Multi-stage Capital Budgeting with Delayed Consumption of Slack

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Abstract

Capital budgeting frequently involves multiple stages at which firms can continue or abandon ongoing projects. In this paper, we study a project requiring two stages of investment. Failure to fund Stage 1 of the investment precludes investment in Stage 2, whereas failure to fund Stage 2 results in early termination. In contrast to the existing literature, we assume that the firm can limit the manager's informational rents with the early termination of the project. In this setting, we find that the firm optimally commits to a capital allocation scheme whereby it foregoes positive NPV projects at Stage 1 (capital rationing), while at Stage 2, depending on the manager's previous report, it sometimes implements projects with a negative continuation NPV but in other situations foregoes implementing projects with positive continuation NPVs.

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I. Introduction

In the presence of privately-informed managers, a firm's capital budgeting policies balance investment efficiency against informational rents, or slack, such as managers' consumption of perquisites. To limit the opportunities for managers to divert capital funds for private benefit, firms use their managerial accounting systems as well as audits of their capital projects. For example, these systems can easily uncover, and thereby preclude, the manager's diversion of capital funds from appropriate investments to private uses such as vacations or excess compensation. However, it is more difficult for these systems to distinguish between investments in necessary and unnecessary assets, provided that both fall within the general class of appropriate purchases. For instance, it may be difficult to detect that a manager is engaging in empire building (acquiring unnecessary hard assets such as research equipment, cars, or offices; or unnecessary soft assets such as staff and consulting services) from which he derives private benefits. Thus, at best the firm's accounting and auditing systems may mitigate a manager's informational rent by constraining the set of assets into which he can divert capital funds for private benefit. One possible implication of restricting investment to specific asset classes is that the private benefit provided to the manager can be consumed over time. For example, the private benefit of lavish offices is consumed over the time that the manager actually uses these offices (as compared to excess compensation or vacations whose benefits are consumed immediately). As a result of constraining a manager's asset diversions, a principal can deny the manager the full consumption of such private benefits by abandoning the project before completion. However, by abandoning the project, the principal may be unable to fully recoup her initial investment. We study the implications of this abandonment option and "delayed slack consumption" on optimal capital allocation schemes.

We consider a multi-stage capital budgeting setting in which a privately-informed manager proposes an initial research budget to develop a new project that the principal either accepts or rejects. If the principal accepts, the funds are made available to the manager who engages in the research required to discover the feasibility and net benefit of implementing the project of interest. If the research stage is successful, the continuation value of the project becomes known to both the principal and manager at the end of Stage 1. The principal determines whether to implement or abandon the project in Stage 2. Consistent with the above discussion, we assume that the manager cannot consume all the slack in Stage 1 that may arise from the investment. As a result, if the project is abandoned at Stage 2 the manager foregoes consuming some of the associated informational rent. Likewise, if the project is abandoned at Stage 2, the principal cannot fully recoup her initial investment. Thus, if the investment is abandoned, a

deadweight loss is incurred. For simplicity, and without loss of generality, we assume that the early abandonment of the project leads to the manager consuming no slack and the principal receiving a zero salvage value for his Stage 1 investment.

Our model of "delayed slack consumption" gives rise to an optimal capital allocation scheme that is qualitatively different from that found in the previous literature but is consistent with empirical findings. For example, we find that while the optimal capital allocation scheme uses a single Stage 1 hurdle rate that results in under-investment or capital rationing at Stage 1 (the research stage), at Stage 2 it applies different hurdle rates depending on the manager's previous Stage 1 cost report and the outcome of his Stage 1 research. In fact, we find that it is optimal for the principal to commit to continue some projects at Stage 2 even if they have a negative continuation value (i.e., Stage 2 over-investment), while it is also optimal to commit to forego other projects that have a positive continuation value (i.e., Stage 2 under-investment). Our results are consistent with the empirical evidence of both over and under-investment across firms found by Richardson (2006) and Driver and Temple (2009). However, our results are most closely related to Poterba and Summers (1995) who find multiple hurdle rates within individual firms, some of which are below and some of which are above their cost of capital.

While our model is descriptive of a large class of capital budgeting problems, we believe that it is particularly descriptive of the capital budgeting process for R&D projects that tend to span multiple stages; require highly-specialized investments whose salvage values, if abandoned early, are significantly lower than their initial costs; and for which the general categories of assets to be funded can be agreed upon ahead of time and audited. For example, R&D investments often require multiple investment stages because additional technological information (regarding cost, reliability and scalability) or market information (regarding availability of suppliers or demand) can only be acquired after constructing a prototype plant or product.[1] Another applicable R&D setting is one where regulatory obligations must be satisfied sequentially; as is the case with newly developed drugs undergoing FDA approval.

II. Literature Review

Our paper builds on the single-period, adverse selection capital budgeting model of Antle and Eppen (1985) in which the project cost is commonly known, but the rate of return is privately known by the

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[1] For a discussion of multi-stage capital budgeting, see Gompers (1995) and Gitman and Mercurio (1982).

manager. Because the manager can immediately consume any allocated funds above and beyond those required for the project, his incentive is always to understate the project's true rate of return. To mitigate the manager's misreporting incentives, the optimal capital allocation scheme provides the same budget for all projects and specifies a single required rate of return or hurdle rate that exceeds the firm's cost of capital, causing the firm to forego some positive NPV projects (i.e., under-invest).[2]

Antle and Fellingham (1990), Fellingham and Young (1990) and Arya et al. (1994) extend the model to a repeated game, where the manager privately observes and reports on a *new and independent* investment opportunity in each period. Similarly, Antle et al. (2006) considers the value of giving the privately-informed manager an option to postpone the investment to a later period at which time he may discover and report on a new and unrelated investment opportunity. In contrast, we consider projects where the Stage 2 investment opportunity only arises if the firm invested in Stage 1; i.e., the former can be viewed as a continuation of the latter. Furthermore, the above-mentioned literature assumes that the manager can immediately consume any slack provided by the principal, whereas we assume that the manager consumes the slack across multiple stages.

Most closely related to our work is the abandonment options literature. Levitt and Snyder (1997) considers a two-stage setting where the manager is subject to moral hazard in Stage 1, and his efforts affect both an interim signal received between the two stages, and the likelihood of the project succeeding at the end of Stage 2.[3] If the principal abandons the project upon receiving the interim signal, she eliminates the possibility of conditioning the manager's compensation on the project outcome, thereby exacerbating the Stage 1 moral hazard problem. Consequently, the principal optimally continues projects with a negative continuation value (over-invests) when the Stage 1 moral hazard problem is sufficiently severe.[4] Unlike Levitt and Snyder (1997), we find instances of both under- and over-investment. Further, because the manager in our model takes no productive actions but has private information, the principal uses her Stage 2 abandonment/continuation decision to control the manager's Stage 1 reporting incentives.

