# “Now or Later?”: When to Deploy Qualification Screening in Open-Bid Auction for Re-Sourcing

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This paper considers a re-sourcing setting in which a qualified supplier (the incumbent) and multiple suppliers which have not yet been qualified (the entrants) compete in an open-bid descending auction for a single-supplier contract. Due to the risk of supplier nonperformance, the buyer only awards the contract to a qualified supplier; meanwhile, the buyer can conduct supplier qualification screening at a cost, to verify whether the entrant suppliers can perform the contract. Conventionally, the buyer would screen entrants before running an auction, i.e., the pre-qualification strategy (PRE). We explore an alternative approach called post-qualification strategy (POST), in which the buyer first runs an auction and then conducts qualification screenings based on the suppliers' auction bids. Our characterization of the dynamic structure of the suppliers' equilibrium bidding strategy enables the calculation of the buyer's expected cost under POST, which is computationally intractable without this characterization. We show analytically that POST is cheaper than PRE when the cost of conducting qualification screening is high, the number of entrant suppliers is large, or the entrants' chance of passing qualification screening is high. To quantify the benefit of POST, we conduct a comprehensive numerical study and find that using the cheaper option between PRE and POST provides significant cost-savings over the conventional PRE-only approach. Furthermore, we leverage a revelation principle for multi-stage games to derive the optimal mechanism as a stronger benchmark for performance comparison. While the optimal mechanism is theoretically optimal, we find that its complexity renders it difficult to implement in practice; but quite strikingly, the simple and practical approach of choosing the cheaper option between PRE and POST captures the majority of the benefit the optimal mechanism can offer over PRE, highlighting the practical benefit of POST.

*Key words*: procurement, supplier asymmetry, open-bid auction, qualification screening
*History*:

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## 1. Introduction

Procurement plays an important role in manufacturers' operations. Toyota spends more than 70% of its revenue on purchasing goods and services (Toyota 2016). According to the 2016 annual survey of manufactures from the Bureau of the Census, United States manufacturers, on average, spend roughly 50% to 60% of their revenue on procurement ( U.S. Department of Commerce

2016). To reduce procurement cost, buying firms often routinely re-evaluate how competitive their existing suppliers are on price. If the current contract price is not very competitive, the buyer may resort to a *re-sourcing* process in which it would invite other potential suppliers in the market (entrant suppliers) *and* the existing supplier (incumbent supplier) to compete for that contract in a procurement auction; this re-sourcing process allows the buyer to obtain a more competitive price by either switching to a less expensive entrant supplier or utilizing the competitive pressure from the entrant suppliers in the auction to reduce the incumbent supplier's price.

While the use of procurement auctions can be effective in reducing prices, the buyer also needs to watch out for potential pitfalls of *supplier nonperformance.* In fact, supplier nonperformance can cause catastrophic damages to the buyer's operations, reputations, and profitability. For example, major automakers had to recall more than 39 millions vehicles in the U.S. due to faulty air bags supplied by Takata (NHTSA 2018). More recently, the Samsung Note 7 Battery Explosion Event due to supplier nonperformance forced Samsung Electronics to recall 2.5 millions Note 7 smartphones globally at an estimated total recall expense around $5 billions (Lee 2016). While supply risks cannot be eliminated, they can be managed, and managing such supplier nonperformance risks is particularly critical during a re-sourcing process. While even the incumbent supplier may fail, as evidenced by the examples we mentioned above, nevertheless, it is reasonable to assume that the buyer may have a good knowledge of the incumbent's capability to deliver the contract, based on previous assessments and past experience. In contrast, the buyer is likely to have less information about the ability of *entrant suppliers* to successfully execute the contract.

One common approach to managing the supplier nonperformance risks in re-sourcing is to integrate an auction with *supplier qualification screening* on the entrant suppliers – an act of verifying the supplier's ability to perform within a reasonable degree of certainty; only the *qualified* supplier, i.e., either the incumbent supplier (because he had already undergone the qualification screening prior to becoming the incumbent) or an entrant supplier who passes the qualification screening, with the most competitive price, wins the contract. This integration leads to an important strategic decision by the buyer in designing the re-sourcing process: should the buyer deploy supplier qualification screenings *before* or *after* the bidding? The conventional approach in practice is the *pre-qualification* strategy (PRE), in which a buyer would first conduct qualification screenings of entrant suppliers, and then only allow *qualified* suppliers to participate in the competitive bidding for the contract. However, qualification screening is a costly process for the buyer because it involves visits to the supplier's facility, purchasing sample products, testing products, etc (Beil 2010). Therefore, PRE may introduce unnecessary qualification expenses on suppliers who turn out to be not competitive. Motivated by this consideration, an alternative proposed strategy is the so-called *post-qualification* strategy (POST), in which qualification screenings are postponed until after the bidding process. Informed by the suppliers' pricing bids, the buyer could choose to vet the capabilities only of suppliers who submitted competitive prices. Hence, compared to PRE, POST may reduce the buyer's expense on qualification screening; but the trade-off is that the resulting equilibrium bid prices will be higher, because suppliers are less inclined to bid aggressively against competitors who have not been qualified (i.e., a supplier can win the contract without placing the lowest bid, as long as all of the lower-bid suppliers fail qualification screenings).

The aforementioned main trade-off between PRE and POST has already been identified in various auction settings (e.g., Wan and Beil (2009), Wan et al. (2012)), but there are some obstacles to applying this managerial insight in practice. For example, Wan and Beil (2009) provide an elegant characterization of the optimal mechanism in the procurement context with qualification screenings, which provides clean managerial insights on the trade-off involved in the timing of conducting qualification screenings. However, optimal mechanisms are complex and difficult to explain to practitioners, and are rarely used in practice. In contrast, Wan et al. (2012) study the similar trade-off in the context of open-descending price-based auction (open-bid auction), a type of auction which allows suppliers to see competitors' bids in real-time and respond by adjusting their own bids dynamically. Our paper also focuses on this auction format due to its popularity in the private sector. For example, auto parts manufacturers use open-bid auctions for sourcing highly engineered commodities (Beil and Duenyas 2012); pharmaceutical companies such as GlaxoSmithKline leverage open-bid auctions to source input materials globally (GlaxoSmithKline 2015); open-bid auction is also a key toolkit provided in SAP Ariba, a procurement and sourcing solution that the leading Enterprice Resource Planning software provider SAP offers to help its clients manage procurement. While Wan et al. (2012) focus on a stylized setting with a single incumbent supplier and a single entrant supplier, in practice, in order to increase competition in the re-sourcing process, buyers usually consider *multiple* entrant suppliers to compete for the contract – a setting that cannot be addressed by Wan et al. (2012). In other words, open-bid auctions are highly prevalent in practice and are often a part of procurement toolkits due to their transparency and simplicity in implementation, and yet, their theoretical properties are not well-understood. *The main focus of our paper, therefore, is to gain insights into the trade-offs involved in open-bid auctions with pre- and post-bidding qualification screening. The specific setting we are considering is one with a single incumbent and multiple entrants, and our primary goal is to characterize the factors that affect which strategy (PRE or POST) is better for the buyer, and even more specifically, how much value would adding the POST option to the procurement toolkits provide for the buyer.* (Henceforth, PRE/POST refer specifically to open-bid auction with pre-bidding/post-bidding qualification screenings whose procedures will be formally introduced in Section 2.)

To assess the value of the POST option, the buyer first needs to develop quantitative models and tools to estimate the expected total procurement cost (qualification screening cost plus contract payment) of both PRE and POST. While PRE is relatively straightforward, the main challenge is the analysis of POST. Specifically, because an entrant supplier may not need to be the lowest bidder to win the contract, it is no longer a dominant strategy for suppliers to drop out at their costs as they would in a standard open-bid auction. This feature makes the open-bid auction with post-bidding qualification screenings a complex dynamic game where suppliers' equilibrium bidding strategies depend on a multi-dimensional state which dynamically evolves as the auction proceeds (e.g., the current auction price, the number of remaining suppliers and their qualification status, and the posterior on the remaining suppliers' cost, etc.). Thus, solving the equilibrium bidding strategy using standard approach (which requires solving several high-dimensional stochastic dynamic programs iteratively until the optimal dynamic policies in all dynamic programs converge) and numerically computing the expected cost of POST, is not computationally tractable. To overcome this challenge, we identify several structural properties of suppliers' equilibrium bidding strategies, and these properties allow us to speed up the numerical computation. Additionally, this equilibrium analysis also provides rich insights on the incentive implications of POST. Therefore, one of our main analytical results deal with determining equilibrium bidding strategies for the entrant suppliers and for the incumbent supplier in POST.

The first analytical contribution of this paper is a full characterization of the suppliers' equilibrium bidding strategy in POST (i.e., Lemma 2 and Theorems 1 and 2). This characterization is very useful because it highlights the different bidding incentives between the entrant suppliers and the incumbent supplier, and provides insights on the economic drivers in POST. It also significantly simplifies the computation of the expected cost of POST, which allows a convenient and efficient comparison between the expected cost of PRE and POST. Hence, the buyer can choose the cheaper option between PRE and POST to implement in her re-sourcing process; this is our proposed approach to the buyer. The second contribution of our work is that we establish analytical conditions under which the buyer should choose POST over the conventional approach PRE: When entrants' probability of passing qualification screening is sufficiently high, when the qualification screening cost is sufficiently high, or when the number of available entrants is sufficiently large (i.e., Proposition 4). These conditions correspond to scenarios where either the benefit of POST's more informed qualification screenings is most pronounced, or the drawback of suppliers' less aggressive bidding behavior in POST is most subdued. Furthermore, using an extensive numerical study, we provide evidence that including POST as a re-sourcing option can provide significant cost-savings compared to the conventional approach of only using PRE, and identify settings in which the benefit of POST is most salient. It is worth noting that our proposed approach (i.e., the cheaper option between PRE and POST) is easy to implement in practice since it uses existing features of procurement auctions which practitioners are already familiar with and combines them in an innovative way. While appealing from a practical standpoint, it naturally raises the question of whether a more sophisticated approach can provide significant benefit over our proposed approach. To that end, we numerically compare our approach with the optimal mechanism (i.e. Proposition 5) which provides a lower bound on the expected cost of any feasible mechanism but is difficult to implement in practice. Our result shows that our proposed approach captures the majority of the cost-savings the optimal mechanism offers compared to the conventional PRE, which highlights the effectiveness of our simple approach.

### 1.1. Related Literature

Our paper is related to the literature in procurement and strategic sourcing (see Elmaghraby (2000) for a comprehensive survey and Beil (2010) for a comprehensive introduction about the procurement process). As pointed out earlier, one of the main challenges of procurement management is to address supply risks in pricing and quality. To deal with these risks, prior research has explored different approaches such as multi-sourcing (e.g., Chaturvedi et al. (2019)), total-cost auctions (e.g., Aral et al. (2019), Stoll and Zöttl (2017)), new supplier recruitment (e.g., Beil et al. (2018)), etc. We focus on the use of supplier qualification screening as a way to control supplier non-performance risks. There is an important stream of work that focuses on this approach. For example, Gillen et al. (2017) study, in a forward auction setting, whether the auctioneer should demand certification of bidders' qualification before or after a second-price sealed-bid auction. In their model, they assume all bidders will pass the qualification for sure at a cost. In our paper, however, the suppliers face the risk of failing the qualification screenings. The paper closest to ours is Wan et al. (2012), in which the authors consider a setting where a qualified incumbent supplier and a *single* not-yet-qualified entrant supplier compete in an open-bid auction for a single-supplier contract from the buyer, and the buyer faces the choice of conducting qualification screening of the entrant before or after the auction. They reveal an important managerial insight to the buyer when post-qualification strategy is used in their setting: The incumbent supplier will inflate the bid, but the entrant supplier will bid down to cost. Our paper captures a more general setting where there are more than one entrant competing with the incumbent for the contract; this generalization captures a more complex scenario that could never happen in Wan et al. (2012): After the incumbent drops out, multiple entrants may remain in the auction. This complication results in richer insights on the impact of post-qualification on suppliers' bidding incentives. For example, we show that the entrant suppliers' bidding strategy is much more nuanced than that characterized in Wan et al.

(2012): The entrants will bid to their costs before the incumbent drops out, but after the incumbent drops out they will inflate their bids above their costs.

There is also another stream of work that employs the optimal mechanism design approach (Myerson 1981) to incorporate supplier qualification screening to the procurement process (e.g., Wan and Beil (2009), Chen et al. (2018), Chaturvedi et al. (2014), Chaturvedi and Martínez-de Albéniz (2011)), etc). While these papers optimally integrate supplier qualification screening with supplier selection in different settings to minimize expected procurement cost, the draw-back is that optimal mechanisms may be difficult to explain to practitioners and hard to implement in practice (Roughgarden and Talgam-Cohen 2019, Rothkopf 2007). We focus on the open-bid auction approach, an important and widely used auction format in practice, and provide insights that are very relevant to practitioners. Having said that, an optimal mechanism analysis of the setting we study provides a lower bound of the procurement cost under any feasible mechanism. Thus, in our paper, we also conduct an optimal mechanism analysis to facilitate a numerical evaluation of the effectiveness of our proposed open-bid auction approach.

### 1.2. Organization of the Paper

In the remainder of the paper, we first introduce the notation and the operational details of PRE and POST, and present the analysis of PRE in Section 2, while the full analysis of POST is postponed in Section 3. Then Section 4 investigates the buyer's optimal choice between PRE and POST. Finally, we assess the benefit of POST numerically in Section 5 and concludes in Section 6.

## 2. Model

Consider a buyer who would like to renew a single-supplier contract. The buyer has an *incumbent* supplier, denoted by 0, who currently charges the buyer $\overline{R}$ for the contract. The incumbent's true cost, denoted by $x_0$, is a random variable with a cumulative distribution function (c.d.f.) $F_0$, a probability density function (p.d.f.) $f_0$, and support $[\underline{R}, \overline{R}]$. The buyer has already identified $N$ ($N \geq 2$) potential suppliers in the market, the *entrants*, and would like to invite them to join the contract competition. These entrants are indexed by $j = 1, 2, \cdots, N$. The entrants' costs $x_j$s are independently and identically distributed with a c.d.f. $F_e$, a p.d.f. $f_e$, and support $[\underline{R}, \overline{R}]$. All suppliers' costs are private knowledge, but their cost distributions are common knowledge. We make the following assumption on entrants' cost.

Assumption 1. *$\frac{F_e}{f_e}$ is nondecreasing and convex.*

Note that most of the log-concave distributions, such as uniform, normal, logistic, exponential, and Weibull distributions, satisfy this condition (see Table 5 of Bagnoli and Bergstrom (2005) for a more detailed list of these log-concave distributions).

The incumbent and an entrant are ex ante asymmetric not only due to their asymmetric cost distributions, but also due to their asymmetric *qualification status*. The incumbent supplier is qualified because he has been supplying the part to the buyer and his technical capabilities have already been validated by the buyer. In contrast, an entrant has not worked with the buyer before and needs to be carefully vetted via a qualification screening process before he can be awarded the contract. The qualification screening process may be lengthy and costly, requiring the buyer to visit the supplier's facility, purchase sample products, test products, etc. Additionally, there is no guarantee that an entrant will always pass the qualification. Following the supplier qualification literature (e.g., Wan et al. (2012)), we model qualification screening cost and uncertainty as follows: It costs the buyer $K$ to conduct the qualification screening on each entrant; if an entrant undergoes qualification screening, he will pass the screening requirements and become qualified with probability $\beta \in (0,1)$, where we assume $\beta$ to be common knowledge. In other words, we consider a setting where the *entrant* suppliers are *ex ante* symmetric, which is mostly appropriate when the buyer's prior belief about those entrant suppliers are similar, and the cost drivers for buyer's qualification screenings are similar. Focusing on this setting also allows us to provide a clean way to highlight the different natures of bidding incentives between the incumbent supplier and the entrant suppliers. We also assume that if an entrant undergoes qualification screening, both the buyer and this entrant observe whether or not this entrant passes the screening.

The buyer's goal is to select a qualified supplier (i.e., either an entrant who passes the qualification or the incumbent) to minimize her expected total procurement cost, i.e., contract payment plus qualification cost.

### 2.1. Pre-Qualification Strategy

In the *pre-qualification strategy* (PRE), the buyer first chooses $N_{pre} \in [0,N]$ and randomly selects $N_{pre}$ out of the $N$ entrants (note that since entrants are ex ante symmetric, it is equivalent to select the entrants 1 to $N_{pre}$) to conduct qualification screenings simultaneously. Then, only the entrants who pass the qualification screenings *and* the incumbent are allowed to compete for the contract in a standard open-bid descending auction which proceeds as follows: The auction price is initially set at the current contract price $\overline{R}$ and then falls continuously until the auction ends when all but one bidder drops out; the last remaining bidder wins the auction and is paid the auction ending price. (Auctions with a continuously falling price are also known as "reverse clock auctions"; see Ausubel and Cramton (2006) for discussions about clock auctions in practice.) Note that, if none of the invited entrants pass the qualification, the incumbent will win the auction directly at price $\overline{R}$; otherwise, since all participants are qualified suppliers, in equilibrium, all of them will stay in the auction until the auction price reaches their true costs (Krishna 2002). Hence, the expected total procurement cost (qualification cost plus contract payment) when the buyer conducts $N_{pre}$ pre-bidding qualification screenings equals

$$\mathcal{PC}_{pre}(N_{pre}) = \sum_{i=0}^{N_{pre}} \mathbf{E}\left[\mathbf{Min2}(\overline{R}, x_0, \ldots, x_i)\right] \frac{N_{pre}!}{i!(N_{pre}-i)!} \beta^i (1-\beta)^{N_{pre}-i} + N_{pre}K, \tag{1}$$

where $\mathbf{Min2}(\ldots)$ corresponds to the second lowest value of its arguments. The buyer chooses the optimal number of entrants for qualification screening, $N^*_{pre}$, to minimize her expected cost under PRE. Hence, the buyer's optimal expected cost under PRE, denoted by $PC_{pre}$, equals:

$$PC_{pre} = \min_{N_{pre} \in \{1,\ldots,N\}} \mathcal{PC}_{pre}(N_{pre}) = \mathcal{PC}_{pre}(N^*_{pre}). \tag{2}$$

## 2.2. Post-Qualification Strategy

An alternative approach to structure the supplier qualification screening and supplier selection is the *post-qualification strategy* (POST) where qualification screenings are conducted post-bidding rather than pre-bidding: The buyer invites *all* suppliers to bid in a (slightly modified) open-bid descending auction (see more details below) before conducting qualification screenings and awarding the contract to one of the qualified suppliers. Specifically, after all suppliers decide whether or not to join the auction, the buyer announces to all participating suppliers which supplier is the incumbent and a fixed price decrement $\Delta := K/\beta$ when incumbent drops out (see more details of this decrement later). Then the auction begins as its descending price clock starts at the current contract price $\overline{R}$ and continuously descends. In this process, all suppliers choose to drop out of the auction or stay at the prevailing price. In contrast to a standard open-bid descending auction as in PRE, when the *incumbent* drops out, the auction price immediately jumps down by $\Delta$ before keeping on continuously descending. The auction ends when there is only one supplier left in the auction, and the *ending auction price* equals the last drop-out price bid (drop-out price for short) if the last drop-out is an entrant, and equals the last drop-out price minus $\Delta$ if the last drop-out is the incumbent (this is because the auction price immediately jumps down by $\Delta$ after the incumbent drops out). During this bidding process, suppliers observe the following: Before the auction starts, the number of suppliers in the auction, which one is the incumbent, and $\Delta$ are all common knowledge; as the auction proceeds and suppliers drop out one after another, each supplier observes the auction price and when other suppliers drop out in real time until he himself drops out (this means that he can perfectly infer the drop-out prices of all suppliers who drop out earlier than him). Note that it is possible that when the auction concludes, the only remaining supplier is an entrant who has not yet been qualified. Hence, while this supplier becomes the *auction winner*, he does *not* automatically become the contract winner. This is where the post-bidding qualification screenings take place: After the auction ends, informed by the suppliers' drop-out prices, the buyer decides which suppliers (if any) and in what sequence to conduct qualification screenings. At the conclusion of the qualification screenings, the contract is awarded to a *qualified* supplier (either the incumbent or an entrant who passes the qualification screening) who has the lowest drop-out price. The contract payment equals the ending auction price if the contract winner is the auction winner; otherwise, the contract payment equals the contract winner's drop-out price.

Given the format of POST, the buyer's (ex post) optimal qualification screening decisions are characterized in the following lemma. (All our proofs can be found in the Online Appendix EC.1.)

LEMMA 1. *In POST, it is ex post optimal for the buyer to keep conducting qualification screening in the reverse order of when the suppliers dropped out, until either the first entrant passes the qualification screening, or all entrants who drop out later than the incumbent fail to pass qualification screenings.*

The buyer's optimal post-auction qualification screening rule is very intuitive: The buyer starts with the supplier with the lowest bid and keeps on conducting qualification screening in the sequence from the low bid to high until she finds the first qualified supplier (either an entrant who passes the qualification screening, or the incumbent). Note that the observation that this screening rule is ex post optimal is driven by the design that the auction price immediately jumps down by $\Delta = K/\beta$ when the incumbent drops out. Indeed, if the incumbent drops out at $p$, this design feature ensures that the drop-out price bids of entrants who drop out later are at least $K/\beta$ lower, which makes it optimal to conduct qualification screening of any of these entrants before awarding the contract directly to the incumbent since the expected benefit (i.e., the reduction in payment when an entrant passes qualification) equals $\beta \times (p - \textit{entrant's bid}) \geq \beta K/\beta = K$, which is larger than the cost of qualification.

Having explained above how POST is operationalized, we now explain an appealing feature of POST which motivates us to propose this format in the first place: Practical applicability. First of all, POST is quite straightforward to deploy mechanically because, compared to the standard open-bid descending auction which is widely adopted in practice, operationally, POST only requires the buyer to ensure that the descending price clock jumps down by $\Delta$ immediately after the incumbent drops out (e.g., if the incumbent drops out at some auction price $p$, then the price clock immediately drops down to $p - \Delta$). Note that this feature is not an arcane concept because one can interpret $p - \Delta$ as a "reserve price" to account for the extra expenditure on verifying entrants' qualification status. Secondly, the payment in POST is very transparent. The final contract winner always pays what he bids except in one scenario when he wins the auction and passes the qualification screening. But even in that scenario, the payment is still very transparent since the contract winner pays the auction ending price (note that this is the same payment rule as in a standard open-bid descending auction widely used in practice). Finally, note that the qualification screening rule the buyer uses in POST is ex post optimal. Thus, the buyer does *not* need to commit to a particular qualification screening rule to the suppliers ex ante which would cause implementation issues as the qualification screenings are not directly observable to all suppliers.

While POST is a practical sourcing strategy, the expected cost of POST may be either higher or lower than PRE depending on the business context. In order to determine the best strategy to use, the buyer needs to compute the expected cost of both options and then choose the less expensive one. While the expected procurement cost for PRE can be easily established in (1) and (2), calculating the expected procurement cost under POST is challenging. We are not aware of any existing literature that provides numerical methods to calculate the expected procurement cost under POST. What makes POST much more complex to analyze than PRE is that suppliers' equilibrium bidding behavior in POST is much more nuanced: The fact that not all auction participants are qualified suppliers means that a supplier does not necessarily need to win the auction in order to win the contract. For example, when an entrant supplier $j$ drops out, if all later drop-out suppliers are entrants and they all fail the qualification screening, then supplier $j$ can still win the contract if he passes qualification screening. Therefore, supplier $j$ may have the incentive to drop out at a price higher than his true cost, which is different from the equilibrium bidding behavior one would expect in a standard open-bid descending auction. As one can imagine, the extent to which supplier $j$ inflates his drop-out price above his cost may depend on all the historic information he observes (e.g., the current auction price, the number of remaining suppliers, and his posterior on the remaining suppliers' cost distributions). Thus, POST induces a complex stochastic dynamic bidding game with ex ante asymmetric bidders. This raises several interesting managerial questions for POST. How should a supplier decide when to drop out based on the information he observes in the bidding process? Would the incumbent and the entrants' equilibrium drop-out strategies be different, and if so, how? How would the nature of qualification screening (e.g., the cost of qualification screening and the entrants' probability to pass the qualification screening) affect suppliers' equilibrium strategies? Finally, returning to our main research question, how to compute the expected procurement cost of POST so that the buyer can compare it with the expected cost of PRE and make a more informed decision on the choice between PRE and POST? We address these questions in the next two sections.

