Supply Chain Coordination with Revenue-Sharing Contracts: Strengths and Limitations*

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Abstract

Under a revenue-sharing contract a supplier charges a retailer a wholesale price per unit plus a percentage of the revenue the retailer generates from the unit. The prevalence of this contract form has recently increased substantially in the video cassette rental industry relative to the more traditional wholesale price contract. Revenue-sharing contracts have been credited with allowing retailers to increase their stock of newly released movies, thereby substantially improving the availability of popular movies. In a general model we study how revenue-sharing contracts improve overall supply chain performance and by how much they improve performance. We compare revenue sharing to other contracts that enhance channel coordination, e.g., buy-back contracts and quantity-flexibility contracts. We show that revenue sharing can coordinate systems (such as a newsvendor problem with price-dependent demand) that those contracts cannot. We demonstrate that revenue sharing can also coordinate systems with multiple competing retailers. Finally, we identify the limitations of revenue-sharing contracts to (at least partially) explain why they are not prevalent in all industries.

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Demand for a movie newly released on video cassette typically starts off high and tapers off rapidly. As a result, a retailer renting video cassettes faces a challenging capacity problem. To illustrate, in a traditional sales agreement between a video retailer and his supplier, the retailer purchases each copy of a tape for about \$65 and collects about \$3 per rental. Hence, the retailer must rent a tape at least 22 times before earning a profit. Unfortunately, peak demand for a given title rarely lasts more than ten weeks, so the retailer simply cannot justify purchasing enough tapes to cover the initial peak demand entirely.

Blockbuster Inc., a large video retailer, was keenly aware of its peak demand problem. The poor availability of new release videos was consistently a major customer complaint (McCollum, 1998). A Time Warner Inc study reported that 20% of customers surveyed said they were unable to rent the movie they wanted on a typical store visit (Shapiro, 1998a). Poor forecasting would be one explanation. But while Blockbuster may underestimate demand for some titles, it is unlikely that Blockbuster consistently underestimated demand across all movies (thereby leading it to purchase too few tapes consistently) since a sudden change in industry demand did not occur. Nor is there any evidence that the availability problem was due to poor execution (*e.g.*, an inefficient process for returning tapes to circulation once they are returned to the store). The best explanation for the availability problem is a misalignment of incentives. If Blockbuster were able to purchase tapes at marginal cost (which is surely well below \$65 per tape), Blockbuster could afford to purchase many more tapes and initial availability would improve dramatically. Of course, selling at cost is unattractive to movie studios.

The solution came from changing the terms of sale. Starting in 1998 Blockbuster entered into revenue-sharing agreements with the major studios. Blockbuster agreed to pay its suppliers a portion (probably in the range of 30-45%) of its rental income in exchange for a reduction in the initial price per tape from \$65 to \$8.[1] If Blockbuster keeps half of the rental income, the break-even point for a tape drops to approximately six rentals. With those terms Blockbuster can clearly afford to increase its purchase quantity and improve

[1] Blockbuster's terms are not public, but Rentrak, a video distributor, offers the following: the studio gets 45% of the revenue, Rentrak 10%, and the retailer 45%. (See www.rentrak.com). Since Blockbuster has agreements directly with the studios, it should have at least as generous terms.

customer service.

The impact of revenue sharing at Blockbuster has been dramatic. Revenue sharing increased rentals by as much as 75% in test markets (Shapiro, 1998b). In the year after instituting revenue sharing Blockbuster increased its overall market share from 25% to 31% and its cash flow by 61% (Pope, 1999). To put that market share gain in perspective, the second largest retailer, Hollywood Entertainment Corp, has a market share of about 5% (Shapiro, 1998a). Blockbuster's success has been so dramatic that independent video stores have filed suit arguing that Blockbuster's favorable terms are driving independent retail store owners out of business (Pope, 1999).

This paper studies the impact of revenue sharing on the performance of a supply chain. While inspired by the success of revenue sharing in the video industry, our model is quite general. It applies to essentially any industry and any link between two levels in a supply chain (*e.g.*, supplier-manufacturer or manufacturer-distributor). It does not matter whether the asset produced at the upstream level is rented at the downstream level (as in the video industry) or sold outright (as in the book industry) or whether demand is stochastic or deterministic.

We begin with the simplest supply chain. A downstream firm, called the retailer, orders q units of an asset from the upstream firm, called the supplier. The supplier produces the q units at a constant marginal cost, and the retailer uses those units to generate revenues over a single selling season. In the marketing literature the revenue function is typically assumed to be derived from a downward sloping, deterministic demand curve (see Lilien, Kotler and Moorthy, 1992), whereas in the operations literature the revenue function is frequently assumed to result from a newsvendor problem with a fixed retail price and stochastic demand (see Tsay, Nahmias and Agrawal, 1998). We work with a general revenue function that encompassed both of those models. It is well known that in this setting the supply chain's profit is less than optimal whenever the supplier charges a wholesale price above marginal cost because then the retailer orders fewer units than optimal (Spengler, 1950). We show that revenue sharing induces the retailer to order the supply chain optimal quantity, coordinating the supply chain. It also can arbitrarily split profits between the two firms. Further, the coordinating contract is independent of the revenue function. Consequently, one contract can coordinate a supply chain with multiple independent retailers.

Two alternative contract types have been proposed to coordinate this supply chain when the revenue function is generated from a newsvendor problem: buy-back contracts (Pasternack, 1985) and quantity-flexibility contracts (Tsay, 1999). Those contracts also coordinate the supply chain and arbitrarily divide profits. We show that revenue sharing and buy-back contracts are equivalent in this setting in the strongest sense. For any buy-back contract there exists a revenue-sharing contract that generates the same cash flows for *any* realization of demand. (The comparable result does not hold between revenue sharing and quantity-flexibility contracts.) However, we also demonstrate that revenue sharing can coordinate settings that buy back and quantity-flexibility contracts do not.

We next consider two extensions to our basic model. In the first we study coordination when demand is stochastic and the retailer chooses his order quantity and his price, *i.e.*, the price-dependent newsvendor problem. Revenue sharing still coordinates the channel and supports an arbitrary division of profits. In the second extension we consider a supply chain with one supplier and multiple competing retailers. The retailers could be Cournot competitors or competing newsvendors (as in Lippman and McCardle, 1997). It has been observed in similar settings that the simple wholesale-price contract can coordinate this system (Mahajan and van Ryzin, 1999, and Bernstein and Federgruen, 1999), but the coordinating wholesale price only allows one split of channel profit. We show that revenue sharing again coordinates this system while supporting alternative profit allocations.

Based on these results, we conclude that revenue-sharing contracts are very effective in a wide range of supply chain settings. Nevertheless, revenue-sharing contracts must have some limitations, otherwise we would expect to observe them in every industry. One limitation to revenue sharing is the additional administrative cost it imposes on the supply chain. With revenue sharing the supplier must monitor the retailer's revenues to verify that the retailer indeed pays the supplier her appropriate share of the earned revenues. The gains from coordination may not always cover these costs. To explore this idea, we study the performance of the supply chain with the simpler wholesale-price contract, which clearly has a lower administrative cost than revenue sharing. If supply chain performance with the wholesale-price contract is relatively close to optimal and if the supplier earns a significant fraction of the supply chain's profit, administrative costs may prevent the adoption of revenue-sharing contracts. We demonstrate that there is considerable variation in the efficiency of the wholesale-price contract (the ratio of supply chain profit with the wholesale-price contract to the optimal profit). We conclude that the administrative cost burden can explain why revenue sharing is not implemented in some settings, but that is not a sufficient explanation for all settings.

We also explore a second limitation based on the hypothesis that retail effort influences demand. In particular, we assume that there are many activities that a retailer performs that increase demand, yet are costly: cleaner stores, more and better-trained staff, *etc.* We show that while revenue sharing helps to coordinate the retailer's quantity decision, it actually works against coordinating the retailer's effort decision. If retail effort has a sufficiently large impact on demand, the supplier is better off using a wholesale-price contract instead of a revenue-sharing contract.

