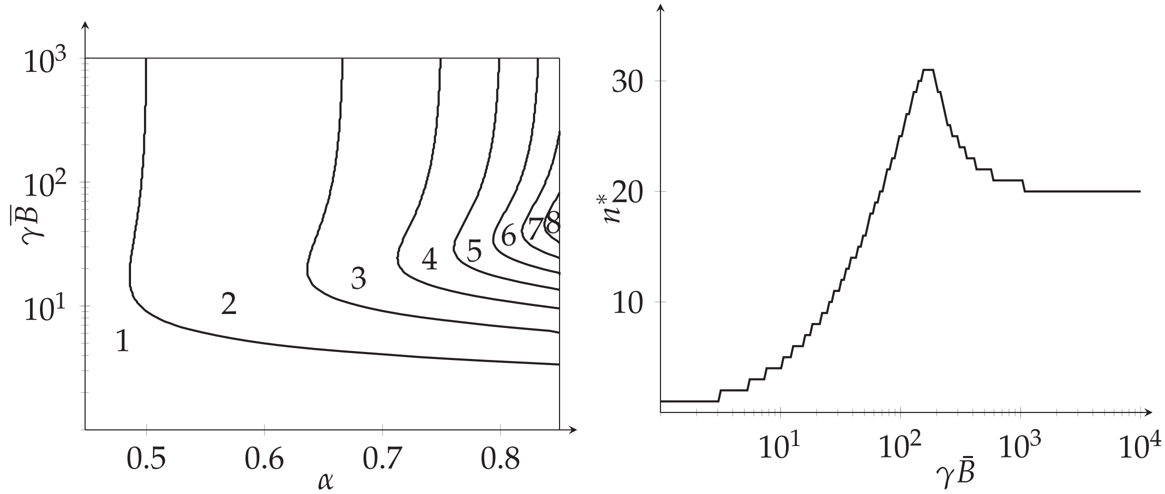
$$\begin{aligned} y^* &= \underset{0<y\leq 1}{\operatorname{argmax}}\{\gamma(1-y)\sum_{m=0}^{n-1} C_{n-1}^{m}(1-y^*)^m (y^*)^{n-1-m}\alpha^m + \frac{\ln y}{b}\} \\ &= \underset{0<y\leq 1}{\operatorname{argmax}}\{\gamma(1-y)(\alpha + y^* - \alpha y^*)^{n-1} + \frac{\ln y}{b}\}. \end{aligned}$$

Based on the FOC, we find that when $n \leq \lfloor \gamma B \rfloor$, $1 > y^*$, the investment decision satisfies: $\gamma B y^*(\alpha + y^* - \alpha y^*)^{n-1} = n$, which implies that y^* is unique and decreasing in B. Since the profit for the principal is $(1-\gamma)n(1-y^*)(\alpha + y^* - \alpha y^*)^{n-1}$, the principal's optimization problem is

$$\begin{aligned} \max_{1\leq n\leq \lfloor \gamma B \rfloor} \quad & (1-\gamma)n(1-y^*)(\alpha + y^* - \alpha y^*)^{n-1} \\ \text{s.t.} \quad & \gamma B y^*(\alpha + y^* - \alpha y^*)^{n-1} = n \text{ and } B \leq \overline{B}. \end{aligned}$$

We solve the above problem numerically and obtain the optimal number of agents receiving positive investment, $n^*(\alpha, \gamma\overline{B})$. Figure 6-(a) illustrates $n^*(\alpha, \gamma\overline{B})$ for a relatively large α. We observe that as the product market becomes less competitive (or as α increases), the principal tends to invest in more agents. Panel (b) displays a slice of $n^*(\alpha, \gamma\overline{B})$ at $\alpha = 0.95$. It shows that for a large α, n^* first increases and then decreases in $\overline{B}$. This pattern resembles Theorem 1, which states that with more resources, the principal may want to reduce the number of invested agents. Therefore, the main results in the baseline model with n capped at two are robust to removing the cap.

[16]This profit structure simplifies our analysis. A quantity competition model can provide the foundation for this assumption. See Appendix A.



(a) $n^*(\alpha, \gamma\bar{B})$

(b) $n^*(0.95, \gamma\bar{B})$

Figure 6: Optimal Number of Agents with Positive Investment

4.3 Nonprofit Principal

Many research grants and entrepreneurship supporting funds are provided by governments and nonprofit organizations. These funding authorities may have objectives other than profit maximization. Most public funding authorities and research institutes allocate grants without acquiring any equity share of startups. The purpose of these grants is mainly maximizing the number of successful innovations or the value of knowledge spillover. For example, National Science Foundation (`www.nsf.org`) and the National Institutes of Health (`www.nih.gov`) evaluate grants based on feasibility (success rate) and quality (consumer surplus or social welfare) (Jacob and Lefgren, 2011; Azoulay and Li, 2020). For some important R&D projects such as COVID-19 vaccines or life-saving drugs (Budish et al., 2015), public funding authorities may value having at least one successful innovation. VCs established by a government or university technology transfer offices typically own equity shares of funded startups (Rothaermel and Thursby, 2005). They can be considered as targeting total social welfare because they care about the profitability of funded startups and also want to achieve some social goals.

Because the principal is not profit-oriented, we set $\gamma = 1$ for simplicity. Based on the baseline model, we consider four alternative objective functions. The expected total number of successful innovations is

$$N(\overline{B}) := 1 - y_1^* + 1 - y_2^*. \tag{15}$$

The probability of having at least one successful innovation is

$$P(\overline{B}) := 1 - y_1^* y_2^*. \tag{16}$$

To compute consumer welfare, we adopt the quantity competition model in Appendix A.1 for the product market. Let q_i is the quantity supplied by agent i, and $p_i = A - gq_{-i} - q_i$ is the equilibrium price of product i. If gent i fails at innovation, $q_i = 0$, where $g \in (0,1)$ measures the intensity of competition. By setting $A = 2$, the equilibrium profit for a monopolistic product is 1, and for two successfully launched products, each agent earns $\alpha = \frac{4}{(2+g)^2}$.

When the market has only one agent, the equilibrium quantity is $\frac{A}{2} = 1$, the equilibrium price is 1, consumer surplus is $\frac{1}{2}$, and the total welfare (profit plus consumer surplus) is $1 + \frac{1}{2} = \frac{3}{2}$. When both agents successfully launch products, the equilibrium quantity of each agent is $\frac{A}{2+g} = \frac{2}{2+g}$, the equilibrium price is $\frac{2}{2+g}$, the consumer surplus is $\frac{4(1+g)}{(2+g)^2}$, and the welfare is $2\alpha + \frac{4(1+g)}{(2+g)^2} = \frac{4(3+g)}{(2+g)^2}$. Hence, the expected consumer surplus is

$$CS(\overline{B}) := \frac{4(1+g)}{(2+g)^2}(1 - y_1^* y_2^*) + \frac{1}{2}[(1-y_1^*)y_2^* + (1-y_2^*)y_1^*]. \tag{17}$$

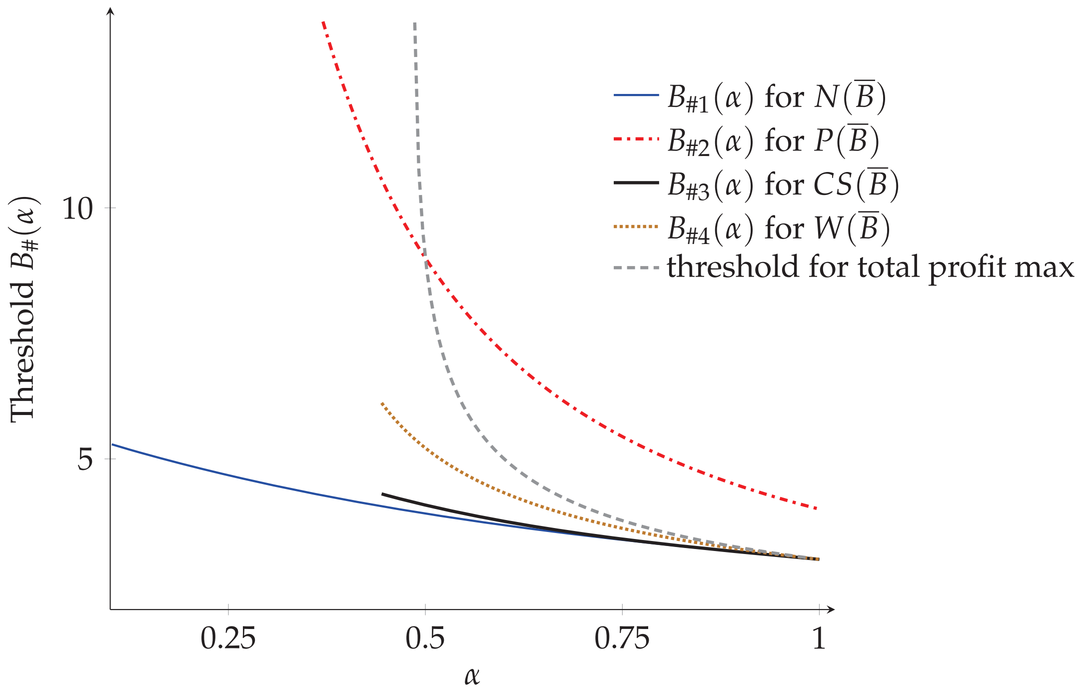
The expected total welfare is

$$W(\overline{B}) := \frac{4(3+g)}{(2+g)^2}(1 - y_1^* y_2^*) + \frac{3}{2}[(1-y_1^*)y_2^* + (1-y_2^*)y_1^*]. \tag{18}$$

Given a resource allocation (b_1, b_2) with $b_1 + b_2 \leq \overline{B}$, the innovation stage equilibrium is characterized by $y_1^* = Y(b_1, y_2^*)$ and $y_2^* = Y(b_2, y_1^*)$. We solve the principal's problem using the objective functions (15)-(18). Here is the result.

Theorem 3. *There exist threshold amounts of resources $B_{\#1}(\alpha)$, $B_{\#2}(\alpha)$, $B_{\#3}(\alpha)$, and $B_{\#4}(\alpha)$, respectively, for the principal's objective functions $N(\overline{B})$, $P(\overline{B})$, $CS(\overline{B})$, and $W(\overline{B})$. For $\iota = 1,2,3,4$, when $\overline{B} \leq B_{\#\iota}(\alpha)$, the principal's optimal resource allocation is $(b_1^*, b_2^*) = (\overline{B}, 0)$; when $\overline{B} > B_{\#\iota}(\alpha)$, the principal's optimal resource allocation is $(b_1^*, b_2^*) = (\frac{\overline{B}}{2}, \frac{\overline{B}}{2})$. Moreover, $B_{\#1}(\alpha)$, $B_{\#2}(\alpha)$, $B_{\#3}(\alpha)$, and $B_{\#4}(\alpha)$ all decrease in α.*

Theorem 3 shows that, although the principal is not profit-oriented from the product market, she will sometimes invest in only one agent under limited resources and intense product market competition. The reason is that the agents are still profit-maximizing, so the potential rent dissipation can discourage them from exerting effort. Unlike the baseline model, we do not observe the non-monotone investment strategy. As the total amount of resources becomes abundant, the principal's investment strategy switches from investing in one agent to two agents but does not switch back because, for all four objectives above, the principal always prefers having two successful innovations more than one. Rent dissipation is not a major concern, so the optimal resource allocation strategy switches only once.



Note: For each threshold, the region below represents that it is optimal to concentrate resources in one agent. For consumer surplus ($B_{\#3}(\alpha)$) and total social welfare ($B_{\#4}(\alpha)$), because the quantity competition model restricts that $g \in [0,1]$, we here only draw the region of $\alpha = \frac{4}{(2+g)^2} \in [\frac{4}{9}, 1]$. The black dotted line is the threshold obtained in Theorem 1 for a profit-oriented principal maximizing total profit.