## 3. Equilibrium Analysis of POST

In this section, we characterize suppliers' equilibrium drop-out strategies in POST, and then use this characterization to derive the buyer's expected procurement cost under POST. Recall that

POST induces a stochastic dynamic game with incomplete information and ex ante asymmetric players. Here, the suppliers have incomplete information because they only know their own true costs but not others'. We use Perfect Bayesian Equilibrium (PBE) as our solution concept. Specifically in the context of POST, all suppliers start with a *common prior* on other suppliers' cost distribution. As the auction proceeds, each supplier observes the descending auction price and when other suppliers drop out until he drops out; this forms a sequence of (weaking) increasing information sets as auction proceeds. A drop-out strategy is a collection of functions that map the supplier's information sets to an auction price at which the supplier should drop out. To ensure *consistency* condition of PBE, for any drop-out strategy profiles of all suppliers, we use Bayes rule to construct the posterior beliefs given the common prior (before the bidding starts) and all suppliers' strategies. Then we say a collection of drop-out strategy profiles along with the beliefs constructed above for this collection of drop-out strategy profiles form a PBE if the strategies are *sequentially rational.* Since the entrants are ex ante symmetric, we focus on the PBE where entrants' equilibrium strategies are symmetric. In the remainder of this section, we first characterize the entrants' and the incumbent's equilibrium drop-out strategies in POST (Sections 3.1 and 3.2), which are then used to characterize the buyer's expected total cost under POST (Section 3.3).

### 3.1. Entrants' Drop-out Strategy

Since the incumbent is a qualified supplier, an entrant faces very different competition before and after the incumbent drops out. Thus, we characterize entrants' equilibrium drop-out strategy *before* and *after* the incumbent drops out separately.

**3.1.1. Entrants' Bidding Before Incumbent Drops Out.** By Lemma 1, an entrant can win the contract only if he drops out later than the incumbent. However, in POST, the auction price immediately drops by $\Delta = \frac{K}{\beta}$ when the incumbent drops out; therefore, to avoid the possibility of winning the contract but losing money, as long as the incumbent is still in the auction, an entrant supplier $j$ should drop out no later than when the auction price reaches his *effective cost* defined as $x_j + \frac{K}{\beta}$. (We assume that $\frac{K}{\beta} < \overline{R} - \underline{R}$ to avoid the trivial situation that no entrant would like to join the auction because all *effective costs* are higher than the auction starting price.) By a similar argument as in the analysis for a standard open-bid descending auction, Lemma 2 establishes the entrants' dominant bidding strategy before the incumbent drops out.

Lemma 2. Before *the incumbent drops out, it is a weakly dominant strategy for the entrants to stay in the auction until the auction price clock reaches their* effective costs.

**3.1.2. Entrants' Bidding After Incumbent Drops Out.** After the incumbent drops out, while the remaining bidding process only involves ex ante symmetric suppliers (i.e., the remaining entrants), their equilibrium drop-out strategy is more complicated than a standard open-bid descending auction because any of the remaining entrant may lose an auction to other remaining entrants but still win the contract if all later drop-out entrants fail to pass the qualification screening; thus, an entrant has an incentive to drop out at a price higher than his cost. For each of the remaining entrant, at what price to drop out depends on not only his true cost, but also how many competing entrants are still in the auction and his posterior on these entrants' costs; of course, the price to drop out should also be dynamically updated as other competing entrants drop out over time. In the remainder of this subsection, *given the incumbent's drop-out price and the number of remaining entrants*, we characterize the symmetric equilibrium (dynamic) bidding strategy for the remaining entrant suppliers after the incumbent drops out.

Formally, consider the scenario where there are $n \in \{1, \cdots, N\}$ remaining entrants after the incumbent drops out. Let $p^n_{(n)}$ denote the auction price right after the incumbent drops out: $p^n_{(n)}$ equals the incumbent's drop-out price minus $\Delta = \frac{K}{\beta}$. Moreover, for all $k = 1, \cdots, n-1$, let $p^n_{(k)}$ denote the $(n-k)^{th}$ drop-out price among the remaining $n$ entrants, and $c^n_{(k)}$ denote the cost of the $(n-k)^{th}$ drop-out entrant. Suppose now that there are only $k$ entrants in the auction, and entrant $j$ is one of them. At this point, the price $b_j$ at which entrant $j$ should drop out should depend on all the pay-off relevant information that he observes so far, which includes not only his cost $x_j$ and the number of remaining entrants $k$, but also his posterior of the remaining entrants' costs; apparently, this posterior is determined by the previous drop-out prices, the current auction price, as well as entrant $j$'s belief of other entrants' drop-out strategy. Below, we first introduce a class of *Active-Bidder-Number Dependent Threshold strategy* (ABN) and then construct an ABN which, along with the remaining entrants' posteriors when the incumbent drops out, forms a symmetric perfect Bayesian Equilibrium in the bidding process after the incumbent drops out.

Definition 1. An ABN is determined by a sequence of $n-1$ functions $\{\widetilde{s}_k(x_j; \overline{c}, \overline{p})\}_{k=2}^n$, where $\widetilde{s}_k$ is strictly increasing and differentiable with respect to $x_j$ and we denote by $\widetilde{s}_k^{-1}(\cdot; \overline{c}, \overline{p})$ its inverse function. For an entrant with cost $x_j$, the ABN specified by $\{\widetilde{s}_k\}_{k=2}^n$ works as follows:

*Set* $k = n$, $\overline{p} = p^n_{(n)}$, and $\overline{c} = p^n_{(n)}$
**if** $k = 1$ **then** *terminate*
**else**
    *compute* a threshold $b_j = \widetilde{s}_k(x_j; \overline{c}, \overline{p})$
    **if** one of the other entrants drops out at an auction price $p^n_{(k-1)} > b_j$ **then**
        *set* $\overline{c} = c^n_{(k-1)} := \widetilde{s}_k^{-1}(p^n_{(k-1)}; \overline{c}, \overline{p})$ and *set* $\overline{p} = p^n_{(k-1)}$
        *set* $k = k-1$, and *go* to 2
    **else if** no entrant drops out before the auction price hits $b_j$ **then**
        *drop* out at $b_j$, and *terminate*
    **end if**
**end if**

The function $\widetilde{s}_k(x_j; \overline{c}, \overline{p})$ maps an entrant's cost type $x_j$ into a drop-out threshold $b_j$, given the number of remaining entrants $k$, the most recent drop-out price $\overline{p}$, and an estimate of the upper bound of other remaining entrants' costs $\overline{c}$. Note that under the supposition that all $n$ entrant suppliers who drop out after the incumbent use the ABN characterized by the same set of functions $\{\tilde{s}_k\}_{k=2}^n$, $\overline{c}$ determined in ABN is consistent with entrants' posterior on other remaining entrants' cost distribution. To see that, recall that by definition, $\widetilde{s}_k$ is increasing in $x_j$; this implies that entrants with higher cost types drop out first. Therefore, when the highest cost entrant among $k$ (for any $2 \leq k \leq n$) remaining entrants drops out at $p^n_{(k-1)}$, all $k-1$ remaining entrants would correctly deduce that the cost of the entrant who just dropped out equals $c^n_{(k-1)} = \widetilde{s}_k^{-1}(p^n_{(k-1)}; \overline{c}, \overline{p})$, and it coincides with the posterior of the upper bound of the support of the $k-1$ remaining entrants' cost distribution. Similarly, when the incumbent drops out with an auction price of $p^n_{(n)}$(recall that it equals the incumbent's drop-out price minus $\Delta$), the upper bound of the support of the $n$ remaining entrants' cost distribution equals $p^n_{(n)}$ as is implied by Lemma 2.

Having explained above that all $n$ remaining entrants using the same ABN ensures the *consistency* condition for PBE, we now construct the ABN that ensures *sequential rationality* condition given entrants' beliefs. Toward that end, recall that we are considering a scenario where there are exactly $n$ remaining entrants when the incumbent drops out, and the auction price (right after a price decrement of $\Delta$ from the incumbent's drop-out price) is $p^n_{(n)}$. Our goal is to find $\{\widetilde{s}_k\}_{k=2}^n$ such that, given that all other remaining entrants employ the ABN specified by $\{\widetilde{s}_k\}_{k=2}^n$, entrant $j$'s best response coincides with the ABN specified by $\{\widetilde{s}_k\}_{k=2}^n$, thus ensuring sequential rationality. Note that entrant $j$'s optimal drop-out problem can be formulated as a stochastic dynamic program. Specifically, denote by $G^k_e(z; \overline{c}) := [F_e(z)/F_e(\overline{c})]^k$ and $g^k_e(z; \overline{c}) := k[F_e(z)]^{k-1} f_e(z)/[F_e(\overline{c})]^k$, respectively, the c.d.f. and p.d.f. of the highest cost out of the $k$ remaining entrants' costs when their costs' upper bound is $\overline{c}$ , and denote by $V^k_e(x_j, p; \overline{c}, \overline{p})$ the maximum profit entrant $j$ can make when there are $k$ active entrants in the auction where the current auction price is $p$, the most recent drop-out occurs when the auction price hits $\overline{p}$, and that the latest drop-out supplier's cost is $\overline{c}$ (recall that assuming other entrants follow the ABN, entrant $j$ can correctly infer that $\overline{c}=\widetilde{s}_{k+1}^{-1}(p_{(k)}^n;\overline{c},\overline{p})$ when $k\leq n-1$ and $\overline{c}=p_{(n)}^n$ when $k=n$). Then, entrant $j$'s optimal drop-out strategy can be obtained by solving the Bellman equations below: For all $k=2,\ldots,n$,

$$V_e^k\left(x_j,p;\overline{c},\overline{p}\right)=\max_{b_j\in\left[x_j,p\right]}\left[\underbrace{\int_{\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right)}^{\widetilde{s}_k^{-1}(p;\overline{c},\overline{p})}V_e^{k-1}\left(x_j,\widetilde{s}_k\left(z;\overline{c},\overline{p}\right);z,\widetilde{s}_k\left(z;\overline{c},\overline{p}\right)\right)g_e^{k-1}\left(z;\widetilde{s}_k^{-1}\left(p;\overline{c},\overline{p}\right)\right)dz}_{\text{when an entrant drops out before entrant } j \text{ (i.e., before auction price reaches } b_j)}\right.$$
$$\left.+\underbrace{\beta G_e^{k-1}\left(\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right);\widetilde{s}_k^{-1}\left(p;\overline{c},\overline{p}\right)\right)(1-\beta)^{k-1}\left(b_j-x_j\right)}_{\text{when no other entrant drops out before entrant } j}\right],\tag{3}$$

$$\text{and}\quad V_e^1\left(x_j,p;\overline{c},\overline{p}\right)=\beta\left(p-x_j\right).\tag{4}$$

Theorem 1 below establishes, by construction, that an ABN forms a pure strategy symmetric PBE for the dynamic bidding game which ensues if the incumbent drops out at $p_{(n)}^n+\Delta$ and with $n$ remaining entrants.

THEOREM 1. *For any $p\in[\underline{R}+\Delta,\overline{R}]$, and any $n=1,\ldots,N$, suppose that the incumbent drops out at price $p$ with $n$ remaining entrants still in the auction, and that it is common knowledge that all remaining entrants' costs are no higher than $p_{(n)}^n$ where $p_{(n)}^n:=p-\Delta$, then the following holds:*

*(i) The ABN specified by a sequence of $n-1$ functions $\{\widetilde{s}_k\}_{k=2}^n$ defined below along with the belief system that is consistent with ABN and the prior that all $n$ entrants' costs are below $p_{(n)}^n$ forms a symmetric PBE: For all $k=2,\ldots,n$, $x_j\in[\underline{R},\overline{c}]$, $\underline{R}\leq\overline{c}\leq\overline{p}\leq\overline{R}$,*

$$\frac{\partial\widetilde{s}_k\left(x_j;\overline{c},\overline{p}\right)}{\partial x_j}=\frac{\beta(k-1)f_e(x_j)\left(\widetilde{s}_k\left(x_j;\overline{c},\overline{p}\right)-x_j\right)}{(1-\beta)F_e(x_j)},\tag{5}$$

*with the boundary condition*

$$\widetilde{s}_k\left(\overline{c};\overline{c},\overline{p}\right)=\overline{p}.\tag{6}$$

*(ii) $\widetilde{s}_k(x_j;\overline{c},\overline{p})$ decreases in $k$ for all $x_j\in[\underline{R},\overline{c})$, decreases in $\overline{c}$, and increases in $x_j$ and $\overline{p}$.*

*(iii) $\widetilde{s}_k(x_j;\overline{c},\overline{p})>x_j$ for all $x_j\in[\underline{R},\overline{c})$.*

*(iv) If $x_j<c_{(k-1)}^n$, then $\widetilde{s}_k(x_j;c_{(k)}^n,p_{(k)}^n)<\widetilde{s}_{k-1}(x_j;c_{(k-1)}^n,p_{(k-1)}^n)$.*

Several remarks are in order. First, note that after the incumbent drops out, by Lemma 2 and the Bayes rule, all remaining entrants infer that their competitors' costs are no higher than $p_{(n)}^n$; thus, Lemma 2 and Theorem 1 jointly characterize entrant suppliers' equilibrium bidding strategy. It is worth noting that we do not need to explicitly characterize the incumbent's equilibrium bidding strategy in order to characterize the entrants' equilibrium bidding strategy. This is because entrants have dominant bidding strategy before the incumbent drops out; once the incumbent drops out, the remaining entrant's expected pay-off depends on the incumbent's bidding strategy only through the incumbent's drop-out price. Second, note that Theorem 1 part (iii) implies that after the incumbent drops out, in equilibrium all entrants will drop out at a price higher than their costs; this is because any of the remaining entrants is incentivized to bid less aggressively because he anticipates that his competing entrants who drop out later than him may not always pass qualification screening. Third, recall that $\widetilde{s}_k(x_j; c^n_{(k)}, p^n_{(k)})$ is entrant $j$'s drop-out threshold when there are exactly $k$ entrants in the auction. Thus, Theorem 1 part (iv) implies that after the incumbent drops out, every time an entrant drops out results in an increase of all remaining entrants' drop-out thresholds. In other words, once the incumbent drops out, all remaining entrants start to increasingly hold back their aggressiveness on bidding as more entrants drop out. As a result, the buyer should not safely assume that intense competition would naturally occur in POST simply because more entrants joined the bidding process. Instead, she should take into account the incentive implications of post-bidding qualification screenings on supplier bidding behavior when evaluating what final contract price to expect from POST. Fourthly, we would like to point out that while the entrant suppliers' optimal drop-out problem is a high-dimensional stochastic dynamic program and is computationally difficult to solve in brute force, we show in Theorem 1 part (i) that instead of solving the dynamic program directly, one only needs to solve a set of one-dimensional first-order ordinary differential equations which can be easily computed numerically. This characterization also facilitates a set of comparative statics results summarized in Theorem 1 part (ii). These comparative statics results are not surprising. For example, when there are a higher number of remaining entrants $k$, there is more competition, so the entrants bid more aggressively by having lower drop-out thresholds. Finally, if the entrants' cost distribution is a uniform distribution, the equilibrium drop-out strategy can be derived in closed-form.

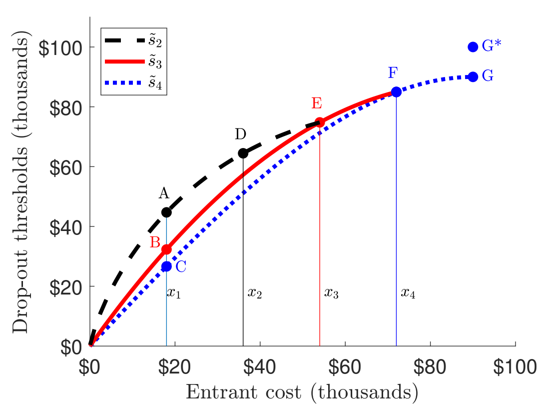
PROPOSITION 1. *When $F_e \sim U[\underline{R}, \overline{R}]$, the ABN in Theorem 1 has a closed-form expression:*

$$\widetilde{s}_k\left(x_j, \overline{c}, \overline{p}\right) = \begin{cases} \frac{\beta(k-1)}{\beta k-1}(x_j - \underline{R}) + \left(\overline{p} - \underline{R} - \frac{\beta(k-1)}{\beta k-1}(\overline{c} - \underline{R})\right)\left(\frac{x_j - \underline{R}}{\overline{c} - \underline{R}}\right)^{\frac{\beta(k-1)}{1-\beta}} + \underline{R}, & \text{if } \beta \neq \frac{1}{k}, \\ (\overline{p} - \underline{R})\left(\frac{x_j - \underline{R}}{\overline{c} - \underline{R}}\right)^{\frac{\beta(k-1)}{1-\beta}} - (x_j - \underline{R})\log\left(\frac{x_j - \underline{R}}{\overline{c} - \underline{R}}\right) + \underline{R}, & \text{if } \beta = \frac{1}{k}. \end{cases} \tag{7}$$

We illustrate below an entrant's equilibrium drop-out strategy after the incumbent drops out.

EXAMPLE 1. Suppose an incumbent competes with four entrants whose costs are uniformly distributed between \$0 and \$100,000. The qualification cost is $K = \$5,000$ and the passing probability is $\beta = 0.5$. In this setting, the buyer sets $\Delta = K/\beta = \$10,000$. Suppose the incumbent drops out first at $p = \$100,000$ and leaves behind four entrants in the auction. We now take the perspective of entrant 1 with cost $x_1 = \$18,000$ to illustrate the entrant's equilibrium drop-out strategy (see Figure 1). As soon as the incumbent drops out (point $G*$ in Figure 1), the auction price immediately drops by $\Delta$ and becomes $\$100,000 - \$5,000/0.5 = \$9,000$ (point $G$ in Figure 1). Then, entrant 1 infers that all other entrants' costs are no higher than $\overline{c} = \$90,000$, and then computes a drop-out threshold according to (7): $\widetilde{s}_4(\$18,000; \$90,000, \$90,000) = \$26,640$ (point $C$ in Figure 1). Suppose that before the auction price reaches entrant 1's threshold, entrant 4 drops out first at $\$84,960$ (point $F$ in Figure 1). Entrant 1 updates his belief about the remaining entrants' cost upper bound to $\overline{c} = \widetilde{s}_4^{-1}(\$84,960; \$90,000, \$90,000) = \$72,000$ (note that this also equals entrant 4's cost), and then re-calculate his drop-out threshold as $\widetilde{s}_3(\$18,000; \$72,000, \$84,960) = \$32,310$ (point $B$ in Figure 1). Suppose that entrant 3 drops out at $\$74,790$ (point $E$ in Figure 1) before the auction price reaches entrant 1's updated drop-out threshold, leaving behind only entrants 1 and 2. This time, the updated cost upper bound becomes $\overline{c} = \widetilde{s}_3^{-1}(\$74,790; \$72,000, \$84,960) = \$54,000$ and entrant 1's drop-out threshold becomes $\widetilde{s}_2(\$18,000; \$54,000, \$84,960) = \$44,710$ (point $A$ in Figure 1). Finally, the other remaining entrant drops out at $\$64,460$ (point $D$ in Figure 1), making entrant 1 the auction winner.

**Figure 1** Illustration of the dynamics of an entrant's equilibrium strategy after the incumbent drops out




Notes: In this example, $F_e$ follows uniform distribution $U[\$0, \$100,000]$, $K = \$5,000$, $\beta = 0.5$. Moreover, the incumbent drops out at $\$100,000$, and the remaining four entrants' costs equal $x_1 = \$18,000$, $x_2 = \$36,000$, $x_3 = \$54,000$, and $x_4 = \$72,000$ respectively. Point $D$ (resp., $E, F, G*$) corresponds to entrant 2's (resp., entrant 3's, entrant 4's, incumbent's) drop-out price. Point $G$ corresponds to the auction price after the incumbent drops out. Point $A$ (resp., $B$, $C$) corresponds to entrant 1's drop-out price when there are 2 (resp., 3, 4) entrants in the auction in this example.

Apparently, the incentive implications on suppliers' drop-out prices also depend on the model parameters predicated on the business context. We provide insight on this in our next result.

Proposition 2. *Fix any $x_j \in [\underline{R}, \overline{c})$, $\overline{c}$, and $\overline{p}$, $\widetilde{s}_k(x_j, \overline{c}, \overline{p})$ decreases in $\beta$ and independent of $K$.*

As the probability of passing qualification $\beta$ increases, there is a higher chance that at least one of the entrants with lower drop-out price would pass qualification and win the contract; thus, all remaining entrants have more incentive to bid more aggressively by lowering their drop-out thresholds. In contrast, since the qualification cost $K$ is the same for all of the remaining competing entrants after the incumbent drops out, it does not affect the competition among these entrants nor their bidding incentives after the incumbent drops out.

### 3.2. Incumbent's Drop-out Strategy

Given the entrants' equilibrium drop-out strategy characterized in the previous subsection, we now analyze the incumbent's equilibrium drop-out strategy. Recall that by Lemma 2, before the incumbent drops out, it is a weakly dominant strategy for entrants to stay in the auction until the auction price reaches their *effective costs*. Therefore, at any auction price $p$, the incumbent infers that all remaining entrants' costs are no higher than $p-\Delta$. Note that once the incumbent drops out, his chance of getting the contract is fully determined by the number of remaining entrants, i.e., the incumbent wins the contract if and only if all remaining entrants fail their qualifications. Therefore, in POST, as the auction price descends, the incumbent needs to decide when to drop out based on the number of the remaining entrants and his belief about the remaining entrants' cost distribution.

Denote by $V_0^m(x_0,p)$ the incumbent's maximum expected profit and denote by $s_m^\star(x_0;p)$ his best response (henceforth, optimal drop-out price) when his cost is $x_0$, the current auction price is $p$, and there are exactly $m$ remaining entrants who follow their equilibrium drop-out strategy characterized in Lemma 2 and Theorem 1. Note that it is without loss of optimality to restrict the set of the incumbent's feasible drop-out price to be $[\max\{\underline{R}+\Delta,x_0\},p]$ because dropping out below $\underline{R}+\Delta$, which equals the lowest possible drop-out price from entrants, is never optimal. Hence, given entrants' equilibrium drop-out strategy, the incumbent's best response can be found by solving the profit optimization problem characterized by the following Bellman equations: For all $m=1,\dots,N$, $x_0\in[\underline{R},\overline{R}]$, $p\in[\max\{\underline{R}+\Delta,x_0\},\infty)$,

$$V_0^m(x_0,p) = \max_{b\in[\max\{\underline{R}+\Delta,x_0\},p]} \Pi_0^m(b;x_0,p) = \Pi_0^m(s_m^\star(x_0;p);x_0,p), \tag{8}$$

where for all $m=1,\dots,N$, $x_0\in[\underline{R},\overline{R}]$, $p\in[\max\{\underline{R}+\Delta,x_0\},\infty)$, $b\in[\max\{\underline{R}+\Delta,x_0\},\infty)$,

$$\Pi_0^m(b;x_0,p) := \int_b^p V_0^{m-1}(x_0,z)g_e^m(z-\Delta;p-\Delta)\,dz + G_e^m(b-\Delta;p-\Delta)(1-\beta)^m(b-x_0), \tag{9}$$

and the boundary conditions are: For all $x_0\in[\underline{R},\overline{R}]$, $p\in[\max\{\underline{R}+\Delta,x_0\},\infty)$,

$$V_0^0(x_0,p) = p-x_0. \tag{10}$$

Theorem 2 below fully characterizes the incumbent's equilibrium drop-out strategy.

Theorem 2. *For all $x_0 \in [\underline{R}, \overline{R}]$, define*

$$m^\star(x_0) := \left\lfloor \min_{b \in \left[\max\{x_0, \underline{R}+\Delta\}, \overline{R}+\Delta\right]} \frac{(1-\beta)F_e(b-\Delta)}{\beta(b-x_0)f_e(b-\Delta)} \right\rfloor;$$

*moreover, for all $x_0 \in [\underline{R}, \overline{R}]$ and $m > m^\star(x_0)$, define*

$$b_m^\star(x_0) := \begin{cases} \underline{R}+\Delta, & \text{if } x_0 \le \underline{R}+\Delta \\ \inf\{b \in \left(x_0, \overline{R}+\Delta\right] : \frac{F_e(b-\Delta)}{f_e(b-\Delta)} - \frac{m\beta(b-x_0)}{1-\beta} < 0\}, & \text{if } x_0 > \underline{R}+\Delta \end{cases},$$
$$p_m^\star(x_0) := \min\{b \in (b_m^\star(x_0), \infty] : \Pi_0^m(b; x_0, \overline{R}+\Delta) - \Pi_0^m(b_m^\star(x_0); x_0, \overline{R}+\Delta) \ge 0\}.$$

*Suppose that the entrants follow the drop-out strategy characterized in Lemma 2 and Theorem 1. Then the following statements hold:*

*(i). $m^\star(x_0), b_m^\star(x_0), p_m^\star(x_0)$ are well-defined, $p_m^\star(x_0) > b_m^\star(x_0) \ge x_0$, $b_m^\star(x_0) > x_0$ when $x_0 \ne \underline{R}+\Delta$, $m^\star(x_0)$ is nondecreasing in $x_0$, $b_m^\star(x_0)$ is nondecreasing in $x_0$ and nonincreasing in $m$, and $p_m^\star(x_0)$ is nonincreasing in $x_0$ and nondecreasing in $m$;*

*(ii). Given the incumbent's cost $x_0$, the number of remaining entrants $m$ and the current auction price $p$, the incumbent's optimal drop-out price is*

$$s_m^\star(x_0; p) = \begin{cases} b_m^\star(x_0) \wedge p, & \text{if } m > m^\star(x_0),\ p \le p_m^\star(x_0) \\ p, & \text{otherwise} \end{cases};$$

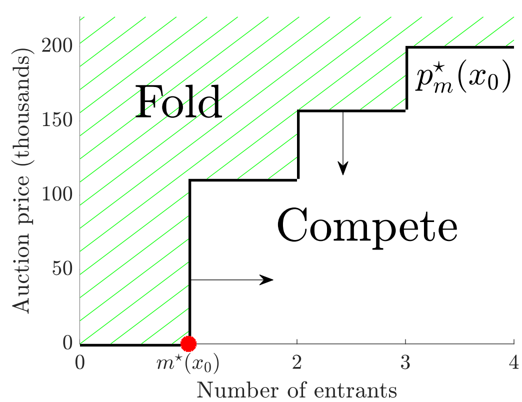
*moreover, $s_m^\star(x_0; p)$ is nondecreasing in $x_0$ and $p$ and nonincreasing in $m$.*

Before explaining the managerial insights revealed in Theorem 2, we would first like to point out that Lemma 2, Theorem 1 and Theorem 2 jointly establish the suppliers' equilibrium drop-out strategies in POST. Indeed, if the entrants follow the symmetric strategy characterzied in Lemma 2 and Theorem 1, Theorem 2 characterizes the incumbent supplier's best response. Conversely, if the incumbent follows the strategy characterized in Theorem 2, Lemma 2 and Theorem 1 characterize the entrants best response: It is a weakly dominant strategy for entrants to follow Lemma 2 before the incumbent drops out; after the incumbent drops out, by Lemma 2 and the Bayes rule, all remaining entrants infer the common posterior that all the remaining entrants' costs are $\Delta$ lower than the incumbent's drop-out price, so the conditions in Theorem 1 are satisfied.

