Optimal Pricing with Speculators and Strategic Consumers

Xuanming Su

Haas School of Business, University of California, Berkeley, CA 94720

Abstract

This paper studies a monopolist firm selling a fixed capacity. The firm sets a price before demand uncertainty is resolved. Speculators may enter the market purely with the intention of resale, which can be profitable if demand turns out to be high. Consumers may strategically choose when to purchase, and they may also choose to purchase from the firm or from the speculators. We characterize equilibrium prices and profits and analyze the long run capacity decisions of the firm. There are three major findings. First, the presence of speculators increases the firm's expected profits even though the resale market competes with the firm. Second, by facilitating resale, the firm can mimic dynamic pricing outcomes and enjoy the associated benefits while charging a fixed price. Third, speculative behavior may generate incentives for the seller to artificially restrict supply and thus may lead to lower capacity investments. We also explore several model extensions that highlight the robustness of our results.

July 2009

Electronic copy available at: http://ssrn.com/abstract=1441533

1 Introduction

Many items are available only in limited quantities. Examples include tickets to a Broadway show, seats at a basketball match, and "hot" electronic gadgets such as Apple's iPhone and Nintendo's Wii video game console. Due to capacity constraints, there is often insufficient supply to meet demand. When scarcity arises, consumers unable to buy over regular channels may be willing to turn to online resellers and internet auction sites (e.g., eBay). Often, these consumers end up paying a significant price premium. Since there is a lucrative arbitrage opportunity, speculative behavior naturally emerges. Unlike "true" consumers, speculators make purchases purely with the intention of reselling them at a profit. The objective of this paper is to introduce a tractable modeling framework that captures such speculative resale and to understand its implications for the firm.

Although ticket scalping has an unglamorous history, the internet resale industry has steadily risen and become increasingly acceptable with the growth of companies such as StubHub (a subsidiary of eBay) and TicketsNow (owned by Ticketmaster). The total revenue from online ticket sales is estimated to reach $4.5 billion by 2012, which is a significant increment over the $22 billion in "regular" sales by US live music and sporting event industries (see Mulpuru and Hult, 2008). These figures are not surprising because online resellers may charge a staggering price premium. For example, even though their face value rarely exceeds $5,000, courtside tickets to the NBA Finals may be priced as high as $35,000 each (see McGinn, 2008). Even the free (but limited) admission tickets to Barack Obama's inauguration were put up for sale, although these postings were soon removed (see Colker, 2008). Scalpers also tried to profit from reselling tickets to Michael Jackson's memorial service, and many questioned the morality behind their actions (see Gundersen and Breznican, 2009). Given rampant speculation, it is not surprising that Ticketmaster, a long-time exclusive provider of tickets to many major live entertainment events, began to introduce its own resale program (see Smith and Silver, 2006). However, speculation is not always profitable: eBay estimates that about 45% of the tickets it has brokered are at or below face value (see Johnson, 2007).

As another example, consider the widespread shortage of Nintendo Wii game consoles in 2007 (see Kane and Wingfield, 2007). Although already into the second year of launch, consumers were unsuccessful at locating the Wii console at local retailers for many months. As December approached, many began to turn to other alternatives and ended up paying substantially higher

Electronic copy available at: http://ssrn.com/abstract=1441533

prices from internet resellers. On eBay auctions, average selling prices for the Wii console remained consistently above $350 from November 2007 to January 2008, occasionally reaching $450, even though the selling price is $249 at retail stores. Since this is a lucrative transaction for resellers, it is not surprising that retailers may also wish to play a part. Amidst the consumer frenzy, some retailers were hoarding their inventory from consumers and instead selling them on eBay at a "Buy It Now" price of $399. Some other retailers chose to increase margins by bundling the much sought-after Wii console with a number of other items such as games and accessories (see Kuchera, 2007). Is Nintendeo simply underpricing the Wii at $249? Why don't they increase production quantities? Our paper sheds new light on these questions.

We study the following model in this paper. There is a monopolist seller with a fixed capacity. We consider two time periods. In the first period, there is a deterministic number of consumers in the market. (These are consumers who, for example, purchase season passes or submit pre-orders.) In the second period, a random number of new consumers arrive. In addition, there is a large pool of "bargain hunters" who are willing to purchase all remaining units at a lower price. There is also a large number of potential speculators who may buy in the first period to sell in the second. Therefore, in the first period, the seller sells to consumers and speculators, and in the second, the seller and speculators jointly serve as suppliers. The seller sets a price at the start of the first period (this fixed price applies in both periods), and the resale price in the second period is competitively determined by the speculators. Note that resale may be profitable if demand is sufficiently large (then speculators can resell units at a sufficiently high price). We assume free entry: speculators enter the market until each earns zero expected profit. Therefore, equilibrium prices and speculative behavior are endogenously determined.

We obtain three main results. First, we show that speculative resale can benefit the seller. The intuition is as follows. With an active resale market, the seller indirectly benefits because speculators "place their bets" upfront. As speculators purchase units before realizing their resale value, the seller not only collects more revenue earlier but also shifts some risk of having leftover inventory to the speculators. However, the downside is that speculators on the resale market compete directly with the firm and may undercut its price. Note that the seller can use the price to control the equilibrium volume of speculative activity: as the price is increased, less speculators will enter the market, and in the extreme case, the seller can charge a sufficiently high price to completely shut speculators out of the market. We find that in many cases, the seller's optimal price leads to some positive amount of speculative resale. Our results suggest that speculative

behavior in equilibrium may have been implicitly encouraged by the seller.

Our second main result is that speculative resale can serve as a proxy for dynamic pricing. The practice of revenue management has been successful in a wide variety of applications, but it may not be feasible in some contexts where equity and fairness across consumers are important considerations and the seller has to charge a fixed price. In such cases, we show that the seller can still enjoy some of the benefits of dynamic pricing by creating (or at least accommodating) resale mechanisms. Effectively, the resale market becomes a mechanism that dynamically adjusts prices to match demand conditions. This increased flexibility extracts consumer surplus and some of the resulting earnings are channeled back to the seller through increased sales to speculators. Therefore, speculators serve as market makers that help the seller achieve a form of "disguised" dynamic pricing. Interestingly, we find that speculators play a completely different role if the seller directly implements dynamic pricing. In that case, speculators act as competitors and it is optimal for the seller to price them out of the market.

Finally, we investigate the seller's long run capacity decision. We find that the seller is less inclined to make capacity investments in the presence of potential speculative behavior. This is because a tight capacity level intensifies scarcity, which helps to sustain profitability on the resale market and thus generates speculative demand. In fact, there are situations where the seller will not expand even if doing so were costless. This result suggests that although speculative resale can improve profits, it may also discourage capacity investments.

The remainder of this paper is organized as follows. Section 2 provides a literature review. Section 3 introduces the model. Sections 4 provides an equilibrium analysis. Section 5 discusses the analogy between speculative resale and dynamic pricing. Section 6 studies long run capacity decisions. Section 7 describes several model extensions. Section 8 concludes. All proofs are presented in the Appendix.

2 Literature Review

This paper is most closely related to the recent literature on strategic consumer behavior in operations management. In this literature, consumers form expectations about future market conditions and strategically respond to them. Specifically, a group of papers study the scenario where consumers may strategically time their purchases in anticipation of future price changes. These papers (e.g., Aviv and Pazgal, 2008; Elmaghraby, Keskinocak, Gulcu, 2008; Jerath, Netessine, Veeraragha-

van, 2007; Levin, McGill, Nediak, 2005; Ovchinnikov and Milner, 2005; Su, 2007) analyze optimal dynamic pricing strategies when consumers strategically wait for markdowns. A more complete review appears in the survey by Shen and Su (2007). There is another set of papers that focus on how strategic consumers anticipate future availability as driven by firms' inventory decisions; these expectations of availability influence their buy-or-wait decision. For example, see Liu and van Ryzin (2008a, 2008b), Su and Zhang (2008), Cachon and Swinney (2009), Lai, Debo and Sycara (2007), Yin, Aviv, Pazgal, Tang (2007). Unlike the above, our current work studies a different type of strategic behavior. While the above papers study strategic *waiting* behavior in anticipation of price changes, we additionally study strategic *speculative* behavior in anticipation of potential resale profits.

There is a number of related papers that study economic mechanisms used to sell a fixed capacity when there is demand uncertainty. Harris and Raviv (1981) use a mechanism design approach to analyze a priority pricing scheme when selling to a fixed number of buyers with unknown valuations. Lazear (1986) studies a clearance sales mechanism. DeGabra (1995) shows that creating scarcity can induce buying frenzies before consumers learn their valuations. Xie and Shugan (2001) consider advance selling to consumers who are uncertain of their valuations at the time of purchase. Dana (1998) studies how to use advance purchase discounts to screen consumers with different levels of uncertainty. Gale and Holmes (1993) and Dana (1999a) consider a model with two different products (e.g., two flights with different departure times) and consumers are uncertain over their relative preferences. When there is limited capacity, Png (1989) points out that offering reservations before consumers learn their valuations insure them against the possibility of being rationed, and Alexandrov and Lariviere (2007) show that it can be optimal to offer reservations for free. While most of the papers above focus on valuation uncertainty, Dana (1999b) considers aggregate demand uncertainty and studies a mechanism where a limited number of units are available at each price and lower priced units are sold first. (This is similar to Wilson, 1988, who does not consider demand uncertainty.) Our paper also considers aggregate demand uncertainty, and more importantly, a key driver of our results is strategic speculation, which is absent in the models above.

There has been some research that studies speculation and resale in markets with limited capacity. Three different approaches have been used and our model differs from all of them.