Figure 7: Switching Thresholds under Different Principal's Objectives

Figure 7 depicts the thresholds characterizing the optimal investment strategies under different principal's objectives derived in Theorems 3 and 1. Compared with the case of a profit-oriented principal, a nonprofit principal tends to invest in a more diverse way. Specifically, the nonprofit principal switches from investing in one agent to two agents with less resources when she aims to maximize the total number of successful innovators ($N(\overline{B})$), consumer surplus ($CS(\overline{B})$), or total welfare ($W(\overline{B})$). This is intuitive because a nonprofit principal does not directly suffer from profit reduction in the product market.

Compared with other nonprofit objectives, the threshold is the highest for the probability of at least one innovator succeeding ($P(\overline{B})$). The objective of $P(\overline{B})$ is special because the principal does not benefit from having more than one successful innovations when maximizing $P(\overline{B})$, but she does when maximizing $N(\overline{B})$, $CS(\overline{B})$, or $W(\overline{B})$. Therefore, the principal is more reluctant to invest in two agents with the objective function $P(\overline{B})$.

Note that the threshold for $P(\overline{B})$ intersects with the one for a profit-maximizing principal. When the product market competition is mild and the amount of resources is small, it is possible that the for-profit principal chooses to invest in two agents while the nonprofit principal invests in one agent. The reason is again that having two products benefits the for-profit principal when $\alpha > \frac{1}{2}$, but does not increase $P(\overline{B})$. However, as rent dissipation occurs ($\alpha < \frac{1}{2}$), the profit-maximizing principal is more likely to concentrate her resources than the nonprofit principal.

In summary, a profit-oriented funding authority will generally invest resources in a more concentrated way than a nonprofit counterpart. When resources are abundant, nonprofit funding authorities will induce more successful innovations and higher social surplus but lower profits. Depending on the policymaker's goal, some areas of innovation should be directed by nonprofit funding authorities and others by private VCs. For example, to develop COVID-19 vaccines, funding decisions should be made by nonprofit entities because the importance of successful innovations and social welfare outweigh the profit during the pandemic.

5 Conclusion

In this paper, we examine how the interplay between allocated resources and market competition crafts an innovator's strategy. Specifically, a downstream innovative agent will exert effort and actively innovate only if he anticipates that the profit from the product market can cover his costs, after considering the uncertainty of innovation. Thus, competition factors in agents' strategic considerations, propagating to the upstream principal's resource arrangement.

We find that, when the product market competition is at the moderate level, the resource allocation strategy exhibits an interesting pattern: the optimal investment diversification first increases and then decreases in the amount of resources. When the amount of resources is small, disseminating resources to multiple agents will discourage agents from exerting effort in the innovation process due to lack of resources. As resources become abundant, although investing in more agents increases the probability that at least one agent successfully innovates, the product competition will erode profits when more agents succeed. As a result, the optimal resource allocation strategy is non-monotone in the resource capacity.

In this regard, our analytical framework and results offer some important policy and managerial implications to both profit-oriented investors and nonprofit funding authorities. Although the principal herself may not be profit-oriented, the principal must be mindful of agents' incentives because the ultimate success of innovation relies on their efforts. The resource capacity subsequently dictates the number of agents that the principal can incentivize effectively. When resources are limited, some degree of concentration is necessary to guarantee the overall success rate of innovation.

For various industries and different stages of R&D, diversifying innovation investment can lead to vastly different outcomes. As funding authorities devote more resources to innovation, it is possible that funding more innovators will lead to worse outcomes in innovation and knowledge transfer because too many similar startups enter the same market. As Lerner (2012) said, "(government) subsidy will increase the profits of entrepreneurs and venture capitalists. But it may lure more entrepreneurs and venture capitalists into the market, so that, unless the supply of good ideas grows, more firms and financiers are chasing after the same ideas. This competition may depress returns, and ultimately discourage entrepreneurs and venture investors."

In establishing innovation policy, the policymaker must carefully consider this possibility. For example, China currently has at least 1,500 incubators supported by the Ministry of Science and Technology, funding approximately 80,000 companies. Most of these companies are in several thrust areas such as advanced equipment manufacturing, new materials, and new-energy vehicles. The product market competition intensity should be an important factor in determining the resource distribution and investment diversity in different areas. Concentrating resources in some areas while diversifying in others may substantially improve the performance of these incubators. For innovation portfolio managers, it is important to allocate resources with a proper level of diversification. Henderson (1994) points out that "successful resource allocation is not simply a matter of picking winners ... Portfolio diversity is the key to success. Increasingly diverse research stimulate productivity, but only up to a point. As in any business, a company can spread itself too thin." From a dynamic perspective, the managers should diversify resources at first to reduce the risk of failure of all projects and concentrate resources in the end to avoid launching too many products onto the market. This strategy is supported by empirical evidence in Klingebiel and Rammer (2014).

We also analyze the case of nonprofit principals. In general, a nonprofit principal prefer more diverse investment portfolio than a for-profit principal because the former cares more about the success of innovation and does not directly bear the loss from rent dissipation. For different areas of innovation, the policymaker can let public authorities, private VCs, or public-private partnerships to be the main investors depending on the policy goals.

For future research, we also plan to explore a complete model with multiple principals. In reality, the innovation activities are often funded by multiple funding authorities including profit-oriented and nonprofit entities, like in the case of COVID-19 vaccine development. This analysis will help the policymaker to compare different ways of stimulating innovation and to evaluate how efficient the decentralized innovation investment outcome is.

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Appendix

(For online publication)

A Product Market Competition

A.1 Quantity competition

Suppose that two successful agents engage in quantity competition. The price of agent i's product is determined by an inverse demand function (Vives, 1984), that is, $P_i(q_i, q_{-i}) = A - gq_{-i} - q_i$. Here, A measures the size of the market, q_i is the quantity produced by agent i, and $g \in (0,1)$ characterizes the degree of substitution between two agents' products. The consumer surplus of the product market is

$$CS = A(q_1 + q_2) - (q_1^2 + 2gq_1q_2 + q_2^2)/2 - p_1q_1 - p_2q_2.$$

Note that an agent is able to launch and produce his product only when his innovation is successful. If there is only one agent succeeding in innovation, he becomes a monopolist with the profit

$$\pi_m = \max_{q_i} \gamma(A - q_i)q_i = \frac{\gamma A^2}{4}.$$

If both agents succeed in the innovation stage, it is straightforward to show that the equilibrium quantities are $q_1^* = q_2^* = \frac{A}{2+g}$. The profit for each agent is

$$\pi_d = \frac{\gamma A^2}{(2+g)^2}$$

As a result, the ratio between the monopoly profit and the duopoly profit can be derived as

$$\alpha = \frac{\pi_d}{\pi_m} = \frac{4}{(2+g)^2}.$$

One can show that when $g \in (0,1)$, quantity competition leads to $\alpha \in (\frac{4}{9}, 1)$.

The quantity competition model can be easily generalized into the case with m competing agents. When there are m agents succeeding in innovation, each will earn a profit $\frac{\gamma a^2}{(2+g(m-1))^2}$, where a is the market potential level and g measures the competition intensity among agents. Specifically, $g = 0$ indicates that these m products are fully independent, and $g = 1$ indicates that they are homogeneous.

In Section 4.2, we use α^{m-1} to capture the profit structure for successful agents. The main results are consistent irrespective of using g or α to measure the competition intensity.

A.2 Price competition

The product market can also be modeled as price competition based on a multinomial-logit (MNL) discrete choice model (Gallego and Topaloglu, 2019). Suppose that two agents engage in innovation to create a specified product for a market of size 1. Let the base quality of a product relative to the outside option be v_0. Apart from the base quality, consumers have heterogeneous preferences for each agent or the outside option. The idiosyncratic random utility terms of consumers follow the i.i.d. Gumbel distribution. When there are k products, consumers make a discrete choice based on the MNL model. The choice probability of product i is $\frac{e^{v_0-p_i}}{1+\sum_{j=1}^{k} e^{v_0-p_j}}$.

When there is only one successful agent in the market, he chooses p by maximizing his profit. The profit of the agent under the monopoly case can be derived as follows:

$$\pi_m = \max_p \frac{\gamma e^{v_0-p} p}{1+e^{v_0-p}} = \gamma \mathcal{L}(e^{v_0-1}),$$

where $\mathcal{L}(y)$ is the root of $xe^x = y$. However, when both agents successfully launch their products, they must compete with each other on their prices. The optimal prices of the two agents are symmetric, which satisfies the following condition:

$$p^* = \operatorname*{argmax}_p \frac{\gamma e^{v_0-p} p}{1+e^{v_0-p^*}+e^{v_0-p}}.$$

Then, we can obtain $e^{v_0} = \frac{e^{p^*}(p^*-1)}{2-p^*}$ with $1 \le p^* \le 2$. Accordingly, the profit for each agent in the competitive case is

$$\pi_d = \frac{\gamma e^{v_0-p^*} p^*}{1+2e^{v_0-p^*}} = \gamma(p^*-1).$$

As a result, the ratio between the monopoly profit and the duopoly profit is captured by:

$$\alpha = \frac{\pi_d}{\pi_m} = \frac{p^*-1}{\mathcal{L}(\frac{e^{p^*-1}(p^*-1)}{2-p^*})}.$$

One can show that when $v_0 \in (-\infty, \infty)$, namely, $p^* \in (1,2)$, it leads to $\alpha \in (0,1)$.

B Proofs

Proof of Lemma 1. By equation (6), for $y_1^* > 0$ and $y_2^* > 0$, we have

$$y^*_{1(2)} = \frac{\sqrt{4(1-\alpha)\alpha^2\gamma b_1 b_2^2 + (b_2(\alpha-1)+b_1(\gamma\alpha^2 b_2-\alpha+1))^2} - \alpha^2\gamma b_1 b_2 \pm (b_2-b_1)(1-\alpha)}{2\alpha(1-\alpha)\gamma b_1 b_2}.$$

Then letting $0 < y^*_{1(2)} < 1$, we can yield the conditions shown in Lemma 1. ∎

Proof of Lemma 2. Given b_{-i}, differentiating equation (6) with respect to b_i yields

$$[\alpha(1-y_{-i})+y_{-i}]\frac{dy_i}{db_i}+(1-\alpha)y_i\frac{dy_{-i}}{db_i}=-\frac{1}{\gamma b_i^2},$$

and

$$[\alpha(1-y_i)+y_i]\frac{dy_{-i}}{db_i}+(1-\alpha)y_{-i}\frac{dy_i}{db_i}=0.$$

By solving the two equations above, we have

$$\frac{dy_i}{db_i}=-\frac{1}{\alpha b_i^2\gamma}\frac{\alpha+(1-\alpha)y_i}{\alpha+(1-\alpha)(y_1+y_2)}<0,$$

and

$$\frac{dy_{-i}}{db_i}=\frac{1}{\alpha b_i^2\gamma}\frac{(1-\alpha)y_{-i}}{\alpha+(1-\alpha)(y_1+y_2)}>0.$$

■

Proof of Proposition 1. We divide the proof into several cases.

Case 1: $\gamma B \leq 2$

Note that when $\gamma B \leq 2$, then $\gamma b_2 \leq 1$. Due to the insufficient amount of resources to incentivize both agents, $y_2^* = 1$ for sure. Now, the profit of the principal only comes from a potentially successful outcome from agent 1's innovation. Similar to Proposition 2, $\tilde{\Pi}(b_2; B)$ is nonincreasing in $b_2 \in [0, \frac{B}{2}]$.

Case 2: $\gamma B > 2$

For $\gamma B > 2$, we perform a special transformation to simplify the exposition of our proof: let $b_2 = \frac{B}{2}[1-\sqrt{1-\frac{1-\alpha}{z}}]$ with $z \in [1-\alpha, \infty)$. This guarantees $b_2 \leq b_1$. Note that $\gamma b_1 > 1$ for sure.