Theorem 2 reveals that when there are multiple entrants, the incumbent's equilibrium drop-out strategy is a state-dependent dynamic strategy characterized by a threshold function $b_m^\star(x_0)$ and two switching curves $p_m^\star(x_0)$ and $m^\star(x_0)$. Specifically, Theorem 2 part (ii) shows that, given the current state of the auction, i.e., the number of remaining entrants $m$ and the current auction price $p$, an incumbent with cost $x_0$ should follow either of the two options: The "Compete" option where the incumbent remains in the auction until the first time the auction price drops below a threshold $b_m^\star(x_0)$, or the "Fold" option where the incumbent immediately drops out at current auction price

$p$ and forfeits the chance of winning the auction. Figure 2 illustrates the incumbent's equilibrium drop-out strategy. The threshold-type strategy in the "Compete" option is not unusual in open-bid auctions; however, when would giving up winning the auction, the "Fold" option, be helpful to the incumbent? Note that with the "Fold" option, the incumbent can still win the contract if the remaining entrants all fail supplier qualification screenings. Thus, the "Fold" option can be optimal when there is a sufficient chance that all $m$ entrants fail qualifications (i.e., when the number of remaining entrants is small, $m \leq m^\star(x_0)$). Moreover, the "Fold" option is optimal when there are sufficient number of remaining entrant suppliers *and* the current auction price is high, i.e., $m > m^\star(x_0)$ and $p \geq p^\star_m(x_0)$. In this case, dropping out at a slightly lower price than $p$ does not significantly reduce the number of entrants who remain in the auction after the incumbent drops out, so the incumbent's chance of ultimately winning the contract does not increase much; in the meantime, a lower drop-out price would definitely decrease the incumbent's profit upon winning the contract. Hence, it is optimal to "Fold" by dropping out at $p$ immediately. Finally, the comparative statics result of $b^\star_m(x_0)$ and $p^\star_m(x_0)$ with respect to $x_0$ established in Theorem 2 part (i) shows that as $x_0$ increases, the region where "Fold" option is better expands (see the arrows in Figure 2 for an illustration). This is because when the incumbent's cost is high, dropping out at a lower price than $p$ is even less appealing since the same absolute reduction in incumbent's bid results in a larger percentage decrease in profit margin upon winning.

**Figure 2** The "Fold" and "Compete" regions of the incumbent's equilibrium drop-out strategy



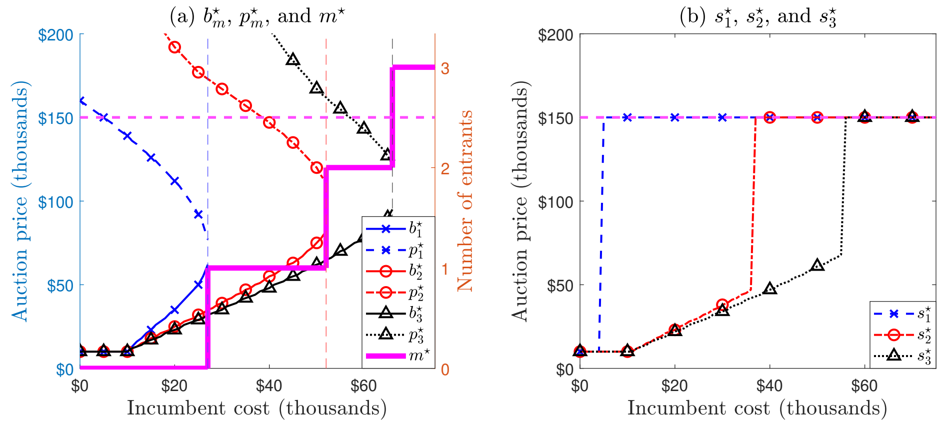

Notes: In this example, $x_0 = \$52,000$, $F_e$ follows normal distribution $N(\$30,000, \$50,000)$ truncated at $\underline{R} = \$0$ and $\overline{R} = \$150,000$, $K = \$6,700$, and $\beta = 0.61$. The arrows show how the boundary of the two regions change as $x_0$ increases.

Our full characterization of the structure of the incumbent's drop-out strategy is very useful for two main reasons. First, from the managerial perspective, the incumbent's *state-dependent dynamic* equilibrium strategy in Theorem 2 is a non-trivial generalization of the *static* equilibrium strategy characterized in Wan et al. (2012): By generalizing Wan et al. (2012)'s model to a more realistic setting with *multiple* entrants, we can tease out how the dynamically evolving competition landscape (e.g., the number of remaining entrants $m$) affects the incumbent's bidding incentives. Specifically, Theorem 2 part (ii) (resp. (i)) shows that the incumbent's optimal drop-out price (resp. threshold $b^{\star}_m(x_0)$) is nonincreasing in $m$ (see Figure 3 for an illustration of how the equilibrium bidding function for all incumbent cost types changes as $m$ varies). Intuitively, as $m$ increases, the incumbent is faced with more competition from the entrants, so he should bid more aggressively. Note that such reduction in incumbent's drop-out price can be quite significant for some cost types. For example, as illustrated in Figure 3, when the incumbent's cost is $\$30,000$, as the number of entrants increases from 1 to 2, the incumbent's drop-out price reduces from $\$150,000$ to $\$38,000$, a 76.67% reduction in drop-out price! Recall that from the buyer's perspective, the main drawback of POST is that inviting entrants to an auction will not always necessarily results in aggressive biddings from the incumbent because the incumbent may take the advantage of his qualified status to behave opportunistically by inflating his bid (the "Fold" option discussed previously). Our observation seems to suggest that such opportunistic behavior may get mitigated significantly as more entrants participate in the auction. This motivates us to numerically explore how the number of entrant suppliers affect the buyer's choice between PRE and POST in more depth in Section 4.

Second, from a technical implementation perspective, Theorem 2 can greatly reduce the computational complexity of the incumbent's strategy. We would like to point out that the dynamic program in (8)-(10) is very difficult to solve computationally because the objective function $\Pi^m_0(b; x_0, p)$ in (8) is not necessarily unimodal in the decision variable $b$! In fact, the lack of unimodal structure is the main reason that the incumbent's equilibrium drop-out strategy has the two separate options "Compete" and "Fold". Fortunately, Theorem 2 part (ii) implies that the optimal solution to (8) can only take two possible values, $b^{\star}_m(x_0)$ and $p$, both of which can be easily computed. (Mathematically, this result is based on our observation that when $\Pi^m_0$ has a local maximum, i.e., $b^{\star}_m(x_0)$, the objective is increasing for all $b < b^{\star}_m(x_0)$ and for all $b > p^{\star}_m(x_0)$ where $p^{\star}_m(x_0)$ is the *smallest* point on the real line to the right of $b^{\star}_m(x_0)$ such that $\Pi^m_0(b^{\star}_m(x_0); x_0, p) = \Pi^m_0(p^{\star}_m(x_0); x_0, p)$.) What is nice about this structure of the optimal solution is that when solving the dynamic program, instead of optimizing a non-convex optimization for each state, we now only need to compare the objective value of two easily computable points. This greatly speeds up the computation of the incumbent's equilibrium drop-out strategy which makes our numerical study in Section 5 feasible.

Next, we investigate how the incumbent's optimal drop-out decision depends on the model parameters (i.e., $K$ and $\beta$). We first study the impact of qualification cost $K$ on the incumbent's optimal drop-out price. Recall that Lemma 2 states that before the incumbent drops out, all entrants will drop out at their *effective costs* $x_j + \Delta$. As $K$ decreases, all entrants' *effective costs*

**Figure 3** Illustration of the incumbent's equilibrium drop-out strategy




Notes: In this example, $F_e$ follows normal distribution $N(\$30,000, \$50,000)$ truncated at $\underline{R} = \$0$ and $\overline{R} = \$200,000$, $K = \$6,700$, and $\beta = 0.61$. The plot on the left illustrates the switching curves that determines incumbent's equilibrium drop-out strategies when there are $m = 1,2,3$ entrants respectively. The plot on the right illustrates the incumbent's equilibrium drop-out price curves when the current auction price is $\$150,000$ and there are $m = 1,2,3$ entrants respectively.

decrease. This means that the incumbent is effectively faced with entrants with more competitive costs; as a result, one may expect the incumbent to bid more aggressively by decreasing his drop-out prices. Interestingly, our next example shows a non-monotonic change of the incumbent's optimal drop-out price as $K$ varies.

EXAMPLE 2. We consider an example with $\underline{R} = \$0$, $\overline{R} = \$100,000$, $F_e$ and $F_0$ follow uniform distribution $U[\$0, \$100,000]$, and $\beta = 0.7$. By abuse of notation, we use $s^\star_m(x_0; p; K)$ to capture the dependence of $s^\star_m(x_0; p)$ on $K$. Suppose there are two entrants and the current auction price is $p = \$100,000$. We can compute an incumbent's optimal drop-out prices under two different qualification costs $K = \$5,000$ and $K = \$25,000$ as follows

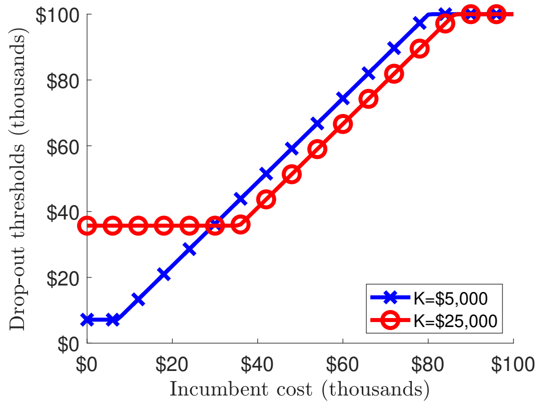
$$s^\star_2(x_0; \$100,000; \$5,000) = \begin{cases} \$7,143, & \text{if } x_0 \in [\$0, \$7,143], \\ 1.273x_0 - \$1,950, & \text{if } x_0 \in [\$7,143, \$80,086], \\ \$100,000, & \text{if } x_0 \in [\$80,086, \$100,000]; \end{cases}$$

and

$$s^\star_2(x_0; \$100,000; \$25,000) = \begin{cases} \$35,714, & \text{if } x_0 \in [\$0, \$35,714], \\ 1.273x_0 - \$9,750, & \text{if } x_0 \in [\$35,714, \$86,213], \\ \$100,000, & \text{if } x_0 \in [\$86,213, \$100,000]. \end{cases}$$

Figure 4 illustrates the optimal drop-out thresholds for all incumbent cost types at $K = \$5,000$ and $K = \$25,000$, respectively. It shows that as the qualification cost increases from $\$5,000$ to $\$25,000$, the incumbent will increase his optimal drop-out price if his cost is $x_0 < \$29,586$ but will decrease or keep the same drop-out price if his cost is $x_0 \geq \$29,586$.

**Figure 4** The impact of qualification cost $K$ on the incumbent's optimal drop-out prices




Note: In this example, $F_e$ follows uniform distribution $U[\$0, \$100,000]$, $\beta = 0.7$, and $m = 2$.

The reason behind this non-monotonic effect of $K$ is due to the trade-off between the "Compete" and "Fold" options identified in Theorem 2. While the entrants become effectively more competitive in costs when $K$ decreases, the incumbent may have different types of reactions depending on his cost type. For an incumbent with a low cost type, he responds by bidding more aggressively. But for an incumbent with a high cost type, lowering his drop-out price will result in a big percentage reduction in his profit margin upon winning but would not significantly reduce the number of entrants who drop out later than him (i.e., his winning probability does not improve significantly). Hence, he will shy away from intense competition by dropping out at a high price when $K$ decreases.

Next, we study how the entrants' qualification passing probability $\beta$ affects the incumbent's optimal drop-out prices. As $\beta$ increases, the entrants are more likely to pass the qualification screening, so the incumbent is faced with more risk of losing the contract. As a result, intuitively, one would expect the incumbent to lower his bid, i.e., the incumbent's optimal drop-out price should decrease in $\beta$. Somewhat surprisingly, our next example shows that this intuition is not always true: For some incumbent cost types, the optimal drop-out prices actually increase in $\beta$.

Example 3. We consider an example with $\underline{R} = \$0$, $\overline{R} = \$100,000$, $F_e$ and $F_0$ follow uniform distribution $U[\$0, \$100,000]$, and $K = \$25,000$. By abuse of notation, we use $s^\star_m(x_0; p; \beta)$ to capture the dependence of $s^\star_m(x_0; p)$ on $\beta$. Suppose there is only one entrant and the current auction price is $p = \$100,000$. We can compute an incumbent's optimal drop-out prices under two different passing probabilities $\beta = 0.3$ and $\beta = 0.7$ as follows

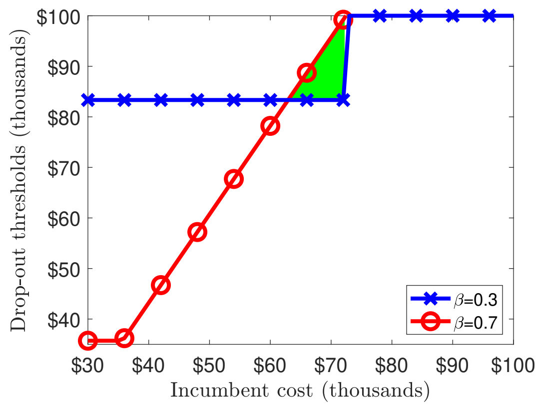
$$s^\star_1(x_0; \$100,000; 0.3) = \begin{cases} \$83,333, & \text{if } x_0 \le \$72,222 \\ \$100,000, & \text{if } x_0 > \$72,222 \end{cases},$$

and

$$s^\star_1(x_0; \$100,000; 0.7) = \begin{cases} \$35,714, & \text{if } x_0 \in [\$0, \$35,714] \\ 1.75x_0 - \$26,785, & \text{if } x_0 \in [\$35,714, \$72,449] \\ \$100,000, & \text{if } x_0 \in [\$72,449, \$100,000] \end{cases}.$$

Figure 5 illustrates the optimal drop-out prices for all incumbent cost types under $\beta = 0.3$ and $\beta = 0.7$. It shows that when the incumbent's cost is $x_0 \in (\$62,925, \$72,222)$, his optimal drop-out price increases as the passing probability increases from 0.3 to 0.7.

**Figure 5** The impact of qualification passing probability $\beta$ on the incumbent's optimal drop-out prices




Note: In this example, $F_e$ follows uniform distribution $U[\$0, \$100,000]$, $K = \$25,000$, and $m = 1$.

The reason behind this non-monotonic effect of $\beta$ is similar to that of $K$. As $\beta$ increases, there is a higher risk of losing the contract if the incumbent drops out at a high price, so the incumbent has the incentive to decrease his drop-out price. However, there is also another competing effect. As $\beta$ increases, all entrants' effective costs decrease. Similar to the rationale behind the "Fold" option, this intensified cost competition incentivizes the incumbent with moderate or high cost to bid less aggressively. When this latter effect dominates the former effect, the incumbent's optimal drop-out price increases in $\beta$.

### 3.3. Buyer's Expected Procurement Cost

Having established the suppliers' equilibrium drop-out strategies, we now derive the buyer's expected procurement cost under POST. Denote by $\mathcal{PC}_{post}(x_0)$ the buyer's procurement cost under POST when the incumbent's cost is $x_0$. The buyer's expected procurement cost under POST equals

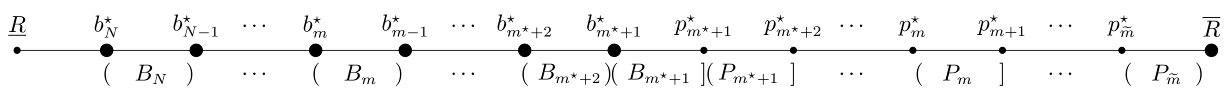
$$PC_{post} = \int_{\underline{R}}^{\overline{R}} \mathbf{E}\left[\mathcal{PC}_{post}(x_0)\right] dF_0(x_0). \tag{11}$$

To derive a closed-form expression for $\mathbf{E}\left[\mathcal{PC}_{post}(x_0)\right]$, we utilize the structural properties of the incumbent's equilibrium drop-out strategy characterized in Theorem 2. Recall that given $x_0$, the incumbent's drop-out strategy is fully characterized by a set of known constants $m^\star(x_0)$, $\{b^\star_m(x_0)\}_{m=m^\star(x_0)+1}^N$ and $\{p^\star_m(x_0)\}_{m=m^\star(x_0)+1}^N$. Moreover, based on the incumbent's drop-out price $\overline{p}$, we can use the incumbent's equilibrium drop-out strategy to infer how many entrant suppliers remain in the auction when the incumbent drops out. For example, if $\overline{p}=b^\star_m(x_0)$, then it means that, with probability one, there are exactly $m$ entrants in the auction when the incumbent drops out. This observation motivates us to divide all possible incumbent drop-out prices into different regions. To that end, we define some intervals as follows (see Figure 6 for an illustration):

$$B_m := (b^\star_m(x_0), b^\star_{m-1}(x_0)) \text{ for } m=m^\star(x_0)+2,\cdots,N, \text{ and } B_m := (b^\star_m(x_0), \overline{R}\wedge p^\star_m(x_0)] \text{ for } m=m^\star(x_0)+1,$$
$$P_m := (p^\star_m(x_0), p^\star_{m+1}(x_0)] \text{ for } m=m^\star(x_0)+1,\cdots,\widetilde{m}(x_0)-1, \text{ and } P_m := (p^\star_m(x_0), \overline{R}) \text{ for } m=\widetilde{m}(x_0),$$

where $\widetilde{m}(x_0) := \max\{m : m^\star(x_0)+1 \le m \le N, p^\star_m(x_0) \le \overline{R}\}$ when $p^\star_{m^\star(x_0)+1}(x_0) \le \overline{R}$, and $\widetilde{m}(x_0) :=$ $\min\{m^\star(x_0), N\}$ otherwise.

**Figure 6** Illustration of the incumbent's drop-out price regions




Note: The dependency of $b^\star_m(x_0), p^\star_m(x_0), m^\star(x_0), \widetilde{m}(x_0)$ on $x_0$ is suppressed for notational simplicity.

Then, based on the incumbent's drop-out price $\overline{p}$, there are four types of scenarios: (a) $\overline{p}=\overline{R}$, (b) $\overline{p}\in P_m$ for $m=m^\star(x_0)+1,\ldots,\widetilde{m}(x_0)$, (c) $\overline{p}\in B_m$ for $m=m^\star(x_0)+1,\ldots,N$, and (d) $\overline{p}=b^\star_m(x_0)$ for $m=m^\star(x_0)+1,\ldots,N$. Based on the incumbent's equilibrium drop-out strategy, one can verify that the following relationship between $\overline{p}$ and $m$ is true: Scenario (a) implies that there are fewer than $\widetilde{m}(x_0)$ entrants who participate in the auction, so the incumbent leaves behind fewer than $\widetilde{m}(x_0)$ entrants in the auction; scenario (b) (resp. (c)) implies that the $m+1^{st}$ (resp. $m^{th}$) lowest entrant drops out at $\overline{p}$, so the incumbent leaves behind $m$ (resp. $m-1$) entrants in the auction; scenario (d) implies that the incumbent drops out at the threshold $b^\star_m(x_0)$ leaving behind $m$ entrants in the auction. By conditioning on the price at which the incumbent drops out, Proposition 3 below provides a closed-form expression for $\mathbf{E}[\mathcal{PC}_{post}(x_0)]$.

PROPOSITION 3. *For all $m=1,\ldots,N$, denote by $X_{(m:N)}$ the $m^{th}$ smallest cost among all $N$ entrants' costs, and let $X_{(N+1:N)} := \overline{R}$. Then, when the incumbent's cost is $x_0$, the buyer's expected cost is*

$$\begin{aligned}\mathbf{E}[\mathcal{PC}_{post}(x_0)] = &\sum_{i=0}^{\widetilde{m}} \frac{N!}{i!(N-i)!}[F_e\left(\overline{R}-\Delta\right)]^i[1-F_e\left(\overline{R}-\Delta\right)]^{N-i}h(i,\overline{R}) \\ &+\sum_{m=m^\star+1}^{\widetilde{m}} \int_{\overline{p}\in P_m} h(m,\overline{p})\Phi'_{m+1}(\overline{p})d\overline{p} \\ &+\sum_{m=m^\star+1}^{N} \int_{\overline{p}\in B_m} h(m-1,\overline{p})\Phi'_m(\overline{p})d\overline{p} + \sum_{m=m^\star+1}^{N} \phi_m h(m,b^\star_m),\end{aligned} \tag{12}$$

*where, for all* $m = m^\star(x_0)+1,\ldots,N$ *and* $\overline{p} \in [\underline{R}, \overline{R}]$*, we define* $h(m,\overline{p}) := \mathbf{E}[\mathcal{PC}_{post}(x_0)|m,\overline{p}]$*,* $\Phi_m(\overline{p}) := \mathbf{P}(X_{(m:N)} \le \overline{p} - \Delta,$ *and* $X_{(j+1:N)} \le p^\star_j(x_0) - \Delta\ \forall j = m,\ldots,\widetilde{m}(x_0))$*, and* $\phi_m := \mathbf{P}(X_{(m:N)} \le b^\star_m(x_0) - \Delta < X_{(m+1:N)},$ *and* $X_{(j+1:N)} \le p^\star_j(x_0) - \Delta\ \forall j = m,\ldots,\widetilde{m}(x_0))$*. (For expositional clarity, the closed-form expressions of* $\Phi_m(\overline{p})$*,* $\phi_m(\overline{p})$*,* $h(m,\overline{p})$ *are derived in the proof.)*

We would like to point out that the distribution of $\overline{p}$ is partly continuous and partly discrete, i.e., $\Phi'_m(\overline{p})$ corresponds to the p.d.f of the incumbent's drop-out price $\overline{p}$ when $\overline{p} \in B_m$ (or when $\overline{p} \in P_{m-1}$), and $\phi_m$ corresponds to the probability that the incumbent's drop-out price $\overline{p}$ equals $b^\star_m(x_0)$. The derivation of $h(m,\overline{p})$ uses the properties of the entrants' equilibrium drop-out strategy and the Revenue Equivalence Theorem (Myerson 1981). Finally, Proposition 3 combined with (11) establishes a closed-form expression of $PC_{post}$.

## 4. Buyer's Choice Between PRE and POST

The expressions for buyer's expected cost under PRE and POST, which are characterized in Section 2.1 and Section 3.3 respectively, allow the buyer to evaluate, ex ante, which qualification strategy is cheaper. In this section, we turn to the buyer's strategic choice between PRE and POST and identify conditions under which the buyer is better off using the POST we propose.

Our analysis and discussions have identified the main cost drivers for the trade-off between the two qualification strategies: On the one hand, POST reduces the number of qualification screenings needed (and hence the qualification cost) because the buyer would only select entrants with low bids for qualification screenings; on the other hand, while there are more suppliers competing in bidding in POST, the fact that not all of them have been qualified yet pose incentives for suppliers to bid less aggressively in the hope that lower bids may fail qualification, which may result in higher contract payment then PRE. The strengths of both driving forces depend on the model primitives, such as how well the entrants are positioned to pass the qualification screening ($\beta$), the economic burden of conducting qualification screenings ($K$), and the level of competition ($N$). Our next result characterizes conditions under which the buyer should choose our proposed POST.