First, there is a stream of work on ticket scalping; see Courty (2003a) for a survey. In this literature, scalpers exploit arbitrage opportunities and make definite profits by buying and then reselling units. For example: Swofford (1999) assumes that scalpers are less risk-averse than

the seller and earn a guaranteed risk premium by serving as middlemen; Courty (2003a) assumes that consumers who arrive later have higher valuations so scalpers make sure profits by reselling to them; and Karp and Perloff (2005) assume that scalpers are able to perfectly price discriminate and extract maximal consumer surplus. In contrast, speculation is a risky prospect in our model: due to aggregate demand uncertainty, speculators may "flip" the unit for a profit as intended, but they may also incur losses. In our model, the uncertainty faced by speculators is an essential ingredient, and speculators make zero expected profit in equilibrium. We further explore the distinction between arbitrage (certain profits) and speculation (possible but uncertain profits) in Section 7.

Next, there are some papers that consider resale markets as a medium for units to change hands between consumers. For example, Courty (2003b) and Geng, Wu, Whinston (2007) develop models where a fixed group of consumers face valuation uncertainty, and consumers who turn out to have low valuations can later resell to consumers with high valuations. However, our model admits a separate population of speculators who have no use for the unit but make purchases because of potential resale profits. Further, we consider free entry of speculators, so the equilibrium volume of speculative transactions is endogenously determined.

Finally, speculators play yet another different role in some other papers. Png (1989) studies a setting where consumers make reservations before learning their valuations; he argues that sellers should not compensate consumers who end up with low valuations, because otherwise speculators with no use for the unit may simply show up to claim the compensation. Similarly, Su and Zhang (2009) suggest that if available guarantees (e.g., sellers compensate consumers when products are out of stock) are excessively generous, speculators may have an incentive to prowl stores for out-of-stock products. In the papers above, speculators do not "speculate" per se: they merely constrain the seller's actions. Since the seller would take care to prevent such behavior, these speculators do not actively participate in the market in equilibrium. However, in our model, speculative resale not only emerges in equilibrium but also has important implications for the seller.

3 Model

In this model, there are three groups of agents. On the supply side, there is a monopolist seller. On the demand side, there are consumers. Finally, there are speculators who have no use for the product; they buy it with the hopes of reselling it at a higher price. We describe these three groups of agents below.

1. Seller The seller has a fixed capacity of K units. For most of the analysis, the capacity K is exogenously given, but we will consider long run capacity decisions later. The seller operates over two time periods, $t = 1, 2$. Let p_t denote the unit price in period t. In the analysis, we shall first focus on a fixed price, i.e., $p = p_1 = p_2$ for both periods, but we will also consider dynamic pricing, where the seller may set an initial price p_1 at $t = 1$ and later adjust it to p_2 at $t = 2$. The seller's goal is to maximize total expected revenue (or equivalently, expected profit, since marginal cost is normalized to zero).

2. Consumers Each consumer demands exactly one unit but may have different valuations for the unit. Consumer valuations are either V_H (high) or V_L (low); let $\Delta \equiv V_H - V_L > 0$. The consumer's valuation refers to his utility from consuming the unit and thus can also be interpreted as the maximum amount he is willing to pay for it. There are three demand segments, comprising of four different types of consumers (referred to as strategic, myopic, new, and low-value consumers in the rest of this paper):

- **Fixed demand.** First, the seller faces a fixed demand segment, comprising of a deterministic number $W < K$ of consumers who are present in the market at $t = 1$. A fraction ϕ of these consumers are *strategic* and the rest are *myopic*. Myopic consumers will purchase at $t = 1$ as long as the price does not exceed their valuations. Strategic consumers rationally anticipate purchase opportunities at $t = 2$ and will buy at $t = 1$ only if the price is sufficiently attractive. All these consumers have high valuations V_H. We denote $\bar{\phi} \equiv 1 - \phi$ for convenience.

- **Random demand.** Another demand segment is uncertain. Depending on market conditions, there is a random number X of *new* consumers who arrive to the market at $t = 2$. Let F denote the distribution of X. Each new consumer will buy at $t = 2$ as long as the price is not greater than his valuation. As above, these consumers have high valuations V_H.

- **Low-value demand.** The final demand segment consists of consumers with low valuations V_L. These *low-value* consumers enter the market only at $t = 2$ and we assume that there is a large number (i.e., greater than K) of them. Hence, at a sufficiently low price (e.g., below V_L), the seller can always sell all remaining capacity. In cases where demand exceeds capacity, we assume efficient rationing, so low-value consumers have the lowest priority. (Equivalently, we may assume that low-value consumers show up only at the end of $t = 2$, after all other consumers have their demands met.)

3. Speculators These are traders who buy the unit at $t = 1$ purely with the intention of reselling it at a higher price later. They have zero valuation for the unit. Speculators face a gamble because they must make purchases at price p_1 before random demand X is realized, and the market clearing price $\tilde{p}$ on the resale market at $t = 2$ may be lower. For convenience, we assume that each speculator trades at most one unit, but we can also think of a speculator who trades multiple units as multiple speculators as long as the trade size is small relative to the entire mass of speculators. We assume *free entry* of speculators. There is a large number of potential speculators who may enter the market until it is no longer attractive to do so, i.e., when the expected profit from speculation is zero. We use S to denote the number of speculators who enter the market. Note that the number of speculators S is endogenously determined.

Summary: Chronology of Events Now, we summarize the timeline in our basic model. At $t = 1$, the seller first sets the price p. Strategic consumers decide whether to buy or wait until $t = 2$, while myopic consumers decide whether to buy or leave the market. At the same time, potential speculators decide whether to enter the market (i.e., buy units for resale). Between the two time points, random demand X is realized and publicly observed. Alternatively, we may assume that a market demand signal is publicly observed and that the signal is sufficiently informative for all parties to perfectly infer demand X. At $t = 2$, the seller and speculators (if any) function as suppliers, selling to potential buyers, i.e., strategic consumers who waited, the random number of new consumers, as well as low-value consumers. Given the seller's price p and random demand realization X, speculators jointly (competitively) determine the market clearing price $\tilde{p}$ on the resale market. Finally, trades take place at these prices. Figure 1 captures the sequence of events in our model and also summarizes some key terms that will be used later.

Applications There are several practical applications of our model. First, consider sports events. For many sports teams, there is a relatively stable group of diehard fans, reasonably assumed to be of a known size. These are represented by the fixed demand segment in our model and they often secure season passes way in advance. As the season progresses and uncertainty unfolds, the popularity of each game is determined and random demand is realized. For popular matches, tickets may be sold out and consumers may have to turn to internet resellers, sometimes paying a substantial premium. Our model can also be applied to study the market for concert tickets. Like the case of sports events, there is also an active resale market where speculators buy and sell

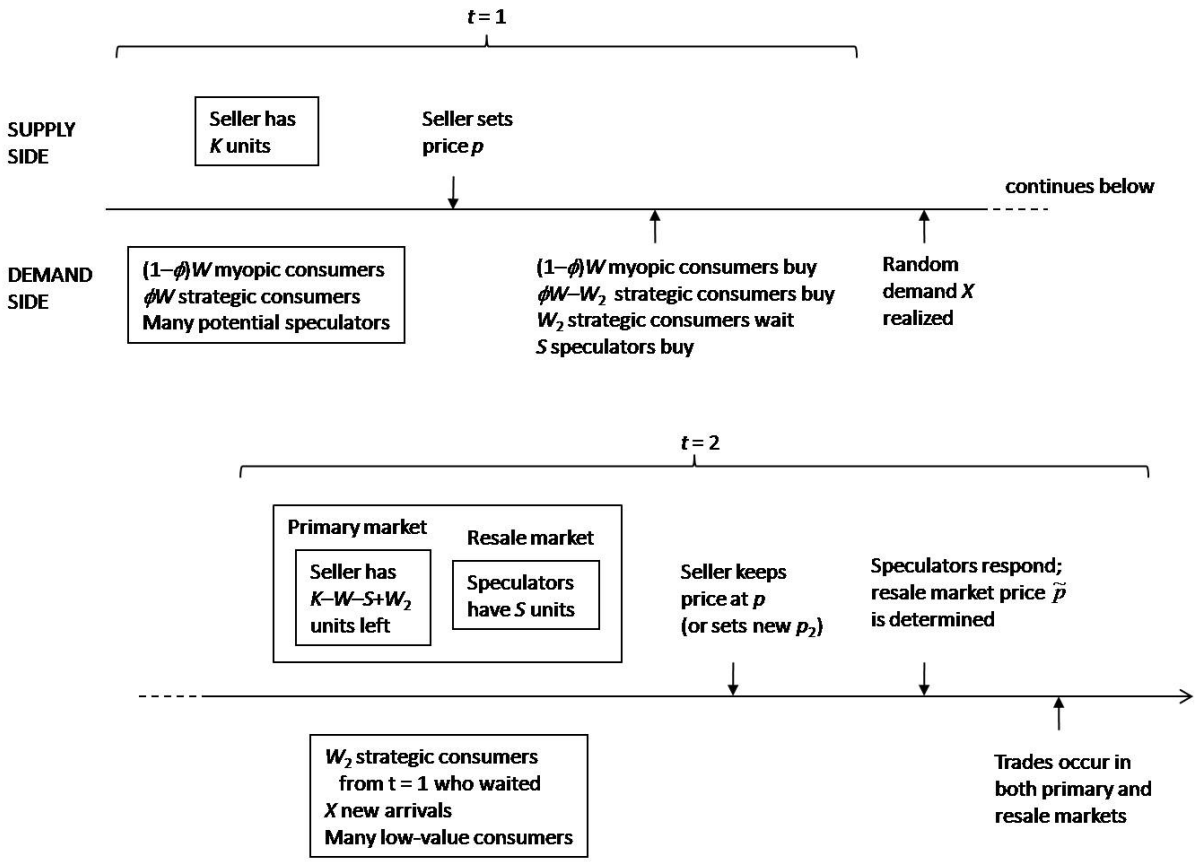


Figure 1: Sequence of Events.

concert tickets, especially for the best seats in the house. Finally, our model can also be applied to newly introduced products, such as electronic gadgets. Due to production constraints, the number of units available for purchase is fixed. Manufacturers sometimes take preorders before the product is launched. Such orders often come from early adopters, and this is captured by the fixed demand segment in our model. However, demand from additional new consumers is unpredictable. There may be scarcity, which creates arbitrage opportunities for speculators.