If $\frac{b_1(z)}{b_2(z)} = 1 + \alpha(\gamma b_1(z) - 1)$, then

$$16z^2+4(\alpha^2 B\gamma-\alpha^2+4\alpha-4)z-(1-\alpha)\alpha^2B^2\gamma^2=0.$$

As $0 < \alpha < 1$, the two roots to the equation above satisfy that $z_1 z_2 = -(1-a)\alpha^2 B^2\gamma^2/16 < 0$. Hence, there is a unique non-negative root denoted as $\hat{z}$. It is easy to verify that $\hat{z} \geq 1-\alpha$. In addition, it can be verified that the condition in Lemma 1 holds only when $z < \hat{z}$.

Case 2.a: $z \geq \hat{z}$

In this case, $y_2^* = 1$, and the profit of the principal will only come from a potentially successful outcome from agent 1's innovation. Then, the profit of the principal is increasing in $z \in [\hat{z}, \infty)$ as agent 1 obtains more resources.

Case 2.b: $z \in [1-\alpha, \hat{z})$

Using the expressions of y_1^* and y_2^* in the proof of Lemma 1, the principal's objective becomes as follows,

$$\tilde{\Pi}(z) = (1-\gamma)(\alpha - \frac{4z}{(1-\alpha)\gamma B} + \frac{\sqrt{\alpha^4 B^2\gamma^2 + 8z(\alpha^2 B\gamma + 2\alpha - 2) + 16z^2}}{\alpha B\gamma}).$$

Denote $d(z) := \alpha^4 B^2\gamma^2 + 8z(\alpha^2 B\gamma + 2\alpha - 2) + 16z^2$. As $\gamma B > 2$, $z \geq 1-\alpha$ and $0 < \alpha < 1$, we have $d(z) \geq 0$. Moreover, we obtain

$$\frac{\tilde{\Pi}''(z)}{1-\gamma} = \frac{64(1-\alpha)}{\sqrt{d^3(z)}\alpha B\gamma}(\alpha^2 B\gamma + \alpha - 1).$$

Case 2.b.1: $\alpha^2 B\gamma + \alpha - 1 \geq 0$

In this case, $\Pi(z)$ is convex in $z \in [1-\alpha, \hat{z})$.

Case 2.b.2: $\alpha^2 B\gamma + \alpha - 1 < 0$

$\tilde{\Pi}(z)$ is concave in $z \in [1-\alpha, \hat{z})$. If $\tilde{\Pi}(z)$ is maximized at z_* with $z_* \in (1-\alpha, \hat{z})$, by the FOC, we have

$$0 = \frac{\tilde{\Pi}'(z_*)}{1-\gamma} = \frac{4}{B\gamma}[\frac{1}{\alpha-1} + \frac{-2+2\alpha+\alpha^2 B\gamma + 4z_*}{\alpha\sqrt{d(z_*)}}].$$

By rearranging the terms, we have

$$\begin{aligned} 0 = & -4(1-\alpha)^4 + 4(1-\alpha)^3\alpha^2 B\gamma + (2\alpha-1)\alpha^4 b^2\gamma^2 \\ & + 8(2\alpha-1)(\alpha^2 B\gamma + 2\alpha - 2)z_* + (32\alpha - 16)z_*^2. \end{aligned}$$

If there is a real solution of z_*, then

$$\begin{aligned} & (8(2\alpha-1)(\alpha^2 B\gamma + 2\alpha - 2))^2 \\ & - 4(-4(1-\alpha)^4 + 4(1-\alpha)^3\alpha^2 B\gamma + (2\alpha-1)\alpha^4 B^2\gamma^2)(32\alpha - 16) \\ = & 256\alpha^2(1 - 3\alpha + 2\alpha^2)(\alpha^2 B + \alpha - 1) \geq 0, \end{aligned}$$

which means that $1 > \alpha \geq \frac{1}{2}$. However, together with $\alpha^2 B\gamma + \alpha - 1 < 0$ and $\gamma B > 2$, there is a contradiction. Therefore $\tilde{\Pi}(z)$ is monotone in $z \in [1-\alpha, \hat{z})$.

In summary, $\tilde{\Pi}(z)$ is convex or monotone in $z \in [1-\alpha, \hat{z})$ and $\tilde{\Pi}(z)$ is increasing in $z \in [\hat{z}, \infty)$. As a result, $\tilde{\Pi}(z)$ is quasiconvex in $z \in [1-\alpha, \infty)$, which implies the claim in the proposition. ■

Proof of Lemma 3. We first show (i). Note that $\frac{\gamma B^2 \Pi'_d(B)}{4(1-\gamma)} = 1 - \frac{(1-\alpha)\sqrt{\gamma B}}{\sqrt{8-8\alpha+\alpha^2\gamma B}}$, and results can be easily derived.

For (ii), denote $\tilde{\mathcal{D}}(B) := B^2\frac{dU_d(B)}{dB} = 1 + \frac{(\alpha-2)\sqrt{B\gamma}}{\sqrt{8-8\alpha+\alpha^2 B\gamma}} - 2\ln\frac{4}{\alpha B\gamma + \sqrt{B\gamma}\sqrt{8-8a+a^2 B\gamma}}$. Then $Sign(\tilde{\mathcal{D}}(B)) =$

$Sign(\frac{dU_d(B)}{dB})$. Moreover, $\tilde{\mathcal{D}}(B=\frac{2}{\gamma})=0$, $\frac{d\tilde{\mathcal{D}}(B)}{dB}|_{B=\frac{2}{\gamma}}=\frac{\gamma}{(2-\alpha)^2}>0$ and

$$\frac{d\tilde{\mathcal{D}}(B)}{dB}=\frac{1}{B}(1-\frac{4(2-5\alpha+3\alpha^2)\sqrt{B\gamma}-\alpha^3(B\gamma)^{3/2}}{(8-8\alpha+\alpha^2B\gamma)^{3/2}}).$$

Solving $\frac{d\tilde{\mathcal{D}}(B)}{dB}=0$ directly provides two solutions as $\frac{1-\alpha}{2\alpha^3\gamma}[4-12\alpha-3\alpha^2\pm(2-a)\sqrt{(2-\alpha)(2-9\alpha)}]$. Then we have the following findings:

(1) When $\alpha\geq 2/9$, there does not exist real solution of B satisfying $\frac{d\tilde{\mathcal{D}}(B)}{dB}=0$. Then as $\tilde{\mathcal{D}}(B=\frac{2}{\gamma})=0$ and $\frac{d\tilde{\mathcal{D}}(B)}{dB}|_{B=\frac{2}{\gamma}}>0$, $U_d(B)$ is always increasing in B.

(2) When $\alpha<2/9$, the two solutions are real. Note that $B_d^*=\frac{8(1-\alpha)}{\gamma(1-2\alpha)}$ is smaller than the larger solution. Therefore when $B\in[\frac{2}{\gamma},B_d^*]$ increases, as $\frac{d\tilde{\mathcal{D}}(B)}{dB}|_{B=\frac{2}{\gamma}}>0$, $\frac{d\tilde{\mathcal{D}}(B)}{dB}$ is either a) first positive and then negative; or b) always non-negative. Case b) leads to $\tilde{\mathcal{D}}(B)>0$, namely $\frac{dU_d(B)}{dB}>0$ for all $B\in[\frac{2}{\gamma},B_d^*]$. For case a), when $B\in[\frac{2}{\gamma},B_d^*]$ increases, $\tilde{\mathcal{D}}(B)$ first increases and then decreases. As $\tilde{\mathcal{D}}(B=\frac{2}{\gamma})=0$, $\mathcal{D}$ is always positive for the increasing part. If $\tilde{\mathcal{D}}(B)<0$ for some $B\in[\frac{2}{\gamma},B_d^*]$, we must have $\tilde{\mathcal{D}}(B_d^*)=\tilde{\mathcal{D}}(\frac{8(1-\alpha)}{\gamma(1-2\alpha)})=-\frac{1}{1-\alpha}+2\ln\frac{2(1-\alpha)}{1-2\alpha}<0$, leading to a contradiction when $\alpha<2/9$. ■

Proof of Proposition 3. For (i), as $\Pi_m(B)$ is always increasing in B, we need to show that $\Pi_d(B)$ is increasing in B when $\Pi_d(B)>\Pi_m(B)$.

(1) If $\alpha\geq\frac{1}{2}$, $\Pi_d(B)$ is increasing in B for sure by Lemma 3.

(2) If $\alpha<\frac{1}{2}$, we need to consider two cases. The condition $\Pi_m(B)=\Pi_d(B)$ implies

$$(1-2\alpha)\gamma^2B^2-2(1-2\alpha)\gamma B+9=0. \tag{19}$$

When $\alpha<6\sqrt{2}-8$, there is no solution of $\gamma B>2$ to the equation above. Because $\Pi_m(\frac{2}{\gamma})>\Pi_d(\frac{2}{\gamma})$, for any given B, $\Pi_m(B)>\Pi_d(B)$.

When $6\sqrt{2}-8\leq\alpha<\frac{1}{2}$, there are two solutions for (19). At optimality, $\Pi_d(B)\geq\Pi_m(B)$ if and only if $\gamma B\in[\frac{1-\alpha-\sqrt{\alpha^2+16\alpha-8}}{1-2\alpha},\frac{1-\alpha+\sqrt{\alpha^2+16\alpha-8}}{1-2\alpha}]$. Recalling Lemma 3 that $\Pi_d(B)$ is increasing in $B\in\frac{1}{\gamma}[2,\frac{8(1-\alpha)}{1-2\alpha}]$. Since $\frac{1-\alpha+\sqrt{\alpha^2+16\alpha-8}}{1-2\alpha}<\frac{8(1-\alpha)}{1-2\alpha}$ for $\alpha\in[6\sqrt{2}-8,\frac{1}{2}]$, $\Pi_d(B)$ is increasing in B when $\Pi_m(B)<\Pi_d(B)$.

As a result, $\max\{\Pi_m(B),\Pi_d(B)\}$ is always increasing in B.

For (ii), let $\Delta(B)=\frac{\Pi_m(B)-\Pi_d(B)}{1-\gamma}=1-\frac{1}{\gamma B}-(\alpha-\frac{4}{\gamma B}+\sqrt{\frac{\gamma\alpha^2B-8\alpha+8}{\gamma B}})$. There is only one point satisfying $\Delta'(B)=0$, which is $\tilde{B}=\frac{72(1-\alpha)}{\gamma(7\alpha^2-32\alpha+16)}$. As a result, $\Delta(B)$ is either quasiconvex or quasiconcave. If $\Delta(B)$ is quasiconvex, the claim holds for sure. If $\Delta(B)$ is quasiconcave, as $\Delta'(B)|_{B=\frac{2}{\gamma}}=\frac{(2+\alpha)\gamma}{4(\alpha-2)}<0$, then $\Delta(B)$ is decreasing in $B\in[\frac{2}{\gamma},\infty)$, which also implies that it is quasiconvex in $B\in[\frac{2}{\gamma},\infty)$. ■

Proof of Theorem 1. By Proposition 3, $b_1^* + b_2^* = \overline{B}$. Applying similar analysis to that of Proposition 3, we can easily derive the result. ■

Proof of Proposition 5. At optimality, $\gamma^*\overline{B} > 2$, otherwise $y^* = 1$. Therefore, we focus on $\gamma \in (\frac{2}{\overline{B}}, 1]$.

Step 0: Auxiliary functions.

Note that y^* only depends on $\gamma\overline{B}$. We denote

$$\pi(x) := 2\alpha(1-y^*)^2 + 2(1-y^*)y^*|_{\gamma\overline{B}=x} = \alpha - \frac{4}{x} + \sqrt{\frac{8-8\alpha+\alpha^2 x}{x}},$$

where $x > 2$. It can be shown that $\pi(2) = 0$ and $\pi'(2) = \frac{1}{2-a} > 0$.

Step 0.a: $\pi(x)$ is quasiconcave in $x \in [2, \infty)$.

Similar to Lemma 3, $\pi(x)$ is increasing in x if $\alpha \geq \frac{1}{2}$; $\pi(x)$ is quasiconcave in x and maximized at $x = \frac{8(1-\alpha)}{1-2\alpha}$, when $\alpha < \frac{1}{2}$.