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[2] Bockem and Schiller (2009) finds report contingent budgets which are used to motivate the manager to engage in costly information gathering. A similar extension is considered in Kim (2006).

[3] We thank an anonymous referee for referring us to Levitt and Snyder (1997).

[4] Dutta and Fan (2009) and Bernardo et al (2009) also study moral hazard models which give rise to instances of over-investment.

The structure of our model is most closely related to that of Pfeiffer and Schneider (2007). In both papers, the principal makes a Stage 1 investment decision based on the manager's report and later makes her Stage 2 continuation decision based on information revealed at Stage 2.[5] The major difference between our paper and Pfeiffer and Schneider (2007) concerns the informational rent. In Pfeiffer and Schneider (2007) the principal incurs an informational rent (and the manager consumes it) only if the project is continued.[6] In our model, the principal incurs the informational rent upfront at the time of the Stage 1 funding. If the project is not continued, there is a deadweight loss in that the principal has already incurred the informational rent but the manager cannot consume it.

Our finding of Stage 2 over-investment is also related to the literature on "escalation errors" (Staw 1976 and Berg et al. 2009). This literature identifies settings in which a person who has previously invested in a project subsequently finds out that the continuation value of the project is negative but still chooses not to abandon it, resulting in over-investment. Kanodia et al. (1989) provides a rational economic explanation for such behavior based on reputational concerns. In that model, abandoning the project would indicate that the person did not have "foresight" at the time he initially invested in the project, causing the labor market to revise downward his value and future outside opportunity wage. The present paper and Kanodia et al. (1989) explain two very different over-investment phenomena. Kanodia et al. (1989) examines a single actor – labor market setting in which it is optimal for *the single actor* to continue a negative NPV project. The present paper examines an optimal contracting, principal-agent setting, in which it is optimal for the *principal* to commit to continue a negative NPV project in order to mitigate the manager's Stage 1 reporting incentives.

III. Model

We study an adverse selection model encompassing two stages. At Stage 1, the manager requests a budget to investigate a potential new project. The cost required to successfully conduct this investigation is privately known by the manager. If funded, the manager's investigative or research work gener-

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[5] The interim signal is privately observed by the manager in Pfeiffer and Schneider (2007), whereas it is publicly observed in our model.

[6] Further, in Pfeiffer and Schneider (2007) the abandonment decision creates information in the sense that it allows the principal to perfectly observe the agent's subsequent effort choice. In contrast, in Arya and Glover (2003) abandonment destroys information about the agent's prior choice of effort. In our model, the Stage 2 decision has no effect on the information available to the principal.

ates a public signal at the end of Stage 1, that reveals the continuation value of implementing the pro-

ject. At the start of Stage 2, after the public signal is revealed, the principal decides whether to abandon

or to implement the project. We label Stage 1 the *research stage* and Stage 2 the *implementation stage*,

as the project is assumed to end without any payoff in Stage 2 unless the principal makes the Stage 2 in-

vestment to implement the project. Consistent with the two stages, we refer to an investment in Stage 1

as "funding" research and an investment in Stage 2 as "implementing" the project.

The timeline is as follows (see Figure 1). At time t=1 a risk-neutral firm (the principal) hires a risk-neutral

manager to oversee the research activity associated with a potential new project. The principal offers a

contract to the manager that fully specifies the compensation scheme and capital budgeting rules (as

discussed below). At t=0, prior to being hired, the manager privately observes the level of funding re-

quired to conduct the research in Stage 1. We assume that the research stage requires a minimum in-

vestment of $c_i \in C = \{c_1, ..., c_n\}$, where $c_i < c_{i+1}\ \forall i$, is privately observed by the manager. Any Stage 1 in-

vestment greater than or equal to the observed c_i generates a contractible signal, $g_j \in G = \{g_1, ..., g_k\}$,

where $g_j > g_{j+1}\ \forall j$, which denotes the net continuation value of implementing the project at t=4.[7] To

highlight the role of the abandonment decision and delayed slack consumption in mitigating inefficien-

cies, we assume that at time t=1, the principal can commit to a t=4 implementation schedule based on

the contractible continuation signal, g_j. The interim signal being contractible allows us to ignore addi-

tional issues that arise when information asymmetry occurs at two different points in time. Without loss

of generality, any Stage 1 investment less than the observed c_i will cause the research stage to fail with

certainty, which we assume to be a contractible event. The probability densities of c_i and g_j are common

knowledge, independent and represented by $f(c_i)$ and $f(g_j) \equiv \rho_j$ respectively.[8] To facilitate the ex-

position, we assume that the discrete costs, c_i, are evenly spaced, with $c_i - c_{i-1} = \delta$ for $i > 1$, $c_1 = \delta$,

and that these costs are uniformly distributed: $f(c_i) = \frac{1}{n}$. The assumptions placed on the probability

distributions of the Stage 1 costs and Stage 2 continuation values significantly simplify the characteriza-

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[7] The ordering of c_i's and g_j's implies that the principal prefers smaller indices for both c_i and g_j.

[8] The model could also accommodate correlated random variables, c and g, although doing so would complicate the interpretation of our results, as one would then have to distinguish between the effects caused by statistical dependence versus those arising from the delayed consumption of slack.

tion of the manager's informational rent. However, we believe that all qualitative results would continue to hold with more general distributions.

We next specify the budgeting and compensation processes. At time $t=2$, the manager uses his private information to submit a report, $\hat{c}_i$, to the principal regarding his research costs and the latter funds the project according to the contract agreed to at time $t=1$: with probability $z(\hat{c}_i) \in [0,1]$ the research is funded and with probability $1 - z(\hat{c}_i)$ it is not. If the project is not funded, the game ends and both parties receive their reservation utility of zero. On the other hand, if the project is funded, the manager receives the contractually-specified budget of $b(\hat{c}_i)$ to fund his research at $t=3$. We assume that the manager's spending is imperfectly audited, as discussed in the Introduction, so that he cannot *immediately* divert any funds in excess of that required to conduct the Stage 1 research (i.e., $b(\hat{c}_i) - c_i$) into consumption. Hence, the manager invests his entire budget in research-related assets; however, only c_i is necessary to conduct the Stage 1 research. The remainder generates no benefit to the firm but provides the manager with personal utility in Stage 2 – so long as the project is implemented.[9] The principal's implementation decision at $t=4$ accords with the contract agreed to at $t=1$: she implements the project with probability $p(\hat{c}_i, g_j) \in [0,1]$, and abandons it with probability $1 - p(\hat{c}_i, g_j)$. We assume that if the project is abandoned at $t=4$, the principal cannot recoup the earlier investment, b_i. On the other hand, if the project is implemented at $t=4$, then at $t=5$, the principal consumes the residual project surplus, $g_j - b(\hat{c}_i)$, and the manager consumes the excess funding, or slack, procured in Stage 1: $b(\hat{c}_i) - c_i$.