We are not the only academics that have been attracted to revenue sharing by the recent media attention. Dana and Spier (1999) consider the use of revenue sharing in a supply chain with a perfectly competitive downstream market and stochastic demand. They demonstrate that a revenue-sharing contract can induce the downstream firms to choose supply chain optimal actions. Our model does not rely on perfect competition or stochastic demand. Pasternack (1999) considers a model in which a supplier sells to a retailer that faces a newsvendor problem. The retailer can purchase units under a traditional contract as well as purchase units under a revenue-sharing agreement, where revenue sharing is modeled as a fixed payment to the supplier per unit sold. The supplier sets the terms of the revenue-sharing contract but the terms of the traditional contract are exogenous. In our model, the supplier offers only revenue sharing based on a percentage of revenue, and we consider a more general setting than the newsvendor problem. For the newsvendor problem, we show that our contract is equivalent to his.

The next section outlines the model. Section 2 investigates channel coordination and the relationship between revenue-sharing contracts and several other contracts. Section 3 extends our results to settings beyond our basic model. Section 4 discusses limitations of revenue-sharing contracts. The final section discusses our results and concludes.

1. Basic Model

Consider a supply chain with two risk neutral firms. The supplier is the upstream firm and the retailer is the downstream firm. The supplier sells an asset to the retailer, which allows the retailer to generate revenues over a selling season. For now, assume that the retailer's expected revenue over the selling season $R(q)$ is solely a function of q, the number of units purchased from the supplier. (Since the firms are risk neutral, there is no need in our analysis to distinguish between expected revenue and realized revenue. To streamline the exposition, all discussion of revenue is assumed to refer to expected revenue.) Assume that $R(q)$ is strictly concave and differentiable for $q \geq 0$. Without loss of generality, normalize the salvage value per unit at the end of the selling season to zero. We make no distinction between the case in which the retailer rents the asset and ends the season with q units and the case in which the retailer sells the asset. The supplier's production cost per unit is $c > 0$. We assume that the product is viable in the market, *i.e.*, $R'(0) > c$, and a finite production quantity is optimal, *i.e.*, $R'(\infty) < c$.

Transactions between the retailer and supplier are governed by a revenue-sharing contract. This contract contains two parameters, ϕ and w. The first, ϕ, is the share of retail revenue the retailer keeps, *i.e.*, given retail revenues $R(q)$, the retailer must transfer $(1-\phi)R(q)$ to the supplier but retains the remaining $\phi R(q)$. It is natural to assume $\phi \in [0, 1]$, even though that restriction is not strictly required. We do not include in our model the administrative costs associated with monitoring revenues and collecting transfers. In other words, we assume the cost of implementation has no impact on the contract the supplier offers or the quantity the retailer purchases. (Implementation costs, of course, may impact whether revenue sharing is adopted at all; see below.) The second parameter in a revenue-sharing contract, $w \geq 0$, is the wholesale price. This is the amount the retailer pays the supplier per unit. Note that a standard wholesale-price contract is a revenue-sharing contract with $\phi = 1$.

In this game the following events occur: the supplier determines and announces the terms of the revenue-sharing contract; the retailer orders q units and pays the supplier wq; the supplier produces and delivers q units; the retailer receives revenue $R(q)$ and transfers $(1-\phi)R(q)$ to the supplier. Each firm maximizes its expected profit given that the other firm does the same. All information in this game is common knowledge to both firms.

2. Analysis

We begin with the integrated channel solution, the decisions that maximize total supply chain profit. We next consider the retailer's order quantity decision for a given revenue-sharing contract and identify the supplier's optimal revenue-sharing contract. We conclude with a comparison between revenue-sharing contracts and two other well known contracts for coordinating supply chains: buy-back contracts and quantity-flexibility contracts.

2.1 Integrated channel

Total supply chain profit given an order quantity q is $\Pi(q)$,

$$\Pi(q) = R(q) - qc.$$

Since $R(q)$ is concave and $R'(0) \geq c$, the optimal order quantity q_I is positive and satisfies

$$R'(q_I) = c. \tag{1}$$

2.2 Actions with a revenue-sharing contract

The retailer's profit function with a revenue-sharing contract is $\pi_r(q)$,

$$\pi_r(q) = \phi R(q) - qw.$$

Assume $R'(0) > w/\phi$, so the retailer's optimal order quantity, $\widehat{q}$, must satisfy

$$\phi R'(\widehat{q}) = w.$$

The retailer's optimal order quantity equals q_I when the wholesale price is $w(\phi) = \phi c$. It follows that the supplier coordinates the channel by selling *below* cost, *i.e.*, $w(\phi) \leq c$. Naturally, a supplier should certainly be skeptical of any scheme that requires him to sell at a loss. As we will see, this is not a problem for the supplier.

If the supplier offers a coordinating contract, $\{\phi, w(\phi)\}$, the retailer's profit is

$$\pi_r(q_I) = \phi R(q_I) - q_I c = \phi \Pi(q_I).$$

Hence, ϕ is not just the fraction of revenue the retailer keeps, but also the fraction of supply chain profit she receives. The supplier captures the remaining profit:

$$\pi_s(q_I, w(\phi), \phi) = (1 - \phi)\Pi(q_I).$$

Thus, with revenue sharing the supplier can maximize total supply chain profit and take any share of that profit for herself. As a result, the supplier is willing to sell below cost because she earns a positive profit and potentially the supply chain's maximum profit.

The mechanics of coordination through revenue sharing are illustrated in Figure 1. Here we show the marginal revenue curve for the integrated channel, $R'(q)$, as well as the marginal cost curve, which is a horizontal line at c. The optimal quantity is found at q_I where these curves intersect, and the profit of the integrated channel is the area a_1 between these curves. The marginal revenue curve for a retailer under a revenue-sharing contract, $\phi R'(q)$, is also shown. It is everywhere below the system's marginal revenue curve. The corresponding marginal cost curve ϕc is similarly below the system cost curve so the optimum remains at q_I. The retailer's profit is the area a_2 and the supplier's profit is a_1 minus a_2. The retailer's profit decreases as ϕ decreases: as ϕ decreases the retailer's cost decreases at rate $cq_I = R'(q_I)q_I$, but the retailer's revenue decreases at a faster rate, $\int_0^{q_I} R'(x)dx$, since $R'(q)$ is a decreasing function. Thus, total supply chain profit is held constant as ϕ decreases (assuming $w(\phi)$ is the wholesale price) but the allocation of profits shifts towards the supplier.

Figure 1 illustrates that coordination with revenue sharing involves shifting down both the marginal revenue curve and the marginal cost curve while maintaining their intersection at q_I. Of course, it is also possible to achieve this objective by shifting only one of the curves. A quantity discount policy does exactly that (Jeuland and Shugan, 1983). As shown by Moorthy (1987) any total cost schedule $c(q)$ that satisfies $c'(q) < R'(q)$ for $q < q_I$, $c'(q_I) = R'(q_I)$ and $c'(q) > R'(q)$ for $q > q_I$, coordinates the channel without altering the retailer's revenue function. To shift profit to the supplier it is sufficient to increase $c'(q)$ for $q < q_I$ while abiding by the condition that $c'(q) < R'(q)$. Although the quantity discount scheme can effectively coordinate the action of a single retailer, it encounters a problem if the supplier sells to more than one independent retailer. If q_I is not constant across retailers (say, because they face heterogenous demand functions), it is unlikely that the same quantity discount schedule coordinates the action of every retailer because the coordinating discount schedule is not independent of the marginal revenue curve. With revenue sharing the coordinating contract is independent of the marginal revenue curve, and so the same revenue sharing contract coordinates the actions of heterogenous retailers.

The particular ϕ chosen depends on the firm's relative bargaining power, but it is clear that they should agree to coordinate the channel. It is always possible to divide a larger pie in such a way that each firm's piece is increased. While the model does not restrict the supplier from expropriating the supply chain's entire profit, that outcome is admittedly neither reasonable nor expected. As ϕ approaches zero, the retailers' profit function becomes quite flat about q_I, which leaves the retailer with little incentive to in fact choose the optimal quantity. (On a percentage basis any deviation may have a large impact on profits, but little impact on an absolute basis.) According to our theory the retailer will choose q_I if he has *some* incentive to choose q_I, but in reality a supplier should think twice before offering the retailer a contract that leaves the retailer with only a small fraction of his revenues.