Proposition 4. *Holding all other parameters constant, the buyer should choose POST if*

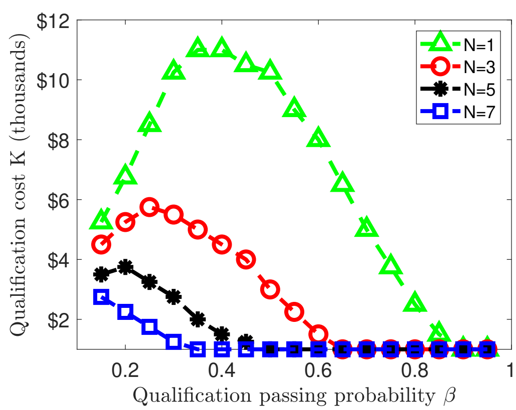
*(a) the likelihood for an entrant to pass qualification screening,* $\beta$*, is sufficiently high;*

*(b) the cost of qualification screening,* $K$*, is sufficiently high;*

*(c) the number of entrants,* $N$*, is sufficiently high.*

The intuitions of the results above are closely related to the cost drivers of the trade-off between PRE and POST identified previously. When entrants have a high chance of passing the qualification screening, $\beta$, suppliers face a higher risk of losing the contract by opportunistically holding back their aggressiveness in bidding; thus, the effect of less aggressive bidding in POST is dampened and makes POST more appealing. When each qualification screening costs a fortune for the buyer, the benefit of conducting more informed qualification screening based on suppliers' bids is more pronounced; thus, the buyer should choose POST. Finally, when $N$ is large, POST has the advantage of tapping into a large pool of competing entrants in the bidding process (which effectively mitigate the magnitude of the incentive for less aggressive bidding in POST) without necessarily conducting a lot of qualification screenings. Figure 7 illustrates the buyer's optimal strategic decision on qualification strategy, where each curve corresponds to a given $N$ and shows the optimal decision boundary between PRE and POST: For a given $N$, the buyer should choose POST if the point that corresponds to $K$ and $\beta$ lies in the region above the decision boundary. Quite strikingly, when the number of available entrants is more than 7 (i.e., $N > 7$), which is not uncommon in many manufacturing industries, POST outperforms PRE for a very high proportion of all of the combinations of $K$ and $\beta$ illustrated here. This seems to suggest that introducing POST in a buyer's sourcing tool can be quite powerful. This also motivates us to assess the magnitude of the benefit that POST offers via a comprehensive numerical study in the next section.

**Figure 7** The buyer's optimal qualification strategy




Notes: The curves correspond to the decision boundaries for different number of all entrant suppliers $N$. Given $N$, the buyer should use POST in the area above the corresponding curve and use PRE in the area below that curve.

## 5. Assessing the Benefit of POST

Recall that our analysis makes it possible for the buyer to compute the expected costs of PRE and POST ex ante and choose the cheaper option to implement; thus, our *proposed approach* to the buyer is to use the cheaper option between PRE and POST. How effective is this approach? To assess the benefit of the POST option, we conduct an extensive numerical study, which consists of 19,635 problem scenarios with a wide range of different model parameters (see Table 1 for the factorial design of our numerical study), to answer the following two questions: (i) how much cost-savings does our proposed approach provide compared with using PRE alone, and when does our proposed approach offer the most cost-savings; (ii) how much does our proposed approach help in bringing down the procurement cost closer to the optimal mechanism which achieves the lowest cost possible in theory but may not be implementable in practice.

**Table 1** Factorial Design of the Numerical Study

| Parameters | Values |
|---|---|
| Cost distribution | $F_0$ and $F_e$ follow uniform distribution $U[\$0, \$100,000]$ |
| Qualification cost $K$ | $\$1,000, \$1,250, \$1,500, \cdots, \$20,000$ |
| Qualification passing probability $\beta$ | $0.15, 0.2, 0.25, \cdots, 0.95$ |
| Number of available entrants $N$ | $1, 2, 3, \cdots, 15$ |
| Number of the incumbent | 1 |

### 5.1. Study 1: Comparing proposed approach to PRE

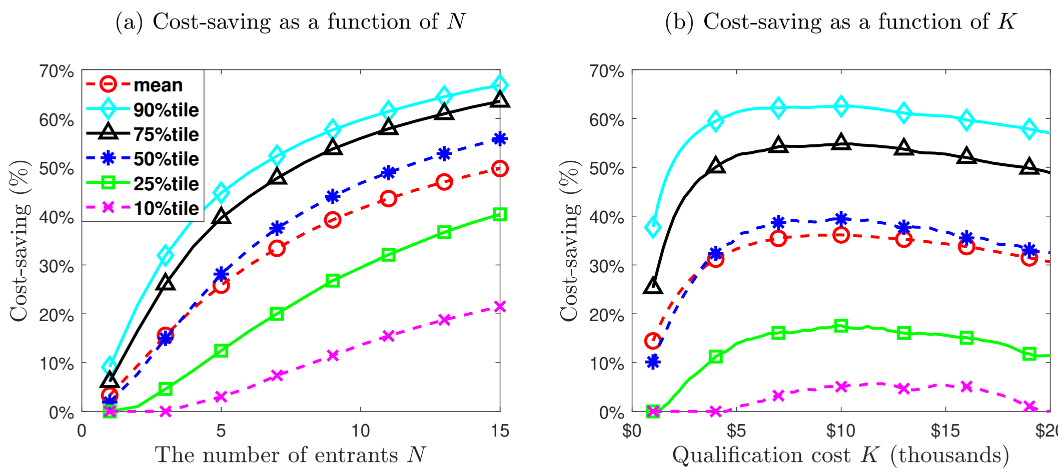
To assess the magnitude of cost-savings that our proposed approach offers compared to the conventional approach PRE, we use the following cost-saving metric:

$$\frac{PC_{pre} - \min\{PC_{pre}, PC_{post}\}}{PC_{pre}}.$$

For each scenario, we compute the expected cost of PRE and POST and calculate the metric defined above (see Online Appendix EC.2 for the detailed computation method and pseudocode); this results in a data set of the cost-saving metric with 19,635 samples and we report key summary statistics of the cost-saving metrics in Table 2. Note that the cost-savings range from 0% (this is when PRE is optimal) to over 60%, which suggests that, depending on the model parameters, the magnitude of the benefit that our proposed strategy can offer over the conventional approach PRE can be very heterogeneous. Having said that, our proposed strategy can save the buyer 32.67% of the procurement cost under PRE on average. More strikingly, even at 10% percentile, our proposed strategy still provides a cost-saving of 2.04%, which is a quite significant cost improvement by industry standards. This suggests that in many practical scenarios, there is a significant cost improvement opportunity to incorporate POST into buyer's re-sourcing toolkit.

Motivated by the heterogeneity of cost-savings across different scenarios, we next investigate when our proposed strategy offers the highest cost-saving compared to PRE. To address this question, in the data set of 19,635 cost-saving metrics we have calculated, we take the subsamples which have the same $N$ (resp. $K$), and plot their summary statistics as a function of $N$ (resp. $K$) in Figure 8. Figure 8(a) shows that as the number of entrants $N$ increases, the cost-saving also increases, which is consistent with the observation in Figure 7 that when the number of entrants

**Figure 8** Relative cost-savings of the proposed strategy over PRE




is sufficiently large, the buyer should use POST. While the number of entrants $N$ affects the cost-saving monotonically, Figure 8(b) shows that the effect of qualification cost $K$ is non-monotone: The cost-saving first increases and then decreases as $K$ increases. When qualification cost is very small, PRE is already very effective, so our proposed strategy does not improve much; when qualification cost is very large, both PRE and POST will end up qualifying a small number of entrant suppliers, so the benefit of our proposed strategy is also limited.[1] In summary, our proposed strategy is most beneficial when the number of available entrants is large, or the qualification cost is moderate.

**Table 2** Summary statistics for the relative cost-savings of the proposed strategy over PRE

| | Cost-saving (%) |
|---|---|
| mean | 32.67 |
| std | 21.03 |
| 90th percentile | 60.14 |
| 75th percentile | 51.43 |
| 50th percentile | 34.35 |
| 25th percentile | 13.03 |
| 10th percentile | 2.04 |

## 5.2. Study 2: Comparing proposed approach to the optimal mechanism

As discussed earlier, both PRE and POST, and hence our proposed approach, are easy to implement in practice with features which the practitioners are very familiar with. While being simple is an appealing feature in practice, one may wonder how well our proposed approach compares with more complicated approaches which may be potentially more cost effective. To assess the additional cost-savings a buyer could achieve in theory on top of our proposed approach, we now assume that

[1] For example, consider the case where $K > \overline{R}\beta$. This implies that $\Delta = K/\beta > \overline{R} - \underline{R}$, so the buyer should not conduct qualification screening in either PRE or POST. As a result, PRE and POST result in the same expected cost.

the buyer has full commitment power and conducts an optimal mechanism analysis to the model we introduced in Section 2. In other words, instead of analyzing a particular mechanism such as POST in Section 3, we seek the optimal mechanism among a broader class of feasible mechanisms where the only constraint we place is that the buyer can only award the contract to *qualified* suppliers. Note that this class of feasible mechanisms subsumes the mechanisms we have discussed in this paper where each supplier is allowed to only place one *single* bid which the buyer can choose to use (i.e., in POST) or ignore (i.e., in PRE) when conducting qualification screenings. However, it also includes mechanisms that are potentially very hard to implement in practice (e.g., after the qualification screenings of some of the entrant suppliers, the buyer allows the incumbent to reduce his bid if an entrant with a lower bid passes qualification screening.) Hence, the optimal mechanism serves as a lower bound of the lowest possible cost the buyer can ever achieve in practice.

To characterize the optimal mechanism, we define the virtual cost for the entrants and the incumbent as $\psi_e(x_i) := x_i + \frac{F_e(x_i)}{f_e(x_i)}$ and $\psi_0(x_0) := x_0 + \frac{F_0(x_0)}{f_0(x_0)}$ respectively.

PROPOSITION 5. *Under the assumption that $\psi_i(x_i)$ is nondecreasing and Assumption 1, an optimal direct, individually rational, and incentive-compatible mechanism that minimizes the buyer's expected total cost is characterized as follows*[2]*:*

- ***Step 1.*** *The buyer asks all suppliers to report their true costs.*
- ***Step 2.*** *Let $i_k$ be the entrant with $k^{th}$ lowest cost among all entrants, and suppose there are $L \leq N$ entrants whose costs are lower than $\psi_e^{-1}(\psi_0(x_0) - \Delta)$. The buyer conducts qualification screenings among entrants according to the sequence $\{i_k\}_{k=1}^{L}$ until either the buyer finds a qualified entrant or the buyer exhausts the list of all $L$ entrants.*
- ***Step 3.*** *At the conclusion of Step 2, if the incumbent is the only qualified supplier, then the buyer awards the contract to the incumbent and pays him*

$$\beta \sum_{l=L+1}^{N} (1-\beta)^{l-L-1} \min\{\overline{R}, \psi_0^{-1}(\psi_e(x_{i_l}) + \Delta)\} + (1-\beta)^{N-L}\overline{R}; \tag{13}$$

*otherwise, the buyer awards the contract to entrant $x_{i_k}$ who passes the qualification screening and pays him*

$$\beta \sum_{l=k+1}^{L} (1-\beta)^{l-k-1} x_{i_l} + (1-\beta)^{L-k}\, \psi_e^{-1}(\psi_0(x_0) - \Delta). \tag{14}$$

[2] Note that a similar optimal mechanism design setting has been investigated in Wan and Beil (2009) and Chen et al. (2018). It is worth-noting that the notion of pre-qualification screening in Wan and Beil (2009) refers to buyer's *endogenous* effort on identifying suppliers with certain qualification probabilities, and is hence different from our setting. Having said that, our optimal mechanism analysis is different to the model in Wan and Beil (2009) in the following way: We consider the case where the suppliers have ex ante asymmetric cost distributions, and the "prequalification probabilities" (per Wan and Beil (2009) terminology) are exogenous (i.e., it equals 1 for the incumbent, and equals $\beta$ for the entrants). Our mechanism design problem is not a special case of the model considered in Chen et al. (2018) because suppliers' cost distributions are asymmetric in our setting.

Note that in the optimal mechanism, it is optimal to conduct supplier qualification screenings post-bidding. This is because, on the one hand, using the suppliers' initial bids helps the buyer make more informed supplier qualification screening decisions and save qualification cost (this is consistent with the finding in Study 1 in that POST performs well compared to PRE in most of the scenarios); on the other hand, the optimal payment rule, under which the buyer commits to ex ante, ensures that she provides the minimum incentive needed for the suppliers to *not* inflate their cost bids (in contrast, the simple payment rule in POST creates incentives for suppliers to inflate their bids compared to PRE). While the optimal mechanism, by definition, results in lower expected total cost for the buyer compared to our proposed approach, we want to point out that the optimal mechanism is difficult to implement due to various practical concerns. For example, the optimal mechanism's payment rule in (13) and (14) is very complicated: The winner's payment not only depends on his own bid but also depends on his competitors' bids in a nontrivial way, which is difficult for the buyer to rationalize to suppliers in the first place in practice. Moreover, the optimal mechanism's qualification screening rule is *not* necessarily ex post optimal for the buyer because, in contrast to Step 2 in Proposition 5, ex post, the buyer may actually have an incentive to *not* screen an entrant with cost $x_j < \psi_e^{-1}(\psi_0(x_0) - \Delta)$ if all lower cost entrants fail qualification screenings[3]. Thus the buyer not only faces the challenge of explaining the rules of the mechanism to the suppliers but also needs to convince the suppliers that she will strictly follow the complicated rules even though following these rules may not be in the buyer's best interest ex post. Another aspect of the implementation challenge of the optimal mechanism relates to the impracticality of truthful bidding. In practice, truthful bidding is hard to induce because both the incumbent and entrants would be hesitant to reveal their true costs for fear of, for example, revealing private cost information which would put them in a disadvantaged position for future business negotiations with the buyer. (We refer interested readers to Roughgarden and Talgam-Cohen (2019) for more discussions on the drawbacks of optimal mechanisms in practice, and Rothkopf (2007) for several more general critiques about why mechanisms that induce truthful bidding may not be practical.) Note that these practical implementation issues do *not* arise in POST: In POST, the payment is essentially a simple pay-as-bid rule (see footnote 1), the qualification screening rule is ex post optimal for the buyer, and the suppliers do not need to reveal their true costs.

[3] Here is an example when the optimal mechanism's qualification screening rule is not ex post optimal. Suppose there is one entrant and one incumbent and both suppliers' cost distributions follow uniform $U[0,\overline{R}]$. Moreover, the entrant's cost $x_1 < \psi_e^{-1}(\psi_0(x_0) - \Delta)$ (namely, $N = L = 1$), and the incumbent's cost $x_0 > \overline{R} - \Delta/2$. By Proposition 5, (13) becomes $\overline{R}$ since $N = L$, and (14) becomes $\psi_e^{-1}(\psi_0(x_0) - \Delta)$. Then, *after seeing both suppliers' bids $x_0$ and $x_1$ (i.e., ex post)*, the buyer's total expected cost under the optimal mechanism is $K + \beta\psi_e^{-1}(\psi_0(x_0) - \Delta) + (1-\beta)\overline{R} = \beta(x_0 + \Delta/2) + (1-\beta)\overline{R} > \overline{R}$. Thus, ex post, she has an incentive to deviate from her committed qualification screening rule by *not* screening the entrant but awarding the contract to the incumbent directly with the payment $\overline{R}$ as in (13).

While the optimal mechanism is hard to implement in practice, it does provide a lower bound of the best achievable cost for the buyer in practice. We can then use this lower bound to numerically evaluate how effective our proposed approach helps bring the cost down to the optimal cost from the conventional approach PRE. To that end, we adopt a metric defined as the fraction of the absolute potential savings the optimal mechanism offers over PRE that can be captured by our proposed approach. To compute this metric, for each scenario, we calculate the savings of our proposed approach over PRE across all scenarios and divide this by the potential savings using the optimal mechanism (see Online Appendix EC.2 for the detailed computation method and pseudocode):

$$\frac{PC_{PRE} - \min\{PC_{PRE}, PC_{POST}\}}{PC_{PRE} - PC_{OPT}}, \tag{15}$$

where $PC_{OPT}$ is the expected cost under the optimal mechanism. Next, we report key summary statistics in Table 3. Consistent with our previous observation, our proposed approach performs very well: Across all scenarios, it captures on average 72.23% of the potential savings from the optimal mechanism which, as we mentioned, would be hard to implement in practice. Thus, we believe that our proposed approach is a powerful procurement toolkit for the buyer.

**Table 3** Summary statistics for the cost-saving recovers of the proposed strategy over the optimal mechanism

| | Cost-saving (%) |
|---|---|
| mean | 72.23 |
| std | 28.10 |
| 90th percentile | 96.11 |
| 75th percentile | 92.99 |
| 50th percentile | 84.45 |
| 25th percentile | 59.95 |
| 10th percentile | 26.07 |

## 6. Conclusions

In this paper, we study the buyer's optimal choice between pre-qualification (PRE) and post-qualification (POST) strategies in a re-sourcing setting where a qualified incumbent and multiple not-yet-qualified entrant suppliers compete for a single-supplier contract in an open-bid auction. While this is an important problem in the academic literature, there is little guidance on how to *quantitatively* choose between PRE and POST in practice. This is primarily because POST in the context of open-bid auction is a very complex dynamic game and it is deemed computationally intractable to compute the buyer's expected cost under POST when multiple entrant suppliers participate in the auction. By providing a full characterization of suppliers' equilibrium bidding strategies in POST, we uncover managerial insights on the entrants' and the incumbent's different bidding incentives in this setting. More importantly, by leveraging the structural properties of suppliers' bidding strategies, we provide the first computationally tractable approach to evaluate the expected cost of POST, which enables practitioners to quantitatively compare the expected costs of PRE and POST ex ante. Hence, we propose the buyer to use the cheaper option between PRE and POST to minimize the expected procurement cost.

While PRE is predominantly used in practice, we derive analytical conditions under which POST is cheaper than PRE, and show via an extensive numerical study that our proposed approach (i.e., the cheaper option between PRE and POST) can significantly reduce the buyer's procurement cost compared to only using PRE for many practical settings. We show that even though our proposed approach is simple and only involves combining features of existing auction methods in a novel way, it captures most of the benefit which a theoretically optimal mechanism (which is quite difficult to implement in practice) offers over PRE. These results provide evidence that POST can be a powerful tool to manage re-sourcing processes in practice. Note that POST is one possible arrangement to incorporate post-bidding qualification screening into open-bid auctions and there may be other practical alternative mechanisms which could provide more cost-savings to the buyer. We leave this as an open research direction.

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## EC.1. Proof of Statements

**Proof of Lemma 1.** Obviously, it is suboptimal to conduct qualification screening on entrants who drop out at higher price than the incumbent. Thus, we only need to focus on entrants who drop out at lower prices (i.e., those who drop out later than the incumbent in our open-bid auction setting) than the incumbent. Suppose there are $n$ entrants in the auction when the incumbent drops out, and the auction price after the incumbent drops out is $p^n_{(n)}$. Suppose, without loss of generality, that these are entrants $j = 1, 2, \ldots, n$ and they drop out in the reverse order in time. Denote by $p^n_{(j)}$ the $(n-j)^{th}$ entrant who drops out after the incumbent (this means that entrant $j+1$ drops out at $p^n_{(j)}$) for $j = 1, \ldots, n-1$. Denote by $\mathbb{W}(b, \mathcal{U})$ the buyer's expected total cost-to-go when the lowest bid among all the *qualified* suppliers is $b$, and the set of not-yet-qualified suppliers who drop out later than the incumbent is $\mathcal{U} \subseteq \{1, \ldots, n\}$. We now show by induction that *it is optimal to screen the supplier with the lowest drop-out price in $\mathcal{U}$ if his drop-out price is lower than $b - \Delta$*. Suppose that $|\mathcal{U}| = 1$. Without loss of generality, assume $\mathcal{U} = \{j\}$. The buyer should screen supplier $j$ if the expected total cost of doing so is lower than not doing so, i.e., when $\delta(b, p^n_{(j-1)}) := K + (1-\beta)b + \beta \min\{b, p^n_{(j-1)}\} - b < 0$. Note that $\delta(b, p^n_{(j-1)})$ is increasing in $p^n_{(j-1)}$ and $\delta(b, p^n_{(j-1)}) = 0$ when $p^n_{(j-1)} = b - \Delta$; this establishes the inductional basis. Suppose the induction hypothesis is true for all $|\mathcal{U}| \leq k-1$, we now show it is true for $|\mathcal{U}| = k$. Suppose $\mathcal{U} = \{j_1, \ldots, j_k\}$. We first prove that, if it is optimal to screen any entrant in $\mathcal{U}$, it is optimal to first screen entrant $j_{l^*}$, where $l^* = \arg\min_{1 \leq l \leq k} j_l$, than to screen entrant $j_{l'}$ for $l' \neq l^*$. Indeed, on sample paths where $j_{l^*}$ passes qualification screening, screening $j_{l^*}$ first and then following the optimal screening policy would result in the same contract payment but lower qualification screening cost compared to screening $j_{l'}$ first and then following the optimal screening policy; on sample paths where $j_{l^*}$ fails qualification screening, both policies yield same total cost. We now show that it is optimal to screen $j_{l^*}$ when $p^n_{(j_{l^*}-1)} < b - \Delta$, but *not* to screen any entrants when $p^n_{(j_{l^*}-1)} \geq b - \Delta$. Note that the expected cost-to-go of not screening any entrants is $b$. Thus, if $p^n_{(j_{l^*-1})} \geq b - \Delta$, then the buyer's expected cost of screening supplier $j_{l^*}$ and then following the optimal policy (which in this case is not screening any other entrant in $\mathcal{U}$ since $p^n_{(j_l)} \geq b - \Delta$ for all $1 \leq l \leq k$) is more expensive than not screening any supplier since $K + \beta \min\{p^n_{(j_{l^*-1})}, b\} + (1-\beta)b \geq b$; if $p^n_{(j_{l^*-1})} < b - \Delta$, then screening supplier $j_{l^*}$ and then following the optimal policy is cheaper than not screening any supplier since $K + \beta \mathbb{W}(\min\{b, p^n_{(j_{l^*-1})}\}, \mathcal{U} - \{j_{l^*}\}) + (1-\beta)\mathbb{W}(b, \mathcal{U} - \{j_{l^*}\}) \leq K + \beta(b - \Delta) + (1-\beta)b = b$. This completes the inductional step. The optimal screening policy established above implies the stated optimal sequence of screening. Moreover, at any time, $b$ would be the minimum of the incumbent's drop-out price and the drop-out price of the first entrant who passes qualification screening. This completes the proof. ■

**Proof of Lemma 2.** Due to the price decrement after the incumbent drops out, an entrant $j$ with cost $x_j$ may lose money if he drops out at a price $\overline{p}$ that is lower than $x_j+\Delta$. Moreover, he is also no better-off by dropping out at a price $b_j > x_j+\Delta$: if the incumbent drops out below $x_j+\Delta$ or above $b_j$, entrant $j$ does not gain by dropping out earlier at $b_j$; but if the incumbent drops out between $b_j$ and $x_j+\Delta$, the entrant is better off staying in the auction longer because otherwise he will lose the contract for sure. ■

**Proof of Theorem 1.** For expositional convenience, we will prove (ii) first and then (i), (iii) and (iv).

**(ii)** We first prove that $\widetilde{s}_k(x_j;\overline{c},\overline{p})$ decreases in $k$ by contradiction. Fix $K$, $\beta$, $\overline{c}$ and $\overline{p}$, suppose, to the contradiction, that $\widetilde{s}_k$ is not decreasing in $k$. Then there exists an $x_j' \in [\underline{R},\overline{c})$ such that $\widetilde{s}_k(x_j';\overline{c},\overline{p}) \geq \widetilde{s}_{k-1}(x_j';\overline{c},\overline{p})$, which, by (5), implies that $(\partial/\partial x_j)\widetilde{s}_k(x_j;\overline{c},\overline{p}) > (\partial/\partial x_j)\widetilde{s}_{k-1}(x_j;\overline{c},\overline{p})$ for all $x_j \geq x_j'$. Thus, $\widetilde{s}_k(x_j;\overline{c},\overline{p}) > \widetilde{s}_{k-1}(x_j;\overline{c},\overline{p})$ for all $x_j \in \left(x_j',\overline{c}\right]$, which contradicts to the fact that $\widetilde{s}_k(\overline{c};\overline{c},\overline{p}) = \widetilde{s}_{k-1}(\overline{c};\overline{c},\overline{p}) = \overline{p}$ due to (6). The sensitivity results for $x_j$, $\overline{c}$ and $\overline{p}$ can be proven similarly.