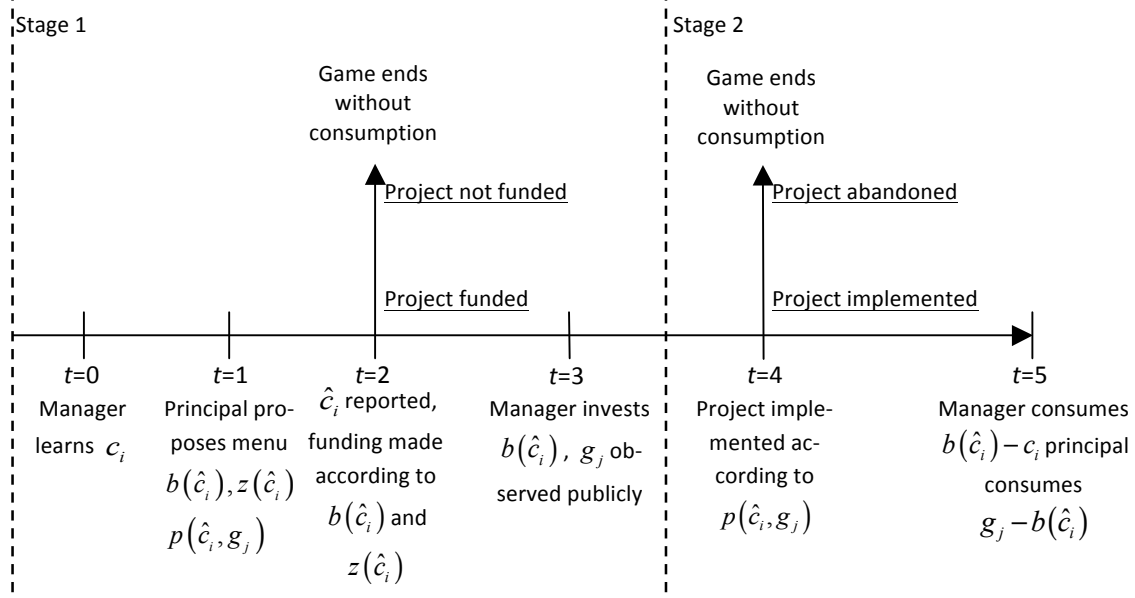[9] Bolton and Dewatripont (1994) make a similar assumption.

Figure 1: Two-stage timeline.

As a managerial example of our setting, consider a firm that is interested in entering a new geographical market with uncertain market potential. The firm decides to delay the large upfront costs required to fully enter the market by hiring a manager to conduct a trial experiment in a representative fraction of the market. To do so, the manager needs to hire people familiar with the new market and invest in facilities. The manager is privately informed as to the cost of running the trial. While the firm can audit the manager's spending, it cannot distinguish between necessary and excessive investments in people and facilities. The manager can only consume the full private benefits provided by excess Stage 1 funding if the trial is successful and the firm decides to fully enter the market. Again, if the project is abandoned, the salvage value of the unconsumed assets is likely to be zero.

Although we have outlined an extensive list of assumptions above, the three critical assumptions required for our results are: (1) the manager does not consume all the slack in Stage 1; (2) the principal cannot recoup all of the unconsumed funding if the project is abandoned; and (3) the principal and manager cannot contract on the price of salvaged assets if the project is abandoned. Assumptions (1) and (2) guarantee that a deadweight loss is incurred if the project is abandoned, whereas (3) assures that the principal cannot fully resolve the Stage 1 adverse selection problem by abandoning the project in Stage 2.

IV. Two Benchmark Models

Before formally stating our problem, we introduce two benchmark models that will help us analyze the efficiency implications of our model (referred to as the **Commitment** model). The **First-Best** (FB) model is identical to the Commitment model, but without adverse selection; that is, we assume that the manager's research cost, c_i, is common knowledge. Accordingly, the principal provides a budget of $b_i = c_i$. Let $c_{h^{FB}}$ be the largest research cost realization that the principal agrees to fund, i.e. $c_{h^{FB}} \leq E[g_j]$ and $c_{h^{FB}+1} > E[g_j]$. At Stage 2, the principal optimally implements all projects with positive continuation values; i.e., a project is implemented if and only if $g_j \geq 0$, regardless of the (sunk) Stage 1 expenditures. As with the definition of $c_{h^{FB}}$, let w^{FB} be the continuation value index such that $g_{w^{FB}} \geq 0$ but $g_{w^{FB}+1} < 0$. Comparing the results from the Commitment and First-Best models will allow us to identify over-investment, under-investment, and the agency costs associated with eliciting the manager's private information. We define under-investment at Stage 1 relative to First-Best as any allocation policy where profitable projects are not funded, i.e., $z_i < 1$ for some $\hat{c}_i \leq c_{h^{FB}}$. We define under-investment at Stage 2 if some projects with a positive continuation value are abandoned, i.e., $p(\hat{c}_i, g_j) < 1$ for some $g_j \geq g_{w^{FB}}$ and i. Over-investment is similarly defined.

To illustrate how our results differ from prior capital budgeting research, we introduce a second benchmark: the **No-Commitment** (NC) model is identical to our Commitment model (including delayed slack consumption), but where the principal *cannot* commit to her Stage 2 implementation rule at the contracting period, *t*=1. Instead, the principal follows a sequentially rational Stage 2 implementation rule that implements all projects with non-negative continuation values ($g_j \geq 0$), regardless of the Stage 1 expenditure. Therefore the No-Commitment solution has the same Stage 2 implementation policy as the First-Best solution, i.e., $g_{w^{NC}} = g_{w^{FB}} \geq 0$. Because the principal cannot commit to an implementation rule when she proposes a contract in the No-Commitment setting, her only available controls are the probability of providing a budget, $z(\hat{c}_i)$ and the size of the allocated budget, $b(\hat{c}_i)$. However, because the sequentially rational Stage 2 implementation rule is independent of the manager's Stage 1 cost report, we can still invoke the Revelation Principle, which implies $\hat{c}_i = c_i$. To simplify what follows, let $z_i \equiv z(c_i)$, $b_i \equiv b(c_i)$, and $p_{i,j} \equiv p(c_i, g_j)$. The optimal solution to the No-Commitment model thus solves:

$$U_P = \max_{b_i, z_i, p_{i,j}} \frac{1}{n} \sum_{i=1}^{n} z_i \left(\sum_{j=1}^{k} \rho_j p_{i,j} g_j - b_i \right)$$

subject to:

$$z_i \sum_{j=1}^{k} \rho_j p_{i,j} (b_i - c_i) \geq z_s \sum_{j=1}^{k} \rho_j p_{s,j} (b_s - c_i) \quad i, s = 1, K, n \qquad \text{(TT}^{\text{NC}}\text{)}$$

$$z_i \sum_{j=1}^{k} \rho_j p_{i,j} (b_i - c_i) \geq 0 \quad i = 1, K, n. \qquad \text{(IR}^{\text{NC}}\text{)}$$

$$p_{i,j} = \begin{cases} 1 & \text{for } j \leq w^{FB} \\ 0 & \text{for } j > w^{FB} \end{cases} \quad \forall i \qquad \text{(SR}^{\text{NC}}\text{)}$$

Sequential rationality ensures that the principal will only implement Stage 2 projects with non-negative continuation values, regardless of the reported Stage 1 cost (SR$^{\text{NC}}$). The (TT$^{\text{NC}}$) constraints ensure that the manager's report is truthful. The (IR$^{\text{NC}}$) constraints assure that the manager is given sufficient funds to conduct his Stage 1 research. Note that the delayed slack consumption plays no role in the principal's Stage 2 implementation decision because she cannot recoup any of the unconsumed budget if she abandons the project at Stage 2. Delayed slack consumption only shows up in the manager's (TT$^{\text{NC}}$) constraints.

Lemma 1: The optimal funding and implementation rules in the No-Commitment benchmark:

(i)   Define a Stage 1 hurdle cost, $c_{h^{NC}}$, with $h^{NC} \in \{1,...,n\}$, such that projects are always funded ($z_i = 1$) when the Stage 1 cost is less than or equal to the hurdle $c_i \leq c_{h^{NC}}$, and are always rejected ($z_i = 0$) when the Stage 1 cost exceeds the hurdle, $c_i > c_{h^{NC}}$.

(ii)   Provides the manager with a fixed Stage 1 budget: $b_i^{NC} = c_{h^{NC}}$, for any funded Stage 1 cost, $c_i \leq c_{h^{NC}}$.

Note that the structure of the optimal Stage 1 funding rule is the same as in the Antle-Eppen (1985) model, because the sequential rationality of the Stage 2 implementation decision makes the Stage 1 funding decision essentially a one-period problem. Comparing our results with those obtained in the No-Commitment model will thus allow us to compare our results with the prior literature and to isolate the roles of delayed slack consumption and commitment in the optimal capital allocation rule.

V. Commitment Model Results

Returning to our **Commitment** model, and again invoking the Revelation Principle, the manager's expected utility upon observing Stage 1 cost c_i is given by:

$$U_A = z_i \sum_{j=1}^{k} \rho_j p_{i,j} \left(b_i - c_i \right).$$

When the manager reports c_i, the principal funds the project in the amount b_i, with probability z_i, and commits to a Stage 2 implementation rule, $p_{i,j}$. Thus, at time $t=1$, we can express the principal's Commitment program as:

$$U_P = \max_{b_i, z_i, p_{i,j}} \frac{1}{n} \sum_{i=1}^{n} z_i \left(\sum_{j=1}^{k} \rho_j p_{i,j} g_j - b_i \right)$$

subject to:

$$z_i \sum_{j=1}^{k} \rho_j p_{i,j} \left(b_i - c_i \right) \geq z_s \sum_{j=1}^{k} \rho_j p_{s,j} \left(b_s - c_i \right) \quad i, s = 1, \mathrm{K}, n \qquad (\mathrm{TT}^C)$$

$$z_i \sum_{j=1}^{k} \rho_j p_{i,j} \left(b_i - c_i \right) \geq 0 \quad i = 1, \mathrm{K}, n. \qquad (\mathrm{IR}^C)$$

Note that because the principal can now commit to a Stage 2 implementation rule, the sequential rationality constraints from the No-Commitment problem can be dropped. The manager's Truth-Telling constraints, (TT^C), ensure that the manager truthfully reports his cost, c_i. The manager's Individual Rationality constraints, (IR^C), guarantee that the manager receives sufficient funds to conduct his Stage 1 research whenever the principal agrees to fund it.[10] Conditional on a Stage 1 cost c_i, the manager's expected utility only depends on the interim information, g_j, to the extent that it affects the probability of implementing the project in Stage 2, or equivalently, the probability that he consumes his informa-

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[10] Without loss of generality, we ignore the possibility that the principal compensates the manager when: (a) the principal does not invest in the research stage and (b) the principal does invest in the research stage, but chooses not implement the project. The proof of this assertion is relatively long but follows from the assumed risk-neutrality of the manager and lack of any additional agency problems.

tional rent. With commitment, the principal has three choice variables to control the manager's incentives: the set of cost reports that receive Stage 1 funding (the set of c_i for which $z_i > 0$); the associated budgets (b_i); and the Stage 2 implementation rule ($p_{i,j}$). Proposition 1 below characterizes the principal's optimal funding and implementation rules, where $p_i \equiv \sum_{j=1}^{k} \rho_j p_{i,j}$ denotes the probability of Stage 2 implementation following a cost observation c_i.

Proposition 1: The optimal funding and implementation rules (denoted by a "*" superscript) of the Commitment model:

(i)   Define a Stage 1 hurdle cost, c_{h^*}, with $h^* \in \{1,...,n\}$, such that projects are always funded ($z_i^* = 1$) when the cost is less than or equal to the hurdle $c_i \leq c_{h^*}$, and are always rejected ($z_i^* = 0$) when the cost exceeds the hurdle, $c_i > c_{h^*}$.

(ii)   Provide the manager with a Stage 1 budget: $b_i^* = c_i + \delta \sum_{q=i+1}^{h^*} \dfrac{p_q^*}{p_i^*}$, that weakly increases in the manager's cost, c_i.

(iii)   Define a Stage 2 continuation value hurdle, $g_{w_i^*}$, for every funded Stage 1 cost, $c_i \leq c_{h^*}$, such that projects are implemented for $g_j \geq g_{w_i^*}$ and are abandoned for $g_j < g_{w_i^*}$.

(iv)   The Stage 2 continuation value hurdle $g_{w_i^*}$ is weakly increasing in the manager's Stage 1 cost.