There is another approach to show that revenue sharing coordinates the supply chain with a single retailer. Proposition 4 in Caldentey and Wein (1999) states that a set of transfer payments coordinates a system if (roughly speaking) each player transfers a constant fraction of its utility to each other player and the fractions sum to one. Define the retailer's utility to be $R(q)$ and the supplier's utility to be $-cq$.[2] With revenue sharing, the retailer transfers to the supplier $(1-\phi)R(q)$ and the supplier transfers to the retailer $-wq$, where a negative transfer implies a payment from the retailer to the supplier. (While awkward, a negative transfer payment is allowed by their theory.) When $w = \phi c$, the ratio of the retailer's transfer to his utility is $(1-\phi)$ and the comparable ratio for the supplier is ϕ. Since those fraction indeed sum to one, the proposition applies when $w = \phi c$.

2.3 Comparison with buy-back contracts

Buy backs are perhaps the most commonly studied contract in the supply chain contracting literature. Under such a contract, the supplier sells units to a retailer at the start of the season for w_b per unit and agrees to purchase left over units at the end of the selling season for b per unit, $b < w_b$. Pasternack (1985) was the first to show that not only does a buy-back contract coordinate the supply chain with a supplier selling to an independent retailer solving a newsvendor problem, it also supports an arbitrary division of profits. His work has since been extended by a number of authors (*e.g.*, Kandel, 1996; Marvel and Peck, 1995; and Donohue, 1996). We now show that a coordinating buy-back contract in a newsvendor problem is equivalent to our coordinating revenue-sharing contract $\{\phi, w(\phi)\} = \{\phi, \phi c\}$.

Following Pasternack (1985), we suppose that there is a single selling period and that

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[2] Technically, Caldentey and Wein (1999) assume voluntary compliance (Cachon and Lariviere, 2000) so the supplier may not fill the retailer's order q_r. However, if the supplier's utility is $-cq_s$ if the supplier's action is $q_s \geq q_r$ and $-2cq_r$ otherwise, $q_s = q_r$ is a dominant strategy. The retailer may act as if the supplier is required to supply q_r.

demand is given by a probability distribution $F(x)$. The retail price is fixed at p. If the supplier offers a buy-back contract $\{b, w_b\}$, the retailer's profit function is

$$\begin{aligned}
\pi_r(q, w_b, b) &= p\left(q - \int_0^q F(x)dx\right) + b\int_0^q F(x)dx - w_b q \\
&= (p-b)\left(q - \int_0^q F(x)dx\right) - (w_b - b)q \qquad (2)
\end{aligned}$$

The retailer's optimal order $\hat{q}$ is found from the critical fractile $F(\hat{q}) = (p-w)/(p-b)$. In contrast, the profit of an integrated system is

$$\Pi(q) = p\left(q - \int_0^q F(x)dx\right) - cq \qquad (3)$$

and the system optimal order quantity is determined by

$$F(q_I) = (p-c)/p. \qquad (4)$$

Suppose the supplier offers:

$$b^* = p(1-\phi) \qquad w_b^* = p(1-\phi) + \phi c.$$

The decentralized channel then faces the same critical fractile as the integrated channel, and the retailer's profit when stocking q^I is $\phi\Pi\left(q^I\right)$. This coordinating buy-back contract thus results in the same split of expected channel profits as the coordinating revenue-sharing contract $\{\phi, \phi c\}$. The relationship between the two is actually deeper. As can be seen from (2), a buy back is equivalent to reducing the retailer's cost of purchasing a unit to $(w_b - b)$ while also reducing the fraction of revenue he keeps to $(p-b)/p$. Since $(w_b^* - b^*) = \phi c$ and $(p - b^*)/p = \phi$, the two contracts result in the same *realized* profits for the retailer and supplier for any realization of demand.

Consequently, for a newsvendor problem with a fixed retail price, the supplier has two ways of implementing revenue sharing. She can either require a percentage of realized revenue or she can demand a fixed payment per unit sold (as in Pasternack, 1999). The approaches are equivalent. Dana and Spier (1999) note that the same is true in their model as well. However, this equivalence does not hold in general. Below we present an example that buy backs cannot coordinate but that proportion-based revenue sharing can.

2.4 Comparison with quantity-flexibility contracts

While buy backs are a special case of revenue sharing, the same is not true for all coordinating supply chain contracts. Consider the quantity flexibility (QF) contract of Tsay and Lovejoy

(1999) and Tsay (1999). Here, the retailer purchases q units for w_Δ per unit at the start of the season and may return up to Δq units at the end of the season for a full refund, $\Delta \in [0,1]$. As with the buy-back contract, let $F(x)$ be the distribution function of demand in the season and fix the retail price at p. The retailer's expected profit is

$$\pi_r(q, w_q, \Delta) = p\left(q - \int_0^q F(x)dx\right) - w_q\left(q - \int_{(1-\Delta)q}^q F(x)dx\right). \tag{5}$$

The retailer's problem is concave in q and the first order condition is

$$p\,(1 - F(q)) = w_q\,(1 - F(q) + (1-\Delta)F((1-\Delta)q))\,.$$

The integrated channel's first order condition is still (4). Thus, the retailer chooses the integrated channel quantity when

$$w_\Delta = \frac{c}{1 - F(q_I) + (1-\Delta)F((1-\Delta)q_I)}.$$

Tsay (1999) shows that as Δ goes to one, w_Δ goes to p and all profits go to the supplier.

While QF and revenue sharing can achieve similar splits of expected profits, several distinctions keep them from being equivalent. For example, with the QF contract $w_\Delta \geq c$ for all values of Δ whereas with revenue sharing $w \leq c$. Additionally, the coordinating price w_Δ is not independent of the demand distribution, whereas $w(\phi) = \phi c$ is. The driver of these differences can be seen by examining (5). The first term is the retailer's revenue. It does not vary with Δ for a fixed q. Any coordinating QF contract leaves the retailer's revenue unchanged. Profits are shifted from the retailer to the supplier by raising the retailer's marginal cost at every point except q_I; hence, $w_\Delta > c$. Under revenue sharing, profits are shifted to the supplier by lowering marginal revenue at every point while coordination is assured by simultaneously reducing marginal cost, necessitating $w\,(\phi) < c$.

3. Model Extensions

We now explore other market settings that can be coordinated through revenue-sharing contracts. We first consider a newsvendor problem with price-dependent demand and show that revenue sharing can coordinate both decisions. We then examine a setting in which retailers compete for customers and demonstrate that a revenue-sharing contract can coordinate the system and support alternative divisions of supply chain profits.

3.1 Price-dependent newsvendor

The setting is the same as in section 2.3 but now the retail price p is also a decision variable. Demand is governed by a known distribution $F(x,p)$ such that for any $p_1 > p_2$, $F(x,p_1) > F(x,p_2)$. Charging a higher retail price consequently leads to a stochastically smaller market. Petruzzi and Dada (1999) provide a recent review of such models. Here, we assume that the integrated supply chain has a unique optimal quantity and price pair, $\{q_I, p_I\}$ that results in a profit of $\Pi(q_I, p_I)$.

To see that revenue sharing can lead an independent retailer to implement $\{q_I, p_I\}$, note that his profit function given a revenue-sharing contract $\{\phi, w\}$ can be written as

$$\pi_r(q,p,w,\phi) = \phi\left(p\left(q - \int_0^q F(x,p)dx\right) - \frac{w}{\phi}q\right).$$

The integrated channel profit function $\Pi(q,p)$ is as given in (3) except that p is now a decision variable. Thus, when $w = \phi c$,

$$\pi_r(q,p,w,\phi) = \phi\Pi(q,p),$$

and the retailer optimizes his profit by ordering the supply chain's optimal quantity q_I and setting the supply chain's optimal price p_I.[3]

We consequently have that the contract that coordinates a channel facing a newsvendor problem with an exogenous retail price also coordinates a newsvendor problem with an endogenous retail price. This result is all the more remarkable because Emmons and Gilbert (1998) have shown that buy-back contract with a fixed buy-back rate b (equivalently, revenue sharing with a fixed per unit payment) *cannot* coordinate such a system. (It is not difficult to show that a quantity-flexibility contract also does not coordinate this system.) Our proportional scheme works because the coordinating contract is independent of the retail price; the decentralized channel picks the same quantity as the integrated system for any retail price. With buy backs, the coordinating contract depends on the retail price. Given

[3] Note that demonstrating coordination does not depend on the specifics of the price-dependent newsvendor problem. A similar argument consequently suffices for any system in which revenue is a function of price and quantity and costs are linear in quantity. For example, the revenue function could be generated from a queuing model in which the demand rate depends on price and average service time, and the latter depends on the chosen capacity/processing rate.

a fixed buy-back rate, the decentralized channel picks the right quantity for only one retail price. The contract consequently lacks the flexibility to coordinate both actions.