**(i)** For any $2 \leq k \leq n$, we denote the objective function in (3) by

$$\begin{aligned}\Pi_e^k\left(b_j;x_j,p;\overline{c},\overline{p}\right) = &\int_{\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right)}^{\widetilde{s}_k^{-1}(p;\overline{c},\overline{p})} V_e^{k-1}\left(x_j,\widetilde{s}_k\left(z;\overline{c},\overline{p}\right);z,\widetilde{s}_k\left(z;\overline{c},\overline{p}\right)\right) g_e^{k-1}\left(z;\widetilde{s}_k^{-1}\left(p;\overline{c},\overline{p}\right)\right)dz \\ &+\beta G_e^{k-1}\left(\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right);\widetilde{s}_k^{-1}\left(p;\overline{c},\overline{p}\right)\right)(1-\beta)^{k-1}\left(b_j-x_j\right).\end{aligned} \tag{EC.1}$$

Take partial derivative with respect to $b_j$ yields

$$\begin{aligned}\frac{\partial \Pi_e^k\left(b_j;x_j,p;\overline{c},\overline{p}\right)}{\partial b_j} &= \left[\frac{F_e\left(\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right)\right)}{F_e\left(\widetilde{s}_k^{-1}\left(p;\overline{c},\overline{p}\right)\right)}\right]^{k-1}\left\{\beta(1-\beta)^{k-1}-(k-1)\frac{f_e(\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}))}{F_e\left(\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right)\right)}\cdot\right.\\ &\qquad\left.\frac{\partial\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right)}{\partial b_j}\left[V_e^{k-1}\left(x_j,b_j;\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right),b_j\right)-\beta(1-\beta)^{k-1}\left(b_j-x_j\right)\right]\right\}\\ &=\left[\frac{(1-\beta)F_e\left(\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right)\right)}{F_e\left(\widetilde{s}_k^{-1}\left(p;\overline{c},\overline{p}\right)\right)}\right]^{k-1}\left\{\beta-\frac{V_e^{k-1}\left(x_j,b_j;\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right),b_j\right)-\beta(1-\beta)^{k-1}\left(b_j-x_j\right)}{\beta(1-\beta)^{k-2}\left(b_j-\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right)\right)}\right\}\end{aligned} \tag{EC.2}$$

where the first equality follows by the definitions of $G_e^{k-1}$ and $g_e^{k-1}$, the second equality holds since, by Implicit Function Theorem and (5),

$$\frac{\partial\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right)}{\partial b_j} = \left(\left.\frac{\partial\widetilde{s}_k\left(x;\overline{c},\overline{p}\right)}{\partial x}\right|_{x=\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right)}\right)^{-1} = \frac{F_e(\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right))}{(k-1)f_e(\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right))}\,\frac{\beta^{-1}(1-\beta)}{b_j-\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right)}. \tag{EC.3}$$

Note that the following is true: for all $b_j \in [x_j,\overline{p})$,

$$b_j - \widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}) > 0. \tag{EC.4}$$

Indeed, if (EC.4) is not true, then, by (5), there exists $b_j' \in [x_j, \overline{p})$ such that $(\partial/\partial x_j)\widetilde{s}_k(\widetilde{s}_k^{-1}(b_j';\overline{c},\overline{p});\overline{c},\overline{p}) \le 0$, which implies that $(\partial/\partial x_j)\widetilde{s}_k(x_j;\overline{c},\overline{p}) < 0$ for all $x_j \in \left(\widetilde{s}_k^{-1}(b_j';\overline{c},\overline{p}),\overline{c}\right]$ and $\widetilde{s}_k(\overline{c};\overline{c},\overline{p}) - \overline{c} < 0$. This means that $\overline{p} = \widetilde{s}_k(\overline{c};\overline{c},\overline{p}) < \overline{c}$ which contradicts with the fact that $\overline{c} \le \overline{p}$.

Next, we show by induction that $\Pi_e^k(b_j;x_j,p;\overline{c},\overline{p})$ is unimodal in $b_j$ and attains its *unconstrained maximizer* at $b_j = \widetilde{s}_k(x_j;\overline{c},\overline{p})$ for all $2 \le k \le n$. When $k=2$, we can simplify (EC.2) to

$$\begin{aligned}
\frac{\partial \Pi_e^2(b_j;x_j,p;\overline{c},\overline{p})}{\partial b_j} &= (1-\beta)\frac{F_e\left(\widetilde{s}_2^{-1}(b_j;\overline{c},\overline{p})\right)}{F_e\left(\widetilde{s}_2^{-1}(p;\overline{c},\overline{p})\right)}\left[\beta - \frac{V_e^1\left(x_j,b_j;\widetilde{s}_2^{-1}(b_j;\overline{c},\overline{p}),b_j\right) - \beta(1-\beta)(b_j-x_j)}{\beta\left(b_j - \widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p})\right)}\right] \\
&= (1-\beta)\frac{F_e\left(\widetilde{s}_2^{-1}(b_j;\overline{c},\overline{p})\right)}{F_e\left(\widetilde{s}_2^{-1}(p;\overline{c},\overline{p})\right)}\left[\beta - \frac{\beta(b_j-x_j) - \beta(1-\beta)(b_j-x_j)}{\beta\left(b_j - \widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p})\right)}\right] \\
&= \beta(1-\beta)\frac{F_e\left(\widetilde{s}_2^{-1}(b_j;\overline{c},\overline{p})\right)}{F_e\left(\widetilde{s}_2^{-1}(p;\overline{c},\overline{p})\right)}\frac{x_j - \widetilde{s}_2^{-1}(b_j;\overline{c},\overline{p})}{b_j - \widetilde{s}_2^{-1}(b_j;\overline{c},\overline{p})},
\end{aligned} \tag{EC.5}$$

where the second equality follows by (3). Note that, by (EC.4) and (EC.5), $(\partial/\partial b_j)\Pi_e^2(b_j;x_j,p;\overline{c},\overline{p})$ is positive when $b_j < \widetilde{s}_2(x_j;\overline{c},\overline{p})$ and is negative when $b_j > \widetilde{s}_2(x_j;\overline{c},\overline{p})$. Therefore, $\Pi_e^2(b_j;x_j,p;\overline{c},\overline{p})$ is unimodal in $b_j$ with a unique maximizer at $b_j = \widetilde{s}_2(x_j;\overline{c},\overline{p})$. This establishes the inductional basis. In the induction step, suppose that $\Pi_e^{k-1}(b_j;x_j,p;\overline{c},\overline{p})$ is unimodal in $b_j$ and has an unconstrained maximizer at $b_j = \widetilde{s}_{k-1}(x_j;\overline{c},\overline{p})$, we need to show $\Pi_e^k(b_j;x_j,p;\overline{c},\overline{p})$ is unimodal in $b_j$ and has an unconstrained maximizer at $b_j = \widetilde{s}_k(x_j;\overline{c},\overline{p})$. Note that it is sufficient to show that $(\partial/\partial b_j)\Pi_e^k(b_j;x_j,p;\overline{c},\overline{p})$ is positive when $b_j < \widetilde{s}_k(x_j;\overline{c},\overline{p})$, and $(\partial/\partial b_j)\Pi_e^k(b_j;x_j,p;\overline{c},\overline{p})$ is negative when $b_j > \widetilde{s}_k(x_j;\overline{c},\overline{p})$. Consider the case when $b_j < \widetilde{s}_k(x_j;\overline{c},\overline{p})$ first. In this case, since $\widetilde{s}_k^{-1}$ is increasing in its first argument, we have $\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}) \le x_j$. By the inductional hypothesis, the unconstrained optimizer of $V_e^{k-1}(x_j,b_j;\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}),b_j)$ is $\widetilde{s}_{k-1}(x_j;\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}),b_j)$ and it satisfies the following:

$$\widetilde{s}_{k-1}(x_j;\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}),b_j) \ge \widetilde{s}_{k-1}(\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p});\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}),b_j) = b_j,$$

where the first inequality follows since $\widetilde{s}_k^{-1}$ is increasing in its first argument and $\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}) \le x_j$. Then, by the inductional hypothesis that $V_e^{k-1}(x_j,b_j;\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}),b_j)$ is unimodal, its constrained maximizer is $b_j$, and $V_e^{k-1}(x_j,b_j;\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}),b_j) = \Pi_e^{k-1}(b_j;x_j,b_j;\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}),b_j) = \beta(1-\beta)^{k-2}(b_j - x_j)$. Thus, in the case when $b_j < \widetilde{s}_k(x_j;\overline{c},\overline{p})$,

$$\frac{\partial \Pi_e^k(b_j;x_j,p;\overline{c},\overline{p})}{\partial b_j} = \beta(1-\beta)^{k-1}\left[\frac{F_e\left(\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p})\right)}{F_e\left(\widetilde{s}_k^{-1}(p;\overline{c},\overline{p})\right)}\right]^{k-1}\frac{x_j - \widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p})}{b_j - \widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p})} > 0.$$

We now consider the case when $b_j > \widetilde{s}_k(x_j;\overline{c},\overline{p})$. In this case, since $\widetilde{s}_k^{-1}$ is increasing in its first argument, $x_j < \widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p})$. By (6) and the fact that $\widetilde{s}_{k-1}^{-1}$ is increasing in its first argument, we then have that $b_j \ge \widetilde{s}_{k-1}(x_j;\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}),b_j)$. This means that the unconstrained maximizer of $\Pi_e^{k-1}(\cdot;x_j,b_j;\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}),b_j)$, $\widetilde{s}_{k-1}(x_j;\widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}),b_j)$, is feasible. This means that

$V_e^{k-1}(x_j, b_j; \widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}), b_j) \geq \Pi_e^{k-1}(b_j; x_j, b_j; \widetilde{s}_k^{-1}(b_j;\overline{c},\overline{p}), b_j) = \beta(1-\beta)^{k-2}(b_j - x_j)$, and thus, for all $b_j > \widetilde{s}_k(x_j;\overline{c},\overline{p})$,

$$\frac{\partial \Pi_e^k\left(b_j; x_j, p; \overline{c}, \overline{p}\right)}{\partial b_j} \leq \beta(1-\beta)^{k-1}\left[\frac{F_e\left(\widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right)\right)}{F_e\left(\widetilde{s}_k^{-1}\left(p;\overline{c},\overline{p}\right)\right)}\right]^{k-1}\frac{x_j - \widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right)}{b_j - \widetilde{s}_k^{-1}\left(b_j;\overline{c},\overline{p}\right)} < 0. \qquad \text{(EC.6)}$$

This completes the induction step. Thus, we have shown, by construction, that after the incumbent drops out at price $p_{(n)}^n + \Delta$ with exactly $n$ entrants still in the auction, the game that ensues among the $n$ entrants have a symmetric Bayesian-Nash equilibrium: specifically, we have shown that following an ABN specified by $\{\widetilde{s}_k\}_{k=2}^n$ characterized in (5) and (6) is a best response given that all other remaining entrants employ the same strategy.

**(iii)** Note that for entrant $j$ who remains in the auction after the incumbent drop out, winning the auction below his cost will cause negative profit according to the payment rule. Therefore, he should drop out before the auction price hits his cost, namely $\widetilde{s}_k(x_j;\overline{c},\overline{p}) \geq x_j$. Note also that $\widetilde{s}_k(x_j;\overline{c},\overline{p}) < \overline{p}$ for $x_j < \overline{c}$ due to (ii) and (6). The result follows by setting $b_j = \widetilde{s}_k(x_j;\overline{c},\overline{p})$ in (EC.4).

**(iv)** Note that after the incumbent drops out, when $k$ entrants remain in the auction, the entrant with a cost $c_{(k-1)}^n$ will drop out at $p_{(k-1)}^n = \widetilde{s}_k(c_{(k-1)}^n; c_{(k)}^n, p_{(k)}^n)$. As a result, for any $x_j < c_{(k-1)}^n$, using $\widetilde{s}_k(c_{(k-1)}^n; c_{(k-1)}^n, p_{(k-1)}^n) = p_{(k-1)}^n$ as the boundary condition for (5) results in the same drop-out threshold as using $\widetilde{s}_k(c_{(k)}^n; c_{(k)}^n, p_{(k)}^n) = p_{(k)}^n$. Thus $\widetilde{s}_k(x_j; c_{(k)}^n, p_{(k)}^n) = \widetilde{s}_k(x_j; c_{(k-1)}^n, p_{(k-1)}^n) < \widetilde{s}_{k-1}(x_j; c_{(k-1)}^n, p_{(k-1)}^n)$ where the inequality follows by (ii). ■

**Proof of Proposition 1.** The boundary condition (6) can be easily verified. Next, we verify that the expressions in (7) satisfies the ordinary differential equations in (5). When $\beta \neq k^{-1}$, (7) indicates that

$$\begin{aligned}
\frac{\partial \widetilde{s}_k\left(x_j,\overline{c},\overline{p}\right)}{\partial x_j} &= \frac{\beta(k-1)}{\beta k - 1} + \left(\overline{p} - \underline{R} - \frac{\beta(k-1)}{\beta k-1}(\overline{c} - \underline{R})\right)\left(\frac{1}{\overline{c} - \underline{R}}\right)^{\frac{\beta(k-1)}{1-\beta}} \frac{\beta(k-1)}{1-\beta}(x_j - \underline{R})^{\frac{\beta k - 1}{1-\beta}} \\
&= \frac{\beta(k-1)}{(1-\beta)(x_j - \underline{R})}\left[\frac{(1-\beta)(x_j - \underline{R})}{\beta k - 1} + \left(\overline{p} - \underline{R} - \frac{\beta(k-1)}{\beta k - 1}(\overline{c} - \underline{R})\right)\left(\frac{x_j - \underline{R}}{\overline{c} - \underline{R}}\right)^{\frac{\beta(k-1)}{1-\beta}}\right] \\
&= \frac{\beta(k-1)}{(1-\beta)(x_j - \underline{R})}\left[\widetilde{s}_k\left(x_j,\overline{c},\overline{p}\right) - x_j\right] = \frac{\beta(k-1) f_e(x_j)}{(1-\beta)F_e(x_j)}\left[\widetilde{s}_k\left(x_j,\overline{c},\overline{p}\right) - x_j\right],
\end{aligned}$$

where the last equality follows since $F_e \sim U[\underline{R}, \overline{R}]$. When $\beta = k^{-1}$, (7) indicates that

$$\begin{aligned}
&\frac{\partial \widetilde{s}_k\left(x_j,\overline{c},\overline{p}\right)}{\partial x_j} = (\overline{p} - \underline{R})\left(\frac{1}{\overline{c} - \underline{R}}\right)^{\frac{\beta(k-1)}{1-\beta}} \frac{\beta(k-1)}{1-\beta}(x_j - \underline{R})^{\frac{\beta k - 1}{1-\beta}} - \log\left(\frac{x_j - \underline{R}}{\overline{c} - \underline{R}}\right) - 1 \\
&= \frac{\beta(k-1)}{(1-\beta)(x_j - \underline{R})}\left[(\overline{p} - \underline{R})\left(\frac{x_j - \underline{R}}{\overline{c} - \underline{R}}\right)^{\frac{\beta(k-1)}{1-\beta}} - \frac{(1-\beta)(x_j - \underline{R})}{\beta(k-1)}\log\left(\frac{x_j - \underline{R}}{\overline{c} - \underline{R}}\right) - \frac{(1-\beta)(x_j - \underline{R})}{\beta(k-1)}\right] \\
&= \frac{\beta(k-1)}{(1-\beta)(x_j - \underline{R})}\left[(\overline{p} - \underline{R})\left(\frac{x_j - \underline{R}}{\overline{c} - \underline{R}}\right)^{\frac{\beta(k-1)}{1-\beta}} - (x_j - \underline{R})\log\left(\frac{x_j - \underline{R}}{\overline{c} - \underline{R}}\right) - (x_j - \underline{R})\right]
\end{aligned}$$

$$= \frac{\beta(k-1)}{(1-\beta)(x_j - \underline{R})}\left[\widetilde{s}_k\left(x_j, \overline{c}, \overline{p}\right) - x_j\right] = \frac{\beta(k-1) f_e(x_j)}{(1-\beta)F_e(x_j)}\left[\widetilde{s}_k\left(x_j, \overline{c}, \overline{p}\right) - x_j\right],$$

where the third equality follows since $\beta = k^{-1}$, and the last equality follows since $F_e \sim U[\underline{R}, \overline{R}]$. ■

**Proof of Proposition 2.** By abuse of notation, we use $\widetilde{s}_k(x_j; \overline{c}, \overline{p}, \beta)$ and $\widetilde{s}_k(x_j; \overline{c}, \overline{p}, K)$ to capture the dependence of $\widetilde{s}_k$ on $\beta$ and $K$, respectively. Consider the sensitivity result on $\beta$ first. Fix $K$, $\overline{c}$ and $\overline{p}$. Note that $(1-\beta)^{-1}\beta(k-1)f_e(x_j)/F_e(x_j)$ strictly increases in $\beta$. Suppose, towards contradiction, that $\widetilde{s}_k$ is not decreasing in $\beta$, then there exist $0 < \beta_1 < \beta_2 < 1$ and some $x_j' \in [\underline{R}, \overline{c})$ such that $\widetilde{s}_k(x_j'; \overline{c}, \overline{p}, \beta_2) \geq \widetilde{s}_k(x_j'; \overline{c}, \overline{p}, \beta_1)$. This along with (5) means that for all $x_j \in (x_j', \overline{c}]$, $(\partial/\partial x_j)\widetilde{s}_k(x_j; \overline{c}, \overline{p}, \beta_2) > (\partial/\partial x_j)\widetilde{s}_k(x_j; \overline{c}, \overline{p}, \beta_1)$. Then, by (6), we arrive at a contradiction since $\overline{p} = \widetilde{s}_k(\overline{c}; \overline{c}, \overline{p}, \beta_2) > \widetilde{s}_{k-1}(\overline{c}; \overline{c}, \overline{p}, \beta_1) = \overline{p}$. The sensitivity result for $K$ can be obtained directly from (5). ■

**Proof of Theorem 2.** First, note that both $\Pi_0^m(b; x_0, p)$ and $(\partial/\partial b)\Pi_0^m(b; x_0, p)$ are well-defined for any $m = 1, \ldots, N$, $x_0 \in [\underline{R}, \overline{R}]$, $p \in (\max\{\underline{R} + \Delta, x_0\}, \infty)$, and $b \in [\max\{x_0, \underline{R} + \Delta\}, \infty]$, and the following holds:

$$\begin{aligned} \frac{\partial \Pi_0^m(b; x_0, p)}{\partial b} &= \frac{\left[F_e\left(b-\Delta\right)\right]^{m-1}(1-\beta)^m}{\left[F_e\left(p-\Delta\right)\right]^m}\left\{-m\left[\frac{V_0^{m-1}(x_0, b)}{(1-\beta)^m} - (b - x_0)\right] f_e\left(b-\Delta\right) + F_e\left(b-\Delta\right)\right\} \\ &\leq \frac{\left[F_e\left(b-\Delta\right)\right]^{m-1}(1-\beta)^m}{\left[F_e\left(p-\Delta\right)\right]^m}\left[-\frac{m\beta}{1-\beta}(b - x_0) f_e\left(b-\Delta\right) + F_e\left(b-\Delta\right)\right] := \zeta^m(b; x_0, p), \end{aligned} \quad \text{(EC.7)}$$

where the inequality is true because $V_0^{m-1}(x_0, b) = \max_{y \in [\max\{x_0, \underline{R}+\Delta\}, b]} \Pi_0^{m-1}(y; x_0, b) \geq \Pi_0^{m-1}(b; x_0, b) = (1-\beta)^{m-1}(b - x_0)$. Note that for $b \in [\max\{x_0, \underline{R} + \Delta\}, \overline{R} + \Delta]$,

$$\zeta^m(b; x_0, p) = (1-\beta)^m\left[F_e\left(b-\Delta\right)\right]^{m-1}\left[F_e\left(p-\Delta\right)\right]^{-m} f_e\left(b-\Delta\right)\left(H(b) - L^m(b; x_0)\right), \quad \text{(EC.8)}$$

where

$$H(b) := \frac{F_e(b-\Delta)}{f_e(b-\Delta)}, \text{ and } L^m(b; x) := \frac{m\beta(b-x)}{1-\beta},$$

while for $b \in (\overline{R} + \Delta, \infty)$,

$$\zeta^m(b; x_0, p) = (1-\beta)^m\left[F_e\left(p-\Delta\right)\right]^{-m} > 0. \quad \text{(EC.9)}$$

We now prove (i) and (ii) of the theorem in sequence.

**(i)** First note that for all $x_0 \in [\underline{R}, \overline{R}]$, $m^\star(x_0)$ is well defined since $\frac{(1-\beta)F_e(b-\Delta)}{\beta(b-x_0)f_e(b-\Delta)} > 0$ for $b \in (\max\{x_0, \underline{R} + \Delta\}, \overline{R} + \Delta]$. Then, fix $x_0$ and $m$ such that $m > m^\star(x_0)$. We next show that $b_m^\star(x_0)$ and $p_m^\star(x_0)$ are well-defined. By the definition of $m^\star$ and the fact that $H(b)$ and $L^m(b; x_0)$ are both continuous in $b$, there exists some $b' \in (\max\{x_0, \underline{R} + \Delta\}, \overline{R} + \Delta)$ such that $H(b') < L^m(b'; x_0)$, and hence, $\zeta^m(b';x_0,p)<0$. Therefore, when $x_0>\underline{R}+\Delta$, the set $\{b\in(x_0,\overline{R}+\Delta]:H(b)-L^m(b,x_0)<0\}$ is not empty. Thus $b^\star_m(x_0)$ is well-defined. We now show below that $p^\star_m(x_0)$ is well-defined. To that end, for all $p\in(\max\{\underline{R}+\Delta,x_0\},\infty)$, $b\in[\max\{x_0,\underline{R}+\Delta\},\infty]$, define

$$D_0^m(b;x_0,p):=\Pi_0^m(b;x_0,p)-\Pi_0^m(b^\star_m(x_0);x_0,p). \tag{EC.10}$$

Note that for all $b\geq\overline{R}+\Delta$, the following inequality holds:

$$\begin{aligned}
D_0^m(b;x_0,\overline{R}+\Delta)&=(1-\beta)^m(b-x_0)-\Pi_0^m(b^\star_m(x_0);x_0,\overline{R}+\Delta)\\
&\geq(1-\beta)^m(b-x_0)-\int_{b^\star_m(x_0)}^{\overline{R}+\Delta}V_0^{m-1}(x_0,z)g_e^m(z-\Delta;p-\Delta)\,dz-(1-\beta)^m(b^\star_m(x_0)-x_0)\\
&\geq(1-\beta)^m(b-b^\star_m(x_0))-\max_{z\in[b^\star_m(x_0),\overline{R}+\Delta]}V_0^{m-1}(x_0,z),
\end{aligned}$$

where the first inequality follows since probability is no larger than one, and the second inequality follows since $g_e^m$ is the p.d.f. of a probability distribution. Since $\lim_{b\to\infty}D_0^m(b;x_0,\overline{R}+\Delta)\geq\lim_{b\to\infty}(1-\beta)^m(b-b^\star_m(x_0))>0$, there exists some $p'\geq\overline{R}+\Delta$ such that $D_0^m(p';x_0,\overline{R}+\Delta)>0$; so $p^\star_m(x_0)$ is well-defined.

Note that for $x_0$ and $m$ that satisfy that $m>m^\star(x_0)$, the result $p^\star_m(x_0)>b^\star_m(x_0)\geq x_0$ follows directly from the definitions of $b^\star_m(x_0)$ and $p^\star_m(x_0)$. Moreover, $b^\star_m(x_0)=\underline{R}+\Delta>x_0$ when $x_0<\underline{R}+\Delta$, and $b^\star_m(x_0)>x_0$ when $x_0>\underline{R}+\Delta$ (this is because $F_e(x_0-\Delta)/f_e(x_0-\Delta)-m\beta(x_0-x_0)/(1-\beta)>0$ when $x_0>\overline{R}+\Delta$). Therefore, $b^\star_m(x_0)>x_0$ when $x_0\neq\underline{R}+\Delta$ is also true.

We now prove the sensitivity results. Note that $\frac{(1-\beta)F_e(b-\Delta)}{\beta(b-x_0)f_e(b-\Delta)}$ is strictly increasing in $x_0$. Therefore, per its definition, $m^\star(x_0)$ is nondecreasing in $x_0$. Next, we consider $b^\star_m(x_0)$. We first fix $x_0$ and vary $m$ such that $m>m^\star(x_0)$. If $x_0\leq\underline{R}+\Delta$, $b^\star_m(x_0)$ is independent of $m$; otherwise (i.e., $x_0>\underline{R}+\Delta$), since $L^m(b,x_0)$ is increasing in $m$, $\inf\{b\in(x',\overline{R}+\Delta]:H(b)-L^m(b,x_0)<0\}$ is nonincreasing in $m$. Thus, $b^\star_m(x_0)$ is nonincreasing in $m$. Next, we fix $m$ and vary $x_0$ such that $m>m^\star(x_0)$. When $x_0\leq\underline{R}+\Delta$, $b^\star_m(x_0)=\underline{R}+\Delta$. When $x_0>\underline{R}+\Delta$, $b^\star_m(x_0)\geq x_0>\underline{R}+\Delta$; moreover, since $L^m(b,x_0)$ decreases in $x_0$, so $\inf\{b\in(x_0,\overline{R}+\Delta:H(b)-L^m(b,x_0)<0\}$ is nondecreasing in $x_0$. Thus, $b^\star_m(x_0)$ is nondecreasing in $x_0$.