4 Equilibrium Analysis

In this section, we study the subgame perfect equilibrium of the game between speculators, consumers, and the seller using standard backward induction. We begin by characterizing the competitive outcomes in the resale market (or secondary market), before analyzing the seller's optimal pricing strategies in the direct-sale market (or primary market). Then, we examine the effect of con-

sumer speculation. We do so by comparing the equilibrium outcomes of our model to a benchmark scenario where there are no speculators.

4.1 Resale Market

The resale market operates at $t = 2$. At this time, the seller may have already sold some units to consumers as well as speculators. Let $K_2 \leq K$ denote the seller's remaining capacity. Further, let S denote the number of speculators who have bought the unit and entered the market. In other words, the total supply (i.e., total number of units available) is $K_2 + S$, with K_2 units on the primary market and S units on the resale market. Further, recall that the random number of new consumers who enter the market at $t = 2$ is X, which is now publicly known. The number of strategic consumers who chose to wait, denoted $W_2 \leq W$, is also publicly known (since this can be inferred from the seller's remaining capacity). Thus, the total high-valuation demand is $W_2 + X$. We stress that the clearing price $\tilde{p}$ on the resale market depends both on total high-valuation demand $W_2 + X$ as well as total supply $K_2 + S$.

On the resale market, speculators set prices competitively. For concreteness, we use the following mechanism to determine equilibrium outcomes. Suppose each of the S speculators individually chooses some price. Given these prices as well as the seller's price p, we have a collection of prices for each of the $K_2 + S$ available units. Based on these prices, available units are arranged in order of increasing price. Similarly, consumers are arranged in order of increasing valuation, with ties broken randomly. Now, each available unit is matched to a corresponding consumer, with lowest priced units being matched to highest valuation consumers. (Since there is a large number of low-value consumers, every unit will be matched to some consumer.) For each matched pair, trades occur at the given price if it does not exceed consumer valuation; otherwise no trades occur and both sides gain nothing. This setup can be interpreted as a posted-price mechanism with efficient rationing: buyers arrive one-by-one in order of decreasing valuations and each may purchase the lowest-priced unit remaining. In fact, the outcome of this mechanism is similar to the double auction (see Wilson, 1985, and McAfee, 1992), which suggests that speculators compete not only with one another but also with the seller. Notice that our mechanism specifies a non-cooperative game between the speculators (since the seller's price is already given). We seek the symmetric, pure-strategy Nash equilibrium of this game. This is the equilibrium price $\tilde{p}$ on the resale market.

The following lemma characterizes the equilibrium resale price.

Lemma 1 *Suppose the seller at $t = 2$ has K_2 units remaining, each priced at $p \in [V_L, V_H]$. Further, suppose there are S speculators, W_2 strategic consumers who remain in the market, and X new consumers who arrive. Then, the equilibrium resale price is*

$$\tilde{p}^* = \begin{cases} V_L, & W_2 + X < S, \\ p, & S \leq W_2 + X < S + K_2, \\ V_H, & W_2 + X \geq S + K_2. \end{cases}$$

This result shows that there are three possible outcomes in the resale market, corresponding to low/moderate/high demand realizations. When demand is low, i.e., $W_2 + X < S$, speculators undercut one another and drive the resale price down to V_L. In this case, all consumers with high-valuations will buy from speculators. There will also be some low-value consumers who buy up the remaining speculator supply. However, the seller who keeps the price at $p > V_L$ will not sell any unit. Next, when demand is moderate, i.e., $S \leq W_2 + X < S + K_2$, speculators maintain the price at p. Since speculators can (slightly) undercut the seller, they will all sell out their units first, and then the remaining high-valuation consumers will buy from the seller, who may have unsold units at the end. Finally, when demand is high, i.e., $W_2 + X \geq S + K_2$, there is ample demand to support a high resale price V_H. However, recall that the seller's price remains at $p < V_H$. Therefore, consumers first buy from the seller; when seller capacity is depleted, they then buy from speculators until total supply is depleted. There may still be some unsatisfied high-valuation demand.

Finally, note that the expected resale price can be written as follows.

$$E\tilde{p} = V_L \cdot F(S - W_2) + p \cdot \Pr\{S - W_2 \leq X \leq K - W\} + V_H \cdot \overline{F}(K - W). \tag{1}$$

This is because $W_2 + X < S$ in the first term is equivalent to $X < S - W_2$, and $W_2 + X \geq S + K_2$ in the last term is equivalent to $X \geq K - W$ (since $K_2 = K - S - W + W_2$). Observe that as the number of speculators S becomes larger, increased competition in the resale market leads to a lower expected resale price $E\tilde{p}$.

4.2 Primary Market

Now we move ahead to consider the primary market at $t = 1$. The seller first chooses the price $p \leq V_H$. The seller's price is a key decision variable because it influences speculators' market entry decisions (i.e., whether to buy the unit for speculation) as well as strategic consumers' purchase decisions. These in turn determine the resale market price as described above.

We begin by considering speculators' decisions. Recall that we assume free entry of speculators. That is, given the seller's price p, speculators will enter until it is no longer profitable to do so. The expected profit from speculation is $E\tilde{p} - p$. Since the expected resale price $E\tilde{p}$ along the equilibrium path is decreasing in the number of speculators, S will increase from 0 until $E\tilde{p} = p$. In other words, from (1), we have

$$p = \frac{V_L \cdot F(S^* - W_2^*) + V_H \cdot \overline{F}(K - W)}{F(S^* - W_2^*) + \overline{F}(K - W)},$$

where S^* denotes the equilibrium number of speculators and W_2^* denotes the number of strategic consumers who wait.

Next, we consider strategic consumers' purchase decisions. These consumers may either purchase immediately at $t = 1$ or wait until $t = 2$. Observe that in any equilibrium with $S^* > 0$, strategic consumers will never be better off buying immediately. Why is this so? Consider the hypothetical situation where strategic consumers can only buy from speculators if they wait. In this case, they will be indifferent between purchasing immediately (at price p) and waiting to buy from the resale market (at price $\tilde{p}$), because as explained above, equilibrium speculation activity will adjust itself until $p = E\tilde{p}$. (Note that rationing will not be a concern here because it occurs only when $\tilde{p} = V_H$, i.e., when the strategic consumer will earn zero ex post surplus anyway.) However, in reality, the strategic consumer who waits may buy from both the seller as well as speculators. Under high demand realizations (i.e., when the resale price is $\tilde{p} = V_H$), there is a chance that the strategic consumer may obtain the product directly from the seller at price $p < V_H$. With the added option of buying from speculators, waiting becomes more attractive. Therefore, it can never be strictly more attractive for strategic consumers to buy immediately. We shall thus proceed with the analysis by taking $W_2^* = \phi W$ whenever $S^* > 0$, i.e., all strategic consumers will wait whenever there is speculative behavior.

Putting speculators' and strategic consumers' decisions together, we make the following observation. When the seller's price is $p < V_H$, we must have

$$p = \frac{V_L \cdot F(S^* - \phi W) + V_H \cdot \overline{F}(K - W)}{F(S^* - \phi W) + \overline{F}(K - W)}; \tag{2}$$

this expression implies that the equilibrium number of speculators S^* is at least as large as the number of strategic consumers ϕW (who all choose to delay their purchase) when $p < V_H$. However, when $p = V_H$, there will be no speculative purchases, i.e., $S^* = 0$, since speculation can never be profitable. Observe that the equilibrium number of speculators S^* increases as the seller's price p

decreases. In other words, a lower price p is required to induce a larger number of speculators S^* to enter the market.

What is the seller's optimal price p at $t = 1$? To answer this question, we write down the seller's profit function

$$\begin{aligned}\Pi(p, S^*) &= p \cdot \{\bar{\phi}W + S^* + E[\min(\max(0, \phi W + X - S^*), K - \bar{\phi}W - S^*)]\} \\ &= p \cdot \{E[\min(\max(\bar{\phi}W + S^*, X + W), K)]\}, \end{aligned} \tag{3}$$

where p and S^* are related according to (2) above. This expression can be interpreted in light of Lemma 1 above. When demand is low, the seller sells only $\bar{\phi}W + S^*$ units at $t = 1$. When demand is moderate, the seller sells $\bar{\phi}W + S^*$ units at $t = 1$ and $\phi W + X - S^*$ units at $t = 2$. Finally, when demand is high, the seller sells out the entire capacity. These three cases yield the profit function (3) above. In this profit function, note that the equilibrium number of speculators S^* is constrained to fall between 0 and $K - \bar{\phi}W$. Therefore, we can find the seller's optimal price p^* by maximizing his expected profit over this range, i.e., $S \in [0, K - \bar{\phi}W]$.

We summarize our equilibrium results in the following proposition.

Proposition 1 *Suppose the seller uses static prices $p = p_1 = p_2$. Then, at $t = 1$, the equilibrium price p^* and number of speculators S^* satisfy the following conditions.*

1. $p^* = V_L + \Delta \cdot \frac{\overline{F}(K-W)}{F(S^*-\phi W)+\overline{F}(K-W)} \in [V_L + \Delta \cdot \overline{F}(K - W), V_H]$.
2. $S^* \in \arg\max_{S \in [0, K-\bar{\phi}W]} \Pi(S)$, *where*

$$\Pi(S) = \frac{V_L \cdot F(S - \phi W) + V_H \cdot \overline{F}(K - W)}{F(S - \phi W) + \overline{F}(K - W)} \cdot E[\min(\max(\bar{\phi}W + S, X + W), K)].$$

Next, at $t = 2$, the equilibrium resale price $\tilde{p}^*(X)$ satisfies

$$\tilde{p}^*(X) = \begin{cases} V_L, & X < S^* - \phi W, \\ p^*, & S^* - \phi W \le X < K - W, \\ V_H, & X \ge K - W. \end{cases}$$

The seller's equilibrium expected profit is $\Pi(S^*)$.