(v)   The Stage 2 implementation probability:

         a.   for any Stage 2 project, $p_{i,j}^*$ is weakly decreasing in the manager's Stage 1 cost, c_i, and increasing in the Stage 2 continuation value, g_j,

b.   for all Stage 2 projects, $p_i^* \equiv \sum_{j=1}^{k} \rho_j p_{i,j}^*$, is weakly decreasing in the manager's Stage 1

cost, c_i.

Proposition 1 (i) finds that the familiar Stage 1 hurdle-rate contracts from the prior capital budgeting lit-erature and the No-Commitment model remain optimal in the presence of Stage 2 commitment and de-layed slack consumption. As in the prior literature, the hurdle is used to discourage the manager from over-stating his cost. The optimal funding probabilities, z_i^*, also follow the earlier models, in that they are binary.[11]

Part (ii) provides the first instance of how our assumptions of delayed slack consumption and commit-ment to Stage 2 implementation result in novel capital allocation rules. We find that the size of the manager's Stage 1 budget, b_i^*, increases in his cost. To derive the optimal Stage 1 budgets, we use standard techniques to establish that only the adjacent upward (TT^C) constraints are binding (preventing a manager with cost c_i from reporting cost c_{i+1}). This gives rise to the recursive equation

$b_i = \frac{p_{i+1}}{p_i}\left(b_{i+1} - c_i\left(1 - \frac{p_i}{p_{i+1}}\right)\right)$, which yields the optimal budgets, $b_i^* = c_i + \delta \sum_{q=i+1}^{h^*} \frac{p_q}{p_i}$. In the prior litera-ture and the No-Commitment model, the budget is constant for all funded cost reports and equal to the highest funded cost, i.e., $b_i^{NC} = c_{h^{NC}}$, and the principal mitigates the manager's informational rent by choosing a hurdle below First-Best, $c_{h^{NC}} < c_{h^{FB}}$. In contrast, the optimal budgets in our Commitment model increase in the manager's cost because, with delayed slack consumption and Stage 2 commit-ment, the manager is interested not only in $b_i - c_i$ as in the No-Commitment model, but also in the

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[11] Our results on the behavior of b_i^*, p_i^* and z_i^* in Proposition 1 are "weak" due to the discreteness of our densi-ties. Our results would be "strict" if we assumed that the supports of Stage 1 and Stage 2 costs were sufficiently dense.

probability with which he is able to consume it (p_i), which the principal controls.[12] Hereafter we will re-

fer to $b_i - c_i$ as "budgetary slack" and to $p_i(b_i - c_i)$ as the "informational rent".

By increasing the manager's budget in his cost report, the principal achieves two efficiency gains over

the NC capital budgeting setting. First, for a given Stage 1 cost hurdle, c_h, the manager's budgetary slack

is less than in the NC and prior models where $b_i^{NC} = c_{h^{NC}}$, $\forall i \leq h^{NC}$. This gives rise to the second benefit

in that it allows the principal to increase the Stage 1 funding hurdle closer to First-Best (see Proposition

2 (ii), below) and thereby reduce Stage 1 under-investment. However, the disadvantage of an increasing

budget is that it provides the manager with additional incentives to overstate his cost. To mitigate this

incentive, the principal commits to a Stage 2 implementation rule, $p_{i,j}$, that *decreases* in the manager's

Stage 1 cost (Prop 1 (v.a)). The manager's situation is similar to an auction where the bidder trades off

the surplus ($b_i - c_i$ in our case) were he to win with a bid b_i , against the probability of winning with that

bid (p_i). Although the probability of "winning" an auction is exogenously determined by the distribution

of bidders but is optimally chosen by the principal in our setting, the tradeoffs the bidder/manager face

are identical.

As (iii) points out, Stage 2 implementation follows a (Stage 1 report-contingent) hurdle policy in that all

projects with a continuation value below the hurdle, $g_{w_i^*}$, are abandoned and projects with a continua-

tion value strictly greater than the hurdle are implemented with probability 1. Because of the assumed

discreteness of g_j , projects whose continuation value is exactly the hurdle may be implemented with a

probability less than 1.

To further isolate the role of delayed slack consumption, the following proposition compares the Stage 1

relative efficiency between our model and both the First-Best and No-Commitment benchmarks.

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[12] Note that, if instead the manager could consume all his informational rent in Stage 1, then the principal would optimally adopt the First-Best Stage 2 implementation rule with constant p_i's, which when plugged into the above recursive equation would imply $b_i = b_{i+1}$, consistent with the constant budget obtained in the extant literature and the No Commitment benchmark of Lemma 1.

Proposition 2: The optimal capital allocation mechanism in the Commitment model results in Stage 1 investment whereby the principal:

(i)   Under-invests in research at Stage 1 relative to the First-Best benchmark.

(ii)   Under-invests *less* in research at Stage 1 than the No-Commitment benchmark.

With delayed slack consumption and Stage 2 commitment, the optimal allocation involves capital ration-ing at Stage 1 ($h^* \leq h^{FB}$), which is consistent with the previous adverse-selection models of capital allo-cation (Antle and Eppen 1985 and others). However, Proposition 2 (ii) finds that this under-investment is less severe in the Commitment model than in the No-Commitment model. To see why, suppose that: (1) there are three Stage 1 cost realizations (c_1 , c_2 , c_3); (2) all Stage 2 projects have non-negative continua-tion values; (3) the principal funds all three Stage 1 managers and implements all Stage 2 projects under First-Best; (4) the Stage 1 hurdle for both the Commitment and the No-Commitment models is c_1 ; and (5) that the principal optimally implements all Stage 2 projects conditional on c_1 in both settings. Be-cause only c_1 is funded, no manager earns rent. Now, consider increasing the Stage 1 hurdle to c_2 . In the No-Commitment model, the principal optimally continues to implement all Stage 2 projects (now for both c_1 and c_2) and must therefore provide a budget of c_2 to both managers. Thus, for the NC model, the increase in informational rent to the c_1 manager from increasing the Stage 1 hurdle is $c_2 - c_1 = \delta$. In the Commitment model, the principal would still implement all Stage 2 projects for the c_1 manager, but could impose a lower implementation probability for the c_2 manager (as shown in Proposition 1). This allows the principal to pay a smaller informational rent to the c_1 manager than in the No-Commitment model, but at the cost of incurring a Stage 2 implementation inefficiency. Thus, the resulting *net* cost of reducing Stage 1 under-investment may be less with commitment than without. In this example, we re-duced the c_1 manager's informational rent by choosing an implementation probability for the c_2 man-ager less than 1, thereby incurring Stage 2 under-investment. Alternatively, if there were Stage 2 pro-jects with negative continuation values, then instead of imposing an inefficient implementation sched-ule on manager c_2 , we could have limited the c_1 manager's misreporting incentives by increasing his implementation probability for one or more of the negative continuation value projects, thereby incur-

ring Stage 2 over-investment. In fact, as we show in Proposition 3, we find that the optimal Stage 2 dis-

tortions may include *both* under-investment and over-investment.