Recall that we have fixed $x_0$ and $m$ such that $m>m^\star(x_0)$. Let $m_0=m^\star(x_0)$ for notational simplicity. So, to prove the sensitivity result for $p^\star_m(x_0)$, it is sufficient to prove the statements below for all $m'=m_0+1,\ldots,m$,

$$\begin{aligned}
&\mathbb{A}_1^{m'}(x_0): &&p^\star_{m'+1}(x_0)\geq p^\star_{m'}(x_0);\\
&\mathbb{A}_2^{m'}: &&p^\star_{m'}(x)\text{ is nonincreasing in }x\text{ for all }x\text{ such that }m'>m^\star(x).
\end{aligned}$$

We will use an inductional argument to prove the sensitivity result for $p^\star_{m'}(x')$ and the structure of the optimal solution. Specifically, consider the inductional hypothesis below: for all $m' = m_0 + 1, \ldots, m$

$$\mathbb{H}^{m'}(x_0): \quad \Pi_0^{m'}(b; x_0, p) \text{ is increasing in } b \text{ on } [\max\{x_0, \underline{R}+\Delta\}, b^\star_{m'}(x_0)] \text{ and } [p^\star_{m'}(x_0), p] \text{ respectively;}$$

The following lemma is useful in our induction argument:

Lemma EC.1. *Fix some $x' \in [\underline{R}, \overline{R}]$. The following statements hold:*

*(a). For all $m' = \{0, \ldots, N\}$, $V_0^{m'}(x', p)$ is nondecreasing in $p$;*

*(b). For all $m' = \{0, \ldots, N-1\}$ and $p \in (\max\{x', \underline{R}+\Delta\}, \infty)$, $V_0^{m'+1}(x', p) \geq (1-\beta) V_0^{m'}(x', p)$;*

*(c). For all $m' = \{0, \ldots, N\}$ and $p \in (\max\{x', \underline{R}+\Delta\}, \infty)$, $(1-\beta)^{-m'} V_0^{m'}(x', p) - (p - x')$ is non-increasing in $x'$ on $[\underline{R}, \overline{R}]$;*

*(d). Fix $m' \in \{1, \ldots, N\}$ such that $m' > m^\star(x')$. Then, for all $p \in (\max\{x', \underline{R}+\Delta\}, \infty)$ and $b \in (b^\star_{m'}(x'), p^\star_{m'}(x'))$, $\Pi_0^{m'}(b^\star_{m'}(x'); x', p) = \Pi_0^{m'}(p^\star_{m'}(x'); x', p) > \Pi_0^{m'}(b; x', p)$.*

Using Lemma EC.1, we prove the last key step in the proof of (i):

Lemma EC.2. *For any $m' = m_0 + 1, \ldots, m$, $\mathbb{H}^{m'}(x_0), \mathbb{A}_1^{m'}(x_0)$ and $\mathbb{A}_2^{m'}$ hold.*

Note that Lemma EC.2 immediately implies that $p^\star_m(x_0)$ is nonincreasing in $x_0$ and nondecreasing in $m$ for all $x_0, m$ such that $m > m^\star(x_0)$.

**(ii)** Note that when $p = \max\{x_0, \underline{R}+\Delta\}$, the feasible set in (8) is a singleton $\{p\}$, so the stated result trivially holds. Next, consider the case when $p > \max\{x_0, \underline{R}+\Delta\}$.

We first consider the case $m \leq m^\star(x_0)$. Fix $x_0$ and $m$ that satisfy such condition. Since $L^m(b; x_0)$ increases in $m$, and $H(b)$ does not depend on $m$, we conclude that $H(b) \geq L^{m'}(b; x_0)$ for all $m' = 1, \ldots, m$ and all $b \in [\max\{x_0, \underline{R}+\Delta\}, \overline{R}+\Delta]$. We now argue by induction that $(\partial/\partial b)\Pi_0^{m'}(b; x_0, p) = \zeta^{m'}(b; x_0, p) \geq 0$ for all $m' = 1, \ldots, m$, $p \in (\max\{x_0, \underline{R}+\Delta\}, \infty)$, and $b \in [\max\{x_0, \underline{R}+\Delta\}, p]$. When $m = 1$, due to (10), the inequality in (EC.7) holds with equality, so $(\partial/\partial b)\Pi_0^1(b; x_0, p) = \zeta^1(b; x_0, p)$. For $b \in [\max\{x_0, \underline{R}+\Delta\}, \overline{R}+\Delta]$, $\zeta^1(b; x_0, p) \geq 0$ due to (EC.8) and $H(b) \geq L^1(b; x_0)$; for $b \in (\overline{R}+\Delta, \infty]$, by (EC.9), $\zeta^1(b; x_0, p) > 0$. This completes the inductional basis. Suppose that the inductional hypothesis holds for some $m'-1$; this means that $\Pi_0^{m'-1}(b; x_0, p)$ is nondecreasing in $b$, so $s^\star_{m'-1}(x_0; p) = p$ and $V_0^{m'-1}(x_0, p) = (1-\beta)^{m'-1}(p - x_0)$. Hence, by (EC.7), $(\partial/\partial b)\Pi_0^{m'}(b; x_0, p) = \zeta^{m'}(b; x_0, p)$. By a similar argument as in the proof of the inductional basis, we conclude that $(\partial/\partial b)\Pi_0^{m'}(b; x_0, p) = \zeta^{m'}(b; x_0, p) \geq 0$, which completes the inductional step. Finally, since $(\partial/\partial b)\Pi_0^m(b; x_0, p) \geq 0$, $s^\star_m(x_0; p) = p$.

Next, we consider the case $m > m^\star(x_0)$. $\mathbb{H}^m(x_0)$ combined with Lemma EC.1 part (d) implies that $\Pi_0^m(b^\star_m(x_0); x_0, p) \geq \Pi_0^m(b; x_0, p)$ for all $b \in [\max\{x_0, \underline{R}+\Delta\}, p^\star_m(x_0)]$ and $\Pi_0^m(b; x_0, p)$ is nondecreasing in $b$ on $(p^\star_m(x_0), \infty)$; this immediately implies the structure of the optimal solution.

Finally, we prove the sensitivity result of $s^\star_m(x_0;p)$. First, it is straightforward to verify by definition that $s^\star_m(x_0;p)$ is nondecreasing in $p$ for any $x_0$ and $m$. Next, fix $p$ and $m$, and take any $x_0 < x_0'$. If $m \le m^\star(x_0')$, then $s^\star_m(x_0';p) = p \ge s^\star_m(x_0;p)$. Otherwise, $m > m^\star(x_0') \ge m^\star(x_0)$, we consider two cases: If $p > p^\star_m(x_0')$, then $s^\star_m(x_0';p) = p \ge s^\star_m(x_0;p)$; otherwise, $p \le p^\star_m(x_0') \le p^\star_m(x_0)$, so $s^\star_m(x_0';p) = b^\star_m(x_0') \wedge p \ge b^\star_m(x_0) \wedge p = s^\star_m(x_0;p)$. Hence, $s^\star_m(x_0;p)$ is nondecreasing in $x_0$. Finally, fix $x_0$ and $p$, and take any $m < m'$. If $m \le m^\star(x_0')$, then $s^\star_m(x_0;p) = p \ge s^\star_{m'}(x_0;p)$. Otherwise, $m > m^\star(x_0') \ge m^\star(x_0)$, we consider two cases: If $p \le p^\star_{m'}(x_0)$, then $s^\star_{m'}(x_0;p) = b^\star_{m'}(x_0) \wedge p \le b^\star_m(x_0) \wedge p \le s^\star_m(x_0;p)$; otherwise, $p > p^\star_{m'}(x_0) \ge p^\star_m(x)$, so $s^\star_m(x_0;p) = p = s^\star_{m'}(x_0;p)$. Hence, $s^\star_m(x_0;p)$ is nonincreasing in $m$. ■

**Proof of Lemma EC.1.** Fix any $x' \in [\underline{R}, \overline{R}]$. For any $m' \in \{1,\dots,N\}$ and any $p \in (\max\{x', \underline{R} + \Delta\}, \infty)$, let $Y_p^{m'}$ denote the random variable with c.d.f. $\mathbf{P}(Y_p^{m'} \le b) := G_e^{m'}(b-\Delta; p-\Delta)$. Note that $Y_p^{m'}$ is stochastically nondecreasing (resp. nonincreasing) in $p$ (resp. $m'$). Note that $\Pi_0^{m'}(b;x',p) = \mathbf{E}[\Psi^{m'}(Y_p^{m'}, x', b)]$ where

$$\Psi^{m'}(z,x',b) := \begin{cases} V_0^{m'-1}(x',z), & \text{if } z \in [b,\infty) \\ (1-\beta)^{m'}(b-x'), & \text{if } z \in (\max\{x', \underline{R}+\Delta\}, b) \end{cases}$$

We prove (a), (b) and (c) by induction on $m'$.

**(a)** When $m' = 0$, $V_0^0(x',p) = p - x'$ by definition, so $V_0^0(x',p)$ is increasing in $p$. Suppose that $V_0^{m'-1}(x',p)$ is nondecreasing in $p$ for some $m' \ge 1$, we now show that $V_0^{m'}(x',p)$ is also nondecreasing in $p$. Indeed, since for all $z > b$, $V_0^{m'-1}(x',z) \ge \Pi_0^{m'-1}(z;x',z) = (1-\beta)^{m'-1}(z-x') > (1-\beta)^{m'}(b-x')$ and $V_0^{m'-1}(x',p)$ is nondecreasing in $p$, we conclude that $\Psi^{m'}(z,x',b)$ is nondecreasing in $z$. Thus, $\Pi_0^{m'}(b;x',p) = \mathbf{E}[\Psi^{m'}(Y_p^{m'}, x', b)]$ is nondecreasing in $p$ as $Y_p^{m'}$ is stochastically nondecreasing in $p$. Since the feasible region in (8) increases in $p$, $V_0^{m'}(x',p)$ is nondecreasing in $p$.

**(b)** When $m' = 0$, $V_0^1(x',p) \ge \Pi_0^1(p;x',p) = (1-\beta)(p-x') = (1-\beta)V_0^0(x',p)$ where the last equality holds by (10). Suppose that $V_0^{m'}(x',p) \ge (1-\beta)V_0^{m'-1}(x',p)$ for some $m' \ge 1$. This means that $\Psi^{m'+1}(z,x',b) \ge (1-\beta)\Psi^{m'}(z,x',b)$. Hence, $\Pi_0^{m'+1}(b;x',p) = \mathbf{E}[\Psi^{m'+1}(Y_p^{m'+1}, x', b)] \ge \mathbf{E}[(1-\beta)\Psi^{m'}(Y_p^{m'+1}, x', b)] \ge (1-\beta)\mathbf{E}[\Psi^{m'}(Y_p^{m'}, x', b)] = (1-\beta)\Pi_0^{m'}(b;x',p)$, where the second inequality follows since $Y_p^{m'}$ is stochastically no larger than $Y_p^{m'+1}$ and $\Psi^{m'}(z,x',b)$ is nondecreasing in $z$ (as we have proven in part (a)). Hence, $V_0^{m'+1}(x',p) \ge (1-\beta)V_0^{m'}(x',p)$.

**(c)** When $m' = 0$, $(1-\beta)^{-0}V_0^0(x',p) - (p-x') = 0$ is independent of $x'$. Suppose $(1-\beta)^{-(m'-1)}V_0^{m'-1}(x',p) - (p-x')$ is nonincreasing in $x'$ for some $m' \ge 1$. This means that $(\partial/\partial x_0)V_0^{m'-1}(x';p) \le -(1-\beta)^{m'-1}$, so

$$\frac{\partial V_0^{m'}(x';p)}{\partial x_0} = \left.\frac{\partial \Pi_0^{m'}(b;x',p)}{\partial x_0}\right|_{b=b^*}$$

$$
\begin{aligned}
&= \int_{b^*}^{p} \frac{\partial V_0^{m'-1}(x',z)}{\partial x_0} g_e^{m'}\left(z-\Delta;p-\Delta\right) dz - G_e^{m'}\left(b^*-\Delta;p-\Delta\right)(1-\beta)^{m'} \\
&\leq -(1-\beta)^{m'} \int_{b^*}^{p} g_e^{m'}\left(z-\Delta;p-\Delta\right) dz - G_e^{m'}\left(b^*-\Delta;p-\Delta\right)(1-\beta)^{m'} = -(1-\beta)^{m'},
\end{aligned}
$$

where $b^*$ denote the optimal solution to $\max_{b\in[\max\{x',\underline{R}+\Delta\},p]} \Pi_0^{m'}(b;x',p)$, the first equality follows by the Envelop Theorem. This inequality implies that $(1-\beta)^{-m'}V_0^{m'}(x',p)-(p-x')$ is nonincreasing in $x'$.

**(d)** Note that $b^\star_{m'}(x') < \overline{R}+\beta^{-1}K$: when $x' \leq \underline{R}+\Delta$, it trivially holds; when $x' > \underline{R}+\beta^{-1}K$, it also holds since $H(b)-L^{m'}(b,x')$ is continuous in $b$. This implies that there exist some $\epsilon > 0$ independent of $p$ such that $(b^\star_{m'}(x'), b^\star_{m'}(x')+\epsilon] \subseteq (\max\{x',\underline{R}+\Delta\},\overline{R}+\Delta]$, *and* for all $p$ and $b \in (b^\star_{m'}(x'), b^\star_{m'}(x')+\epsilon]$,

$$
\frac{\partial \Pi_0^{m'}(b;x',p)}{\partial b} \leq \left[F_e\left(b-\Delta\right)\right]^{m'-1} \left[\frac{1-\beta}{F_e(p-\Delta)}\right]^{m'} f_e\left(b-\Delta\right)\left(H(b)-L^{m'}(b;x')\right) < 0, \tag{EC.11}
$$

where the first inequality follows by (EC.7), and the second inequality holds by considering three separate cases: when $x' < \underline{R}+\Delta$, it trivially holds; when $x' = \underline{R}+\Delta$, it must hold because otherwise, by convexity of $H(b)$, $H(b) \geq L^{m'}(b;x')$ for all $b \in [\underline{R}+\Delta, \overline{R}+\Delta]$ which contradicts with $m' > m^\star(x')$; when $x' > \underline{R}+\Delta$, it holds by the definition of $b^\star_{m'}(x')$ and the fact that $H(b)-L^{m'}(b,x')$ is continuous on $[x',\overline{R}+\Delta]$. Hence, (EC.11) implies that $D_0^{m'}(b;x',\overline{R}+\Delta) < 0$ for $b \in (b^\star_{m'}(x'), b^\star_{m'}(x')+\epsilon]$. Since $D_0^{m'}(b;x',\overline{R}+\Delta)$ is continuous in $b$, we conclude that

$$
\Pi_0^{m'}(b^\star_{m'}(x');x',\overline{R}+\Delta) = \Pi_0^{m'}(p^\star_{m'}(x');x',\overline{R}+\Delta) > \Pi_0^{m'}(b;x',\overline{R}+\Delta), \forall b \in (b^\star_{m'}(x'), p^\star_{m'}(x')).
$$

Note that the following identity holds: for any $p' \in (b^\star_{m'}(x'),\infty)$ and $p \in (\max\{x',\underline{R}+\Delta\},\infty)$,

$$
\begin{aligned}
D_0^{m'}(p';x',p) &= \int_{b^\star_{m'}(x')}^{p'} \frac{\partial \Pi_0^{m'}(b_0;x',p)}{\partial b_0} db_0 = \left[\frac{F_e\left(\overline{R}\right)}{F_e\left(p-\Delta\right)}\right]^{m'} \int_{b^\star_{m'}(x')}^{p'} \frac{\partial \Pi_0^{m'}(b_0;x',\overline{R}+\Delta)}{\partial b_0} db_0 \\
&= \left[F_e\left(p-\Delta\right)\right]^{-m'} D_0^{m'}(p';x',\overline{R}+\Delta),
\end{aligned}
$$

where the second equality follows by the first equality in (EC.7); the desired result follows by noting that $D_0^{m'}(p';x',p)=0 \Leftrightarrow D_0^{m'}(p';x',\overline{R}+\Delta)=0$ and $D_0^{m'}(p';x',p)<0 \Leftrightarrow D_0^{m'}(p';x',\overline{R}+\Delta)<0$, for any $p' \in (b^\star_{m'}(x'),\infty)$ and $p \in (\max\{x',\underline{R}+\Delta\},\infty)$. ■

**Proof of Lemma EC.2.** We first state a useful claim whose proof is relegated to the end for expositional clarity.

CLAIM EC.1. *For any $m' \in \{1,\ldots,N\}, x' \in [\underline{R},\overline{R}]$ that satisfies $m' > m^\star(x')$,*

$$
\begin{cases}
\zeta^{m'}(b;x',p) > 0, \forall b \in [\max\{x',\underline{R}+\Delta\}, b^\star_{m'}(x')) \cup (b^{\star\star}_{m'}(x'),\infty] \\
\zeta^{m'}(b;x',p) < 0, \forall b \in (b^\star_{m'}(x'), b^{\star\star}_{m'}(x'))
\end{cases}, \tag{EC.12}
$$

*where* $b^{\star\star}_{m'}(x') := \sup\{b \in (x', \overline{R}+\Delta] : H(b) - L^{m'}(b, x') < 0\}$*. Moreover, for any* $m'' \in \{1, \ldots, N\}, x'' \in [\underline{R}+\Delta, \overline{R}+\Delta]$ *such that* $m'' \geq m'$, $x'' \leq x'$*, the following inequalities hold:*

$$p^{\star}_{m''}(x') > b^{\star\star}_{m''}(x') \geq b^{\star}_{m'}(x') \geq b^{\star}_{m''}(x'), \tag{EC.13}$$

$$p^{\star}_{m'}(x') > b^{\star\star}_{m'}(x') \geq b^{\star}_{m'}(x''), \quad \text{and} \quad b^{\star\star}_{m'}(x'') \geq b^{\star\star}_{m'}(x') \tag{EC.14}$$

The proof of Lemma EC.2 proceeds in two steps: first, we prove that for any $m' = m_0 + 1, \ldots, m$, $\mathbb{H}^{m'}(x_0)$ implies $\{\mathbb{A}^{m'}_1(x_0), \mathbb{A}^{m'}_2\}$; second, we use an induction argument to show that $\{\mathbb{H}^{m'}(x_0)\}^{m}_{m'=m_0+1}$ hold.

*Step 1: For any* $m' = m_0 + 1, \ldots, m$, $\mathbb{H}^{m'}(x_0) \Rightarrow \{\mathbb{A}^{m'}_1(x_0), \mathbb{A}^{m'}_2\}$.

We prove $\mathbb{A}^{m'}_1(x_0)$ by contradiction. Suppose, towards contradiction, that $p^{\star}_{m'+1}(x_0) < p^{\star}_{m'}(x_0)$. First note that the identity $V^{m'+1}_0(x_0, p^{\star}_{m'+1}(x_0)) = \Pi^{m'+1}_0(b^{\star}_{m'+1}(x_0); x_0, p^{\star}_{m'+1}(x_0))$ holds due to the following reason: $\Pi^{m'+1}_0(b^{\star}_{m'+1}(x_0); x_0, p^{\star}_{m'+1}(x_0)) \geq \Pi^{m'+1}_0(b; x_0, p^{\star}_{m'+1}(x_0))$ for all $b \in [b^{\star}_{m'+1}(x_0), p^{\star}_{m'+1}(x_0)]$ by Lemma EC.1 part (d) and $\Pi^{m'+1}_0(b; x_0, p^{\star}_{m'+1}(x_0))$ is increasing in $b$ on $[\max\{x_0, \underline{R}+\Delta\}, b^{\star}_{m'+1}(x_0)]$ (this is because $(\partial/\partial b)\Pi^{m'+1}_0(b; x_0, p^{\star}_{m'+1}(x_0)) = \zeta^{m'+1}(b; x_0, p^{\star}_{m'+1}(x_0)) > 0$ for all $b \in [\max\{x_0, \underline{R}+\Delta\}, b^{\star}_{m'+1}(x_0)]$, where the equality follows by (EC.7), $\mathbb{H}^{m'}(x_0)$ and (EC.13), and the inequality follows by (EC.12)). Then, we have

$$\begin{aligned}
V^{m'}_0(x_0, p^{\star}_{m'+1}(x_0)) &\leq \frac{V^{m'+1}_0(x_0, p^{\star}_{m'+1}(x_0))}{1-\beta} = \frac{\Pi^{m'+1}_0(b^{\star}_{m'+1}(x_0); x_0, p^{\star}_{m'+1}(x_0))}{1-\beta} \\
&= \frac{\Pi^{m'+1}_0(p^{\star}_{m'+1}(x_0); x_0, p^{\star}_{m'+1}(x_0))}{1-\beta} = (1-\beta)^{m'}(p^{\star}_{m'+1}(x_0) - x_0) \\
&= \Pi^{m'}_0(p^{\star}_{m'+1}(x_0); x_0, p^{\star}_{m'+1}(x_0)) < \Pi^{m'}_0(b^{\star}_{m'}(x_0); x_0, p^{\star}_{m'+1}(x_0)) \leq V^{m'}_0(x_0, p^{\star}_{m'+1}(x_0)),
\end{aligned}$$

where the first inequality holds by Lemma EC.1 part (b), the second equality holds by Lemma EC.1 part (d), the second inequality holds by $\mathbb{H}^{m'}(x_0)$ and $b^{\star}_{m'}(x_0) < p^{\star}_{m'+1}(x_0) < p^{\star}_{m'}(x_0)$ (the first inequality follows by (EC.13)), and the last inequality holds as $p^{\star}_{m'+1}(x_0) > b^{\star}_{m'}(x_0)$; and this is a contradiction. So $\mathbb{A}^{m'}_1(x_0)$ holds.

To prove $\mathbb{A}^{m'}_2$, take any $x', x \in [\underline{R}, \overline{R}]$ such that $m' > m^{\star}(x)$ and $x' < x$. The following holds:

$$\begin{aligned}
\Pi^{m'}_0(p^{\star}_{m'}(x); x', p) - \Pi^{m'}_0(b^{\star}_{m'}(x'); x', p) &\leq \Pi^{m'}_0(p^{\star}_{m'}(x); x', p) - \Pi^{m'}_0(b^{\star}_{m'}(x); x', p) \\
= \int_{b^{\star}_{m'}(x)}^{p^{\star}_{m'}(x)} \frac{\partial \Pi^{m'}_0(b; x', p)}{\partial b} db &\leq \int_{b^{\star}_{m'}(x)}^{p^{\star}_{m'}(x)} \frac{\partial \Pi^{m'}_0(b; x, p)}{\partial b} db = 0,
\end{aligned}$$

where the first inequality holds since $\Pi^{m'}_0(b; x', p)$ is decreasing in $b$ on $(b^{\star}_{m'}(x'), b^{\star\star}_{m'}(x'))$ (by (EC.7) and (EC.12)) and $b^{\star}_{m'}(x) \in [b^{\star}_{m'}(x'), b^{\star\star}_{m'}(x')]$ (by (EC.14)), the second inequality holds since $(\partial/\partial b)\Pi^{m'}_0(b; x, p) \geq (\partial/\partial b)\Pi^{m'}_0(b; x', p)$ by (EC.7) and Lemma EC.1 part (c), and the last equality follows by Lemma EC.1 part (d). Then, $\mathbb{H}^{m'}(x_0) \Rightarrow \mathbb{A}^{m'}_2$.

*Step 2:* $\{\mathbb{H}^{m'}(x_0)\}_{m'=m_0+1}^{m}$ *holds.*

We prove by induction on $m'$. When $m' = m_0+1$, note that since $V_0^0(x_0,z) = z - x_0$ by (10), when $m_0 = 0$, $s^\star_{m_0}(x_0;z) = z$, and by (ii), when $m_0 \geq 1$, the following identify holds:

$$V_0^{m_0}(x_0,z) = (1-\beta)^{m_0}(z - x_0). \tag{EC.15}$$

Thus, $(\partial/\partial b_0)\Pi_0^{m_0+1}(b;x_0,p) = \zeta^{m_0+1}(b;x_0,p)$. So $\mathbb{H}_1^{m_0+1}(x_0)$ holds by (EC.12) and (EC.14). Suppose $\mathbb{H}_1^{m'-1}(x_0)$ holds for some $m'-1 \geq m_0+1$, we need to show that $\mathbb{H}^{m'}(x_0)$ holds. By Step 1, $\mathbb{H}_1^{m'-1}(x_0) \Rightarrow \mathbb{A}_1^{m'-1}(x_0)$, so $p^\star_{m'}(x_0) \geq p^\star_{m'-1}(x_0)$. Note also that $b^\star_{m'}(x_0) \leq b^\star_{m'-1}(x_0)$ by (EC.13). Hence, for all $b_0 \in [\max\{x_0, \underline{R}+\Delta\}, b^\star_{m'}(x_0)) \cup (p^\star_{m'}(x_0), p]$, $(\partial/\partial b_0)\Pi_0^{m'}(b_0;x_0,p) = \zeta^{m'}(b_0;x_0,p) > 0$, where the equality follows since the inequality in (EC.7) holds with equality by $\mathbb{H}^{m'-1}(x_0)$, and the inequality follows by (EC.12) in Claim EC.1. Hence, $\mathbb{H}^{m'}(x_0)$ holds. This completes the proof in Step 2. To complete the whole proof, we now prove Claim EC.1.