It is useful to consider the seller's two extreme alternatives. On one hand, it is possible for the seller to sell out the entire capacity at $t = 1$. The seller can charge a sufficiently low price $p = V_L + \Delta\overline{F}(K - W)$, which induces a sufficiently large number of speculators $S^* = K - \bar{\phi}W$ to

completely deplete inventory. At the other extreme, it is possible for the seller to completely shut speculators out of the market (i.e., $S^* = 0$). The seller can do so by charging a sufficiently high price $p = V_H$. In general, the seller's optimal price falls somewhere in between.

We have the following proposition.

Proposition 2 *There exists some $\hat{K}$ such that when $K > \hat{K}$, we have $S^* > 0$ in equilibrium.*

The above proposition shows that the seller will prefer to attract speculators with low prices when capacity is sufficiently large, or equivalently, when demand is relatively low. In such cases, the marginal value of each unit of capacity at $t = 2$ is low. Even though there may be high-valuation consumers who are willing to pay up to V_H for these units, this is quite unlikely. Therefore, the seller is better off selling the units at some price $p < V_H$ to speculators at $t = 1$. By doing so, the seller immediately collects revenue and transfers the risk of unsold inventory to speculators. In contrast, when capacity is small, there is a higher chance of selling each remaining unit to a high-valuation consumer. Then, the seller is better off charging high prices (i.e., $p = V_H$) and keeping speculators out of the market.

More generally, our results demonstrate that the presence of potential speculators can benefit the seller of a capacity-constrained good. To illustrate this point, consider the benchmark scenario where there are no potential speculators in the market. In this case, the seller may either charge $p = V_L$ and earn a profit of KV_L, or charge $p = V_H$ and earn a profit of $V_H \cdot E\min(W + X, K)$. The former alternative is dominated by $\Pi(K - \bar{\phi}W)$ since the seller can sell out capacity at a price $p > V_L$ when there are speculators, and the latter alternative is simply $\Pi(0)$, i.e., when the seller uses $p = V_H$ to price speculators out of the market. Clearly, both alternatives combined are no better than $\max_{S\in[0,K-\bar{\phi}W]} \Pi(S)$, which is the seller's equilibrium expected profit in the presence of speculators.

We conclude this section with a somewhat controversial statement. The equilibrium analysis suggests that any speculation activity $S^* > 0$ must have had the seller's "approval." This is because the seller has the ability to price all speculators out of the market. In other words, when there are speculators in the market, the seller must have explicitly chosen to accommodate such behavior. As we have explained above, equilibrium speculation activity can benefit the seller when capacity is sufficiently large. Otherwise, the seller will simply increase his price and there will be no speculation activity in equilibrium.

5 Dynamic Pricing Strategies

In this section, we consider dynamic pricing strategies. Although the seller charges the price p_1 at $t = 1$, he is now free to set a different price p_2 at $t = 2$ after the random demand realization X is publicly known. Dynamic pricing strategies have been used in a wide variety of industries. For example, it is well-known that prices of airline tickets fluctuate on a regular basis. In retailing, markdowns are commonly used, especially for products with weaker-than-expected demand. For some other applications, such as baseball tickets, dynamic pricing is relatively new; for example, the San Francisco Giants plan to introduce dynamic pricing in 2009 although price changes will be kept minimal (see McCauley, 2008). In such cases, sellers have to exert special care to make sure that such practices are acceptable to customers. Regardless of whether it is a well-accepted practice or merely a hypothetical thought experiment, our analysis below considers the implications of dynamic pricing.

5.1 Dynamic Pricing Without Speculators

As a starting point, we first analyze the seller's dynamic pricing strategies in the absence of speculators. As before, we employ backward induction and start by considering the seller's pricing decision at $t = 2$. The seller's optimal choice depends on his remaining capacity as well as the number of strategic consumers who chose to wait. We have the following lemma.

Lemma 2 *Suppose there are no speculators in the market. Further, suppose the seller has K_2 units of capacity remaining and there are W_2 strategic consumers who chose to wait. Let X be the number of new consumers who enter the market at $t = 2$. Then, the seller's optimal price is*

$$p_2^* = \begin{cases} V_L, & X + W_2 < \frac{V_L}{V_H} \cdot K_2, \\ V_H, & X + W_2 \geq \frac{V_L}{V_H} \cdot K_2. \end{cases}$$

The reason behind the above decision is simple. The seller may either sell all K_2 remaining units at V_L or sell only to high-valuation consumers (i.e., the W_2 strategic consumers who waited and the X new consumers who enter at $t = 2$) at V_H. The latter is preferred if X is sufficiently large.

Our next lemma considers the effect of the seller's pricing decision at $t = 1$ on outcomes along the equilibrium path.

Lemma 3 *Suppose the seller chooses* $p_1 = V_L + \Delta \cdot \overline{F}\left(\frac{V_L}{V_H}(K-W) - \frac{\Delta}{V_H}W_2\right)$ *for some* $W_2 \in [0, \phi W]$. *Then, along the equilibrium path, at* $t = 1$, *all myopic consumers will buy,* W_2 *strategic consumers will wait (the rest of the strategic consumers will buy); at* $t = 2$, *the seller's optimal price* p_2^* *is as given in Lemma 2. Further, the seller's expected equilibrium profit is given by*

$$\Pi(W_2) = F(z)KV_L + V_H \int_z^{K-W} (x+W)dF(x) + \overline{F}(K-W)KV_H,$$

where $z \equiv \frac{V_L}{V_H}(K-W) - \frac{\Delta}{V_H}W_2$.

Notice that the seller can use the initial price p_1 to influence the purchase behavior of strategic consumers. As the initial price is decreased, the seller induces a larger proportion of strategic consumers to buy early. Interestingly, the next result shows that it is optimal for the seller to charge a high initial price so that all strategic consumers will wait.

Proposition 3 *Suppose there are no speculators. Then, the seller's optimal initial price is* $p_1^* = V_H$. *In equilibrium, at* $t = 1$ *myopic consumers buy but all strategic consumers wait. At* $t = 2$, *the seller's optimal price is*

$$p_2^* = \begin{cases} V_L, & X < \frac{V_L}{V_H}(K-W) - \frac{\Delta}{V_H}\phi W, \\ V_H, & X \geq \frac{V_L}{V_H}(K-W) - \frac{\Delta}{V_H}\phi W. \end{cases}$$

The seller's equilibrium expected profit is

$$\overline{\phi} W V_H + E[\min(\max((K - \overline{\phi}W)V_L, (X + \phi W)V_H), (K - \overline{\phi}W)V_H)].$$

The optimal dynamic pricing strategy described in Proposition 3 above has an intuitive interpretation. At $t = 1$, the seller should charge a high initial price $p_1^* = V_H$. At $t = 2$, the seller should maintain the same price if demand is strong; otherwise, a markdown to V_L is preferable. This type of pricing format is commonly observed in practice.

So far, we consider the seller's dynamic pricing strategies in the absence of speculators. Next, we introduce potential speculators and show that our main findings above remain valid.

5.2 Dynamic Pricing With Speculators

Consider the scenario where the seller practices dynamic pricing in the presence of speculators. Similarly as before, our analysis follows backward induction. The reasoning is similar to that in Sections 4 and 5.1. We defer the details to the appendix (see Lemmas 4 and 5) and proceed directly to our main result.

Proposition 4 *When there are potential speculators and the seller practices dynamic pricing, the equilibrium outcome is identical to the case where there are no potential speculators. It is optimal for the seller to shut speculators out of the market using a sufficiently high initial price* $p_1^* = V_H$. *In equilibrium, myopic consumers buy at* $t = 1$ *but all strategic consumers wait until* $t = 2$.

Surprisingly, we find that the presence or absence of speculators makes no difference when the seller practices dynamic pricing. This is because, even if there were *potential* speculators, the seller will charge sufficiently high initial prices $p_1^* = V_H$ so that speculation is not profitable. Then, it is as if there were no speculators in the market. Furthermore, Proposition 4 also shows that consumers behave the same way regardless of whether there are potential speculators: myopic consumers buy immediately but strategic consumers wait. (Unsurprisingly, such equilibrium behavior also emerges in the fixed-price scenario analyzed in Section 4.) We may thus view the two dynamic pricing scenarios (with and without speculators) in the same light. Henceforth, we no longer distinguish between these two cases.

However, there is a stark contrast between Proposition 4 here (with dynamic pricing) and Proposition 2 above (when the seller charges a fixed price in both periods). With dynamic pricing, it is optimal for the seller to charge a sufficiently high price $p_1 = V_H$ in order to "price out" all potential speculators. Interestingly, this does not occur under a fixed price; in that case, provided that capacity is large enough, the seller's initial price $p_1 < V_H$ is sufficiently low such that speculation will always emerge in equilibrium, i.e., $S^* > 0$. Therefore, our results suggest that when there is a fixed price, the seller may *accommodate* speculative behavior, but when dynamic pricing is feasible, he will instead *eliminate* speculation by charging a sufficiently high price to shut resellers out of the market.

The intuition behind our contrasting results proceeds as follows. Speculators play different roles in our two scenarios. Under dynamic pricing, speculators are competitors to the seller in the following sense. At $t = 2$, speculators have the flexibility to react to the seller's price p_2 and thus can always undercut the seller. In equilibrium, given that there are S speculators, ϕW strategic consumers who wait, and X new consumers who arrive, the seller's high-valuation demand at $t = 2$ is only $\phi W + X - S$ because speculators are always able to attract these high-valuation consumers first. Therefore, the seller is better off preemptively shutting speculators out of the market. The seller prefers to retain monopoly power at $t = 2$ and can do so by simply charging a sufficiently high price at $t = 1$ so that there will be no speculative resale.