Step 0.b: $k(x) := \frac{\pi(x)}{\pi'(x)} + x$ is increasing in $x \in [2, x_*)$, where

$$x^* = \begin{cases} \frac{8(1-\alpha)}{1-2\alpha}, & 0 < \alpha < \frac{1}{2} \\ \infty, & 1 \geq \alpha \geq \frac{1}{2}. \end{cases}.$$

Solving $k'(x) = 0$ directly yields one solution as $\frac{18(1-\alpha)}{1-\alpha-2\alpha^2}$. We observe that when $\alpha \geq \frac{1}{2}$ the solution is negative; when $0 < \alpha < \frac{1}{2}$, $\frac{18(1-\alpha)}{1-\alpha-2\alpha^2} > \frac{8(1-\alpha)}{1-2\alpha}$. Then, $k'(x) \neq 0$ for all $x \in [2, x_*)$. Then, $k'(2) > 0$, and we have the claim.

Step 1: $\Pi_d(\gamma; \overline{B})$ is strictly quasiconcave in $\gamma \in (\frac{2}{\overline{B}}, 1)$.

We can write the profit of the principal as follows,

$$\Pi_d(\gamma, B) := (1-\gamma)\pi(\gamma B).$$

Step 1.a: $\Pi_d(\gamma; \overline{B})$ is strictly quasiconcave in $\gamma \in (\frac{2}{\overline{B}}, \frac{x_*}{\overline{B}})$, where x_* is defined in Step 0.b.

Note that

$$\frac{\Pi'(\gamma; \overline{B})}{\pi'(\gamma\overline{B})} = -k(\gamma\overline{B}) + \overline{B},$$

where function $k(\cdot)$ is defined in Step 0.b. From Step 0.a, $\pi'(x) > 0$ when $x < x_*$. Then when $\gamma \in (\frac{2}{\overline{B}}, \frac{x_*}{\overline{B}})$, $sign(\frac{\Pi'(\gamma;\overline{B})}{\pi'(\gamma\overline{B})}) = sign(\Pi'(\gamma; \overline{B}))$. From Step 0.b, $\frac{\Pi'(\gamma;\overline{B})}{\pi'(\gamma\overline{B})}$ is decreasing in $\gamma \in (\frac{2}{\overline{B}}, \frac{x_*}{\overline{B}})$; in particular, $\frac{\Pi'(\gamma;\overline{B})}{\pi'(\gamma\overline{B})}$ is first positive and then negative when γ increases. Therefore, $\Pi(\gamma; \overline{B})$ is strictly quasiconcave in $\gamma \in (\frac{2}{\overline{B}}, \frac{x_*}{\overline{B}})$, and there is a unique γ satisfying $\Pi'(\gamma; \overline{B}) = 0$.

Step 1.b: $\Pi_d(\gamma; \overline{B})$ is decreasing in $\gamma \in (\frac{x_*}{\overline{B}}, 1)$, where x_* is defined in Step 0.b.

From Step 0.a, $\pi'(x) \leq 0$ when $x \geq x_*$. Then, when $\gamma \in (\frac{2}{\overline{B}}, 1)$, we have $\Pi'(\gamma; \overline{B}) = -\pi(\gamma\overline{B}) + \pi'(\gamma\overline{B})\overline{B}(1-\gamma) < 0$.

Based on Step 1.a and Step 1.b, we conclude that $\Pi(\gamma, \overline{B})$ is strictly quasiconcave in $\gamma \in (\frac{2}{\overline{B}}, 1)$.

Step 2: Property of γ^*.

From Step 1, at optimality,

$$\overline{B} = k(\gamma^* \overline{B}).$$

By solving this equation above, we have

$$\begin{aligned}\overline{B} =& \frac{(1+\gamma^* - \alpha\gamma^*)\sqrt{\alpha^2(2-\gamma^*)^2 + (1+\gamma^*)^2 - 2\alpha(2-\gamma^* + (\gamma^*)^2)}}{\alpha^2(\gamma^*)^2} \\ & - \frac{1 - 2\alpha + 2(1-\alpha^2)\gamma^* + (1-\alpha)^2(\gamma^*)^2}{\alpha^2(\gamma^*)^2}\end{aligned}$$

As $0 < \gamma^* < 1$ and $0 < \alpha < 1$, and we can find that the RHS is nondecreasing in α and decreasing in γ^*. Then we have the claim shown in Proposition 5. ■

Proof of Theorem 2. Recall that

$$\Pi_m^*(B) = (1 - \frac{1}{\sqrt{B}})^2.$$

By setting $z = B\gamma$, we have

$$\Pi_d^*(B) = \max_{2<z<B} \{\Pi_d(z; B) := \frac{B-z}{Bz}(\alpha z + \sqrt{z}\sqrt{8 - 8\alpha + \alpha^2 z} - 4)\}.$$

Let $z_*(B) = \underset{2<z<B}{\operatorname{argmax}}\{\Pi_d(z; B)\}$.[17] Denote $\Delta(B) := \Pi_d^*(B) - \Pi_m^*(B)$. Then we obtain

$$\Delta'(B) = \frac{d\Pi_d^*(z; B)}{dB}\Big|_{z=z_*} - \frac{d\Pi_m^*(B)}{dB} = \frac{-3 - \sqrt{B} + \alpha z_* + \sqrt{z_*}\sqrt{8 + \alpha(-8 + \alpha z_*)}}{B^2}, \tag{20}$$

where, in the first equality, the envelop theorem is used. Note that when $B \to 2$, due to $2 < z_* < B$, $z_* \to 2$. Hence, we derive $\Delta'(2) = \frac{1}{4}(1 - \sqrt{2}) < 0$ and $\Delta(2) = -\Pi_m^*(2) < 0$.

Step 0: Auxiliary functions.

Define $g(x) := \frac{27+27\sqrt{x}-31x+9x^{3/2}}{16x(\sqrt{x}-1)}$ with $x \in [2, \infty)$. Then we have

$$g'(x) = \frac{27 - 27\sqrt{x} - 27x + 11x^{3/2}}{16x^2(\sqrt{x}-1)^2}.$$

Therefore, $g(x)$ is decreasing in $x \in [2, 9]$ and increasing in $x \in (9, \infty)$. In addition, one can verify that $g(2) \approx 2.16069 > 1$, $g(9) = \frac{1}{4}$ and $\lim_{x\to\infty} g(x) = \frac{9}{16}$. As a result, when solving $g(x) = t$, (1) if $t \le \frac{1}{4}$, there do not exist solutions of $x > 2$; (2) if $\frac{1}{4} < t < \frac{9}{16}$, there exist two solutions of $x > 2$;

[17]In the proof, all z_* will be a function of B. We omit "(B)" if there is no confusion.

and (3) if $\frac{9}{16} \leq t \leq 1$, there only exists one solution of $x > 2$.

Step 1: Main results.

Assume that at B_*, $\Delta'(B_*) = 0$. From (20), we have

$$z_*(B_*) = \frac{-36 + 9\alpha - 24\sqrt{B_*} + 15\alpha\sqrt{B_*} - 4B_* + 7\alpha B_* + \alpha B_*^{3/2}}{-32 + 16\alpha - 2\alpha^2 + 2\alpha^2 B_*}. \tag{21}$$

Moreover, $z_*(B_*)$ satisfies the following condition,

$$\begin{aligned} 0 =& \frac{d\Pi_d(z;B)}{dz}|_{z=z_*,B=B_*} \\ =& \frac{4B_*(\sqrt{8+\alpha(-8+\alpha z_*)} - \sqrt{z_*} + \alpha\sqrt{z_*}) - z_*^{3/2}(4 - 4\alpha + \alpha^2 z_*(B_*) + \alpha\sqrt{z_*}\sqrt{8+\alpha(-8+\alpha z_*)})}{B_* z_*^2\sqrt{8+\alpha(-8+\alpha z_*)}}. \end{aligned}$$

Substituting the expression of z_* in (21) into the equation above, we can find that

$$\alpha = \frac{27 + 27\sqrt{B_*} - 31B_* + 9B_*^{3/2}}{16B_*(\sqrt{B_*} - 1)} = g(B_*). \tag{22}$$

where $g(\cdot)$ is defined in Step 0.

Step 1.a: There is at most one continuous region of B such that the principal invests in two agents.

From the result in Step 0, there is at most two points of $B > 2$, satisfying $\Delta'(B) = 0$, namely (22). Recall that $\Delta'(2) < 0$ and $\Delta(2) < 0$. Therefore there is at most one continuous region of $B > 2$ such that $\Delta(B) > 0$.[18]

Step 1.b: $\alpha \leq \frac{1}{4}$

When $a \leq \frac{1}{4}$, from Step 0, there do not exist solutions of $B > 2$ satisfying $\Delta'(B) = 0$. As $\Delta'(2) < 0$ and $\Delta(2) < 0$, $\Delta(B) < 0$ for all $B > 2$. Therefore when $\alpha \leq \frac{1}{4}$, at optimality, the principal invests in one agent.

Step 1.c: $\alpha \geq \frac{1}{2}$.

For any fixed $z > 2$, we have

$$\Pi_d(z;\infty) = \lim_{B\to\infty} \Pi_d(z;B) = \alpha + \frac{\sqrt{z}\sqrt{8 - 8\alpha + \alpha^2 z} - 4}{z}.$$

Then, we obtain that $\lim_{z\to\infty} \Pi_d(z;\infty) = 2\alpha \geq \lim_{B\to\infty} \Pi_m(B) = 1$. Therefore, when $B \to \infty$, at optimality, the principal invests in two agents. Therefore, due to Step 1.a, when B increases, at optimality, the principal first invests in one agent and then switches to investing in two agents.

[18] For example, suppose that there are two points of $B > 2$, satisfying $\Delta'(B) = 0$, which are denoted as $B_1 < B_2$. Since $\Delta'(2) < 0$, the sign of $\Delta'(B)$ for $B \in (2, B_1)$,(B_1, B_2),(B_2, ∞) will be one of the four cases: $(-,-,-)$,$(-,+,+)$, $(-,-,+)$ or $(-,+,-)$. As $\Delta(2) < 0$, it is easy to find that (1) for the first case, $\Delta(B) < 0$ for all $B \in [2, \infty)$; (2) for the second and third cases, if $\Delta(B) > 0$ for some $B \in [2, \infty)$, then $\Delta(\tilde{B}) > 0$ for all $\tilde{B} \geq B$; and (3) for the fourth case, as $\Delta(\tilde{B}) < 0$ for all $B \in [2, B_1)$ and $\Delta(\tilde{B})$ is quasiconcave in $B \in [B_1, \infty)$, and there is one continuous region of B satisfying $\Delta(B) > 0$.

Step 1.d: $\frac{1}{2} > \alpha > \frac{1}{4}$.

From Step 0, there are two points satisfying $\Delta'(B) = 0$ (namely (22)), which are denoted as B_1 and B_2. Note that $B_1 < 9 < B_2$.

Since $\lim_{B\to\infty} \Pi_m(B) = 1$, and $\Pi_d(z; B) < 1$ for sure.[19] Then, when $B \to \infty$, at optimality, the principal invests in one agent. By Step 1.a, that is to say, the principal adopts the strategy of investing in two agents only in a closed region of $B > 2$. Therefore, if $\Delta(B) > 0$ for some B, the sign of $\Delta'(B)$ for $B \in (2, B_1)$, (B_1, B_2) and (B_2, ∞) must be $(-, +, -)$. That is to say, $\Delta(B)$ must be maximized at B_2 when $\Delta(B) > 0$ for some B. Based on the above discussion, to examine whether $\Delta(B) > 0$ for some B, we need only check whether $\Delta(B_2) > 0$.