**Proof of Claim EC.1.** Take any $m' \in \{1,\dots,N\}, x' \in [\underline{R},\overline{R}]$ that satisfies $m' > m^\star(x')$. Similar to how we prove $b^\star_m(x_0)$ is well-defined at the beginning of the proof of Theorem 2 part (i-b), we can show that $b^{\star\star}_{m'}(x')$ is well-defined. Since $L^m(b;x)$ is increasing in $m$ and decreasing in $x$, $m' > m^\star(x')$ implies that $m' > m^\star(x'')$ for all $x'' \leq x'$, so $b^{\star\star}_{m'}(x'')$ is well-defined; similarly, $m'' > m^\star(x)$ for all $m'' \geq m'$ and $b^{\star\star}_{m''}(x')$ is well-defined. Since $H(b)$ is convex and increasing, and $L^{m'}(b;x')$ is linear and increasing in $b$, it is easy to verify that

$$\begin{cases} H(b) - L^{m'}(b;x') > 0,\ \forall b \in [\max\{x', \underline{R}+\Delta\}, b^\star_{m'}(x')) \cup (b^{\star\star}_{m'}(x'), \overline{R}+\Delta] \\ H(b) - L^{m'}(b;x') < 0,\ \forall b \in (b^\star_{m'}(x'), b^{\star\star}_{m'}(x')) \end{cases}, \tag{EC.16}$$

and, for any $x'' \leq x'$, $m'' \geq m'$,

$$b^\star_{m''}(x') \leq b^\star_{m'}(x') \leq b^{\star\star}_{m'}(x') \leq b^{\star\star}_{m''}(x'), \tag{EC.17}$$

$$b^\star_{m'}(x'') \leq b^\star_{m'}(x') \leq b^{\star\star}_{m'}(x') \leq b^{\star\star}_{m'}(x''). \tag{EC.18}$$

Hence, (EC.12) holds by (EC.8), (EC.9) and (EC.16). Note that (EC.12) holds when we replace $m'$ by any $m'' \geq m'$. Thus, by (EC.7), for all $m'' \geq m'$, $\Pi_0^{m''}(b;x',p)$ is decreasing in $b$ on $(b^\star_{m''}(x'), b^{\star\star}_{m''}(x'))$ which, combined with the definition of $p^\star_{m''}(x')$, implies that $p^\star_{m''}(x') > b^{\star\star}_{m''}(x')$. Therefore, for any $m'' \geq m'$, (EC.13) holds by (EC.17). Similarly, for all $x'' < x'$, due to (EC.18) and $\Pi_0^{m'}(b;x',p)$ is decreasing in $b$ on $(b^\star_{m'}(x'), b^{\star\star}_{m'}(x'))$, we conclude (EC.14) holds. ■

**Proof of Proposition 3.** Fix some $x_0 \in [\underline{R},\overline{R}]$. For notational simplicity, we suppress the dependency of $b^\star_m(x_0)$, $p^\star_m(x_0)$, $m^\star(x_0)$ and $\widetilde{m}(x_0)$ on $x_0$ in the proof. Note that in equilibrium, $\overline{p} \in [b^\star_N, \overline{R}]$. Hence, we only need to consider four cases: (a) $\overline{p} = \overline{R}$, (b) $\overline{p} \in P_m$ for $m = m^\star+1,\dots,\widetilde{m}$, (c) $\overline{p} \in B_m$ for $m = m^\star+1,\dots,N$, and (d) $\overline{p} = b^\star_m$ for $m = m^\star+1,\dots,N$. We now show that each of the four terms in (12) corresponds to buyer's procurement cost on sample paths that satisfy the conditions (a)-(d).

Case (a) Per the definition of $\widetilde{m}$, we have $\overline{R} \geq p_m^\star$ for all $m = m^\star + 1, \ldots, \widetilde{m}$. Therefore, per Theorem 2 part (ii), the incumbent will drop out at $\overline{R}$ if and only if there are $i \leq \widetilde{m}$ entrants joining the auction. When the incumbent drops out at $\overline{R}$ and $i \leq \widetilde{m}$ entrants join the auction, the expected procurement cost equals $h(i, \overline{R})$. Hence, the first term of (12) corresponds to the buyer's cost for sample paths that satisfy (a).

Case (b) Since the auction price descends, per Theorem 2 part (ii), $\{\overline{p} \in P_m\}$ happens if and only if the following conditions hold:

(P1) The incumbent does not drop out at $\overline{R}$ (i.e., at least $\widetilde{m}+1$ entrants join the auction);

(P2) The incumbent does not drop out in $\{P_j\}_{j=m+1}^{\widetilde{m}}$ (i.e. $X_{(j+1:N)} + \Delta \leq p_j^\star$ for all $j = m+1, \ldots, \widetilde{m}$);

(P3) The incumbent drops out at $\overline{p} \in P_m$ (i.e., $X_{(m+1:N)} + \Delta = \overline{p}$).

which implies that $\Phi'_{m+1}(\overline{p})$ is the p.d.f. of $\overline{p}$ when $\overline{p} \in P_m$. Moreover, P3 implies that there are exactly $m$ entrants remaining in the auction when the incumbent drops out at $\overline{p}$. Hence, the second term of (12) corresponds to the buyer's cost for sample paths that satisfy (b).

Case (c) Since the auction price descends, per Theorem 2 part (ii), $\{\overline{p} \in B_m\}$ happens if and only if the following conditions hold:

(B1) The incumbent does not drop out at $\overline{R}$ (i.e., at least $\widetilde{m}+1$ entrants join the auction);

(B2) The incumbent drops out neither in $\{P_j\}_{j=m^*+1}^{\widetilde{m}}$ (i.e. $X_{(j+1:N)} + \Delta \leq p_j^\star$ for all $j = m^\star+1, \ldots, \widetilde{m}$) nor in $\{B_j\}_{j=m^*+1}^{m-1}$ (i.e. $X_{(j+1:N)} + \Delta \leq b_j^\star$ for all $j = m^\star+1, \ldots, m-1$);

(B3) The incumbent drops out at $\overline{p} \in B_m$ (i.e., $X_{(m:N)} + \Delta = \overline{p}$).

Note that $b_j^\star < p_j^\star$ and $b_j^\star$ is nonincreasing in $j$ per Theorem 2 part (i). Note also that $\overline{p} \leq b_{m-1}^\star$ when $\overline{p} \in B_m$. We conclude that given (B1), the conditions (B2) and (B3) are equivalent to the sample paths where $X_{(m:N)} + \Delta = \overline{p}$ and $X_{(j+1:N)} + \Delta \leq p_j^\star(x_0)$ for all $j = m, \ldots, \widetilde{m}$. This implies that $\Phi'_m(\overline{p})$ is the p.d.f. of $\overline{p}$ when $\overline{p} \in B_m$. Moreover, (B3) implies that there are exactly $m-1$ entrants remaining in the auction when the incumbent drops out at $\overline{p}$. Hence, the third term of (12) corresponds to the buyer's cost for sample paths that satisfy (c).

Case (d) Since the auction price descends, per Theorem 2 part (ii), $\{\overline{p} = b_m^\star\}$ happens if and only if the following conditions hold:

(b1) The incumbent does not drop out at $\overline{R}$ (i.e., at least $\widetilde{m}+1$ entrants join the auction);

(b2) The incumbent drops out neither in $\{P_j\}_{j=m^*+1}^{\widetilde{m}}$ (i.e. $X_{(j+1:N)} + \Delta \leq p_j^\star$ for all $j = m^\star+1, \ldots, \widetilde{m}$) nor in $\{B_j\}_{j=m^*+1}^{m-1}$ (i.e. $X_{(j+1:N)} + \Delta \leq b_j^\star$ for all $j = m^\star+1, \ldots, m-1$);

(b3) The incumbent drops out at $\overline{p} = b_m^\star$ (i.e., $X_{(m:N)} + \Delta \leq b_m^\star < X_{(m+1:N)} + \Delta$).

By a similar argument as in Case (c), we conclude that given (b1), the conditions (b2) and (b3) are equivalent to the sample paths where $X_{(m:N)} + \Delta \leq b_m^\star < X_{(m+1:N)} + \Delta$ and $X_{(j+1:N)} + \Delta \leq p_j^\star(x_0)$

for all $j=m,\dots,\widetilde{m}$. This implies that $\phi_m$ is the probability that $\overline{p}=b^\star_m$. Moreover, b3 implies that, with probability one, there are exactly $m$ entrants remaining in the auction when the incumbent drops out at $\overline{p}$. Hence, the fourth term of (12) corresponds to the buyer's cost for sample paths that satisfy (d).

This completes the proof of (12). To complete the derivation of the closed-form expression of $\mathbf{E}[\mathcal{PC}_{post}(x_0)]$, we derive the closed-form expressions of $\Phi_m(\overline{p})$, $\phi_m(\overline{p})$, and $h(m,\overline{p})$ in Lemma EC.3.

LEMMA EC.3. *Under the same assumptions stated in Proposition 3, the following identities hold : if $m\le\widetilde{m}$*

$$\begin{aligned}\Phi_m(\overline{p}) &= \sum_{\mathcal{T}_m}\frac{N!}{\prod_{j=0}^{\widetilde{m}-m+1}t_j!(N-\sum_{j=0}^{\widetilde{m}-m+1}t_j)!}[F_e(\overline{p}-\Delta)]^{t_0}\left[F_e(p^\star_m-\Delta)-F_e(\overline{p}-\Delta)\right]^{t_1}\\ &\quad\prod_{j=2}^{\widetilde{m}-m+1}[F_e(p^\star_{m+j-1}-\Delta)-F_e(p^\star_{m+j-2}-\Delta)]^{t_j}[1-F_e(p^\star_{\widetilde{m}}-\Delta)]^{N-\sum_{j=0}^{\widetilde{m}-m+1}t_j},\\ \phi_m &= \sum_{\mathcal{T}'_m}\frac{N!}{\prod_{j=0}^{\widetilde{m}-m+1}t_j!(N-\sum_{j=0}^{\widetilde{m}-m+1}t_j)!}[F_e(b^\star_m-\Delta)]^{t_0}[F_e(p^\star_m-\Delta)-F_e(b^\star_m-\Delta)]^{t_1}\\ &\quad\prod_{j=2}^{\widetilde{m}-m+1}[F_e(p^\star_{m+j-1}-\Delta)-F_e(p^\star_{m+j-2}-\Delta)]^{t_j}[1-F_e(p^\star_{\widetilde{m}}-\Delta)]^{N-\sum_{j=0}^{\widetilde{m}-m+1}t_j},\end{aligned}$$

*where* $\mathcal{T}_m:=\{t_0,t_1,\cdots,t_{\widetilde{m}+1-m}: t_0\ge m, t_0+t_1\ge m+1,\dots,t_0+t_1+\cdots+t_{\widetilde{m}-m}\ge\widetilde{m}, N\ge t_0+t_1+\cdots+t_{\widetilde{m}+1-m}\ge\widetilde{m}+1\}$ *and* $\mathcal{T}'_m:=\{(t_0,t_1,\cdots,t_{\widetilde{m}+1-m})\in\mathcal{T}_m: t_0=m\}$*; otherwise (i.e., $m>\widetilde{m}$),*

$$\begin{aligned}\Phi_m(\overline{p}) &= \sum_{i=m}^{N}\frac{N!}{i!(N-i)!}[F_e(\overline{p}-\Delta)]^i[1-F_e(\overline{p}-\Delta)]^{N-i},\\ \phi_m &= \frac{N!}{m!(N-m)!}[F_e(b^\star_m-\Delta)]^m[1-F_e(b^\star_m-\Delta)]^{N-m}.\end{aligned}$$

*Moreover, $h(0,\overline{p})=h(1,\overline{p})=\overline{p}$, and for $m\ge2$,*

$$\begin{aligned}h(m,\overline{p}) &= m\beta\int_{\underline{R}}^{\overline{p}-\Delta}\left(x+\frac{F_e(x)}{f_e(x)}\right)\left[1-\beta G_e\left(x;\overline{p}-\Delta\right)\right]^{m-1}g_e\left(x;\overline{p}-\Delta\right)dx\\ &\quad+(1-\beta)^m\overline{p}+\Delta[1-(1-\beta)^m].\end{aligned}\tag{EC.19}$$

**Proof of Lemma EC.3.** Note that when $m>\widetilde{m}$, $\Phi_m(\overline{p})=\mathbf{P}(X_{(m:N)}<\overline{p}-\Delta)$, and the identity follows by the formulae of the c.d.f of order statistics. When $m\le\widetilde{m}$, let $Y_0$ denote the number of entrants whose cost is below $\overline{p}-\Delta$, and for all $i=1,\dots,\widetilde{m}+1-m$, let $Y_i$ denote the number of entrants whose cost is below $p^\star_{i+m}(x_0)-\Delta$. Let $T_0=Y_0$ and $T_i=Y_i-Y_{i-1}$ for all $i=1,\dots,\widetilde{m}+1-m$. Then, $\Phi_m(\overline{p})=\mathbf{P}(Y_0\ge m, Y_1\ge m+1,\dots,Y_{\widetilde{m}-m}\ge\widetilde{m}, N\ge Y_{\widetilde{m}+1-m}\ge\widetilde{m}+1)=\mathbf{P}(T_0\ge m, T_0+T_1\ge m+1,\dots,\sum_{i=0}^{\widetilde{m}-m}T_i\ge\widetilde{m}, N\ge\sum_{i=0}^{\widetilde{m}-m+1}T_i\ge\widetilde{m}+1)$, and the stated identity follows. The identities for $\phi_m$ can be established by a similar argument.

We now derive $h(m,\overline{p})$. When $m=0$, the incumbent wins the auction (and the contract) when the last entrant drops out at $\overline{p}$, so $h(m,\overline{p})=\overline{p}$. When $m=1$, the incumbent drops out with one entrant in the auction, so $h(m,\overline{p}) = K+\beta(\overline{p}-\Delta)+(1-\beta)\overline{p}=\overline{p}$. We now prove that (EC.19) holds when $m\geq 2$. Note that the buyer's expected procurement cost includes her expected contract payment and expected qualification cost. Given that there are $m$ entrants in the auction after the incumbent drops out, the expected qualification cost equals $K\sum_{i=1}^{m} i\beta(1-\beta)^{i-1}+mK(1-\beta)^m = \Delta[1-(1-\beta)^m]$. The buyer's expected contract payment equals her expected contract payment to the incumbent, which equals $(1-\beta)^m\overline{p}$, and her expected contract payment to the entrants. Hence, we only need to show that the buyer's expected payment to the entrants equals the first term in (EC.19). Assume, without loss of generality, that when the incumbent drops out at $\overline{p}$ with $m$ entrants still in the auction, these $m$ entrants correspond to entrant $1,\dots,m$. Denote by $u_j(x_j)$, $w_j(x_j)$, and $t_j(x_j)$ the equilibrium expected payoff, contract winning probability, and equilibrium expected payment of entrant $j$ with cost $x_j$. Then, we have

$$t_j(x_j)=u_j(x_j)+x_jW_j(x_j)=x_jW_j(x_j)+\int_{x_j}^{\overline{p}-\Delta}W_j(z_j)dz_j$$

where the first equality follows since $u_j(x_j)=t_j(x_j)-x_jW_j(x_j)$, and the second equality follows by the Revenue Equivalence Theorem (Myerson 1981). Note that by Theorem 1,

$$\begin{aligned}
W_j(x_j) &= \sum_{k=1}^{m}\beta(1-\beta)^{k-1}\mathbf{P}(x_j \text{ is the } k^{th} \text{ smallest cost among } x_1,\dots,x_m)\\
&= \sum_{k=1}^{m}\beta(1-\beta)^{k-1}\frac{(m-1)!}{(k-1)!(m-k)!}[G_e^1(x_j;\overline{p}-\Delta)]^{k-1}[1-G_e^1(x_j;\overline{p}-\Delta)]^{m-k}\\
&= \beta[1-G_e^1(x_j;\overline{p}-\Delta)+(1-\beta)G_e^1(x_j;\overline{p}-\Delta)]^{m-1}\\
&= \beta(1-\beta G_e^1(x_j;\overline{p}-\Delta))^{m-1}.
\end{aligned} \tag{EC.20}$$

Hence, the buyer's expected payment to all entrants equals the first term in (EC.19):

$$\begin{aligned}
\sum_{j=1}^{m}\mathbf{E}[t_j(x_j)] &= \sum_{j=1}^{m}\int_{\underline{R}}^{\overline{p}-\Delta} t_j(x_j)g_e^1(x_j;\overline{p}-\Delta)dx_j\\
&= \sum_{j=1}^{m}\int_{\underline{R}}^{\overline{p}-\Delta}\left(x_j+\frac{F_e(x_j)}{f_e(x_j)}\right)W_j(x_j)g_e^1(x_j;\overline{p}-\Delta)dx_j\\
&= m\beta\int_{\underline{R}}^{\overline{p}-\Delta}\left(x+\frac{F_e(x)}{f_e(x)}\right)(1-\beta G_e^1(x;\overline{p}-\Delta))^{m-1}g_e^1(x;\overline{p}-\Delta)dx,
\end{aligned}$$

where the second equality holds by integration by parts and the last equality holds by (EC.20).■

**Proof of Proposition 4. (a)** We need to show that for any $N$ and $K$, when $\beta=1$, $PC_{pre}\geq PC_{post}$. Let $N_{pre}^*$ denote the optimal number of entrants to screen in PRE when $\beta=1$. Then by (1),

$$PC_{pre}=\mathcal{PC}_{pre}(N_{pre}^*)=\mathbf{E}\left[\mathbf{Min2}(\overline{R},x_0,x_1,\dots,x_{N_{pre}^*})\right]+N_{pre}^*K.$$

Note that when $\beta = 1$, POST is equivalent to a standard open-bid descending auction where entrants' costs are increased by $\Delta := K/\beta = K$. Thus,

$$\begin{aligned} PC_{post} &= \mathbf{E}\left[\mathbf{Min2}(\overline{R}, x_0, x_1 + K, \ldots, x_N + K)\right] + K\mathbf{P}(x_0 > K + \min\{x_1, \ldots, x_N\}) \\ &\le \mathbf{E}\left[\mathbf{Min2}(\overline{R}, x_0 - K, x_1, \ldots, x_{N^*_{pre}})\right] + 2K, \end{aligned} \tag{EC.21}$$

where the inequality follows since $N^*_{pre} \le N$ and probability is no larger than 1. Hence, it is obvious that $PC_{post} \le PC_{pre}$ when $N^*_{pre} \ge 2$. We next show that $PC_{post} \le PC_{pre}$ when $N^*_{pre} \le 1$.

If $N^*_{pre} = 0$, $PC_{pre} = \overline{R} \ge PC_{post}$ where the inequality holds since not conducting any post-bidding screening in POST is not necessarily optimal but ensures that the buyer would pay no more than $\overline{R}$. If $N^*_{pre} = 1$, then $PC_{pre} = \mathbf{E}\left[\mathbf{Min2}(\overline{R}, x_0, x_1)\right] + K = \mathbf{E}[\max\{x_0, x_1\}] + K$ and

$$\begin{aligned} PC_{post} &\le \mathbf{E}\left[\mathbf{Min2}(\overline{R}, x_0, x_1 + K)\right] + K\mathbf{P}(x_0 > x_1 + K) \\ &\le \mathbf{E}\left[\mathbf{Min2}(\overline{R}, x_0 - K, x_1)\right] + K + K\mathbf{P}(x_0 - K > x_1) \\ &= \mathbf{E}[\max\{x_0 - K, x_1\}] + K\mathbf{P}(x_0 - K > x_1) + K = PC_{pre}, \end{aligned}$$

where the first inequality follows since the RHS represents the buyer's cost in POST when only screening entrant 1 which is not necessarily optimal.

**(b)** Note that $\mathcal{PC}_{pre}(n)$ is discrete convex in $n$. This means that if $\mathcal{PC}_{pre}(0) \le \mathcal{PC}_{pre}(1)$, then $\mathcal{PC}_{pre}(0) \le \mathcal{PC}_{pre}(n)$ for all $n \ge 1$, and hence $N^*_{pre} = 0$. Fix $\beta, N$. Define $\bar{K} = \beta(\overline{R} - \mathbf{E}[\max\{x_0, x_1\}])$. We now show that for all $K \ge \bar{K}$, $PC_{post} \le PC_{pre}$. Note that in this case, $PC_{pre} = \mathcal{PC}_{pre}(0) = \overline{R}$ because

$$\mathcal{PC}_{pre}(1) - \mathcal{PC}_{pre}(0) = \beta\mathbf{E}[\max\{x_0, x_1\}] + (1-\beta)\overline{R} + K - \overline{R} = K - \bar{K} \ge 0.$$

Thus, $PC_{post} \le PC_{pre}$ holds by a similar argument as in **(a)**.

**(c)** Fix $\beta, K$, we only need to show that $\lim_{N\to\infty} PC_{post} < \lim_{N\to\infty} PC_{pre}$. To that end, we first show that $\lim_{N\to\infty} PC_{post} \le \underline{R} + \Delta$. Note that the expected cost of POST is cheaper than its conditional expected cost when the incumbent's cost is $x_0 = \overline{R}$, i.e.,

$$PC_{post} \le \mathcal{PC}_{post}(\overline{R}) = \sum_{m=0}^{N} \frac{N!}{i!(N-i)!}[F_e(\overline{R} - \Delta)]^m [1 - F_e(\overline{R} - \Delta)]^{N-m} h(m, \overline{R}).$$

Note that by (EC.19) and integration by parts, we can rewrite $h(m, \overline{R})$ as follows:

$$\begin{aligned} h(m, \overline{R}) &= -\int_{\underline{R}}^{\overline{R}-\Delta} \psi_e(x) d[1 - \beta G_e(x; \overline{R} - \Delta)]^m + (1-\beta)^m \overline{R} + \Delta[1 - (1-\beta)^m] \\ &= \underline{R} - \psi_e(\overline{R} - \Delta)(1-\beta)^m + \int_{\underline{R}}^{\overline{R}-\Delta} [1 - \beta G_e(x; \overline{R} - \Delta)]^m d\psi_e(x) + (1-\beta)^m \overline{R} + \Delta[1 - (1-\beta)^m], \end{aligned}$$

where $\psi_e(x) = x + F_e(x)/f_e(x)$. Thus, combining the two equations above, we have

$$\lim_{N\to\infty} PC_{post} \le \lim_{N\to\infty} \mathcal{PC}_{post}(\overline{R}) = \underline{R} + \Delta - (\psi_e(\overline{R}-\Delta) + \overline{R} + \Delta) \lim_{N\to\infty} [1-\beta F_e(\overline{R}-\Delta)]^N + \int_{\underline{R}}^{\overline{R}-\Delta} \lim_{N\to\infty} [1-\beta F_e(\overline{R}-\Delta) G_e(x;\overline{R}-\Delta)]^N d\psi_e(x) = \underline{R} + \Delta, \tag{EC.22}$$

where the first equality follows by elementary algebra and the dominated convergence theorem.