In contrast, under static pricing, speculators help the seller bring about a form of "disguised" dynamic pricing. Even though the seller charges a fixed price across both periods, speculators respond to market conditions and higher demand states are reflected through higher resale prices. Effectively, the resale market becomes a dynamic mechanism that automatically adjusts prices to suit demand conditions. This increased flexibility extracts consumer surplus and some of the speculators' profits are transferred back to the seller in the form of increased sales at $t = 1$. Even though the seller does not practice dynamic pricing per se, he enjoys some of its benefits through selling to speculators (i.e., increased speculative demand).

In summary, speculators act as *competitors* when the seller practices dynamic pricing, but serve as *market makers* when the seller charges a fixed price. It is thus not surprising that the seller exhibits different reactions to speculators under these two scenarios.

5.3 Competitive Resale Markets Versus Monopolistic Dynamic Pricing

In this section, we propose the view that competitive resale markets can serve as a proxy for monopolistic dynamic pricing. Our analysis has shown that both mechanisms offer the seller some flexibility to improve profits. However, dynamic pricing may not be feasible in some scenarios, as we explain below. In such circumstances, when the seller is constrained to charged a fixed price, our results suggest that the seller can still reap some of the benefits of dynamic pricing by maintaining an active resale market.

Dynamic pricing is most likely not a viable alternative when fairness and equity across customers are important considerations. An inevitable consequence of dynamic pricing is that consumers will be charged different prices for the same product. Although different prices are typically levied at different times, consumers are still likely to react negatively. Consider the following examples.

1. On a particular visit to her favorite fashion boutique, a customer notices that the price of an exact same dress she bought last month has been marked down by 50%. She demands a refund but store policy does not permit that.

2. While talking to a fellow passenger sitting beside him, a traveler finds out that his neighbor has paid only \$300 for the plane ticket (the traveler paid \$800 himself). He did not particularly enjoy that flight.

3. On the day of a football match, a season pass holder notices that the ticket office is offering

a 50% discount for tickets to all future matches. Reason? The team has been performing poorly and management is using dynamic pricing to boost demand.

Although the above examples are hypothetical, it is not difficult to envision how consumer uproar might emerge as a result of so-called dynamic pricing. Generally, consumers expect to pay the same price for the same product, and they will treat any price differentials as being entirely unfair (see Ho and Su, 2009). Therefore, although dynamic pricing strategies can boost bottom-lines, there may be associated non-pecuniary costs. When maintaining customer goodwill is a top priority, dynamic pricing may no longer be a possible alternative.

Our analysis suggests that when dynamic pricing is infeasible, the seller can still increase profits by establishing (or at least accommodating) secondary markets for speculative resale. For example, Ticketmaster maintains an online reseller website where users can buy and sell event tickets. Many sports teams have also authorized ticket resale through online exchanges such as StubHub and RazorGator. Our results suggest that in these cases, the seller benefits from ticket resale. Through selling to resellers, the seller reaps some of the benefits of dynamic pricing even though the face value of tickets are set in advance. Further, notice that firms are relieved from public scrutiny since consumers generally blame the scalpers when demand drives prices up too high. In this sense, secondary markets also help firms by serving another function: they shield firms from potential consumer antagonism.

To conclude this section, we highlight a special case in which charging a fixed price in the presence of speculators yields the same profit level as compared to using the optimal dynamic pricing strategy. In this case, competitive resale markets can be viewed as a perfect substitute for dynamic pricing mechanisms. We present the special case in the next proposition.

Proposition 5 *Suppose that (i) all consumers are strategic, and (ii) there are exactly two possible demand realizations. Then, the seller's expected profit when he charges a fixed price (in the presence of speculators) is the same as when he practices dynamic pricing.*

This proposition provides two conditions under which speculative resale is a perfect substitute for dynamic pricing. Each of these two conditions highlights an area where resale markets and speculative behavior fall short of attaining the full benefits of dynamic pricing, as explained next. First, under the optimal dynamic pricing strategy, the seller can extract full surplus from myopic consumers at $t = 1$ by charging $p_1^* = V_H$. However, this is no longer an option if the seller sets a fixed price and keeps it low enough to support speculative activity. Therefore, consistent with

condition (i) of our result above, resale markets can function as a perfect substitute for dynamic pricing only when there are no myopic consumers. Next, Proposition 5 above also suggests that speculative resale does not serve the seller as well as dynamic pricing under moderate demand realizations. Why is this so? Observe that under dynamic pricing, the seller acts as a monopolist at $t = 2$, but with secondary markets, speculators determine the resale price competitively. Intuitively, more surplus is left for consumers in the latter case. This difference arises under moderate demand realizations but vanishes in extreme cases: when demand is very high, speculators and the monopolist can earn V_H on every unit; similarly, when demand is very low, the common price will be V_L. Therefore, consistent with condition (ii) above, when there are only two possible demand realizations, a seller who sets a fixed price can earn the same profit as a seller who uses dynamic pricing. Such binary outcomes may be a good approximation of some situations. For sporting events, consumer demand may depend on the weather (e.g., rain or shine) or on the team's performance at a previous game (e.g., win or lose). Our results suggest that when conditions (i) and (ii) of Proposition 5 hold, the seller is likely to prefer authorizing resale mechanisms over implementing dynamic pricing.

6 Long Run Capacity Decisions

In this section, we consider long run capacity decisions. The seller first chooses capacity K and then sells these units at a fixed price $p = p_1 = p_2$ as described in Section 4. Below, we demonstrate that the presence of speculators can generate some counter-intuitive effects on the firm's optimal choice of capacity K. We have the following result.

Proposition 6 *Suppose the seller uses the optimal fixed price and the capacity is K. Let $S^*(K)$ denote the number of speculators in equilibrium. Then, a marginal increase in capacity will lower the seller's expected profit, if and only if*

$$\frac{F(S^*(K) - \phi W)^2}{f(S^*(K) - \phi W)} \geq \frac{\overline{F}(K - W)^2}{f(K - W)}. \tag{4}$$

The above proposition suggests that expanding capacity does not always add value to the firm. This is because excessive capacity may threaten the survival of resale markets, and consequently, the firm's speculative demand may be reduced or even eliminated. In such circumstances, the seller may not increase capacity even when capacity expansion is free. As an illustration, consider an example in which random demand X follows a uniform distribution between 0 and some

$M > K - W$, so $f(\cdot) \equiv 1/M$. Then, from (4), we know that increased capacity leads to decreased profits if $S^*(K) - \phi W$ exceeds $M - (K - W)$. In other words, if existing capacity K is large enough, i.e., $K > (M + W) - (S^*(K) + \phi W)$, then the seller will not expand capacity even if doing so were costless.

We stress that the result above (i.e., negative effect of capacity) depends critically on the presence of speculators. Consider the situation where there are no speculators and the seller continues to charge a fixed price. Observe that the seller may choose between pricing at V_L or V_H, and expected profit in these two cases are KV_L and $V_H \cdot E\min(X + W, K)$ respectively. In both cases, the seller's profit is increasing in capacity K, so capacity expansion will always be beneficial. In fact, moving one step further, we have the following result.

Proposition 7 *Suppose the seller charges a fixed price. Then, the presence of potential speculators always leads to a lower capacity choice K.*

This proposition suggests that speculative behavior creates incentives for the seller to keep capacity low. As discussed above, speculators help the seller achieve a "disguised" form of dynamic pricing. Scarcity associated with low capacity levels helps to maintain profitability for speculators, which in turn increases sales and improves profits for the seller. It is therefore not surprising that speculative resale over secondary markets can generate perverse incentives for the seller to artificially restrict capacity.

Our finding offers a potential explanation for the prevalence of scarcity and consequently speculative behavior when "hot products" are newly introduced into the market. In this new-product setting, the capacity decision K can be interpreted as an upfront production decision, which determines the number of units available for purchase. During the initial phases of product introduction, the seller may not be able to adjust prices freely. For example, Apple's decision to slash the price of iPhones in 2007 attracted significant consumer uproar (see Wingfield, 2007). The seller is often restricted to use a fixed price and hence, as shown above, may find it profitable to maintain the resale market; one way to do so is to keep availability low. When there is scarcity, the seller may be better off as a result of speculative resale. Such "scarcity" strategies can be applied to the case of the Nintendo Wii described in the introduction, as well as another earlier example of Sony's Playstation 2 in 2000 (see Stock and Balachander, 2005, who suggest an alternative view that scarcity can help to signal high quality to uninformed consumers). In these examples, capacity constraints and production shortfalls may benefit the manufacturer. Consistent with our model

predictions, rampant stockouts did occur and speculative behavior did emerge.

7 Extensions

In this section, we describe several model extensions and show that the main insights from our basic model remain unchanged.

7.1 Selling Through Intermediaries

In many practical situations, the seller sells to end-consumers through an intermediary. We describe how our model can be adapted to this scenario. For example, many concert promoters (sellers) rely on companies such as Ticketmaster (intermediary) as a platform to reach consumers over the internet. In this case, Ticketmaster charges each end-consumer a convenience fee ν and remits the entire face value of the ticket to the promoter. We can apply our model to this setting by using $V_H - \nu$ and $V_L - \nu$ in place of V_H and V_L. This is because after accounting for the convenience fee, consumers' net valuations are essentially reduced by an equivalent amount. Therefore, we can use a similar analysis, so our main results remain valid. In particular, we conclude that speculative resale can benefit the seller even in the presence of an intermediary.