3.2 Competing retailers

We now extend our basic model and allow for more than two firms. Specifically, we suppose that there is still a single supplier but that sales occur through n distinct locations. Denoting the vector of stocking levels as $\bar{q} = \{q_1, \ldots, q_n\}$, the revenue at location i is $R_i(\bar{q})$ and the revenue for the entire system is:

$$R(\bar{q}) = \sum_{i=1}^{n} R_i(\bar{q}).$$

We assume that $R_i(\bar{q})$ is continuous and that $\partial^2 R_i / \partial q_i \partial q_j \leq 0$ for all $j \neq i$. Locations i and j are thus substitutes since increasing the quantity at location j reduces the marginal revenue (and hence total revenue) at i. Let $\bar{q}_i^0 = \{0, \ldots, 0, q_i, 0, \ldots, 0\}$ for $i = 1, \ldots, n$. We also assume that there exists a finite q_i^δ such that $\partial R_i(\bar{q}_i^0) / \partial q_i < \delta$ for all $q_i \geq q_i^\delta$ for any $\delta > 0$. That is, marginal revenue at i permanently drops below any positive number at a finite quantity level even if all other locations stock nothing.

A unit costs $c > 0$ regardless of the location where it is stocked. Denote the system optimal vector of quantities as $\bar{q}^I = \{q_1^I, \ldots, q_n^I\}$. We assume $R_i(\bar{q})$ is unimodal in q_i and that $R(\bar{q})$ is sufficiently well-behaved that $\bar{q}^I$ is unique with $q_i^I > 0$ for all i. Previous assumptions assure that q_i^* is finite for all i. Assume that $\bar{q}^I$ can be found from first order conditions and thus satisfies the following system of equations:

$$R_i^i\left(\bar{q}^I\right) + \sum_{j \neq i} R_j^i\left(\bar{q}^I\right) = c \quad i = 1, ..., n, \tag{6}$$

where $R_j^i(\bar{q}) = \frac{\partial R_j(\bar{q})}{\partial q_i}$. Let $\Pi\left(\bar{q}^I\right) = R\left(\bar{q}^I\right) - c \sum_{i=1}^{n} \bar{q}_i^I$ be the integrated system profit. In a decentralized system, one of n independent retailers runs each location. Retailer i sets q_i to maximize his own profit without coordinating his decision with other retailers.

Although we have imposed some mathematical structure, our model is general enough to capture a wide variety competitive situations. For example, if $R_i(\bar{q})$ is a deterministic function such as

$$R_i(\bar{q}) = q_i \left(1 - q_i - \beta \sum_{j \neq i} q_j\right) \tag{7}$$

for $0 \leq \beta < 1$, we have a model of Cournot competition (Tirole, 1988, Tyagi, 1999). Alternatively, we could have competing newsvendors as studied by Parlar (1988) and Lippman and

McCardle (1997). Also, while our formulation treats the retailers as competing in quantity, an extension to price competition is straightforward.

We assume that the supplier can charge each retailer a unique price $w_i > 0$ and look for a Nash equilibrium in order quantities. Because a rational retailer will never set a quantity that pushes his marginal revenue below his acquisition cost, the game is unchanged if we restrict retailer i to choosing an order quantity from the interval $[0, q_i^{w_i}]$. Combined with our earlier assumptions, this assures the existence of a pure strategy Nash equilibrium q^N (see Theorem 1.2 of Fudenberg and Tirole, 1991). Assuming that each element of q^N is positive, it can be found from the following system of equations:

$$R_i^i\left(\bar{q}^N\right) = w_i \quad i = 1, ..., n. \tag{8}$$

In comparing (8) with (6), one sees that an immediate consequence of decentralization is that the individual retailer does not account for the externality $\sum_{j \neq i} R_j^i(\bar{q})$ he imposes on the rest of the system. Consequently, $\bar{q}^I$ cannot be a Nash equilibrium if the supplier were to transfer at marginal cost; since $R_i^i\left(\bar{q}^I\right) > c$, retailer i would deviate to a higher quantity.

The supplier, of course, can reduce the incentive to raise the order quantity by raising the wholesale price. Suppose the supplier charges $\bar{w}^I = \left\{w_1^I, \ldots, w_n^I\right\}$ where:

$$w_i^I = c - \sum_{j \neq i} R_j^i\left(\bar{q}^I\right) \quad i = 1, ..., n.$$

The two systems of equations (8) and (6) are now equivalent and $\bar{q}^*$ is a Nash equilibrium. The scheme works by charging retailer i for the marginal cost he imposes on the system – both in production and externalities – at the system optimal quantity. Thus the non-linear system cost that retailer i ignores:

$$\int_0^{q_i^I} \sum_{j \neq i} R_j^i\left(q_1^I, \ldots, q_{i-1}^I, z_i, q_{i+1}^I, \ldots, q_n^I\right) dz_i$$

is imposed on him via a linear proxy:

$$q_i \sum_{j \neq i} R_j^i\left(\bar{q}^I\right).$$

Because the cost imposed on retailer i is only an approximation, one cannot guarantee that $\bar{q}^I$ is a unique equilibrium when $\bar{w}^I$ is charged.

Because $R_j^i(\bar{q}) \leq 0$, w_i^I is greater than the marginal cost of production. Bernstein and Federgruen (1999) present a related model in which coordination can also be achieved by linear prices above the marginal cost of production. One consequence of pricing above cost is that the supplier can both coordinate the system and earn a positive profit when selling to competing retailers. However, w_i^I can only support one division of profits:

$$\begin{aligned}\pi_s\left(\bar{q}^I, \bar{w}^I\right) &= \sum_{i=1}^{n} q_i^I \sum_{j\neq i} -R_j^i\left(\bar{q}^I\right) \\ \pi_{r_i}\left(\bar{q}^I, \bar{w}^I\right) &= R_i\left(\bar{q}^I\right) - q_i^I w_i^I \quad i = 1, ..., n.\end{aligned}$$

Bernstein and Federgruen (1999) propose using lump sum transfer to achieve alternative profit allocations. We now show that this can also be achieved through revenue sharing.

Suppose the supplier now offers retailer i a revenue-sharing contract $(\phi, w_i(\phi))$ for $\phi \in [0, 1]$. Assuming a constant ϕ across all retailers is solely for convenience. It is straightforward to verify that the sufficient conditions for the existence of a pure strategy Nash equilibrium still hold and that an equilibrium now must satisfy

$$\phi R_i^i\left(\bar{q}^N\right) = w_i(\phi) \quad i = 1, ..., n.$$

Hence if $w_i(\phi) = \phi w_i^I = \phi\left(c - \sum_{j\neq i} R_j^i\left(\bar{q}^I\right)\right)$, $\bar{q}^I$ can again be supported as a Nash equilibrium. Retailer i's profit is now

$$\pi_{r_i}\left(\bar{q}^I, \phi, \phi\bar{w}^I\right) = \phi\left(R_i\left(\bar{q}^I\right) - q_i^I w_i\right) = \phi\pi_{r_i}\left(\bar{q}^I, \bar{w}^I\right).$$

It can be shown that $\pi_{r_i}\left(\bar{q}^I, \bar{w}^I\right) \geq 0$, so the retailer is willing to participate for any $\phi \geq 0$. The supplier's profit is now

$$\pi_s\left(\bar{q}^I, \phi, \phi\bar{w}^I\right) = (1-\phi)\,\Pi\left(\bar{q}^I\right) + \phi\pi_s\left(\bar{q}^I, \bar{w}^I\right),$$

a convex combination of the integrated system profit and what she would earn without revenue sharing.