Proposition 3: The principal's optimal contract in the Commitment model results in the following Stage

2 distortions relative to the First-Best and the No-Commitment models:

(i)   Weak under-investment on average.

(ii)   Weak over-investment for small Stage 1 costs.

(iii)   Weak under-investment for large Stage 1 costs.

Recall that in both the First-Best and No-Commitment solutions, the principal's Stage 2 implementation

rules are efficient in that they ignore the sunk costs from Stage 1.[13] However, in the Commitment model,

the principal uses the Stage 2 implementation rules to trade-off her project profits (g_j) and the manag-

er's compensation. To see this, consider the principal's unconstrained optimization problem after substi-

tuting the optimal budgets from Proposition 1 (ii) into the principal's objective function:

$$U_P = \max_{p_{i,j}} \frac{1}{n} \sum_{i=1}^{h^*} \sum_{j=1}^{k} \left(\rho_j p_{i,j} g_j - c_i - \delta \frac{\sum_{s=i+1}^{h^*} \sum_{m=1}^{k} \rho_m p_{s,m}}{\sum_{m=1}^{k} \rho_m p_{i,m}} \right).$$

From the principal's objective function it is straightforward that the more Stage 1 projects that are

funded, the more slack the principal has to provide the managers, as the last term in brackets increases

in h^*. Furthermore, changing the implementation probability $p_{i,j}$ has three effects for the principal, one

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[13] Over-investment from the principal's point of view and the society's point of view are identical under First-Best.
In the No-Commitment model and in our model, over-investment from the principal's point of view occurs when
any project with a negative continuation value is implemented. This need not be over-investment from a social
welfare point of view because the loss to the principal from continuing a negative continuation value project may
be compensated by the slack which the agent gains from such implementation. We define over-investment from
the principal's point of view. Note that this is consistent with what empiricists are able to document, as the slack
arising from earlier stage investments cannot be observed. We thank an anonymous referee for raising this issue.

direct and two indirect. These three effect can be seen in the principal's marginal payoff to raising the implementation probability $p_{i,j}$, holding fixed the number of funded managers, h^*:

$$\frac{\partial U_P}{\partial p_{i,j}} = \frac{\rho_j}{n}\left(g_j + \delta\sum_{s=i+1}^{h^*}\frac{p_s}{p_i^2} - \delta\sum_{t=1}^{i-1}\frac{1}{p_t}\right). \tag{1}$$

By changing $p_{i,j}$, the principal changes the probability with which she implements a project with continuation value g_j. For a positive continuation value, increasing $p_{i,j}$ directly increases the principal's utility by reducing the level of Stage 2 under-investment. The two indirect effects change the principal's utility by affecting the budgetary slack $b_i - c_i = \delta\sum_{s=i+1}^{h^*}\frac{p_s}{p_i}$ required to satisfy the manager's truth-telling constraints. From (1), the first indirect effect, $\delta\sum_{s=i+1}^{h^*}\frac{p_s}{p_i^2}$, represents the amount of manager i's budgetary slack that the principal can reduce via a marginal increase in $p_{i,j}$, while leaving the manager's reporting incentives unchanged. However, the final term in (1), $-\delta\sum_{t=1}^{i-1}\frac{1}{p_t}$, represents a second, countervailing indirect effect. In equilibrium the upward truth-telling conditions are binding. This implies that when increasing the probability of implementation for manager c_i, the principal must also increase the informational rents paid to for all managers whose cost is less than c_i. That is, by increasing the implementation probability for one manager, the payoff to over-reporting increases for all managers with lower Stage 1 cost, therefore the principal must also increase the rents paid to them to maintain their truth-telling incentives.

Taking these three effects together illustrates how the optimal allocation rule may result in Stage 2 over-investment for relatively low Stage 1 cost realizations and under-investment for relatively high Stage 1 cost realizations. For example, because there are no managers with a lower Stage 1 cost than manager c_1, the second indirect effect is absent. As a result, the FOC for manager c_1 demonstrates how the first indirect effect can lead to over-investment since $\frac{\partial U_P}{\partial p_{1,j}} = \frac{\rho_j}{n}\left(g_j + \delta\sum_{s=2}^{h}\frac{p_s}{p_1^2}\right)$ can be positive, resulting in

$p_{1,j} > 0$ — even when $g_j < 0$. Similarly, for manager c_h there are no managers with a higher Stage 1 cost, hence, the first indirect effect is absent. This FOC shows how the second indirect effect can lead to

under-investment since $\dfrac{\partial U_P}{\partial p_{h,j}} = \dfrac{\rho_j}{n}\left(g_j - \delta\sum\limits_{t=1}^{h-1}\dfrac{1}{p_t}\right)$ can be negative, resulting in $p_{h,j} = 0$ − even when

$g_j > 0$. Proposition 3 (i) shows that on average, the effect of increasing the slack of other managers

dominates the effect of reducing the slack of any one manager so that the principal commits to under-

investment at Stage 2. Notice however that if either the principal could not commit to her Stage 2 im-

plementation rule or if the manager consumed all of his informational rents in Stage 1 then

$\dfrac{\partial U_P}{\partial p_{i,j}} = \dfrac{\rho_j}{n}g_j$, in which case there would be no indirect effects at Stage 2 and the principal would in-

stead follow the sequential rational implementation rule used both in the First-Best and No-

Commitment models, $p_{i,j} = \begin{cases} 1 & g_j \geq 0 \\ 0 & g_j < 0 \end{cases}$.

Recall that in our model there is a deadweight loss from abandonment. The first indirect effect, which

gives rise to over-investment, is entirely caused by this deadweight loss. To see this formally, suppose

the principal could recoup the budgetary slack $b_i - c_i$ if she abandons the project. The objective function

then becomes $U_P = \max\limits_{b_i, z_i, p_{i,j}} \dfrac{1}{n}\sum\limits_{i=1}^{n} z_i\left(\sum\limits_{j=1}^{k}\rho_j p_{i,j}g_j - b_i + (1-p_i)(b_i - c_i)\right)$, and the associated FOC then be-

comes $\dfrac{\partial U_P}{\partial p_{i,j}} = \dfrac{\rho_j}{n}\left(g_j - \delta\sum\limits_{q=1}^{i-1}1\right)$, precluding over-investment.