7.2 Speculation and Arbitrage

The analysis above considers *speculators* who buy units at $t = 1$ for possible resale profits at $t = 2$; in other words, they may incur a loss (e.g., if demand turns out to be weak). In practice, there may be *arbitrageurs* who are able to exploit price differentials (between primary and resale markets) at $t = 2$ and make certain profits. Within the context of our model, we define arbitrageurs as responsive scalpers who buy from the seller at $t = 2$ and then immediately put units up for resale. Note that there will be no price differentials under dynamic pricing, so arbitrageurs may emerge only when the seller charges a fixed price.

We find that the introduction of arbitrageurs does not affect our results. When the seller charges a fixed price $p < V_H$, there are three possible demand outcomes (low/moderate/high) as explained immediately after Lemma 1. Arbitrageurs do not appear when demand is weak or moderate. However, when demand turns out to be strong enough to support a high resale price $\tilde{p} = V_H$, arbitrageurs will spring into action, enter the market, and extract all consumer surplus (so consumers are worse off). Nevertheless, in all cases, prices on the primary and resale markets as

well as the seller's revenues remain unchanged. Therefore, our main conclusions also carry through.

7.3 Different Beliefs

Our basic model assumes that speculators share the same correct belief about the demand distribution F as the seller. In practice, they may not have the necessary information or means to form an accurate assessment of demand uncertainty. To capture heterogeneity in speculators' beliefs, consider the following setup. Suppose the population of speculators can have different beliefs $\hat{F} \in \mathcal{F}$ and we assume that this set of beliefs can be stochastically ordered from pessimistic to optimistic. Note that whenever a particular speculator is willing to enter the market, all other more optimistic speculators are also willing to enter. With this setup, in any equilibrium, we may equivalently assume that all speculators share the same beliefs as the marginal speculator (i.e., the one who is indifferent over whether to enter the market). For example, if there are $S^* = 100$ speculators in equilibrium, the same outcome would still be an equilibrium if all speculators share the same beliefs as the 100-th most optimistic speculator, because given these beliefs, individual speculators will find it profitable to enter the market only as long as the number of speculators does not exceed 100. Therefore, when speculators hold different beliefs, the equilibrium possesses a similar structure as that in our basic model. Furthermore, observe that the speculators who enter the market may suffer from a form of the "winner's curse." They have the most optimistic beliefs and thus are likely to over-buy and incur losses as a whole. In this case, speculation may benefit consumers because such optimism may exert downward pressure on the resale price.

7.4 Uncertain Valuations

In our model, we assume that all consumers have a deterministic valuation. However, previous research has considered uncertain valuations. Air travel and entertainment events provide excellent motivating examples. In these cases, there may be last-minute changes or emergencies that render consumers unable to enjoy the full benefit of their purchases. A simple but useful construct to capture such possibilities is to assume that consumers may have either high valuations V_H (with probability λ) or low valuations V_L (with probability $1-\lambda$), and that these valuations are realized at $t = 2$. This approach was used by Xie and Shugan (2001) in the context of advance selling. We shall consider Bernoulli valuations for strategic, myopic, and new consumers, but continue to assume that the low-value demand segment has deterministic valuation V_L. Using this model extension, note that the seller is worse off when there is uncertainty in valuations because average valuations

have decreased. Consider the consumers arriving at $t = 1$ who face uncertain valuations. While these consumers at $t = 1$ were willing to pay up to V_H in our basic model, they are now willing to pay at most $\mu < V_H$, where $\mu \equiv \lambda V_H + (1 - \lambda)V_L$ denotes the expected valuation. Furthermore, with this extension, our basic model can be re-interpreted to cover the case where there are N new consumers at $t = 2$ with uncertain valuations. Suppose these consumers have independent Bernoulli random valuations as described above. Then, the total number of new consumers (with high valuations) X follows a Binomial(N, λ) distribution. We may then use our basic model and proceed with the analysis as before. The main results are unaffected.

8 Conclusion

In this paper, we have studied a model of pricing with speculative resale. The firm has a finite capacity but faces uncertain aggregate demand, so the market value of each unit may fluctuate as market conditions unfold. As a result, speculators may make purchases purely with the intention of resale, and such behavior has important implications for the firm.

One of our main conclusions is that speculative behavior may enhance profits. Intuitively, speculators help firms in two ways. When demand is low, speculators help share the burden of unsold inventory. When demand is high, speculators help extract consumer surplus while isolating firms from consumer resentment: in such cases, consumers unable to secure their desired items from the firm often end up paying a price premium but they blame the "scalpers." Although firms do not directly collect resale profits, the associated benefit is transferred to them through increased sales. For example, when tickets to a basketball match are sold on StubHub for $35,000, the lucrative profit margin attracts increased speculative demand and boosts revenues even though the home team does not directly collect the $35,000.

Our analysis then proceeds to draw an analogy between speculative resale and dynamic pricing. Dynamic pricing allows the firm to freely adjust prices to match demand. Similarly, in resale markets, strategic behavior on the part of speculators implies that equilibrium prices reflect market conditions. Both are useful mechanisms that provide some form of price flexibility. In fact, we show that there are situations where resale markets can provide the full benefits of dynamic pricing: firms can achieve the profit levels from optimal dynamic pricing strategies even when the price is held constant, as long as there are opportunities for speculators to enter the market. This observation suggests that resale mechanisms can serve as a proxy for dynamic pricing, especially

in situations where price discrimination may be perceived in a negative light.

We also study the firm's long run capacity decisions. Although costless capacity investments are generally beneficial to the firm, we identify an interesting effect of speculative behavior on the value of capacity. Specifically, we show that in the presence of speculators, increasing capacity may lead to lower profits. In such cases, the firm will not expand capacity even if it were costless to do so. Such tendencies may explain the widespread scarcity of items for which speculative resale is common (e.g., Nintendo's Wii game console). Interestingly, this effect disappears once resale is prohibited. Our results caution that although speculative resale can generate additional income, it may also discourage capacity investments, which can have implications beyond our analysis.

There are many interesting questions that remain to be answered. First, it would be useful to study resale mechanisms in greater detail. What selling strategies (e.g., posted prices, auctions) should resellers use? How do these price dynamics evolve as the "deadline" (e.g., the day of the football match) approaches? Note that this is essentially a competitive revenue management problem between the speculators and the firm. Next, consider the perspective of consumers. For how long should they search before turning to the resale market? What price should they accept and when? Answers to these questions may provide useful rules of thumb for consumers. Finally, it would be worthwhile to explore contracting issues within the "revenue chain." How should intermediaries structure their revenue model? Should they charge a fixed fee or a fractional commission? Should they charge buyers or sellers or both? We believe that the topic of pricing with speculative resale presents many fruitful opportunities for future research.

Appendix: Proofs

Proof of Lemma 1 We consider the three cases separately.

First, when $W_2 + X < S$, speculator supply alone already exceeds high-valuation demand. Therefore, any resale price $\tilde{p} > V_L$ can not be an equilibrium of the double auction game. Any speculator can earn a higher payoff by slightly undercutting the price $\tilde{p}$. In this case, the equilibrium resale price must be $\tilde{p} = V_L$.

Next, when $S \leq W_2 + X < S + K_2$, high-valuation demand is less than total supply. This implies that any resale price $\tilde{p} > p$ can not be an equilibrium: at any such price, some speculator will not be able to sell since consumers can always find another unit priced at p from the seller. At the same time, high-valuation demand exceeds total speculator supply. Thus, any resale price

$\tilde{p} < p$ can not be an equilibrium because any speculator can slightly increase the price (but keeping it below p) and still be guaranteed of selling it. Therefore, the equilibrium resale price is $\tilde{p} = p$.

Finally, when $W_2 + X \geq S + K_2$, high-valuation demand exceeds total supply. In the double auction game, it is dominant for all speculators to price at V_H since there is abundant demand. Therefore, the resale price is $\tilde{p} = V_H$. ■

Proof of Proposition 1 The first condition for $t = 1$ follows from (2). The second condition follows from expressing the seller's profit maximizing choice of p, as in (3), in terms of S. This also yields the seller's equilibrium expected profit $\Pi(S^*)$. Finally, the equilibrium resale price at $t = 2$ follows from Lemma 1 with $W_2^* = \phi W$, as discussed in the text. ■

Proof of Proposition 2 Consider the seller's expected profit when he prices speculators out of the market. In this case, he earns $V_H \cdot E \min(W + X, K)$. Next, consider the other extreme case where the seller sells out the entire capacity at $t = 1$. In this case, the seller earns $K(V_L + \Delta \cdot \overline{F}(K - W))$. Observe that there must be some $\hat{K}$ such that the latter exceeds the former whenever $K \geq \hat{K}$. Then, by continuity of the profit function, we must have $S^* > 0$ in equilibrium. ■

Proof of Lemma 3 Given price $p_1 \leq V_H$, it is clear all myopic consumers will buy. Further, using Lemma 2, note that when $\hat{W}_2$ strategic consumers choose to wait, the expected second period price is $Ep_2 = V_L + \Delta \cdot \overline{F}\left(\frac{V_L}{V_H}(K - W) - \frac{\Delta}{V_H}\hat{W}_2\right)$, which is increasing in W_2. There is no equilibrium if $Ep_2 > p_1$ (then $\hat{W}_2$ will decrease) or if $Ep_2 < p_1$ (then $\hat{W}_2$ will increase). Thus, in equilibrium, we must have $Ep_2 = p_1$, so we have W_2 strategic buyers who wait at $t = 1$. The pricing decision at $t = 2$ follows from Lemma 2. In equilibrium, the seller's expected profit is

$$\begin{aligned}\Pi(W_2) = p_1(W - W_2) \quad &+ \quad F\left(\frac{V_L}{V_H}(K - W) - \frac{\Delta}{V_H}W_2\right) \cdot (K - W + W_2)V_L \\ &+ \quad V_H \int_{\frac{V_L}{V_H}(K-W) - \frac{\Delta}{V_H}W_2}^{K-W} (x + W_2)dF(x) \\ &+ \quad \overline{F}(K - W) \cdot (K - W + W_2)V_H,\end{aligned}$$

where the first term comes from initial sales to consumers at $t = 1$, and the final three terms come from sales to other consumers (for low/moderate/high demand realizations). Using the fact that $p_1 = Ep_2 = V_L + \Delta \cdot \overline{F}\left(\frac{V_L}{V_H}(K - W) - \frac{\Delta}{V_H}W_2\right)$ and substituting $z = \frac{V_L}{V_H}(K - W) - \frac{\Delta}{V_H}W_2$ yields the expression in the lemma. ■