Given α, $B_2(\alpha) > 9$ will be uniquely determined by (22), which is increasing in α. By (21) and (22), we can replace α and z_* by $B_2(\alpha)$ in the expression of $\Delta(B_2, \alpha)$ and thus

$$\Delta(B_2; \alpha) = \frac{(\sqrt{B_2} - 1)(9 - 18\sqrt{B_2} + B_2)}{8(3 + \sqrt{B_2})B_2}.$$

If $\Delta(B_2; \alpha) > 0$, as $B_2 > 9$, we have that

$$B_2 > 9(17 + 12\sqrt{2}),$$

namely,

$$\alpha > g(x)|_{x=9(17+12\sqrt{2})} = 6\sqrt{2} - 8,$$

where the condition of α is due to (22). That is, when $\alpha \leq 6\sqrt{2} - 8$, $\Delta(B) < 0$ all $B > 2$. When $\frac{1}{2} > \alpha > 6\sqrt{2} - 8$, due to (22), $B_2(\alpha) > 9(17 + 12\sqrt{2})$, we obtain that $\Delta(B_2; \alpha) > 0$. That is, when $\frac{1}{2} > \alpha > 6\sqrt{2} - 8$, in some regions, investing in two agents can be optimal.

In summary, we have the claim shown in Theorem 2. ■

Proof of Theorem 3. Without loss of generality, let $b_1 \geq b_2$.

Number of successful innovations

The principal's problem is

$$N(\overline{B}) := \max_{b_1 \geq 0, b_2 \geq 0} \; 1 - y_1^* + 1 - y_2^*$$
$$\text{s.t.} \;\; b_1 + b_2 \leq \overline{B}, \; y_1^* = Y(b_1, y_2^*), \; y_2^* = Y(b_2, y_1^*).$$

Step 1: At optimality, the principal invests in one or two equally.

We divide the proof into several cases given $b_1 + b_2 = B$.

Case 1: $B \leq 2$

[19] When both agents succeed, the principal's profit is not larger than 1 (i.e., $2\alpha < 1$); when only one agent succeeds, the profit is 1.

In this case, a potentially successful outcome only comes from agent 1's innovation. As $N_d(B - b_2, b_2) = 1 - \frac{1}{B-b_2}$, we must allocate all resources to agent 1.

Case 2: $B > 2$

Similar to the proof of Proposition 1, we perform a special transformation to simplify the exposition of the proof: $b_2 = \frac{B}{2}[1 - \sqrt{1 - \frac{1-\alpha}{z}}]$ with $z \in [1-\alpha, \infty)$. There is $\hat{z} \geq 1-\alpha$ such that $y_2 < 1$ only when $z < \hat{z}$.

Case 2.a: $z \geq \hat{z}$

In this case, $y_2^* = 1$ and similar to case 1, $N_d(z) := N_d(B - b_2(z), b_2(z))$ is increasing in $z \in [\hat{z}, \infty)$.

Case 2.b: $z \in [1-\alpha, \hat{z})$

Using the expressions of y_1^* and y_2^* in the proof of Lemma 1, the principal's objective becomes

$$N_d(z) = \frac{2-\alpha}{1-\alpha} - \frac{\sqrt{d(z)}}{(1-\alpha)\alpha B}$$

where $d(z) := \alpha^4 B^2 + 8z(\alpha^2 B + 2\alpha - 2) + 16z^2$. Note that

$$d'(z) = 8(2\alpha + \alpha^2 B + 4z - 2) \geq 8(2\alpha + 2\alpha^2 + 4(1-\alpha) - 2) = 16(\alpha^2 - \alpha + 1) > 0,$$

where the first inequality is due to $B \geq 2$ and $z \geq 1-\alpha$. Then $N_d(z)$ is decreasing in $z \in [1-\alpha, \hat{z})$.

In summary, $N_d(z)$ is quasiconvex in $z \in [1-\alpha, \infty)$, which implies that the principal invests in one or invests equally in two.

Step 2: Use up resources under two strategies.

If the principal invests equally in two agents with $B = b_1 + b_2 > 2$, the equilibrium failure rate is given by (10), namely,

$$y_1 = y_2 = y^* = \frac{4}{\alpha B + \sqrt{B}\sqrt{8 - 8\alpha + \alpha^2 B}}. \tag{23}$$

The principal's objective is

$$N_d(B; \alpha) = 1 - y_1^* + 1 - y_2^* = \frac{(\alpha - 2)B + \sqrt{B(8 - 8\alpha + \alpha^2 B)}}{(\alpha - 1)B}.$$

The expected number of successful innovations, N_d, is increasing in α and B as $B > 2$ and $\alpha \in (0, 1)$. Therefore, the principal will exhaust her resources if investing in two equally, i.e., $(b_1^*, b_2^*) = (\overline{B}/2, \overline{B}/2)$.

If the principal invests in one agent with $B > 1$, by (2), the failure rate is $y^* = 1/B$. The principal chooses B by

$$\max_{B \leq \overline{B}} N_m(B) = 1 - y^* = 1 - \frac{1}{B}.$$

Obviously, $N_m(B)$ increases in B, so $(b_1^*, b_2^*) = (\overline{B}, 0)$ when the principal invests in one agent.

Step 3: Comparison between two strategies.

Next, to derive the globally optimal allocation strategy for the principal, we need to compare $N_d(\overline{B};\alpha)$ and $N_m(\overline{B})$. Given $\overline{B} \geq 2$, the difference between $N_d(\overline{B};\alpha)$ and $N_m(\overline{B})$ is

$$\Delta N(\overline{B};\alpha) = N_d - N_m = \frac{\alpha - 1 - \overline{B} + \sqrt{\overline{B}(8 - 8\alpha + \alpha^2 \overline{B})}}{(\alpha - 1)\overline{B}}.$$

One can show that $\Delta N(\overline{B};\alpha)$ increases in $\overline{B}$ and α as $B > 2$ and $\alpha \in (0,1)$, which implies the proposition. Given $\overline{B} \geq 2$, if $\Delta N(\overline{B};\alpha) = 0$, $B_\# = \frac{3+\sqrt{\alpha^2+8}}{1+\alpha} > 3$, which is decreasing in α. $\square$

Probability that at least one innovator succeeds

The principal's problem is

$$P(\overline{B}) := \max_{b_1 \geq 0, b_2 \geq 0} \ 1 - y_1^* y_2^*$$
$$\text{s.t.} \ \ b_1 + b_2 \leq \overline{B}, \ y_1^* = Y(b_1, y_2^*), \ y_2^* = Y(b_2, y_1^*).$$

Step 1: At optimality, the principal invests in one or two equally.

We divide the proof into several cases given $b_1 + b_2 = B$.

Case 1: $B \leq 2$

In this case, a potential successful outcome only comes from agent 1's innovation. As $P_d(B - b_2, b_2) = 1 - \frac{1}{B - b_2}$, we must allocate all resources to agent 1.

Case 2: $B > 2$

Similar to the proof of Proposition 1, we perform a special transformation to simplify the exposition of the proof: $b_2 = \frac{B}{2}[1 - \sqrt{1 - \frac{1-\alpha}{z}}]$ with $z \in [1-\alpha, \infty)$. There is $\hat{z} \geq 1 - \alpha$, such that $y_2 < 1$ only when $z < \hat{z}$.

Case 2.a: $z \geq \hat{z}$

In this case, $y_2^* = 1$, and similar to case 1, $P_d(z) := P_d(B - b_2(z), b_2(z))$ is increasing in $z \in [\hat{z}, \infty)$.

Case 2.b: $z \in [1 - \alpha, \hat{z})$

Using the expressions of y_1^* and y_2^* in the proof of Lemma 1, the principal's objective becomes as follows,

$$P_d(z) = \frac{2 - 4\alpha + \alpha^2}{2(1-\alpha)^2} + \frac{\sqrt{d(z)} - 4z}{2(1-\alpha)^2 B}$$

where $d(z) := \alpha^4 B^2 + 8z(\alpha^2 B + 2\alpha - 2) + 16z^2$. By the FOC, one can verify that when $0 \leq \alpha < \frac{1}{2}$ and $2 < B \leq \frac{1-\alpha}{\alpha^2}$, $\sqrt{d(z)} - 4z$ is nondecreasing in $z > 1 - \alpha$; otherwise, $\sqrt{d(z)} - 4z$ is nonincreasing in $z > 1 - \alpha$.

In summary, $P_d(z)$ is quasiconvex in $z \in [1-\alpha, \infty)$, which implies that the principal invests in one or invest in two equally.

Step 2: Use up resources under two strategies.

If investing in two equally with resource $B > 2$, the objective of the investor is

$$P_d(B;\alpha) = 1 - y_1^* y_2^* = \frac{-4 - 4\alpha(B-1) + 2B + \alpha^2 B + \alpha\sqrt{B(8 - 8\alpha + \alpha^2 B)}}{2(1-\alpha)^2 B},$$

which can be shown that it is increasing in α and B as $B > 2$ and $\alpha \in (0,1)$. Therefore the principal will exhaust her resources if investing in two equally, i.e., $(b_1^*, b_2^*) = (\overline{B}/2, \overline{B}/2)$.

If investing in one with resource $B > 1$, the objective of the investor is

$$P_m(B) = 1 - \frac{1}{B},$$

which is increasing in $B > 1$. So $(b_1^*, b_2^*) = (\overline{B}, 0)$ when the principal invests in one agent.

Step 3: Comparison between two strategies.

Next, to derive the globally optimal allocation strategy for the principal, we need to compare $P_d(\overline{B};\alpha)$ and $P_m(\overline{B})$. If $P_d(B_\#) = P_m(B_\#)$, $B_\# = \frac{(1+\alpha)^2}{\alpha^2} > 4$, which is decreasing in α. Moreover,

$$\frac{d[P_d(\overline{B};\alpha) - P_m(\overline{B})]}{d\overline{B}}\Big|_{\overline{B}=B_\#} = \frac{\alpha^4}{(3-\alpha)(1+\alpha)^3} > 0.$$

Then it implies the proposition. □

Consumer surplus

Since $\alpha = \frac{4}{(2+g)^2}$ and $g \in (0,1)$, we have $\alpha \in (\frac{4}{9}, 1)$ and $g = \frac{2}{\sqrt{\alpha}} - 2$. The principal's problem is

$$\begin{aligned} CS(\overline{B}) := \max_{b_1 \geq 0, b_2 \geq 0} \ & \frac{4(1+g)}{(2+g)^2}(1 - y_1^* y_2^*) + \frac{1}{2}[(1-y_1^*)y_2^* + (1-y_2^*)y_1^*] \\ \text{s.t.} \ & b_1 + b_2 \leq \overline{B},\ y_1^* = Y(b_1, y_2^*),\ y_2^* = Y(b_2, y_1^*). \end{aligned}$$

Step 1: At optimality, the principal invests in one or two equally.

We divide the proof into several cases given $b_1 + b_2 = B$.

Case 1: $B \leq 2$

A potentially successful outcome only comes from agent 1's innovation. As $CS_d(B - b_2, b_2) = (1 - \frac{1}{B - b_2})/2$, we must allocate all resources to agent 1.

Case 2: $B > 2$

Similar to the proof of Proposition 1, we perform a special transformation to simplify the exposition of the proof: $b_2 = \frac{B}{2}[1 - \sqrt{1 - \frac{1-\alpha}{z}}]$ with $z \in [1-\alpha, \infty)$. There is $\hat{z} \geq 1 - \alpha$ such that

$y_2 < 1$, only when $z < \hat{z}$.

Case 2.a: $z \geq \hat{z}$

In this case, $y_2^* = 1$, and similar to case 1, we obtain that $CS_d(z) := CS_d(B - b_2(z), b_2(z))$ is increasing in $z \in [\hat{z}, \infty)$.

Case 2.b: $z \in [1 - \alpha, \hat{z})$

Using the expressions of y_1^* and y_2^* in the proof of Lemma 1, the principal's objective becomes

$$CS_d(z) = -\frac{\sqrt{\alpha}(4 + \sqrt{\alpha} - 3\alpha - \alpha^{3/2} + \alpha^2)}{2(\alpha^{3/2} + \alpha - \sqrt{\alpha} - 1)} - \frac{2z}{(1 + \sqrt{\alpha})^2 B} + \frac{1 - 3\sqrt{\alpha} - \alpha + \alpha^{3/2}}{2(1 - \sqrt{\alpha})(1 + \sqrt{\alpha})^2 \alpha B}\sqrt{d(z)},$$

where $d(z) := \alpha^4 B^2 + 8z(\alpha^2 B + 2\alpha - 2) + 16z^2$. The second term is decreasing z. Due to $\alpha \in (\frac{4}{9}, 1)$, we have $\frac{1-3\sqrt{\alpha}-\alpha+\alpha^{3/2}}{2(1-\sqrt{\alpha})(1+\sqrt{\alpha})^2\alpha} < 0$, and as shown in Proposition **??**, we have $d'(z) > 0$. As a result, the third term is also decreasing z. Then, $CS_d(z)$ is decreasing in $z \in [1 - \alpha, \hat{z})$.