Thus our results for a single retailer carry over to competing retailers. Our analysis complements Dana and Spier (1999) who examine revenue sharing with competing newsvendors. They assume perfect competition so retailers earn zero profits regardless of the contract offered. We assume an oligopoly in which retailers earn positive returns as long as $\phi > 0$.

There are several difference between the single and multiple retailer settings that are worthy of mention. First, since the transfer price in the oligopoly case must account for competitive externalities, $w_i^I(\phi)$ is greater than the cost of production for $\phi > c/\left(c - \sum_{j\neq i} R_j^i\left(\bar{q}^I\right)\right)$. Since $w_i^I(1) > c$, allowing the retailers to keep all of their revenue does not allow them to receive all system profits. One would have to have $\phi = \Pi\left(\bar{q}^I\right)/\left[\Pi\left(\bar{q}^I\right) - \pi_s\left(\bar{q}^I, \bar{w}^I\right)\right] > 1$ to drop the supplier's profit to zero. In the single retailer case, the coordinating wholesale price never exceeds c and transferring at cost allows the retailer to capture all profits. Next, the coordinating revenue-sharing contract is no longer independent of the system's revenue curve. Now, the terms offered retailer i depend on the revenue function of every other location. This is again driven by the need to account for competition. Finally, revenue sharing is no longer equivalent to the coordination scheme of Caldentey and Wein (1999). They assume exchanges are made between every agent. We assume all exchanges involve the supplier. Their proposal thus requires a greater number of transactions.

4. Limitations of revenue sharing

To this point we have presented a very favorable picture of revenue sharing. We have shown that is a powerful tool capable of coordinating a variety of supply chain problems. Now we argue the opposite case and offer some caveats on when to implement revenue sharing. We focus on administrative costs and retailer moral hazard.

4.1 Administrative costs

Essential to implementing revenue sharing is the ability for the supplier to *ex post* verify the retailer's revenue, which we have supposed is costless. But that need not be the case in practice. At a minimum, the channel would incur the cost of linking the supplier's and retailer's information system. More likely, the supplier would have to monitor closely how the downstream firm manages the assets it has purchased.[4]

In many ways, a video retailer like Blockbuster is an ideal candidate for revenue sharing. First, the assets (*i.e.*, the video tapes) have a limited number of uses. Second, the chain has essentially uniform prices and rental policies. Third, the individual stores already have the technology in place to capture relevant information (*e.g.*, whether all copies of a movie are out) and report it to corporate level. Thus a movie studio should be able to monitor Blockbuster's revenue from a given title by merely linking to its corporate system. Contrast this with the problem faced by the maker of machine tools selling equipment to a job shop. To implement revenue sharing, the tool maker must know just how many lots were

[4] The analysis of buy backs (Pasternack, 1985) and QF contracts (Tsay, 1999) similarly ignore administrative expenses. Since these contracts also require monitoring retail sales, their costs should be comparable to revenue sharing.

processed through its equipment and the transaction price for each lot. Overcoming such administrative costs could easily swamp any increase in the supplier's earnings.

In general, a supplier must balance the costs of running revenue sharing with the profit sacrificed by using a non-coordinating contract. The simplest such contract is a wholesale-price contract in which the supplier sets a fixed per-unit wholesale price and does not share in the retailer's revenue. Selling the product outright is then the only way the supplier earns a profit. We now consider supply chain performance when the supplier sets the wholesale price to maximize her own profit in both single and multi-retailer settings.

4.1.1 The single retailer case

Suppose there is a single retailer. Given a wholesale-price contract, the retailer's optimal order quantity is the unique solution to

$$R'(q) - w = 0, \tag{9}$$

if $R'(0) > w$, otherwise the optimal order quantity is zero. Since $R'(q)$ is strictly decreasing, there exists a function $w(q)$ such that q is the retailer's optimal order quantity when the supplier charges the wholesale price $w(q)$. From (9) it must be that $w(q) = R'(q)$. The supplier's profit can then be expressed as $\pi_s(q)$,

$$\pi_s(q) = q(w(q) - c) = q\left(R'(q) - c\right),$$

and

$$\pi_s'(q) = w(q) - c + qw'(q) = R'(q) + qR''(q) - c. \tag{10}$$

The supplier's profit function is unimodal in q if $R'(q) + qR''(q)$ is decreasing in q. This is equivalent to assuming that the elasticity of the retailer's order decreases in q, so successive percentage decreases in the wholesale price bring about smaller and smaller increases in sales. For tractability, we assume that condition holds.[5] Let q^* be the supplier's optimal quantity to induce, *i.e.*, q^* is the solution to $\pi_s'(q) = 0$ and $w(q^*)$ is the supplier's optimal wholesale-price contract.

Since $qR''(q) < 0$, comparing (10) with (1) immediately reveals that $q^* < q_I$. Thus total supply chain performance is not optimal with the wholesale-price contract. Further, since

[5] If $R(q)$ is the revenue function implied by a newsvendor problem, Lariviere and Porteus (2000) present conditions on the hazard rate of demand for this to be true.

$w(q)$ is decreasing and $R'(q_I) = c$, it follows (not too surprisingly) that the optimal wholesale price $w(q^*)$ is greater than marginal cost, which is in sharp contrast to the optimal wholesale price under a revenue-sharing contract.

Whether the supplier finds a wholesale-price contract attractive depends on two factors: How much the decentralized channel earns and what part of those earnings she captures. A useful measure of the former is the efficiency of the wholesale-price contract:

$$\frac{\pi_s(q^*) + \pi_r(q^*)}{\Pi(q_I)},$$

The efficiency of a contract is the percentage of the optimal profit achieved under that contract. A measure of the latter is the supplier's profit share, which is the ratio of the supplier's profit to the supply chain's profit, $\pi_s(q^*)/\Pi(q^*)$. A wholesale-price contract is attractive to the supplier if the efficiency and the profit share are close to one.

Since the optimal wholesale price is $w(q^*) = c - q^* R''(q^*)$, the curvature of the marginal revenue curve $R'(q)$ plays an important role in determining the contract's efficiency and profit share. This is shown in Figure 2. At the optimal solution $R'(q^*) - c = -q^* R''(q^*)$, since $R'(q^*) = w$. Thus, in the optimal solution $-q^* R''(q^*)$, which is the height of the triangle label a_2, equals the height of the rectangle labeled a_3. (The triangle a_2 is formed by the tangent of the marginal revenue curve at q^*.) The supplier's profit equals the area of the rectangle a_3, $q^*(w(q^*) - c)$. The triangle a_2 is an approximation for the retailer's profit. It underestimates the retailer's earnings if $R'(q)$ is convex and it overestimates the retailer's profit if $R'(q)$ is concave. Since the area of the triangle is half of the area of the rectangle, the supplier's profit share is less (more) than $2/3^{rds}$ if the marginal revenue is convex (concave).

A similar analysis allows us to estimate the efficiency of the system. The loss in supply chain profit from a wholesale-price contract is:

$$\int_{q^*}^{q_I} (R'(z) - c)\, dz.$$

The corresponding region is labeled a_4 in the diagram. An approximation for this loss is the triangle formed by dropping the tangent to $R'(q^*)$ from q^* down to where it crosses the horizontal at c. This happens at $2q^*$. The area of the resulting triangle is again equal to half of supplier's profit. It is less than the area of a_4 if $R'(q)$ is convex but greater if $R'(q)$ is concave. It is straightforward to see that this also implies that $q^* > q_I/2$ ($< q_I/2$) when marginal revenue is concave (convex). Consequently, coordinating the system increases total profit by more (less) than 50% of the supplier's profit if marginal revenue is convex (concave). It increases by exactly 50% of the supplier's profit if marginal revenue is linear.

Interestingly, Rentrak, a video-cassette distributor, claims that a retailer should increase his order quantity by a factor of four when switching from the traditional wholesale-price contract to their revenue-sharing contract (see www.rentrak.com). If we assume optimal contracts are implemented, then the marginal revenue curve in that industry must be quite convex. (Recall that for a linear marginal revenue curve $2q^* = q_I$.) If the marginal revenue curve is quite convex, then efficiency could be substantially lower than 75%. In that case revenue sharing can significantly increases the profit of both firms in the supply chain.