Proof of Proposition 3 Consider the seller's profit function $\Pi(W_2)$ from Lemma 3. It is easy to show that

$$\frac{d\Pi}{dW_2} = (W - W_2) \cdot \frac{\Delta}{V_H} \cdot f\left(\frac{V_L}{V_H}(K - W) - \frac{\Delta}{V_H}W_2\right) > 0.$$

Therefore, $\Pi(\phi W) \geq \Pi(W_2)$ for every $W_2 \in [0, \phi W]$. Finally, note that the seller can attain even higher profits by choosing $p_1 = V_H$ since myopic consumers will buy as long as $p_1 \leq V_H$. Given $p_1 = V_H$, the equilibrium price p_2 at $t = 2$ follows from Lemma 2. Finally, in the profit expression, the first term comes from sales to the $\overline{\phi}W$ myopic consumers at $t = 1$ at price $p_1^* = V_H$, and the second term comes from sales at $t = 2$. Since there are $K - \overline{\phi}W$ units remaining and the number of high-valuation consumers is $X + \phi W$ (i.e., new consumers as well as strategic consumers who waited), we have the result. This completes the proof. ■

Lemma 4 *Suppose there are S speculators in the market. Further, suppose the seller has K_2 units of capacity remaining and there are W_2 strategic consumers who wait. Let X be the number of new consumers who enter the market at $t = 2$. Then, the seller's optimal price and the resale price satisfy*

$$p_2^* = \tilde{p}^* = \begin{cases} V_L, & X + W_2 - S < \frac{V_L}{V_H} \cdot K_2, \\ V_H, & X + W_2 - S \geq \frac{V_L}{V_H} \cdot K_2. \end{cases}$$

Proof of Lemma 4 Using Lemma 2, we know the resale market price will be V_L when $X+W_2 < S$ and V_H when $X + W_2 \geq S + K_2$. However, when $S \leq X + W_2 < S + K_2$, the resale market will respond to and match the seller's price (so consumers will first buy from speculators); if the seller charges $p_2 = V_H$, he will sell $X + W_2 - S$ units, but if he charges $p_2 = V_L$, all K_2 units will be sold. The seller prefers the latter when $X + W_2 - S < \frac{V_L}{V_H} \cdot K_2$. These observations yield p_2^* and $\tilde{p}^*$ as given in the lemma. ■

Lemma 5 *Suppose the seller chooses $p_1 = V_L + \Delta \cdot \overline{F}\left(\frac{V_L}{V_H}(K - W) - \frac{\Delta}{V_H}Y\right)$ for some $Y \in [-(K - W), \phi W]$. Let $W_2 \in [0, \phi W]$ denote the number of strategic consumers who wait (the rest will buy), and let $S \in [0, K - W]$ denote the number of speculators who enter the market. Then, we must have $W_2 - S = Y$ along any equilibrium path. Further, the seller's expected equilibrium profit is given by*

$$\Pi(Y) = F(z)KV_L + V_H \int_z^{K-W} (x + W)dF(x) + \overline{F}(K - W)KV_H,$$

where $z \equiv \frac{V_L}{V_H}(K - W) - \frac{\Delta}{V_H}Y$.

Proof of Lemma 5 This lemma follows the same argument as Lemma 3. Note from Lemma 4 that along the equilibrium path, $p_2^* = \tilde{p}^*$ for all demand realizations X, so $Ep_2^* = E\tilde{p}^*$. Therefore, as in Lemma 3, Ep_2^* is increasing in W_2 and $E\tilde{p}^*$ is decreasing in S, so the number of strategic consumers who wait W_2 and the number of speculators S who enter, in any equilibrium, must satisfy $W_2 - S = Y$. This is exactly the point where each speculator makes zero expected profit and when each strategic consumer is indifferent between buying and waiting. Finally, the seller's expected profit can be computed similarly as in Lemma 3. ■

Proof of Proposition 4 This result follows from Lemmas 4 and 5 and the same argument as in the proof of Proposition 3. ■

Proof of Proposition 5 By our assumptions, we have $\phi = 1$ and the random demand X may be x_L with probability π_L and x_H with probability π_H, with $x_H > x_L$ and $\pi_L + \pi_H = 1$. If $K - W < x_L$ the result follows trivially. If $K - W > x_H$ we essentially have $K - W = x_H$ since some excess units will never be used. Thus, we focus on the case with $x_L \leq K - W \leq x_H$.

First, consider charging a fixed price in the presence of speculators. The seller faces two options. One option is to charge V_H. In this case, the expected revenue is $V_H(\pi_L(W + x_L) + \pi_H K)$. The second option is to charge a lower price and sell out at $t = 1$. The required price is $\pi_L V_L + \pi_H V_H$. This is because when the seller sells out at $t = 1$, the resale price at $t = 2$ will be V_H with probability π_H and V_L with probability π_L. Now, speculators are willing to buy only if $p \leq \pi_L V_L + \pi_H V_H$. In this case, the seller's expected revenue is $(\pi_L V_L + \pi_H V_H)K$. Combining both options, the seller's expected revenue when charging a fixed price is $\max\{V_H(\pi_L(W + x_L) + \pi_H K), (\pi_L V_L + \pi_H V_H)K\}$.

Next, consider dynamic pricing. As shown in Section 5.2 (particularly, Proposition 4), we may assume that there are no speculators and the seller charges $p_1^* = V_H$. Suppose $(W + x_L)V_H \geq KV_L$. Then, from Lemma 2, the seller sets $p_2 = V_H$ with probability 1 and the expected revenue is $V_H(\pi_L(W + x_L) + \pi_H K)$. On the other hand, suppose $(W + x_L)V_H < KV_L$. Then, from Lemma 2, the seller sets $p_2 = V_H$ when $X = x_H$ and $p_2 = V_L$ when $X = x_L$. Thus, expected revenue is $\pi_H K V_H + \pi_L K V_L = (\pi_L V_L + \pi_H V_H)K$. Combining the two cases, the seller's expected revenue is again $\max\{V_H(\pi_L(W + x_L) + \pi_H K), (\pi_L V_L + \pi_H V_H)K\}$. Therefore, he earns the same profit under a fixed price and under dynamic pricing. ■

Proof of Proposition 6 Recall from (3) that the seller's expected revenue under the optimal static pricing policy, denoted $R(K)$, can be characterized in terms of a maximization problem:

$$R(K) = \max_p \Pi(p; K),$$

where $\Pi(p; K) \equiv p \cdot \{E[\min(\max(W + S^*(p; K), X + W), K)]\}$ and $S^*(p; K)$ is the equilibrium number of speculators with static price p and capacity K. The marginal value of the K-th unit of capacity is denoted $MV(K) = dR(K)/dK$. Using the envelope theorem, we obtain

$$MV(K) = \frac{\partial}{\partial K}\Pi(p^*(K); K),$$

where $p^*(K)$ denotes the optimal fixed price with capacity K. We will show that $MV(K) \leq 0$ if and only if (4) holds.

Consider a slight increase in the current capacity from K to $K + \epsilon$ for some small $\epsilon > 0$. In this entire proof, we keep price fixed at $p = p^*(K)$. Recall from (2) that when the static price is p and when the capacity is K, the equilibrium number of speculators $S^*(p; K)$ satisfies

$$p = \frac{V_L \cdot F(S^*(p; K) - \phi W) + V_H \cdot \overline{F}(K - W)}{F(S^*(p; K) - \phi W) + \overline{F}(K - W)}.$$

Therefore, for capacities K and $K + \epsilon$ at the same price p, we have

$$\begin{aligned}
\frac{V_L \cdot F(S^*(p; K) - \phi W) + V_H \cdot \overline{F}(K - W)}{F(S^*(p; K) - \phi W) + \overline{F}(K - W)} &= \frac{V_L \cdot F(S^*(p; K + \epsilon) - \phi W) + V_H \cdot \overline{F}(K + \epsilon - W)}{F(S^*(p; K + \epsilon) - \phi W) + \overline{F}(K + \epsilon - W)}, \\
\frac{F(S^*(p; K) - \phi W)}{F(S^*(p; K) - \phi W) + \overline{F}(K - W)} &= \frac{F(S^*(p; K + \epsilon) - \phi W)}{F(S^*(p; K + \epsilon) - \phi W) + \overline{F}(K + \epsilon - W)}, \\
\frac{F(S^*(p; K + \epsilon) - \phi W)}{F(S^*(p; K) - \phi W)} &= \frac{\overline{F}(K + \epsilon - W)}{\overline{F}(K - W)} \equiv 1 - \psi_\epsilon, \qquad (5)
\end{aligned}$$

for some small $\psi_\epsilon > 0$.