In summary, $CS_d(z)$ is quasiconvex in $z \in [1 - \alpha, \infty)$, which implies that the principal invests in one or invest equally in two.

Step 2: Use up resources under two strategies.

When the principal invests in one agent with resources $B > 1$, the expected consumer surplus is

$$CS_m(B) = \frac{1}{2} - \frac{1}{2B},$$

which is increasing in B. Hence, when the principal only invests in one agent, the principal will grant all available resources $\overline{B}$ to this agent, and the consumer surplus is $CS_m(\overline{B})$.

When the principal invests equally in two agents with resources B, the equilibrium failure rate is given by (23). The expected consumer surplus is

$$CS_d(B; g) = (1 - y^*)^2 \frac{4(1 + g)}{(2 + g)^2} + (1 - y^*)y^*.$$

Note that fixing g, we have

$$\frac{\partial C_d(B; g)}{\partial y^*} = -\frac{4 + 4g + g^2(2y^* - 1)}{(2 + g)^2} < -\frac{4 + 4g - g^2}{(2 + g)^2} < 0.$$

Because $CS_d(B; g)$ decreases in y^* and y^* decreases in B, $CS_d(B; g)$ increases in B. Therefore, the principal chooses $(b_1^*, b_2^*) = (\overline{B}/2, \overline{B}/2)$.

Also, note that fixing B, we have

$$\frac{dCS_d(B; \alpha)}{dg} = -\frac{4g(1 - y^*)^2}{(2 + g)^3} - \frac{4 + 4g + g^2(2y^* - 1)}{(2 + g)^2}\frac{\partial y^*}{\partial g}.$$

The first term is negative and the second term is also negative as y^* decreases in α, or equivalently increases in g. Then it implies that fixing B, $CS_d(B; g)$ is decreasing in g.

Step 3: Comparison between two strategies.

Next, to derive the globally optimal allocation strategy for the principal, we need to compare $CS_d(\overline{B};\alpha)$ and $CS_m(\overline{B})$. If $CS_d(B_\#;\alpha) = CS_m(B_\#)$ and $B_\# > 2$, then $B_\#$ is unique as below,

$$B_\# = \frac{1}{2} + \frac{1}{2 - 8\sqrt{\alpha} + 4\alpha} + \frac{6\sqrt{\alpha}}{3\sqrt{\alpha} + \alpha + \alpha^{3/2} - 1} + \frac{(1 - 3\sqrt{\alpha} - \alpha + \alpha^{3/2})\sqrt{32\sqrt{\alpha} - 16\alpha + \alpha^2 - 8}}{(1 - 4\sqrt{\alpha} + 2\alpha)(3\sqrt{\alpha} + \alpha + \alpha^{3/2} - 1)}.$$

Furthermore, note that $\lim_{\overline{B}\to 2} CS_d(\overline{B};\alpha) = 0 < \lim_{\overline{B}\to 2} CS_m(\overline{B})$ and $\lim_{\overline{B}\to\infty} CS_d(\overline{B};\alpha) = (1-y^*)^2\frac{4(1+g)}{(2+g)^2} + (1-y^*)y^*|_{y^*=0} = \frac{4(1+g)}{(2+g)^2} > \lim_{\overline{B}\to\infty} CS_m(\overline{B}) = \frac{1}{2}$. Combining it with $CS_d(B_\#) = CS_m(B_\#)$ yields a unique solution, we conclude that: when $\overline{B} > B_\#$, $CS_d(\overline{B};\alpha) > CS_m(\overline{B})$; otherwise, $CS_d(\overline{B};\alpha) \leq CS_m(\overline{B})$. Moreover, since fixing $\overline{B}$, $CS_m(\overline{B})$ does not depend on α and $CS_d(\overline{B};\alpha)$ is increasing in α (or equivalently decreasing in g). As a result, $B_\#$ will be decreasing in α. □

Total welfare

The principal's problem is

$$\begin{aligned} W(\overline{B}) := \max_{b_1\geq 0, b_2\geq 0} \; & \frac{4(3+g)}{(2+g)^2}(1 - y_1^* y_2^*) + \frac{3}{2}[(1-y_1^*)y_2^* + (1-y_2^*)y_1^*] \\ \text{s.t.} \; & b_1 + b_2 \leq \overline{B},\; y_1^* = Y(b_1, y_2^*),\; y_2^* = Y(b_2, y_1^*). \end{aligned}$$

Step 1: At optimality, the principal invests in one or two equally.

We divide the proof into several cases given $b_1 + b_2 = B$.

Case 1: $B \leq 2$

Note that when $B \leq 2$, then $b_2 \leq 1$. Due to the insufficient amount of resources to incentivize both agents to innovate, $y_2^* = 1$ for sure. The profit of the principal will only come from a potentially successful outcome from agent 1's innovation. Similar to Proposition 2, $W(b_2; B)$ is nonincreasing in $b_2 \in [0, \frac{B}{2}]$.

Case 2: $B > 2$

Similar to the proof of Proposition 1, we perform a special transformation to simplify the exposition of the proof: $b_2 = \frac{B}{2}[1 - \sqrt{1 - \frac{1-\alpha}{z}}]$ with $z \in [1-\alpha, \infty)$. There is $\hat{z} \geq 1 - \alpha$ such that $y_2 < 1$, only when $z < \hat{z}$.

Case 2.a: $z \geq \hat{z}$

In this case, $y_2^* = 1$, and the profit of the principal will only come from a successful outcome from agent 1's innovation. Then the objective of the principal is increasing in $z \in [\hat{z}, \infty)$ as agent 1 obtains more resources.

Case 2.b: $z \in [1-\alpha, \hat{z})$

Using the expressions of y_1^* and y_2^* in the proof of Lemma 1, the principal's objective becomes

$$W(z) = \frac{1}{2(1-\sqrt{\alpha})(1+\sqrt{\alpha})^2\alpha B}[-\alpha^{3/2}(-4-3\sqrt{\alpha}+\alpha+3\alpha^{3/2}+\alpha^2)B$$
$$-4(3+\sqrt{\alpha})\alpha z + (3-\sqrt{\alpha}(1+3\sqrt{\alpha}+\alpha))\sqrt{d(z)}],$$

where $d(z) := \alpha^4B^2 + 8z(\alpha^2B + 2\alpha - 2) + 16z^2 \geq 0$ and we have used $g = \frac{2}{\sqrt{\alpha}} - 2$. Denote $w(z) := -4(3+\sqrt{\alpha})\alpha z + (3-\sqrt{\alpha}(1+3\sqrt{\alpha}+\alpha))\sqrt{d(z)}$. Then the problem of the investor is same as maximizing $w(z)$. Note that

$$w''(z) = \frac{64(1-\alpha)}{\sqrt{d^3(z)}}(3-\sqrt{\alpha}-3\alpha-\alpha^{3/2})(\alpha^2B+\alpha-1).$$

Case 2.b.1: $(3-\sqrt{\alpha}-3\alpha-\alpha^{3/2})(\alpha^2B+\alpha-1) \geq 0$

In this case, $w(z)$ is convex in $z \in [1-\alpha, \hat{z})$.

Case 2.b.2: $(3-\sqrt{\alpha}-3\alpha-\alpha^{3/2})(\alpha^2B+\alpha-1) < 0$

$w(z)$ is concave in $z \in [1-\alpha, \hat{z})$. If $w'(z) = 0$, we have two solutions z_1 and z_2 satisfying

$$z_1 + z_2 = 1 - \alpha - \frac{\alpha^2 B}{2},$$

and

$$z_1 - z_2 = -\frac{(3+\sqrt{\alpha})\alpha\sqrt{-(-9+24\sqrt{\alpha}-13\alpha-4\alpha^{3/2}+2\alpha^2)(-1+\alpha+\alpha^2B)}}{9-15\sqrt{\alpha}-2\alpha+2\alpha^{3/2}}.$$

We show that in Case 2.b.2, z_1 and z_2 cannot be real or larger than $1-\alpha$.

To obtain a real solution, the following inequality should be satisfied:

$$-(-9+24\sqrt{\alpha}-13\alpha-4\alpha^{3/2}+2\alpha^2)(-1+\alpha+\alpha^2 b) \geq 0.$$

Combining with $[3-\sqrt{\alpha}(1+3\sqrt{\alpha}+\alpha)](\alpha^2B+\alpha-1) < 0$, $B > 2$ and $\alpha \in (\frac{4}{9}, 1)$, the inequality above holds only when $\alpha \in (\frac{4}{9}, \frac{1}{2})$ and $B \in (2, \frac{1-\alpha}{\alpha^2})$. Moreover, when $\alpha \in (\frac{4}{9}, \frac{1}{2})$ and $B \in (2, \frac{1-\alpha}{\alpha^2})$, it can be verified that $z_1, z_2 < 1-\alpha$. Therefore, $w(z)$ is monotone in $z \in [1-\alpha, \hat{z})$.

In summary, $W(z)$ is convex or monotone in $z \in [1-\alpha, \hat{z})$ and $W(z)$ is increasing in $z \in [\hat{z}, \infty)$. As a result, $W(z)$ is quasiconvex in $z \in [1-\alpha, \infty)$, which implies the claim shown in the proposition.

Step 2: Use up resources under two strategies.

When the principal invests in one agent with resources $B > 1$, the expected welfare is

$$W_m(B) = \frac{3}{2} - \frac{3}{2B},$$

which is increasing in B. Hence, when the principal only invests in one agent, the principal will grant all available resources $\overline{B}$ to this agent, and the consumer surplus is $W_m(\overline{B})$.

When the principal invests equally in two agents with resources B, the equilibrium failure rate is given by (23). The expected welfare is

$$W_d(B;g) = (1-y^*)^2\frac{4(3+g)}{(2+g)^2} + 3(1-y^*)y^*.$$

Note that fixing g,

$$\frac{\partial W_d(B;g)}{\partial y^*} = \frac{-12+4g+3g^2-16gy^*-6g^2y^*}{(2+g)^2} < \frac{-12+4g+3g^2}{(2+g)^2} < 0.$$

Because $W_d(B;g)$ decreases in y^* and y^* decreases in B, and $W_d(B;g)$ increases in B. Therefore, the principal chooses $(b_1^*, b_2^*) = (\overline{B}/2, \overline{B}/2)$.

Also, note that fixing B, we have

$$\frac{dW_d(B;\alpha)}{dg} = -\frac{4(4+g)(1-y^*)^2}{(2+g)^3} + \frac{-12+4g+3g^2-16gy^*-6g^2y^*}{(2+g)^2}\frac{\partial y^*}{\partial g}.$$

The first term is negative and the second term is also negative as y^* decreases in α, or equivalently increases in g. Then it implies that fixing B, $W_d(B;g)$ is decreasing in g.

Step 3: Comparison between two strategies.

Next, to derive the globally optimal allocation strategy for the principal, we need to compare $W_d(\overline{B};\alpha)$ and $W_m(\overline{B})$.