To illustrate these results, suppose

$$R'(q) = 1 - q^{\alpha},$$

for $\alpha > 0$ and $q \in [0, 1]$. Such a marginal revenue curve results if, for example, the supply chain faces a deterministic inverse demand curve $P(q) = 1 - q^{\alpha}/(\alpha + 1)$. Note that the marginal revenue curve is convex for $\alpha < 1$, linear for $\alpha = 1$, and concave for $\alpha > 1$. Furthermore, it satisfies our assumption that $R'(q) + qR''(q)$ is decreasing, which guarantees a unique optimal contract for the supplier. Figure 2 is based on this example.

The optimal quantity for the supplier to induce under a wholesale-price contract is

$$q^* = \left(\frac{1-c}{1+\alpha}\right)^{1/\alpha}.$$

The optimal quantity for an integrated channel is $q_I = (1-c)^{1/\alpha}$. The resulting profits are

$$\begin{aligned} \pi_r(q^*) &= \left(\frac{\alpha}{1+\alpha}\right)\left(\frac{1-c}{1+\alpha}\right)^{\frac{1+\alpha}{\alpha}} \\ \pi_s(q^*) &= \alpha\left(\frac{1-c}{1+\alpha}\right)^{\frac{1+\alpha}{\alpha}} \\ \Pi(q_I) &= \left(\frac{\alpha}{1+\alpha}\right)(1-c)^{\frac{1+\alpha}{\alpha}} \end{aligned}$$

We see that the profit share is $(1+\alpha)/(2+\alpha)$ and the efficiency is

$$\frac{\pi_s(q^*) + \pi_r(q^*)}{\Pi(q_I)} = \frac{2+\alpha}{(1+\alpha)^{\frac{1+\alpha}{\alpha}}}.$$

Efficiency is a decreasing function of α, *i.e.*, efficiency improves as the marginal revenue curve becomes more concave. As $\alpha \to 0$, efficiency approaches $2/e \approx 0.73$, and as $\alpha \to \infty$, efficiency approaches one and the system is coordinated in the limit. However, it approaches coordination rather slowly. For example, with $\alpha = 10$, which is displayed in Figure 3, efficiency is 86% even though the marginal revenue curve is quite concave. What changes much more quickly is the profit share. At $\alpha = 10$, the supplier now captures 91.7% of the supply chain's profit.

To summarize, the potential profit gain from coordination in a supply chain with a single retailer depends on the shape of the marginal revenue curve. A convex marginal revenue curve generally leads to worse performance; the decentralized system stocks less than half of the integrated system quantity and efficiency is frequently less than 75%. Supply chain efficiency is generally higher when the marginal revenue curve is concave (although it may still be less than 90%). We conclude that the gains from coordinating the system decrease (revenue sharing is less attractive) as the marginal revenue function becomes more concave. However, implementing revenue sharing may still be worthwhile especially if the supply has a pre-existing infrastructure to track revenues.

4.1.2 The multiple retailer case

In Section 3.2, we showed that the supplier can coordinate the system with a wholesale-price contract and earn a positive profit. That, however, is not the wholesale-price contract that would maximize her profit. We now explore how supply chain efficiency varies with the level of competition among retailers through an example.

Suppose there are n symmetric retailers and the revenue function in market i for $i = 1, \ldots, n$ is as given in (7). This structure allows two measures of competition, the parameter β and number of retailers n, with an increase in either implying more intense competition. For a fixed β and n, there is a unique equilibrium such that $q_i^N = (1 - w) / (2 + \beta (n - 1))$. The integrated channel in contrast has $q_i^I = (1 - c) / (2 + 2\beta (n - 1))$. $R_j^i (\bar{q}) = -\beta q_j$ for all $j \neq i$, so the system is coordinated at a price of

$$w^I = c + \frac{\beta (n - 1) (1 - c)}{2 + 2\beta (n - 1)}.$$

One can show that w^I is increasing β and n. As competition increases by either measure, a higher wholesale price is required to moderate competition. Since the total amount the centralized channel sells for a given β is increasing in n, a greater number of retailers in the system thus shifts more profit to the supplier if she were to price at w^I.

The supplier, however, will not price at w^I. From her perspective, w^I is too low. Somewhat remarkably, her optimal wholesale price $w^* = (1 + c) / 2$ is independent of both β and

n.[6] The gap between w^I and w^* is $(1-c)/(2+2\beta(n-1))$ and drops to zero as n gets large. Indeed, if β is close to one, the difference between the two wholesale prices is quite small for even low values of n. This suggests that the supply chain may not suffer much loss when the supplier prices to maximize her own profit. The efficiency of the channel when the supplier charges w^* is

$$1-\frac{1}{(2+\beta(n-1))^2}.$$

If β equals zero, the system reduces to n independent linear markets and the efficiency of the system is 75%. More remarkably, if $\beta > 0$, efficiency improves rapidly as the number of retailers increases. For example, if β equals 1/3, then efficiency is over 85% with just three retailers while five retailers brings efficiency over 90%. Double β to 2/3, and efficiency with three and five retailers is 91% and 95.4%, respectively. Tyagi (1999) shows that for essentially any demand structure the supplier's profit always increases as more Cournot competitors are added but does not consider the efficiency of the supply chain. van Ryzin and Mahajan (2000) do consider system efficiency for an inventory problem in which stocking levels of substitute products are set by distinct firms. They similarly find that efficiency improves rapidly as the number of competitors increases.

Contrasting this example with that of the single retailer case suggests that competition in the retail market may have a greater impact on supply chain efficiency under a wholesale-price contract than the nature of the revenue function. Thus revenue sharing should be less attractive to the supplier when several competitors serve the market. This is particularly true if there are limited economies of scale in administering revenue sharing so that each retailer added to the system requires a significant additional administrative cost.

4.2 Retailer effort, moral hazard, and revenue sharing

We now consider another consideration that may work against the use of a revenue-sharing contract, retailer moral hazard. We have thus far assumed that revenue depends on the retailer's order quantity and perhaps the retail price. In reality, the retailer influences revenue through many other actions: *e.g.*, advertising, service quality and merchandizing, to name just a few. We now consider the impact of those alternative decisions on the effectiveness

6 Tyagi (1999) presents a necessary and sufficient condition for the supplier's optimal wholesale price to be independent of the number of retailers.

of revenue-sharing contracts. In particular, we assume that demand is influenced by retailer effort, which we take as a proxy for a variety of decisions. Naturally, effort is costly; better service and cleaner stores do not come for free. In addition, we assume that effort is non-contractable, which means that the supplier and the retailer cannot write a contract that specifies the retailer's effort level. Equivalently, we assume that the firms choose not to contract on effort because the cost of specifying, monitoring and enforcing retailer effort is too high. That is a reasonable assumption in our setting because we presume that retailer effort is the aggregation of many decisions.

Others have examined the impact of retail effort on channel performance. Chu and Desai (1995) study a model in which costly retailer effort improves customer satisfaction, and higher customer satisfaction has both short and long term benefits. Lariviere and Padmanabhan (1997) and Desiraju and Moorthy (1997) examine retailer effort under alternative information structures. None of these consider revenue-sharing contracts. In the franchising literature, Lal (1990) has both franchisee and franchisor moral hazard and finds only the latter warrants the franchisor sharing in the franchisee's revenue. Gallini and Lutz (1992) and Desai and Srinivasan (1995) suppose that the franchisee can increase demand and evaluate the use of revenue sharing as a way for the franchisor to signal private information about the value of the franchise. None of these papers considers revenue sharing in the absence of the franchisee paying a lump sum to the franchisor.

4.2.1 Model

We begin with a model similar to the base model of Section 1, except now the retailer chooses an order quantity q and an effort level e. Decisions are made after observing the terms the supplier offers, $\{\phi, w\}$. Expected revenue is $R(q, e)$, which is continuous, differentiable, strictly increasing in e, and concave in q. The retailer incurs a cost $g(e)$ to choose effort level e, where $g(e)$ is continuous, increasing, differentiable and convex with $g(0) = 0$.