To further show how the direct and indirect effects discussed above interact, we next consider the ef-

fect of an exogenous shock to a single Stage 2 continuation value g_j on the implementation probabili-

ties. The above discussed first-order condition $\partial U_P \big/ \partial p_{i,j}$ illustrates the direct effect on the optimal $p_{i,j}$

of a change in the continuation value g_j. However, in equilibrium, varying $p_{i,j}$ affects the budgetary

slack $b_i - c_i$ and consequently the manager's (TT^C) constraints. As a result, a change in any *one* g_j may

have an indirect effect on *all* implementation probabilities. This comparative static analysis is illustrated

in the following proposition.

Proposition 4: For a fixed h^*, increases (decreases) in a continuation value, g_j to $g_j{}'$ such that $g_{j+1} < g_j{}' < g_{j-1}$, will result in larger (smaller) optimal implementation probabilities, $p_i^*{}'$, relative to p_i^*, for all $i \leq h^*$.

Increasing the continuation value g_j to $g_j{}'$ changes the principal's Stage 2 expected payoff whenever $g_j \geq g_{w_i^*}$, i.e., whenever the manager observes a cost c_i for which this specific Stage 2 project is actually implemented. To capitalize on the increased continuation value, intuition suggests that the principal should weakly increase $p_{i,j}$, which has two indirect effects. To illustrate these two effects consider the following example. Assume there are three potential Stage 1 costs, c_1, c_2 and c_3 as well as four potential Stage 2 continuation values g_1, g_2, g_3 and g_4. Further assume that the optimal $p_{i,j}$'s are represented in the following table.

	g_1	g_2	g_3	g_4
c_1	$p_{1,1} = 1$	$p_{1,2} = 1$	$p_{1,3} = 1$	$p_{1,4} = 0$
c_2	$p_{2,1} = 1$	$p_{2,2} = 1$	$p_{2,3} = 0$	$p_{2,4} = 0$
c_3	$p_{3,1} = 1$	$p_{3,2} = 0$	$p_{3,3} = 0$	$p_{3,4} = 0$

Table 1: Implementation probabilities in an example

Now assume that g_3 increases to $g_3{}'$. This makes increasing $p_{2,3}$ worthwhile to the principal due to the direct effect. However, increasing $p_{2,3}$ now makes it more valuable for manager c_1 to misrepresent himself as a c_2 manager. To maintain manager c_1's truth-telling incentive, the principal weakly increases $p_{1,4}$ (as long as g_4 is not too negative).[14] In addition, increasing $p_{2,3}$ relaxes the upward (TT^C) constraint for manager c_2, which the principal exploits by weakly increasing $p_{3,2}$. Therefore, the principal weakly increases all $p_{i,j}$'s following the increase in g_3.

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[14] If $g_3 > 0$ and $g_4 < 0$, then this change results in optimal over-investment for c_1.

VI. Conclusion

Capital investment decisions often involve multiple stages. Such investments are particularly prone to problems of adverse selection, because the firm must initially rely on managers with relevant prior experience or expertise. These informed managers thus stand to collect informational rents. We highlight the role that abandonment options and the delayed consumption of slack have on the optimal design of capital allocation schemes. These features result in an optimal allocation scheme that has several interesting features. First, the budgets allocated at Stage 1 increase in the manager's cost report, unlike the previous literature. Second, while we find a single Stage 1 hurdle rate that exceeds the firm's cost of capital, as in the previous literature, we find that Stage 2 optimally exhibits multiple hurdle rates (some above and some below the firm's cost of capital) that depend on the manager's Stage 1 cost report and the outcome of the Stage 1 research. The multiple hurdle rates in Stage 2 are consistent with findings in Poterba and Summers (1995).

Our model makes a number of assumptions. For example, we assume that the interim information available at Stage 2 is independent of the manager's Stage 1 cost. If instead, the manager's cost information is correlated with the Stage 2 continuation cost, then the principal could further reduce the manager's informational rents by exploiting this statistical relation. However, to the extent that the Stage 2 information does not *perfectly* reveal the manager's cost, then the same tensions examined in this study would continue to hold.

Our formulation also assumes that the manager only derives utility from excessive investment in assets if the project is implemented in Stage 2 and that the salvage value of the Stage 1 investment is zero if the project is abandoned. While these assumptions are relatively strong, they are not critical to our results. All of our results continue to hold provided that (1) at least part of the manager's slack consumption depends on the project implementation and that (2) at least part of the remaining slack cannot be recouped by the principal if the project is abandoned. In other words, a necessary condition for our findings is that there be a deadweight loss associated with the Stage 1 funding (unconsumed excess assets), and that the deadweight loss depend on the manager's reported cost. Thus in addition to the traditional

tradeoff between production efficiencies and informational rents, our principal is also concerned with the deadweight loss attributed to the cancelation of previously funded projects.[15]

Another assumption is that while the manager privately observes the Stage 1 cost, the Stage 2 information is publicly observed. As long as the manager cannot leave the firm after having observed the Stage 2 information (as in Pfeiffer and Schneider 2007), he would not earn any rents on that private information and our results would continue to hold. If instead the manager could leave, the principal would design the contract to limit the additional informational rents, thereby introducing additional distortions. Alternatively, if the principal privately observes the continuation value, her ability to commit to a multi-stage funding rule would be limited, possibly leading to the use of implicit contracts.

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[15] Trading off productive efficiency and deadweight loss, although not informational rent, arises in the implicit contracting literature (e.g., MacLeod 2003 and Rajan, and Reichelstein 2009).

VII. Appendix

Proofs

Proposition 1:

(i Stage 1 hurdle) Note that the manager's utility does not depend on $p_{i,j}$ *per-se*, but only on the total

implementation probability $p_i = \sum_{j=1}^{k} \rho_j p_{i,j}$, therefore in evaluating the manager's expected utility, it suf-

fices to consider the total implementation probabilities, p_i, instead of the individual probabilities, $p_{i,j}$.

Thus, we can write the expected utility of a manager who observes and reports c_i as $z_i p_i (b_i - c_i)$. We

first show that if $z_i p_i > 0$ then $b_{i-1} \leq b_i$, which will allow us to establish that $z_i \in \{0,1\}$ and that the op-

timal contract employs a hurdle rule for Stage 1 funding decisions.

Assuming $z_i p_i > 0$, adding the (TT^C) constraints for i and $i-1$, yields: $z_{i-1} p_{i-1} (c_i - c_{i-1}) \geq z_i p_i (c_i - c_{i-1})$ or

$z_i p_i \leq z_{i-1} p_{i-1}$. To satisfy (TT^C) for i, we must have: $z_i p_i (b_i - c_i) \geq z_{i-1} p_{i-1} (b_{i-1} - c_i)$,

thus $(b_i - c_i) \geq (b_{i-1} - c_i)$, or equivalently, $b_i \geq b_{i-1}$.