Denote $h = \frac{4}{\alpha\overline{B}+\sqrt{\overline{B}}\sqrt{8-8\alpha+\alpha^2\overline{B}}}$. As $\overline{B} \geq 2$, $h \in (0,1)$. Furthermore, $\overline{B} = -(2/(y(-a-y+ay)))$, then there is a one-to-one correspondence between h and $\overline{B}$. Hence, we obtain

$$W_d(\overline{B};\alpha) - W_m(\overline{B}) = \frac{1}{4}[(8\sqrt{\alpha}+4\alpha-6) + (12-16\sqrt{\alpha}-5\alpha)h + (8\sqrt{\alpha}+\alpha-9)h^2].$$

The right hand is decreasing in h as $\alpha \in (4/9, 1)$, so $W_d(\overline{B};\alpha) - W_m(\overline{B})$ is increasing in $\overline{B}$. Thus there is a threshold $B_\#$ such that when $\overline{B} > B_\#$, $W_d(\overline{B};\alpha) > W_m(\overline{B})$; otherwise, $W_d(\overline{B};\alpha) \leq W_m(\overline{B})$. Moreover, since fixing $\overline{B}$, $W_m(\overline{B})$ does not depend on α, and $W_d(\overline{B};\alpha)$ is increasing in α (or equivalently decreasing in g). Hence, $B_\#$ will be decreasing in α. ∎

C Other Extensions

C.1 Costly Resources

Suppose that the principal incurs a marginal cost, $c \in [0,1]$, of allocating more resources. If the principal allocates $B = b_1 + b_2 \leq \overline{B}$ resources, she incurs a cost cB. For financial resources of VCs, the cost can be risk-free returns from not investing in startups; For incubators, granting the right to use office space and equipment could incur additional cost of maintenance and depreciation.

In this setting, the principal's payoff becomes

$$R(c) = \max_{b_1 \geq 0, b_2 \geq 0} \{\Pi(b_1, b_2) - c(b_1 + b_2)\} \; s.t. \; b_1 + b_2 \leq \overline{B}, \tag{24}$$

where $\Pi(b_1, b_2)$ is given by (7). We can show the following result:

Proposition 6. *As the cost parameter c increases, the optimal amount of total resources $B^* = b_1^* + b_2^*$ allocated to two agents decreases.*

Proof.

$$R(k) = \max_{B \leq \overline{B}} \{\Pi(B) - kB\}$$

where $\Pi(B)$ is the profit given total resources allocated being B. Clearly, $R(k)$ is convex on k, because it is a supreme of affine functions. Then, by the envelope theorem, we obtain $\frac{dR(k)}{dk} = -B^*(k)$, and it is increasing in k. ■

Given the optimally allocated B^*, the resource allocation rule follows Theorem 1 by replacing $\overline{B}$ by B^*. Hence, our main results hold in the case of costly resources.

C.2 General Innovation Success Function

In this section, we show that the qualitative results of the optimal resource allocation strategy are robust under two general classes of innovation success functions. Given a fixed profit-sharing rate, the principal makes resource allocation decisions by maximizing the total profit from the product market. For simplicity, we set $\gamma = 1$.

Class 1: hump-shaped equilibrium effort

Let $p(b, x) = 1 - e^{-\psi(b)x}$, where ψ is an increasing function with $\psi(0) = 0$ and $\psi(\infty) = \infty$. The innovate success function (1) is a special case with $\psi(b) = b$.

If the principal invests in a single agent with resources b, the failure rate is $y = e^{-\psi(b)x}$. Given b, the agent solves $\max_y \left\{1 - y + \frac{\ln y}{\psi(b)}\right\}$, and thus, $y^* = \frac{1}{\psi(b)}$ for $\psi(b) \geq 1$. The equilibrium effort level, $x^*(b) = \frac{\ln \psi(b)}{\psi(b)}$, exhibits the hump-shaped pattern shown in Figure 1. The profit from the product market is $\Pi_m(\psi(b)) = 1 - \frac{1}{\psi(b)}$, which is increasing in b.

If the principal invests in two agents with the same amount $\frac{b}{2}$, the failure rate is $y = e^{-\psi(\frac{b}{2})x}$. The equilibrium failure rate of one agent is

$$y^* = \underset{0<y\leq 1}{\operatorname{argmax}} (1 - y)[\alpha(1 - y^*) + y^*] + \frac{\ln y}{\psi(\frac{b}{2})} \text{ for } \psi(\frac{b}{2}) \geq 1.$$

The total product market profit is

$$\Pi_d(\psi(\frac{b}{2})) = 2\alpha(1-y^*)^2 + 2y^*(1-y^*) = \frac{-2+\alpha\psi(\frac{b}{2}) + \sqrt{\psi(\frac{b}{2})}\sqrt{4-4\alpha+\alpha^2\psi(\frac{b}{2})}}{\psi(\frac{b}{2})}.$$

Proposition 7. *With $p(b,x) = 1 - e^{-\psi(b)x}$, the principal will use up her resources ($b_1 + b_2 = \bar{B}$). When $\alpha < \frac{1}{2}$ and there exists a B such that $\psi(B) \in (1, \frac{1}{1-2\alpha})$ and $\psi(\frac{B}{2}) > \frac{2\psi(B)}{1+\sqrt{\psi(B)}\sqrt{2(1-\alpha)-(1-2\alpha)\psi(B)}}$, the optimal resource allocation strategy exhibits the following non-monotone pattern: when $\overline{B}$ is sufficiently small or large, it only invests in one agent; when $\overline{B}$ is at some moderate level, it invests in two agents.*

Proof. For expositional convenience, denote $\mu = \psi(b)$ and $\nu = \psi(\frac{b}{2})$. The total profit from investing in one agent is $\Pi_m(\mu) = 1 - \frac{1}{\mu}$. The total profit from investing in two agents equally is

$$\Pi_d(\nu) = \frac{-2+\alpha\nu+\sqrt{\nu}\sqrt{4-4\alpha+\alpha^2\nu}}{\nu}.$$

Step 1: We first show that $\max\{\Pi_m(\psi(b)), \Pi_d(\psi(\frac{b}{2}))\}$ is increasing in b.

When $0 < \alpha < \frac{1}{2}$, $\Pi_d(\nu)$ is maximized at $\nu^* = \frac{4-4\alpha}{1-2\alpha}$ with $\Pi_d(\nu^*) = \frac{1}{2-2\alpha}$. Let $b_\# = 2\psi^{-1}(\nu^*)$. Then $\max\{\Pi_m(\psi(b)), \Pi_d(\psi(\frac{b}{2}))\}$ is increasing in $b \in [0, b_\#]$ by the envelope theorem. When $b > b_\#$, $\Pi_m(\psi(b)) \geq \Pi_m(\psi(\frac{b}{2})) \geq \Pi_m(\psi(b_\#/2)) = \Pi_m(\nu^*) = \frac{3-2\alpha}{4(1-\alpha)} \geq \frac{1}{2-2\alpha} = \Pi_d(\nu^*) \geq \Pi_d(\psi(\frac{b}{2}))$. Thus when $b \in [b_\#, \infty]$, $\max\{\Pi_m(\psi(b)), \Pi_d(\psi(\frac{b}{2}))\} = \Pi_m(\psi(b))$, which is also increasing in b.

When $\alpha \geq \frac{1}{2}$, both $\Pi_m(\psi(b))$ and $\Pi_d(\psi(\frac{b}{2}))$ is increasing in b. Therefore, $\max\{\Pi_m(\psi(b)), \Pi_d(\psi(\frac{b}{2}))\}$ is increasing in $b \in [0, b_\#]$ by the envelope theorem.

As a result, the principal will use up her resources.

Step 2: Conditions for non-monotone optimal resource allocation strategy.
Condition (1): When $b \to \infty$, the investor invests in one agent.

As $b \to \infty$, $\mu \to \infty$, $\Pi_m(\mu) = 1$; $\nu \to \infty$, $\Pi_d(\nu) = 2\alpha$. Hence, it requires that $\alpha < \frac{1}{2}$.
Condition (2): When b is small, the investor invests in one agent.

Clearly, when $\mu = 1$, $\nu < 1$. As $\mu \to 1$, $\Pi_m(\mu) > 0$ and $\Pi_d(\nu) = 0$. Therefore, if $b \in [\psi^{-1}(1), 2\psi^{-1}(1)]$, $\Pi_m(\mu) > \Pi_d(\nu)$, which requires that $\overline{B} > \psi^{-1}(1)$.
Condition (3): When b is at an intermediate region, the investor will invest in two agents.

This condition requires that $\Pi_m(\mu) < \Pi_d(\nu)$, namely $1 - \frac{1}{\mu} < \frac{-2+\alpha\nu+\sqrt{\nu}\sqrt{4-4\alpha+\alpha^2\nu}}{\nu}$.

These three conditions together require the conditions in the proposition. ■

Class 2: monotone equilibrium effort

Let $p(b,x) = \phi(b)(1-e^{-\kappa x})$, where $\kappa > 1$ and ϕ is increasing in its support with $\phi(0) = 0$ and $\phi(\infty) = 1$. Denote $y = e^{-\kappa x}$.

If the principal invests in a single agent with resource b, the agent solves $\max_y \left\{\phi(b)(1-y)+\frac{\ln y}{\kappa}\right\}$, and thus $y^* = \frac{1}{\kappa\phi(b)}$ for $\phi(b) \geq \frac{1}{\kappa}$. The equilibrium effort level, $x^*(b) = \frac{1}{\kappa}[\ln\kappa + \ln\phi(b)]$, is monotonically increasing in b. The profit from the product market is $\Pi_m(\phi(b)) = \phi(b) - \frac{1}{\kappa}$, which is increasing in b.

If the principal invests in two agents with the same amount $\frac{b}{2}$, in equilibrium, we have that

$$y^* = \underset{0<y\leq 1}{\operatorname{argmax}}\, \phi(\frac{b}{2})(1-y)[\alpha\phi(\frac{b}{2})(1-y^*) + 1 - \phi(\frac{b}{2})(1-y^*)] + \frac{\ln y}{\kappa} \text{ for } \phi(\frac{b}{2}) \geq \frac{1}{\kappa}.$$

The total product market profit is

$$\begin{aligned}\Pi_d(\psi(\frac{b}{2})) =& 2\phi(\frac{b}{2})(1-y^*)[\alpha\phi(\frac{b}{2})(1-y^*) + 1 - \phi(\frac{b}{2})(1-y^*)] \\ =& \frac{1}{\kappa}\left\{\kappa\phi(\frac{b}{2})(1-\phi(\frac{b}{2})+\alpha\phi(\frac{b}{2})) - 2 + \phi(\frac{b}{2})\sqrt{\kappa}\sqrt{4-4\alpha+(1-\phi(\frac{b}{2})+\alpha\kappa\phi(\frac{b}{2}))^2}\right\}.\end{aligned}$$

Proposition 8. *With $p(b,x) = \phi(b)(1-e^{-\kappa x})$, the principal will use up her resources ($b_1 + b_2 = \bar{B}$). When $\alpha < \frac{\kappa-1}{2\kappa}$, and there exists a B satisfying $\phi(B) \in (\frac{1}{\kappa}, \frac{1}{4(1-\alpha)}(1+\sqrt{\frac{8-8\alpha+\kappa}{\kappa}}))$ and $\phi(\frac{B}{2}) > \frac{\kappa\phi(B)+1}{2(1-\alpha)(\kappa\phi(B)-1)}(1-\sqrt{\frac{2-2\alpha+\kappa-2\kappa\phi(B)+2\alpha\kappa\phi(B)}{\kappa}})$, the optimal resource allocation strategy exhibits the following non-monotone pattern: when $\overline{B}$ is sufficiently small or large, it only invests in one agent; when $\overline{B}$ is at some moderate level, it invests in two agents.*

Proof. For expositional convenience, denote $\mu = \phi(b)$ and $\nu = \phi(b/2)$. The total profit from investing in one agent is $\Pi_m(\mu) = \mu - \frac{1}{\kappa}$. The total profit from investing in two agents equally is

$$\Pi_d(\nu) = \frac{-2 + \nu(1-\nu+\alpha\nu)\kappa + \nu\sqrt{\kappa}\sqrt{4-4\alpha+(1-\nu+\alpha\nu)^2\kappa}}{\kappa}.$$

Step 1: We first show that $\max\{\Pi_m(\phi(b)), \Pi_d(\phi(\frac{b}{2}))\}$ is increasing in b.