We have already demonstrated that revenue sharing can coordinate a supply chain with a single retailer and arbitrarily divide profits when demand is independent of retail effort. We now show that revenue sharing cannot coordinate the channel and allow the supplier a positive profit if retailer effort affects demand. Let $\Pi(q, e)$ be the integrated channel's profit function,

$$\Pi(q, e) = R(q, e) - g(e) - qc.$$

Suppose the maximization of $\Pi(q, e)$ is well behaved and unimodal in q and e. Given that $\Pi(q, e)$ is continuous and differentiable, the optimal integrated solution, $\{q_I, e_I\}$, must satisfy the first order conditions:

$$\frac{\partial \Pi(q_I, e_I)}{\partial q} = \frac{\partial R(q_I, e_I)}{\partial q} - c = 0 \tag{11}$$

$$\frac{\partial \Pi(q_I, e_I)}{\partial e} = \frac{\partial R(q_I, e_I)}{\partial e} - g'(e_I) = 0 \tag{12}$$

The retailer's profit function is

$$\pi_r(q, e) = \phi R(q, e) - g(e) - qw.$$

Coordination requires that q_I be optimal if the retailer chooses effort e_I. Under revenue sharing, that holds when

$$\frac{\partial \pi_r(q_I, e_I)}{\partial q_I} = \phi \frac{\partial R(q_I, e_I)}{\partial q} - w = 0,$$

which, from (11), requires that $w = \phi c$. The coordinating contract thus has not changed, which is intuitive because we have so far assumed that the retailer's effort is fixed at the optimal level. Given the above wholesale price, the retailer's profit function is

$$\pi_r(q^I, e) = \phi R(q^I, e) - g(e) - q^I \phi c.$$

The retailer's profit is strictly concave in e, and the optimal effort satisfies the first order condition. But from (12),

$$\frac{\partial \pi_r(q^I, e^I)}{\partial e} = \phi \frac{\partial R(q^I, e^I)}{\partial e} - g'(e^I) < 0,$$

so the retailer's optimal effort is less than e^I if $\phi < 1$. In other words, the retailer chooses the optimal effort only if $\phi = 1$. In that case the channel is coordinated only if the supplier sells at marginal cost, leaving her with no profit.

Revenue sharing does not coordinate the supply chain because it fails to induce the correct retailer effort decision. It is useful to contrast this result with those for the price-dependent newsvendor. There, a revenue-sharing contract coordinates the price and quantity decisions. The chief difference between the two is that in the price-dependent newsvendor the cost of expanding demand (foregone revenue on units that would have been sold anyway) is captured in the revenue function. The supplier then bears her share of the cost. Here, the cost of expanding demand $g(e)$ falls only on the retailer. As the supplier share of revenue increases, the retailer's incentive to exert demand-enhancing effort decreases. Thus, the supplier can coordinate the channel for a given effort level, but that coordination causes the retailer to choose an effort level that is lower than optimal. Coordinating effort is possible if the supplier could assume part of the effort cost but the retailer then has every reason to misrepresent the true cost incurred. Corbett and DeCroix (1999) make a similar argument.

4.2.2 What is a supplier to do?

Revenue sharing does not coordinate the supply chain when retail effort matters, but the supplier only cares about channel coordination indirectly. The supplier's primary objective is the maximization of her profit. Therefore, the supplier may still offer a revenue-sharing contract if that contract does better than an alternative. The natural alternative is the standard wholesale-price contract.

To provide tractability, we assume specific functional forms for the revenue and effort cost functions. Suppose the retailer faces the following inverse demand curve[7]

$$P(q,e) = 1 - q + 2\tau e$$

where $\tau \in [0,1]$ is a constant parameter. Retail effort has a greater impact on demand as τ increases. For a given quantity q, the marginal change in the quantity clearing price with respect to retail effort is increasing in τ. Expected revenue is then $R(q,e) = qP(q,e)$. Suppose the effort cost function is $g(e) = e^2$. Thus, the retailer's expected profit function is

$$\pi_r(q,e) = \phi R(q,e) - e^2 - qw.$$

The upper bound on τ ensures that the problem is jointly concave in q and τ.

Let $e(q)$ be the retailer's unique optimal effort,

$$e(q) = \phi\tau q.$$

Naturally, the retailer's optimal effort is increasing in his share of revenue, ϕ. Given $e(q)$, the retailer's profit function can be written as

$$\begin{aligned} \pi_r(q,e(q)) = \pi_r(q) &= \phi R(q,e(q)) - e(q)^2 - qw \\ &= q\left[\phi - q(\phi - \phi^2\tau^2) - w\right], \end{aligned}$$

and the optimal quantity is found to be

$$q(w,\phi) = \frac{\phi - w}{2(\phi - \phi^2\tau^2)},$$

[7] Desai and Srinivasan (1995), Lariviere and Padmanabhan (1997), and Desiraju and Moorthy (1997) all use functionally equivalent formulations.

assuming $w < \phi$, otherwise $q(w, \phi) = 0$. Solving for the retailer's optimal profit yields

$$\pi_r(q(w, \phi)) = \frac{(\phi - w)^2}{4(\phi - \phi^2 \tau^2)}.$$

The integrated channel solution is obtained from the retailer's solution with marginal cost pricing, *i.e.*, $w = c$ and $\phi = 1$. Note that the integrated channel charges a retail price of

$$p_I = \frac{1 + c(1 - 2\tau^2)}{2\,(1 - \tau^2)},$$

which exhibits a peculiar behavior with respect to the cost of production c. The retail price is increasing in the cost of production, as one would expect, if $\tau < 1/\sqrt{2}$ but is decreasing in c if $\tau > 1/\sqrt{2}$. An increase in the marginal cost has two impacts on the integrated channel's problem. For a fixed effort level, it decreases the quantity at which marginal revenue equals marginal cost, leading to a higher price. It also induces a lower effort level, resulting in a smaller market and hence a lower price for any quantity. If the impact of effort is significant (*i.e.*, $\tau > 1/\sqrt{2}$), the effort effect dominates the quantity effect, and the retail price falls.

Returning to the decentralized system, the supplier's profit function is

$$\pi_s(w, \phi) = (1 - \phi)R(q(w, \phi)) + q(w, \phi)(w - c)$$

The supplier's profit is concave in w,

$$\frac{\partial^2 \pi_s(w, \phi)}{\partial w^2} = -\frac{1 - \phi\,(1 - 2\tau^2)}{2\phi^2\,(1 - \phi\tau^2)^2} < 0,$$

so the optimal wholesale price is

$$w(\phi) = \frac{\phi\,((1 - \tau^2)\,\phi + c(1 - \phi\tau^2))}{1 + \phi(1 - 2\tau^2)}.$$

The supplier profit function simplifies to

$$\pi_s(w(\phi), \phi) = \frac{(1 - c)^2}{4\,(1 + \phi\,(1 - 2\tau^2))}.$$

Examining $\pi_s(w(\phi), \phi)$, one sees that the supplier's profit is increasing in ϕ if $\tau > 1/\sqrt{2}$, otherwise the supplier's profit is decreasing in ϕ. Consequently, the optimal contract is a wholesale-price contract ($\phi = 1$) if $\tau > 1/\sqrt{2}$, otherwise the supplier's optimal contract is a revenue-sharing contract with $\phi = 0$. We again have that effort effects dominate when $\tau > 1/\sqrt{2}$, and the supplier prefers a wholesale-price contract which minimizes the distortion in the retailer's effort decision. In those cases, the supplier prefers a smaller share of a larger pie. When retail effort has only a minimal impact on demand, quantity effects dominate. The supplier prefers to use revenue sharing to extra a large share of the supply chain profit.

5. Discussion

Our analysis demonstrates that revenue sharing is a very attractive contract. Given a single supplier and retailer it coordinates the supply chain and arbitrarily divides the resulting profits for essentially any reasonable revenue function. The supplier sells at a wholesale price below her production cost, but her participation in the retailer's revenue more than offsets the loss on sales. We have shown that the widely studied buy-back contract of Pasternack (1985) is a special case of our proportional revenue-sharing contract and that our contract can coordinate problems that buy backs cannot. In particular, since a coordinating revenue-sharing contract is independent of the retail price, it can coordinate a newsvendor problem with price-dependent demand. We have also addressed competition among retailers, showing that coordination is still possible although the ability to divide profits may be limited.