Next, we show $\lim_{N\to\infty} PC_{pre} > \underline{R} + \Delta$. Define a function $M_2 : [0, N] \to \mathbb{R}_+$ as follows

$$M_2(i) = \ (\lceil i \rceil - i)\, \mathbf{E}\left[\mathbf{Min2}(\overline{R}, x_0, \cdots, x_{\lfloor i \rfloor})\right] + (i - \lfloor i \rfloor)\, \mathbf{E}\left[\mathbf{Min2}(\overline{R}, x_0, \cdots, x_{\lceil i \rceil})\right].$$

Note that for all $i \in \mathbb{Z}$, $M_2(i) = \mathbf{E}[\mathbf{Min2}(\overline{R}, x_0, \cdots, x_i)]$, and for all $i \notin \mathbb{Z}$, by definition, $M_2(i)$ is the linear interpolation of $M_2(\lfloor i \rfloor)$ and $M_2(\lceil i \rceil)$. Since $\mathbf{E}[\mathbf{Min2}(\overline{R}, x_0, \cdots, x_i)]$ is discrete convex in $i$, $M_2(i)$ is convex. Hence, for any $N$,

$$\begin{aligned} PC_{pre} &= \sum_{i=0}^{N^*_{pre}} E\left[\mathbf{Min2}(\overline{R}, x_0, \cdots, x_i)\right] \frac{N^*_{pre}!}{i!(N^*_{pre}-i)!} \beta^i (1-\beta)^{N^*_{pre}-i} + N^*_{pre} K \\ &= \sum_{i=0}^{N^*_{pre}} M_2(i) \frac{N^*_{pre}!}{i!(N^*_{pre}-i)!} \beta^i (1-\beta)^{N^*_{pre}-i} + N^*_{pre} K \\ &\ge M_2\left(\sum_{i=0}^{N^*_{pre}} i \frac{N^*_{pre}!}{i!(N^*_{pre}-i)!} \beta^i (1-\beta)^{N^*_{pre}-i}\right) + N^*_{pre} K = M_2(N^*_{pre}\beta) + N^*_{pre} K, \end{aligned} \tag{EC.23}$$

where the inequality follows through Jensen's Inequality, the convexity of $M_2$, and the fact that $N^*_{pre}\beta \in [0, N^*_{pre}]$. Note that $M_2(N^*_{pre}\beta) + N^*_{pre}K > \underline{R} + \Delta$. Indeed, if $N^*_{pre}\beta \ge 1$, $M_2(N^*_{pre}\beta) + N^*_{pre}K > \underline{R} + N^*_{pre}K \ge \underline{R} + \Delta$. Otherwise, when $N^*_{pre}\beta < 1$, we have

$$\begin{aligned} M_2(N^*_{pre}\beta) + N^*_{pre}K &= N^*_{pre}\beta \mathbf{E}\left[\mathbf{Min2}(\overline{R}, x_0, x_1)\right] + (1 - N^*_{pre}\beta)\mathbf{E}\left[\mathbf{Min2}(\overline{R}, x_0)\right] + N^*_{pre}K \\ &= N^*_{pre}\beta\left(\mathbf{E}\left[\max\{x_0, x_i\}\right] + \Delta\right) + (1 - N^*_{pre}\beta)\overline{R} \\ &> N^*_{pre}\beta\left(\underline{R} + \Delta\right) + (1 - N^*_{pre}\beta)(\underline{R} + \Delta) = \underline{R} + \Delta. \end{aligned} \tag{EC.24}$$

Hence, by (EC.22), (EC.23) and (EC.24), $\lim_{N\to\infty} PC_{pre} \ge M_2(N^*_{pre}\beta) + N^*_{pre}K > \underline{R} + \Delta \ge \lim_{N\to\infty} PC_{post}$. ■

**Proof of Proposition 5.** We first formally define the class of feasible mechanisms we consider in this paper, then formulate the optimal mechanism design problem, and finally derive the optimal mechanism.

*Step 1. The class of feasible mechanisms.* To introduce the class of feasible mechanisms we consider in this paper, consider a multi-stage dynamic game with $N+2$ players, i.e., one buyer, one incumbent and $N$ entrants, who interact in $N$ stages. In each stage, the following four events occur in sequence:

**E1.** A signal vector is drawn and each player observes a component of this signal vector. Specifically, in the first stage, each supplier observes his production cost; in later stages, if a qualification screening occurs in the previous stage, say entrant $i$ is screened, then both the buyer and supplier $i$ observes whether he passes the qualifications screening; otherwise, no signal is observed.

**E2.** Each supplier chooses a message to send to the buyer (e.g., their bids).

**E3.** The buyer chooses a message to send to each supplier. Specifically, the buyer, either fully or partially, discloses to the suppliers information that relates to suppliers' previous messages and qualification screening outcome; of course, the buyer can also choose to disclose no information.

**E4.** Each player takes an action. Specifically, the buyer decides whether to conduct qualification screening of one more entrant and, if the answer is yes, she decides which entrant to screen; otherwise, the buyer ends the process by awarding the contract and making payments; the suppliers decide, in each stage, whether or not to keep participating in the mechanism.

Note that based on the dynamics outlines above, a dynamic game is well-defined for any given combination of *non-anticipated rules*: information disclosure rule (in **E3**), qualification screening rule (in **E4**), contract allocation rule (in **E4**) and payment rule (in **E4**). By choosing and committing to a combination of the four rules, the buyer can induce different equilibrium outcomes. Thus, the buyer's optimal mechanism design problem is to choose a mechanism (i.e., the combination of information disclosure, investigation, allocation and payment rules) to minimize her expected procurement cost.

*Step 2. Formulation of the optimal mechanism design problem.* To analyze the optimal mechanism design problem, we invoke the Revelation Principle established in the first equality of equation (4) of Theorem 1 in Sugaya and Wolitzky (2018). This version of Revelation Principle establishes that any conditional probability perfect Bayesian equilibrium (please refer to Section 3.2 of Sugaya and Wolitzky (2018) for a formal definition of this solution concept; intuitively, it can be viewed as a type of perfect Bayesian Equilibrium which not only requires consistency of on-equilibrium beliefs and sequential rationality of player's actions, but also requires that both on-equilibrium and off-equilibrium beliefs are derived from a common conditional probability system on the set of complete histories of play) of the mechanism we described above can be implemented by a canonical mechanism where communication between players and the mediator takes the following form: non-mediator players (suppliers in our setting) communicate only their private information to the mediator (buyer in our setting), and the mediator communicates only recommended actions to the players. Note also that, after the first stage, the suppliers in our setting do not observe private signal after the first stage (i.e., whether supplier $i$ passes qualification screening is also observed by the buyer). This observation combined with the Revelation Principle implies that the choice of information disclosure rule is irrelevant in finding the optimal canonical mechanism; hence, it is without loss of generality to optimize within the class of three-step direct mechanisms defined below.

Definition EC.1. Let $\Xi := (\xi_1, \dots, \xi_N) \in \{0,1\}^N$ where $\xi_j = 1$ if and only if entrant $j$ passes qualification screening. For any $Q \subseteq \{1, \dots, N\}$, define $\Xi^Q := (\xi_j)_{j\in Q}$. A three-step direct mechanism in our context is determined by a screening policy $\pi$, an allocation rule $A$ and a payment rule $M$. In the first step, all suppliers report their costs to the buyer (they can choose to be not truthful); in the second step, the buyer investigates the suppliers according to the screening policy $\pi$; in the third step, the buyer allocates the contract and makes payment according to $A$ and $M$. Specifically:

- **Screening policy** $\pi$**.** The screening policy determines, in each stage, which supplier (if any) to screen based on all the information the buyer observes up to that stage (i.e., suppliers' reported cost and qualification screening outcome of screened entrants). Mathematically, $\pi : (\pi_1, \dots, \pi_N)$ where for any stage $k$, $\pi_k : [\underline{R}, \overline{R}]^{N+1}(\otimes_{j\in\mathcal{Q}_k}\{0,1\}) \to \{\emptyset\} \cup (\{1, \dots, N\} - \mathcal{Q}_k)$ where $\mathcal{Q}_k$ is the set of suppliers who have been investigated before round $k$. Let $\mathcal{Q}_k^\pi$ denote the set of suppliers who have been investigated before round $k$ under a screening policy $\pi$. Denote by $\Pi$ the class of all screening policies. Define $Q^\pi(\mathbf{x}) := \mathcal{Q}^\pi_{N+1}(\mathbf{x})$ for all cost vectors $\mathbf{x} = (x_0, x_1, \dots, x_N) \in [\underline{R}, \overline{R}]^{N+1}$. The dependency of $Q^\pi(\mathbf{x})$ on $\pi$ and $\mathbf{x}$ will be suppressed for notational simplicity whenever there is no confusion.
- **Allocation rule A.** $A(\mathbf{x}, \Xi^Q) := [A_i(\mathbf{x}, \Xi^Q)] \in [0,1]^{N+1}$, where $A_i(\mathbf{x}, \Xi^Q)$ is the probability that supplier $i$ wins the contract, given the bid vector $\mathbf{x}$ and the revealed screening outcomes $\{\xi_i : i \in Q(\mathbf{x})\}$. We suppress the dependency on $\pi$ for notational clarity.
- **Payment rule M.** $M(\mathbf{x}, \Xi^Q) := [M_i(\mathbf{x}, \Xi^Q)] \in \mathbb{R}^{N+1}$, where $M_i(\mathbf{x}, \Xi^Q)$ is the expected payment from the buyer to supplier $i$, given the bid vector $\mathbf{x}$ and the revealed screening outcomes $\{\xi_i : i \in Q(\mathbf{x})\}$. We suppress the dependency on $\pi$ for notational clarity.

Let $\mathcal{M} := (\pi, A, M)$ denote a three-step direct mechanism. We denote by $\hat{U}_i(z_i; x_i)$ supplier $i$'s expected profit under $\mathcal{M}$ when his cost is $x_i$ and he bids $z_i$ in step 1, and other suppliers bid their true costs in step 1:

$$\hat{U}_i(z_i; x_i) = \mathbf{E}_{\mathbf{x}_{-i},\Xi}[M_i(z_i, \mathbf{x}_{-i}, \Xi^{Q^\pi(z_i,\mathbf{x}_{-i})}) - A_i(z_i, \mathbf{x}_{-i}, \Xi^{Q^\pi(z_i,\mathbf{x}_{-i})})x_i].$$

Let $U_i(x_i) := \hat{U}_i(x_i; x_i)$. Then supplier $i$'s individual rationality, $\mathbf{IR}^i$, and incentive compatibility constraints, $\mathbf{IC}^i$, can be formulated as follows: for all $x_i \in [\underline{R}, \overline{R}]$,

$$(\mathbf{IC}^i)\ \ U_i(x_i) = \max_{z_i\in[\underline{R},\overline{R}]} \hat{U}_i(z_i; x_i), \qquad (\mathbf{IR}^i)\ \ U_i(x_i) \geq 0.$$

Hence, the mechanism design problem can be formulated as the following mathematical program:

$$\min_{\mathcal{M}=(\pi,A,M)} \mathbf{E}\left[\sum_{i=0}^{N} M_i(\mathbf{x},\Xi^{Q^\pi(\mathbf{x})}) + |Q^\pi(\mathbf{x})|K\right] \tag{EC.25}$$

$$s.t. \quad \sum_{i=0}^{N} A_i(\mathbf{x},\Xi^{Q^\pi(\mathbf{x})}) = 1 \tag{EC.26}$$

$$A_i(\mathbf{x},\Xi^{Q^\pi(\mathbf{x})}) = 0 \text{ for all } i \notin \{0\}\cup\{j : j \in Q^\pi(\mathbf{x}) \text{ and } \xi_j = 1\} \tag{EC.27}$$

$$(\mathbf{IR}^i) \text{ and } (\mathbf{IC}^i) \text{ hold for all } i = 0,\ldots,N \tag{EC.28}$$

where $|Q^\pi(\mathbf{x})|$ in (EC.25) is the cardinality of $Q^\pi(\mathbf{x})$, (EC.26) requires that the contract must be allocated to one of the suppliers because the incumbent is qualified already and is always a viable supplier, (EC.27) corresponds to the constraint that only suppliers who are qualified can be awarded the contract. Following a standard derivation (see Myerson (1981)), (EC.28) is equivalent to the following conditions:

$$\mathbf{E}_{\mathbf{x}_{-i},\Xi}[A_i(\mathbf{x},\Xi^{Q^\pi(\mathbf{x})})] \text{ in nonincreasing in } c_i \text{ for all } i=0,\ldots,N \text{ and for all } x_i \in [\underline{R},\overline{R}] \tag{EC.29}$$

$$\mathbf{E}_{\mathbf{x}_{-i},\Xi}[M_i(\mathbf{x},\Xi^{Q^\pi(\mathbf{x})})] = U_i(\overline{R}) + \mathbf{E}_{\mathbf{x}_{-i},\Xi}\left[A_i(\mathbf{x},\Xi^{Q^\pi(\mathbf{x})})x_i + \int_{x_i}^{\overline{R}} A_i(z_i,\mathbf{x}_{-i},\Xi^{Q^\pi(z_i,\mathbf{x}_{-i})})dz_i\right] \tag{EC.30}$$

$$U_i(\overline{R}) \geq 0, \text{for all } i=0,\ldots,N \tag{EC.31}$$

By plugging (EC.30) into (EC.25) and algebraic manipulation, the objective function becomes:

$$\sum_{i=0}^{N} U_i(\overline{R}) + \mathbf{E}_{\mathbf{x},\Xi}\left[A_0(\mathbf{x},\Xi^{Q^\pi(\mathbf{x})})\psi_0(x_0) + \sum_{i=1}^{N} A_i(\mathbf{x},\Xi^{Q^\pi(\mathbf{x})})\psi_e(x_i)\right] + \mathbf{E}_{\mathbf{x},\Xi}[|Q^\pi(\mathbf{x})|]K.$$

Because $U_i(\overline{R})$ is simply a constant, the cost-minimizing buyer will set $U_i(\overline{R}) = 0$ for all $i$ while still satisfying (EC.31). Then, the buyer's mechanism design problem (MD) can be written as:

$$MD: \min_{\mathcal{M}=(\pi,A,M)} \mathbf{E}_{\mathbf{x},\Xi}\left[A_0(\mathbf{x},\Xi^{Q^\pi(\mathbf{x})})\psi_0(x_0) + \sum_{i=1}^{N} A_i(\mathbf{x},\Xi^{Q^\pi(\mathbf{x})})\psi_e(x_i)\right] + \mathbf{E}_{\mathbf{x},\Xi}[|Q^\pi(\mathbf{x})|]K \tag{EC.32}$$

$$s.t. \quad \text{The constraints (EC.26), (EC.27), (EC.29) and (EC.30) hold.}$$

*Step 3. Derivation of the optimal mechanism.* We first relax constraint (EC.29) in MD and solve for the optimal solution; then we verify that the solution satisfies (EC.29).

For the relaxed MD problem, we use backward induction to characterize the solution. First, given any screening policy $\pi$, we solve for the optimal allocation and payment rule. Note that given $\pi$, the second term in (EC.32) is a constant. Thus, the optimal allocation rule $A^*$ should minimize the first term (EC.32) while satisfying constraints (EC.26) and (EC.27). Thus, the optimal allocation rule $A^*$ is as follows: The contract is allocated to one of the suppliers $i \in \{0\}\cup\{j : j \in Q^\pi(\mathbf{x}) \text{ and } \xi_j = 1\}$

who has the lowest virtual cost. The payment rule $M^*$ characterized below satisfies (EC.30): For any supplier $i$, if he gets the contract, his payment equals

$$M_i^*(\mathbf{x}, \Xi^{Q^\pi(\mathbf{x})}) = x_i + \int_{x_i}^{\overline{R}} \mathbf{E}_{\Xi\{1,\dots,N\}-Q^\pi(\mathbf{x})}[A_i^*(z_i, \mathbf{x}_{-i}, \Xi^{Q^\pi(z_i,\mathbf{x}_{-i})})|\Xi^{Q^\pi(\mathbf{x})})]dz_i \tag{EC.33}$$

where the expectation is taken over the random variables $\xi_j$ for all entrants $j$ who have not be screened; otherwise, his payment is zero. Next, we plug $A^*$ and $M^*$ in the relaxed MD and solve for the optimal screening policy $\pi^*$, which yields the following stochastic optimization problem

$$\pi^* = \arg\min_{\pi\in\Pi} \mathbf{E}_{\mathbf{x},\Xi}\left[\min\left\{\psi_0(x_0), \min_{i\in Q^\pi(\mathbf{x}),\xi_i=1} \psi_e(x_i)\right\} + |Q^\pi(\mathbf{x})|K\right].$$

Let $i_k, k = 1,\dots,N$ denote the entrant with the $k^{th}$ lowest cost. Similar to the argument in the proof of Lemma 1, the optimal screening rule is one where only entrants whose costs are below $\psi_e^{-1}(\psi_0(x_0) - \Delta)$ are potentially screened. Suppose there are $L$ such entrants. Then, under the optimal screening rule $\pi^*$, the buyer conducts qualification screening from $i_1$ to $i_L$ until either the buyer finds a qualified entrant or the buyer exhausts the list of all $L$ entrants. Then, by $A^*$, if none of the $L$ entrants pass qualification screening, the incumbent gets the contract; otherwise, the contract is allocated to the first qualified entrant. Finally, for the payment rule, if the incumbent wins the contract, by (EC.33), his payment equals:

$$\begin{aligned}
M_0^*(\mathbf{x}, \Xi^{Q^\pi(\mathbf{x})}) &= x_0 + \int_{x_0}^{\overline{R}} \mathbf{E}_{\Xi\{1,\dots,N\}-Q^\pi(\mathbf{x})}[A_0^*(z_0, \mathbf{x}_{-0}, \Xi^{Q^\pi(z_0,\mathbf{x}_{-0})})|\Xi^{Q^\pi(\mathbf{x})})]dz_0 \\
&= x_0 + \int_{x_0}^{\min\{\overline{R},\psi_0^{-1}(\psi_e(x_{i_{L+1}})+\Delta)\}} dx \\
&\quad + \sum_{l=L+1}^{N-1} \int_{\min\{\overline{R},\psi_0^{-1}(\psi_e(x_{i_l})+\Delta)\}}^{\min\{\overline{R},\psi_0^{-1}(\psi_e(x_{i_{l+1}})+\Delta)\}} (1-\beta)^{l-L} dx + \int_{\min\{\overline{R},\psi_0^{-1}(\psi_e(x_{i_N})+\Delta)\}}^{\overline{R}} (1-\beta)^{N-L} dx \\
&= \beta \sum_{l=L+1}^{N} (1-\beta)^{l-L-1} \min\{\overline{R}, \psi_0^{-1}(\psi_e(x_{i_l}) + \Delta)\} + (1-\beta)^{N-L}\overline{R};
\end{aligned}$$

otherwise, i.e., if entrant $i_k$ where $1 \le k \le L$ wins the contract, his payment equals:

$$\begin{aligned}
M_{i_k}^*(\mathbf{x}, \Xi^{Q^\pi(\mathbf{x})}) &= x_{i_k} + \int_{x_{i_k}}^{\overline{R}} \mathbf{E}_{\Xi\{1,\dots,N\}-Q^\pi(\mathbf{x})}[A_{i_k}^*(z_{i_k}, \mathbf{x}_{-i_k}, \Xi^{Q^\pi(z_{i_k},\mathbf{x}_{-i_k})})|\Xi^{Q^\pi(\mathbf{x})})]dz_{i_k} \\
&= x_{i_k} + \sum_{l=k}^{L-1} \int_{x_{i_l}}^{x_{i_{l+1}}} (1-\beta)^{l-k} dx + \int_{x_{i_L}}^{\psi_e^{-1}(\psi_0(x_0)-\Delta)} (1-\beta)^{L-k} dx \\
&= \beta \sum_{l=k+1}^{L} (1-\beta)^{l-k-1} x_{i_l} + (1-\beta)^{L-k}\, \psi_e^{-1}(\psi_0(x_0) - \Delta).
\end{aligned}$$

We complete the proof by noting that the mechanism characterized above satisfies (EC.29) by $A^*$ and $\pi^*$ and the assumption that $\psi_i$ and $\psi_e$ are both nondecreasing. ■

## EC.2. Numerical Computation and Pseudocode

**PRE.** Note that in our numerical studies, all suppliers' cost distributions are uniformly distributed between $\underline{R}$ and $\overline{R}$. Hence, if $i$ entrants pass qualification screening, the expected contract payment equals the mean of the second order statistics of $i+1$ identical and independent uniform random variables between $\underline{R}$ and $\overline{R}$, i.e., $\underline{R}+\frac{2\times(\bar{R}-\underline{R})}{i+2}$. The following pseudocode summarizes how we calculate the expected cost of PRE for any given $K, \beta, N$ in our numerical study.

*input* $K$, $\beta$, and $N$<br>
**for** $n=0\ :\ N$ **do**<br>
    **for** $i=0\ :\ n$ **do**<br>
        *set* $P_i^n=\underline{R}+\frac{2\times(\overline{R}-\underline{R})}{i+2}$<br>
    **end for**<br>
    *set* $P^n=\sum_{i=0}^{n}P_i^n\cdot\frac{n!\beta^i(1-\beta)^{n-i}}{i!(n-i)!}+nK$<br>
**end for**<br>
*set* $PC_{PRE}=\min\{P^n\}_{n=0}^{N}$

**POST.** We use Monte Carlo simulation to calculate the buyer's expected cost under POST. Note that in the equilibrium we characterize in this paper, any realization of suppliers' costs result in a unique incumbent drop-out price, $\overline{p}$, and the number of entrants remaining after the incumbent drops out, $n$; moreover, once $\overline{p}$ and $n$ are determined, the buyer's total expected cost equals (EC.19). Thus, the main step for calculating POST involves determining $\overline{p}$ and $n$ for given suppliers' cost realizations, which is outlined in lines 3 - 14 in the pseudocode below. Specifically, consider the scenario where there are $m$ entrants who decide *not* to drop out at $\overline{R}$ (i.e., their true costs are lower than $\overline{R}-\Delta$). Suppose these $m$ entrants correspond to the first $m$ entrants and their costs are $x_1,\ldots,x_m$, then we claim that the incumbent's drop out price $\bar{p}$ equals the the $m^{th}$ largest elements in the following set:

$$\mathcal{J}_m(x_0):=\{x_1+\Delta,\ldots,x_m+\Delta,s_1^\star(x_0,\overline{R}),\ldots,s_m^\star(x_0,\overline{R})\} \tag{EC.34}$$

To see why this is true, consider two cases

1. If $s_m^\star(x_0,\overline{R})=\overline{R}$, the incumbent drops out at $\bar{p}=\overline{R}$, which is the $m^{th}$ largest element in $\mathcal{J}_m(x_0)$ because Theorem 2 part (ii) implies that $s_j^\star(x_0,\overline{R})=\overline{R}$ for all $j<m$.
2. Otherwise, $s_m^\star(x_0,\overline{R})<\overline{R}$, so by Theorem 2 (also illustrated in Figure 6), the incumbent will drop out $\bar{p}=b_{\tilde{j}}^\star(x_0)$ where $\tilde{j}:=\max_j\{x_j+\Delta\geq b_j^\star(x_0)\}$. Note that Theorem 2 part (ii) implies that $s_m^\star(x_0,\overline{R})=b_m^\star(x_0)<\overline{R}\leq p_m^\star(x_0)$; this also implies that $s_j^\star(x_0,\overline{R})=b_j^\star(x_0)$ for all $j<m$ and that $\{s_j^\star(x_0,\overline{R})\}_{j=1}^m$ are non-increasing in $j$ due to Theorem 2 part (i). Hence, $\bar{p}$ equals the $m^{th}$ largest elements in $\mathcal{J}_m(x_0)$ as well.

The following pseudocode summarizes how we compute the expected cost of POST for any given $K,\beta,N$: We first evaluate the expected cost conditioning on exactly $m$ entrants' cost are below

$\overline{R}-\Delta$ for all $m=0,\dots,N$ by a Monte Carlo simulation with $T$ samples (in our numerical study, we use $T=5000$), and then calculate the expected cost of POST by taking the weighted average across $m$.

*input* $K$, $\beta$, $N$, and $T$
**for** $m=0\ :\ N$ **do**
  **for** $t=1\ :\ T$ **do**
    randomly *draw* $m$ entrants' costs from uniform distribution $U[\underline{R},\overline{R}-\Delta]$
    randomly *draw* the incumbent's cost $x_0$ from uniform distribution $U[\underline{R},\overline{R}]$
    *calculate* $\{s_k^\star(x_0,\overline{R})\}_{k=1}^m$ according to (ii) of Theorem 2
    *set* $\overline{p}$ according to the $m$th largest element in $\mathcal{J}_m(x_0)$ defined in (EC.34)
    **if** $\overline{p}=x_k+\Delta$ **then**
      *set* $n=k-1$
    **else if** $\overline{p}=s_k^\star(x_0,\overline{R})$ **then**
      *set* $n=k$
    **end if**
    *set* $P_t^m=h(n,\overline{p})$ according to (EC.19)
  **end for**
  *set* $P^m=\frac{N!}{m!(N-m)!}F_e^m(\overline{R}-\Delta)\overline{F}_e^{N-m}(\overline{R}-\Delta)\frac{\sum_{t=1}^T P_t^m}{T}$
**end for**
*set* $PC_{POST}=\sum_{m=0}^N P^m$

**Optimal Mechanism.** We use Monte Carlo simulation to calculate the expected cost under the optimal mechanism. The following pseudocode summarizes how we compute the expected cost for any given $K,\beta,N$ by conducting Monte Carlo simulation with $T$ samples (in our numerical study, we use $T=5000$).

*input* $K$, $\beta$, $N$, and $T$
**for** $t=1\ :\ T$ **do**
  **for** $i=0\ :\ N$ **do**
    randomly *draw* $x_i$ from uniform distribution $U[\underline{R},\overline{R}]$
  **end for**
  *calculate* $L$ and $\{i_k\}_{k=1}^L$ according to Step 2 of Proposition 5
  *set* $P_0^t$ according to (13)
  **for** $k=1\ :\ L$ **do**
    *set* $P_{i_k}^t$ according to (14)
  **end for**
  *set* $P^t=\sum_{k=1}^L\beta(1-\beta)^{k-1}(P_{i_k}^t+kK)+(1-\beta)^L(P_0^t+LK)$
**end for**
*set* $PC_{OPT}=\sum_{t=1}^T\frac{P^t}{T}$