If $\kappa > 4$ and $\alpha \in [0, \frac{\kappa-4}{2\kappa-4}]$, $\Pi_d(\nu)$ is maximized at $\nu^* = \frac{4+\kappa-4\alpha}{2(1-\alpha)\kappa} \in (\frac{1}{\kappa}, 1]$ with $\Pi_d(\nu^*) = \frac{1}{2-2\alpha}$. Let $b_\# = 2\phi^{-1}(\nu^*)$. Then $\max\{\Pi_m(\phi(b)), \Pi_d(\phi(\frac{b}{2}))\}$ is increasing in $b \in [0, b_\#]$ by the envelope theorem. When $b > b_\#$, $\Pi_m(\phi(b)) \geq \Pi_m(\phi(\frac{b}{2})) \geq \Pi_m(s(b_\#/2)) = \Pi_m(\nu^*) = \frac{1}{2-2\alpha} + \frac{1}{\kappa} \geq \Pi_d(\nu^*) \geq \Pi_d(\phi(\frac{b}{2}))$. Thus when $b \in [b_\#, \infty]$, $\max\{\Pi_m(\phi(b)), \Pi_d(\phi(\frac{b}{2}))\} = \Pi_m(\phi(b))$, which is also increasing in b.

When $\kappa \leq 4$ and $\alpha \notin [0, \frac{\kappa-4}{2\kappa-4}]$, since both $\Pi_m(\phi(b))$ and $\Pi_d(\phi(\frac{b}{2}))$ are increasing in b, we have $\max\{\Pi_m(\phi(b)), \Pi_d(\phi(\frac{b}{2}))\}$ is increasing in $b \in [0, b_\#]$ by the envelope theorem.

As a result, the principal will use up her resources.

Step 2: Conditions for non-monotone optimal resource allocation strategy.
Condition (1): When $b \to \infty$, the investor invests in one agent.

Note that, when $b \to \infty$, namely $\mu \to 1$, then $\Pi_m^0 = 1 - \frac{1}{\kappa}$. When $b \to \infty$, $\nu = 1$, then

$\Pi_d^0 = \frac{-2+\alpha\kappa+\sqrt{\kappa}\sqrt{4-4\alpha+\alpha^2 k}}{\kappa}$ Then the nontrivial result requires that $\Pi_m^0 > \Pi_d^0$, namely $1-\frac{1}{\kappa} > \frac{-2+\alpha k+\sqrt{\kappa}\sqrt{4-4\alpha+\alpha^2 k}}{\kappa}$.

Condition (2): When b is small, the investor invests in one agent.

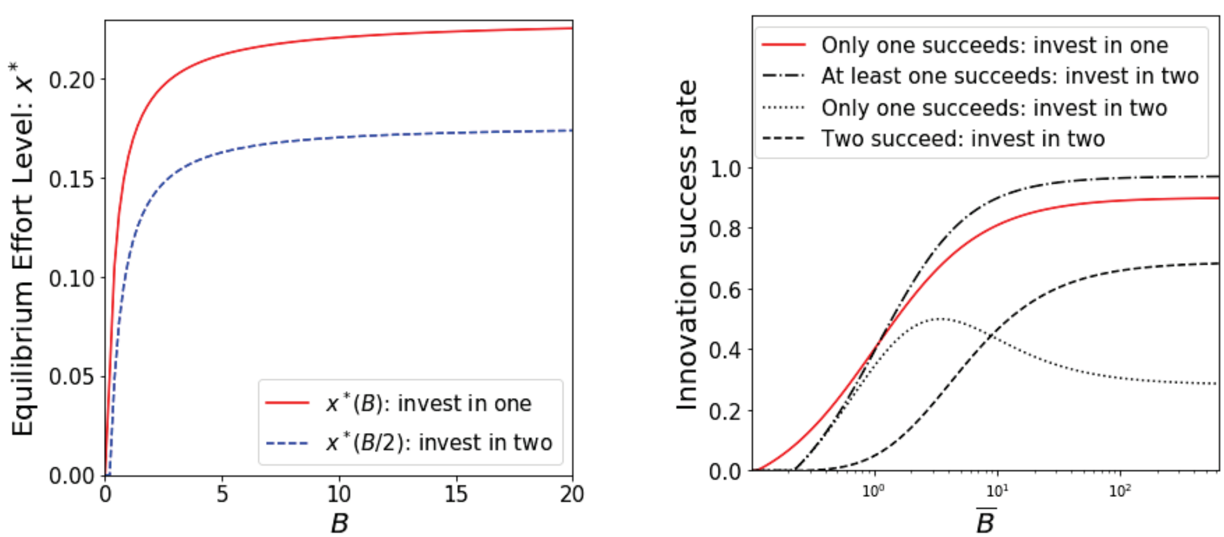
Clearly when $\mu = \frac{1}{\kappa}$, $\nu < \frac{1}{\kappa}$. As $\mu \to \frac{1}{\kappa}$, $\Pi_m(\mu) > 0$ and $\Pi_d(\nu) = 0$. Therefore, if $b \in [\phi^{-1}(\frac{1}{\kappa}), 2\phi^{-1}(\frac{1}{\kappa})]$, $\Pi_m(\mu) > \Pi_d(\nu)$, which requires that $\overline{B} > \phi^{-1}(\frac{1}{\kappa})$.

Condition (3): When b is at an intermediate region, the investor will invest in two agents.

This condition requires that $\Pi_m < \Pi_d$, namely $\mu - \frac{1}{\kappa} < \frac{-2+\nu(1-\nu+\alpha\nu)\kappa+\nu\sqrt{\kappa}\sqrt{4-4\alpha+(1-\nu+\alpha\nu)^2\kappa}}{\kappa}$.

These three conditions together require the conditions in the proposition. ■

Note that the equilibrium effort depicted in Figure 8-(a) has a different shape from that in Figure 1, but the equilibrium success rates depicted in Figure 8-(b) exhibit the same features as in Figure 4. Hence, the intuition of Theorem 1 holds in this case.



(a) (b)

Figure 8: Equilibrium Effort and Success Rates with $\alpha = 0.5$, $\phi(b) = \frac{b}{1+b}$ and $\kappa = 10$.

C.3 Competition between Principals

We study a simple case with competing principals in the resource allocation stage. Consider two principals, a leader (L) and follower (F), with the same amount of resources $\overline{B}^L = \overline{B}^F = \overline{B}$. The leader chooses the resource allocation, (b_1^L, b_2^L), first with $b_1^L + b_2^L \leq \overline{B}$. After observing (b_1^L, b_2^L), the follower chooses (b_1^F, b_2^F) with $b_1^F + b_2^F \leq \overline{B}$. agent 1 receives resources $b_1 = b_1^L + b_1^F$, and agent 2 receives resources $b_2 = b_2^L + b_2^F$. Following the outcome of the resource allocation stage, (b_1, b_2), two agents continue with the innovation stage and product market competition as described in the baseline model with innovation success rate (1) of each agent.

Let $\pi_i(b_1, b_2) = \pi_i(b_1^L + b_1^F, b_2^L + b_2^F)$ denote the expected profit of agent i. Agent i's profit will be shared by the two principals according to the proportion of their investment. Given (b_1^L, b_2^L), the follower chooses her resource allocation as

$$\left(b_1^F(b_1^L, b_2^L), b_2^F(b_1^L, b_2^L)\right) = \underset{b_1^F \geq 0, b_2^F \geq 0}{\operatorname{argmax}} \sum_{i=1}^{2} \frac{b_i^F}{b_i^L + b_i^F} \pi_i(b_1^L + b_1^F, b_2^L + b_2^F).$$

The optimal resource allocation strategy of the leader is as follows:

$$(b_1^{L*}, b_2^{L*}) = \underset{b_1^L \geq 0, b_2^L \geq 0}{\operatorname{argmax}} \sum_{i=1}^{2} \frac{b_i^L}{b_i^L + b_i^F(b_1^L, b_2^L)} \pi_i\left(b_1^L + b_1^F(b_1^L, b_2^L), b_2^L + b_2^F(b_1^L, b_2^L)\right).$$

We numerically find the equilibrium, $(b_1^{L*}, b_2^{L*}, b_1^F(b_1^{L*}, b_2^{L*}), b_2^F(b_1^{L*}, b_2^{L*}))$, of this leader-follower resource allocation problem. Let $\Pi^*(\overline{B}^L, \overline{B}^F)$ denote the total profit of the product market in equilibrium with $\overline{B}^L$ and $\overline{B}^F$. Define $\Delta = \Pi^*(\overline{B}, 0)/\Pi^*(\overline{B}, \overline{B})$ as the ratio of the product market profit with only the leading investor and the profit with both the leader and the follower. Figure 9 depicts the values of Δ over different $\overline{B}$ and α.

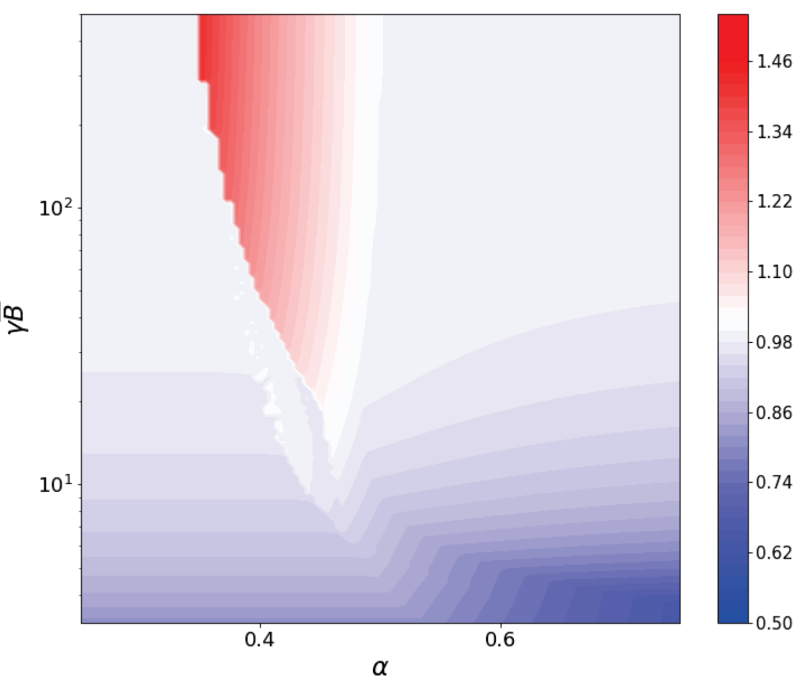


Figure 9: Ratio of the Total Market Profit between without and with an Additional Investor: Δ.

When $\Delta < 1$, introducing a new investor to fund innovation will increase the expected profit of the product market ($\Pi^*(\overline{B}, 0) < \Pi^*(\overline{B}, \overline{B})$). Intuitively, because the total amount of resources is doubled with two principals, the success rate of product development will increase, like the expected product market profit. However, this outcome is not always the case. In Figure 9, we find a red region with $\Delta > 1$, which means that having an additional investor will reduce the total profit of the product market ($\Pi^*(\overline{B}, 0) > \Pi^*(\overline{B}, \overline{B})$).

This phenomenon occurs with $\alpha < \frac{1}{2}$ and $\overline{B}$ being sufficiently large. $\alpha < \frac{1}{2}$ indicates that when both agents succeed, the business-stealing effect will cause rent dissipation. According to Theorem 1, with $\alpha < \frac{1}{2}$ and a sufficiently large $\overline{B}$, a single investor will only invest in one agent, say agent 1. However, when there are two investors, the follower may support agent 2 because she can obtain a larger share of agent 2's profit than that of agent 1. Consequently, if both investors have sufficient resources, the product market is more likely to end up with competition.

Hence, introducing new investors and funding can intensify the product market competition and reduce the profits of investors and innovators. Many researchers have found that public R&D funding program crowds out investment by private VCs. For example, Wallsten (2000) find that SBIR grants crowd out private R&D spending dollar for dollar; Cumming and MacIntosh (2006) show that the Labour Sponsored Venture Capital Corporation of the Canadian government reduces the overall size of the VC pool in Canada. Our model is consistent with the empirical evidence of crowding out. We leave the full analysis of competing principals for future research.