With so much going for it, one might argue that revenue sharing should be ubiquitous. We present some reasons why it is not. First, we try to identify conditions under which the gains from revenue sharing over a simpler wholesale-price contract may not cover revenue-sharing's additional administrative expense. For a bilateral monopoly, we show that the performance of a wholesale-price contract depends on the shape of the marginal revenue curve and that the efficiency of the system under a wholesale-price contract improves as marginal revenue becomes more concave. However, even with a concave marginal revenue curve, the gains from coordination may be substantial. Competition between retailers appears to have a much greater impact in improving the efficiency of the system.

We also demonstrate that the revenue sharing may not be attractive if the retailer's actions influence demand. Specifically, we assume that the retailer can increase demand by exerting costly effort and that retail effort is non-contractable. Since revenue-sharing contracts reduce the retailer's incentive to undertake effort relative to a wholesale-price contract, the supplier may prefer offering a wholesale-price contract. In other words, while revenue-sharing contracts are effective at coordinating the retailer's purchase quantity decision, they work against the coordination of the retailer's effort decision. When demand is sufficiently influenced by retail effort, revenue-sharing contracts should be avoided.

Other factors beyond those we have considered may influence the decision to offer revenue sharing. In particular, a retailer may carry substitute or complementary products from other suppliers. If one supplier offers revenue sharing and the other does not in the substitute case, the retailer could be predisposed to favor the supplier that allows the retailer to keep all revenue by, for example, recommending the product to undecided consumers. In the case of complements (say, personal computers and printers), the retailer may discount the product offered under revenue sharing to spur sales of the other product. Here revenue sharing may result in a product being used as a loss leader. We leave these issues to future research.

We began this paper with a discussion of the video cassette rental industry, so we close with it as well. Our model suggests that in a wholesale-price contract the optimal wholesale price should be set above marginal cost, but with revenue sharing the wholesale price should be set below marginal cost. Consistent with that result, the wholesale price in the video industry fell from \$65 per tape to \$8 per tape when revenue sharing was introduced. A wholesale price of \$8 is plausibly below marginal cost (production, royalties, transportation, handling, *etc.*), so the industry may have adopted a channel coordinating contract.

The adoption of revenue sharing in the video industry is also consistent with the limitations we identified for revenue sharing. The first limitation is that administrative costs should be sufficiently low. Almost all video stores have systems of computers and bar codes to track each tape rental, so it should not be difficult for the suppliers to monitor and verify revenues. Further, it is unlikely that retail effort has a sufficient impact on demand. In a video rental store, the retailer merely displays boxes of available tapes from which customers make their selections. Unlike home appliance or automobile retailing (to name just two examples), customers do not make their video selection after substantial consultation with a retail salesperson (which requires effort). Hence, we feel that the video rental supply chain is particularly suited for revenue sharing. Although there are limits to these contract, we suspect that other industries have yet to discover the virtues of revenue sharing.

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Figure 1: A coordinating revenue sharing contract: $\phi = 1/3$

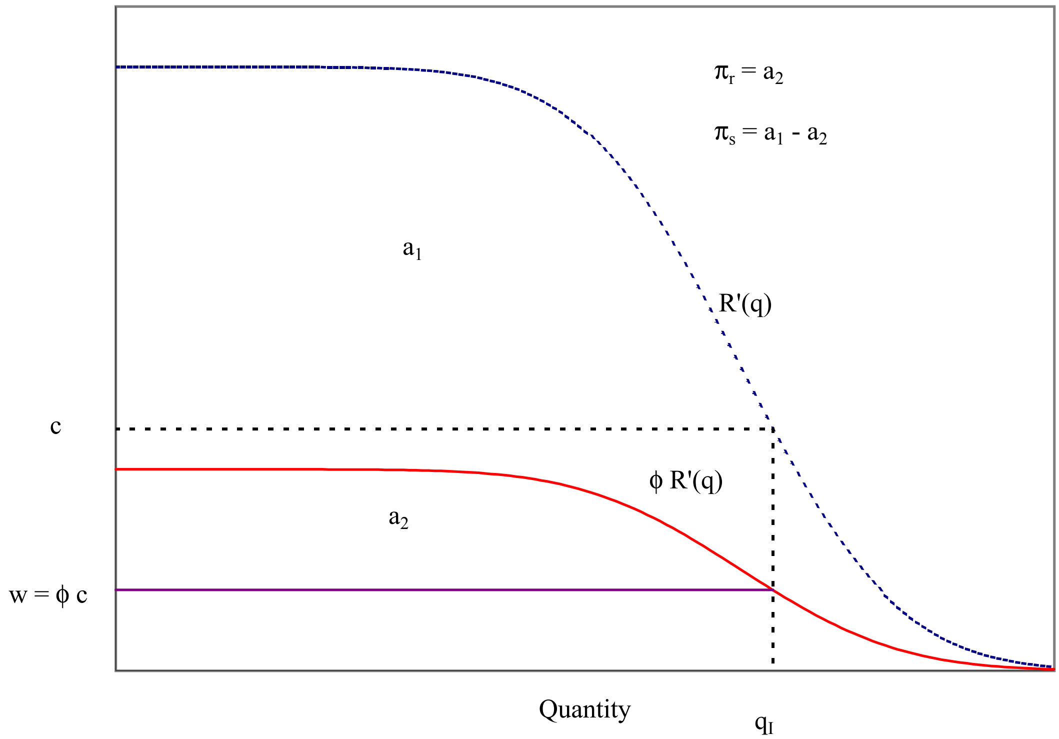

Figure 2: Optimal Wholesale Price Only Contract with $\alpha = 1/4$;
efficiency = 74%

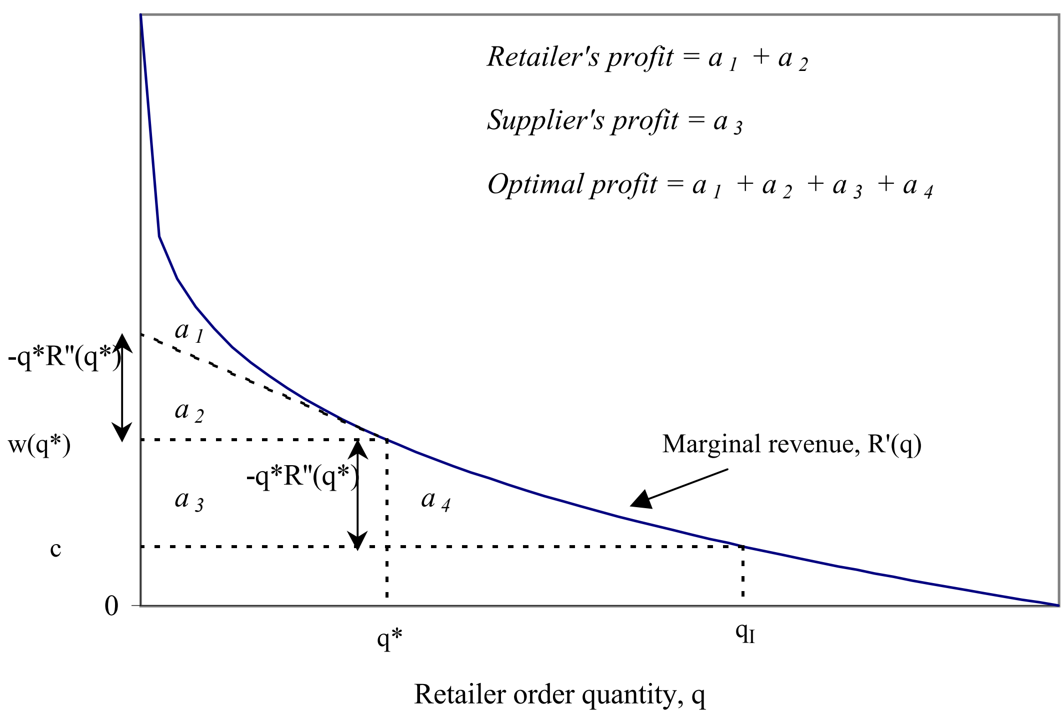

Figure 3: Optimal Wholesale Price Only Contract with α = 10;
efficiency = 86%