Next, since the equilibrium profit with price p and capacity K is

$$\Pi(p; K) = p \cdot \{E[\min(\max(\bar{\phi} W + S^*(p; K), X + W), K)]\},$$

we have

$$\Pi(p; K + \epsilon) - \Pi(p; K) = p \cdot \{[S^*(p; K + \epsilon) - S^*(p; K)] \cdot F(S^*(p; K) - \phi W) + \epsilon \cdot \overline{F}(K - W) + o(\epsilon)\}.$$

Now we are ready to calculate $MV(K)$. We have

$$\begin{aligned} MV(K) &= \lim_{\epsilon\to 0}\frac{\Pi(p;K+\epsilon)-\Pi(p;K)}{\epsilon} \\ &= p\cdot\lim_{\epsilon\to 0}\frac{[S^*(p;K+\epsilon)-S^*(p;K)]\cdot F(S^*(p;K)-\phi W)+\epsilon\cdot\overline{F}(K-W)}{\epsilon} \\ &= p\cdot\lim_{\epsilon\to 0}\frac{1}{\epsilon}\left\{\frac{F(S^*(p;K+\epsilon)-\phi W)-F(S^*(p;K)-\phi W)}{f(S^*(p;K)-\phi W)}\cdot F(S^*(p;K)-\phi W)\right. \\ &\qquad\left. -\frac{\overline{F}(K+\epsilon-W)-\overline{F}(K-W)}{f(K-W)}\cdot\overline{F}(K-W)\right\} \\ &= p\cdot\left\{\lim_{\epsilon\to 0}\frac{-\psi_\epsilon}{\epsilon}\right\}\cdot\left\{\frac{F(S^*(p;K)-\phi W)^2}{f(S^*(p;K)-\phi W)}-\frac{\overline{F}(K-W)^2}{f(K-W)}\right\}, \end{aligned}$$

which is less than zero if and only if the last term is positive, where the last equality above follows from (5). This completes the proof. ■

Proof of Proposition 7 As in the proof of Proposition 6, let $R(K)$ denote the seller's profit under capacity K and let $MV(K) = dR(K)/dK$ denote the marginal value of the K-th unit of capacity. Similarly, let $MV_0(K)$ denote the same quantity, but in the absence of speculators. We have

$$MV_0(K) = \begin{cases} V_H\overline{F}(K-W), & V_H\cdot E\min(X+W,K)\geq KV_L, \\ V_L, & V_H\cdot E\min(X+W,K)<KV_L. \end{cases}$$

Next, since the seller prefers to expand capacity as long as the marginal value of capacity exceeds the marginal cost, it suffices to show that the marginal value of the K-th unit of capacity is always higher in the absence of speculators, i.e., $MV(K)\leq MV_0(K)$ for every K. We first derive a useful bound:

$$\begin{aligned} MV(K) &= \frac{\partial}{\partial K}\Pi(p^*(K);K) \\ &= p^*(K)\frac{\partial}{\partial K}\{E[\min(\max(\bar{\phi}W+S^*(p;K),X+W),K)]\} \\ &\leq p^*(K)\overline{F}(K-W) \\ &\leq V_H\overline{F}(K-W), \end{aligned}$$

where the first inequality holds because it follows from (2) that an increase in K also leads to a decrease in $S^*(p;K)$. Now, we consider two cases. When $KV_L\leq V_HE\min(X+W,K)$, we have $MV_0(K)=V_H\overline{F}(K-W)$. Therefore, $MV(K)\leq V_H\overline{F}(K-W)=MV_0(K)$. For the second case, when $KV_L>V_HE\min(X+W,K)$, we must have $V_L\geq V_H\overline{F}(K-W)$ since $E\min(X+W,K)\geq K\overline{F}(K-W)$. It then follows that $MV(K)\leq V_H\overline{F}(K-W)\leq V_L=MV_0(K)$. This completes the proof. ■

References

[1] Alexandrov, A., M.A. Lariviere. 2007. Are reservations recommended? Working paper.

[2] Aviv, Y., A. Pazgal. 2008. Optimal pricing of seasonal products in the presence of forward-looking consumers. *MSOM.* 10(3): 339-359.

[3] Cachon, G.P., R. Swinney. 2009. Purchasing, pricing and quick response in the presence of strategic consumers. *Mgmt. Sci.* 55(3): 497-511.

[4] Volker, D. 2008. EBay removes offers of inauguration tickets. *Los Angeles Times.* November 14.

[5] Courty, P. 2003a. Some economics of ticket resale. *Jour. Econ. Perspectives.* 17(2): 85-97.

[6] Courty, P. 2003b. Ticket pricing under demand uncertainty. *Jour. Law & Econ.* 46: 627-652.

[7] Dana, J.D. 1998. Advance-purchase discounts and price discrimination in competitive markets. *Jour. Pol. Econ.* 106(2): 395-422

[8] Dana, J.D. 1999a. Using yield management to shift demand when th peak time is unknown. *RAND Jour. Econ.* 30(3): 456-474.

[9] Dana, J.D. 1999b. Equilibrium price dispersion under demand uncertainty: the roles of costly caapcity and market structure. *RAND Jour. Econ.* 30(4): 632-660.

[10] DeGraba, P. 1995. Buying frenzies and seller-induced excess demand. *RAND Jour. Econ.* 26(2): 331-342.

[11] Elmaghraby, W., A. Gulcu, P. Keskinocak. 2008. Designing optimal preannounced markdowns in the presence of rational customers with multiunit demands. *MSOM.* 10(1): 126-148.

[12] Gale, I., T. Holmes. 1993. Advance-purchase discounts and monopoly allocation of capacity. *Amer. Econ. Rev.* 83(1): 135-146.

[13] Geng, X., R. Wu, A.B. Whinston. 2007. Profiting from partial allowance of ticket resale. *Jour. Mktg.* 71: 184-195.

[14] Gundersen, E., A. Breznican. 2009. Jackson tickets aren't for sale, but not for lack of trying. *USA Today.* Jul 6.

[15] Harris, M., A. Raviv. 1981. A theory of monopoly pricing schemes with demand uncertainty. *Amer. Econ. Rev.* 71(3): 347-365.

[16] Ho, T.H., X. Su. 2009. Peer-induced fairness in games. *Amer. Econ. Rev.* Forthcoming.

[17] Jerath, K., S. Netessine, S.K. Veeraraghavan. 2007. Last-minute selling and opaque selling. Working paper.

[18] Johnson, G. 2007. Ticket scalping comes to a head. *Los Angeles Times.* March 10. A1.

[19] Kane, Y.I., N. Wingfield. 2007. Nintendo plays it a Wii bit cautious. *Wall Street Journal.* Dec 7, p. B1.

[20] Karp, L., J. Perloff. 2005. When promoters like scalpers. *Jour. Econ. Mgmt. Strategy.* 14(2): 477-508.

[21] Kuchera, B. 2007. Retail chain scalping Wii allotment on eBay. *Ars Technica.* Dec 20. Available at: http://arstechnica.com/news.ars/post/20071220.

[22] Lai, G., L.G. Debo, K. Sycara. 2007. Impact of price matching policy on pricing, inventory investment and profit with strategic consumers. Working paper.

[23] Lazear, E. 1986. Retail pricing and clearance sales. *Amer. Econ. Rev.* 76(1): 14-32.

[24] Levin, Y., J. McGill, M. Nediak. 2005. Optimal dynamic pricing of perishable items by a monopolist facing strategic consumers. Working paper.

[25] Liu, Q., G. van Ryzin. 2008a. Strategic capacity rationing to induce early purchases. *Mgmt. Sci.* 54(6): 1115-1131.

[26] Liu, Q., G. van Ryzin. 2008b. Strategic capacity rationing when customers learn. Working paper.

[27] McAfee, R.P. 1992. A dominant strategy double auction. *Jour. Econ. Theory.* 56(2): 434-450.

[28] McCauley, J. 2008. Giants try 'dynamic pricing.' *Oakland Tribune.* December 2.

[29] McGinn, D. 2008. The biggest game in town. *The Boston Globe.* September 21.

[30] Mulpuru, S., P. Hult. 2008. *The future of online secondary ticketing.* Forrester Research, Cambridge, MA.

[31] Ovchinnikov A., J.M. Milner. 2005. Strategic response to wait-or-buy: revenue management through last minute deals in the presence of customer leaning. Working paper.

[32] Png, I.P.L. 1989. Reservations: customer insurance in the marketing of capacity. *Mktg. Sci.* 8(3): 248-264.

[33] Shen, Z.J., X. Su. 2007. Customer behavior modeling in revenue management and auctions: a review and new research opportunities. *Prod. & Oper. Mgmt.* 16(6): 713-728.

[34] Smith, E., S. Silver. 2006. Ticketmaster adapts to rivals' online threat. *Wall St. Jour.* September 12.

[35] Stock, A., S. Balachander. 2005. The making of a "hot product": A signaling explanation of marketers' scarcity strategy. *Mgmt. Sci.* 51(8): 1181-1192.

[36] Su, X. 2007. Intertemporal pricing with strategic customer behavior. *Mgmt. Sci.* 53(5): 726-741.

[37] Su, X., F. Zhang. 2008. Strategic customer behavior, commitment, and supply chain performance. *Mgmt. Sci.* 54(10): 1759-1773.

[38] Su, X., F. Zhang. 2009. On the value of commitment and availability guarantees when selling to strategic consumers. *Mgmt. Sci.* 55(5): 713-726.

[39] Swofford, J. 1999. Arbitrage, speculation and public policy toward ticket scalping. *Public Finance Rev.* 27(5): 531-540.

[40] Wilson, C. 1988. On the optimal pricing policy of a monopolist. *Jour. Pol. Econ.* 96(1): 164-176.

[41] Wilson, R. 1985. Incentive efficiency of double auctions. *Econometrica.* 53(5): 1101-1115.

[42] Wingfield, N. 2007. Apple price cuts on new iPhone shakes investors. *Wall St. Jour.* September 6.

[43] Xie, J., S.M. Shugan. 2001. Electronic tickets, smart cards, and online prepayments: when and how to advance sell. *Mktg. Sci.* 20(3): 219-243.

[44] Yin, R., Y. Aviv, A. Pazgal, C.S. Tang. 2007. Optimal markdown pricing: implications of inventory display formats in the presence of strategic customers. Working paper.