Note that projects are funded in Stage 1 with probability 1 or 0 $(z_i \in \{0,1\})$ because the principal's prob-

lem is linear in z_i.

To complete the proof to Proposition 1 (i) and establish the existence of a hurdle research cost, c_h: sup-

pose a manager reporting $\hat{c}_i$ is funded, then $z_i = 1$, implying $p_i > 0$ (otherwise the allocated budget is

wasted). Since $z_i p_i$ was shown to decrease in i, we have $z_{i-1} p_{i-1} > 0$. Therefore if a manager receives

Stage 1 funding, all managers with lower reported costs also receive funding, which establishes the ex-

istence of a Stage 1 hurdle rule. This together with $0 < z_i p_i \leq z_{i-1} p_{i-1}$ implies that $p_i \leq p_{i-1}$.

(ii Optimal budgets) We first show that expected rents are strictly decreasing in the reported cost, $\hat{c}_i$.

We then prove that the upward (TT^C) constraints must bind in equilibrium, which enables us to derive

the optimal budgets.

From (TT^C) for any i,l such that $p_i, p_l > 0$ and $c_i < c_l$ we have: $p_i (b_i - c_i) \geq p_l (b_l - c_i) > p_l (b_l - c_l)$. That

is, the manager's expected rent is decreasing in his cost observation if (TT^C) holds. If c_h is the greatest

research cost observation for which $p_h > 0$, then it must be that $b_h = c_h$; i.e., when the manager truth-

fully reports the largest allowable research cost, he earns no expected rents. If not, the principal could

increase her expected payoff by decreasing all b_i for $i < h$ by $b_h - c_h > 0$. All (IR^C) constraints continue

to be satisfied since we have already shown that $p_h > 0 \Rightarrow p_i (b_i - c_i) \geq p_h (b_h - c_h) \ \forall i < h$, and the

(TT^C) constraints are unaffected since all budgets are decreased equally.

We next show that only the adjacent upward (TT^C) constraints bind at optimality. First, it is straightforward to establish that satisfying the adjacent upward constraints implies that all other upward constraints are satisfied provided that the budgets are monotonic (which was shown above). In addition, it is also straightforward to show that, given that the p_i's are weakly decreasing, satisfying the upward constraints implies that the downward constraints are also satisfied. Accordingly, we restrict our attention to adjacent upward constraints. If $p_i > 0$, then if the upward adjacent (TT^C) constraint binds for i,

we must have $p_i(b_i - c_i) = p_{i+1}(b_{i+1} - c_i)$ or equivalently, $b_i = \dfrac{p_{i+1}}{p_i}\left(b_{i+1} - c_i\left(1 - \dfrac{p_i}{p_{i+1}}\right)\right)$. Solving iteratively,

we obtain: $b_i = c_i + \delta \displaystyle\sum_{q=i+1}^{h} \dfrac{p_q}{p_i}$ as stated in part (ii).

To see that the adjacent upward (TT^C) constraints must bind, suppose that the (TT^C) constraint for a manager truthfully reporting c_i does not bind; i.e., $p_i\left(b_i - c_i\right) - p_{i+1}\left(b_{i+1} - c_i\right) = \varepsilon > 0$. Consider a change wherein we reduce the budget for a manager with cost c_i by $\gamma = \dfrac{\varepsilon}{p_i}$ so that his adjacent upward (TT^C) constraint now binds. The change relaxes the adjacent upward (TT^C) constraint of a manager with c_{i-1} but leaves unchanged the adjacent upward (TT^C) constraints of all other managers. Since all the upward adjacent (TT^C) constraints continue to hold it is again straightforward to show that the downward constraints continue to be satisfied. Further, the (IR^C) constraint for manager $i+1$ assures that his utility is at least 0, therefore $p_i\left(b_i - c_i\right) - \varepsilon = p_{i+1}\left(b_{i+1} - c_i\right) > p_{i+1}\left(b_{i+1} - c_{i+1}\right) \geq 0$, hence the new contract continues to satisfy manager i's (IR^C) constraint. Finally, we previously showed in the proof of part (i) that b_i is weakly increasing in the manager's Stage 1 cost report.

(iii-v)

We show that the desired properties of $p_{i,j}$ follow from the first order conditions (FOC).[16] Using the optimal budgets from part (ii), we can express the principal's problem as $\dfrac{1}{n}\displaystyle\sum_{i=1}^{h}\sum_{j=1}^{k}\left(\rho_j p_{i,j} g_j - c_i - \sum_{s=i+1}^{h}\dfrac{p_s}{p_i}\delta\right)$.

Differentiating with respect to $p_{i,j}$ yields:

$\dfrac{\partial U^h}{\partial p_{i,j}} = \dfrac{\rho_j}{n}\left[g_j + \displaystyle\sum_{s=i+1}^{h}\dfrac{p_s}{p_i^2}\delta - \sum_{t=1}^{i-1}\dfrac{1}{p_t}\delta\right]$. The principal will optimally set $p_{i,j}$ such that $\dfrac{\partial U^h}{\partial p_{i,j}} = 0$. If equality

cannot be met with $p_{i,j} \in [0,1]$, then the principal sets $p_{i,j}^* = 1$ if $\dfrac{1}{\delta}g_j > \displaystyle\sum_{t=1}^{i-1}\dfrac{1}{p_t} - \sum_{s=i+1}^{h}\dfrac{p_s}{p_i^2}$, and $p_{i,j}^* = 0$ if

$\dfrac{1}{\delta}g_j < \displaystyle\sum_{t=1}^{i-1}\dfrac{1}{p_t} - \sum_{s=i+1}^{h}\dfrac{p_s}{p_i^2}$. Since $\dfrac{g_j}{\delta}$ is decreasing in j, if $p_{i,j}^* = 1$, then $p_{i,l}^* = 1$ for $l < j$, that is, the opti-

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[16] A proof that the first-order approach yields a maximum is available from the authors upon request.

mal Stage 2 implementation probabilities are weakly decreasing (increasing) in the Stage 2 index j (continuation value g_j), which establishes **(iii)** and the second part of **(v.a)**. From above, we know that $p_i \leq p_{i-1}$ which proves **(iv)** and **(v.b)** and, together with the Stage 2 hurdle rule, implies that $p_{i,j}$ is weakly decreasing in i, which proves the first part of **(v.a)**.

Proposition 2:

To facilitate what follows, we exploit the result that the $p_{i,j}$'s are decreasing in j. Therefore, if for any given funded c_i, $p_i \leq \rho_1$, the principal only implements g_1. Likewise, if for any given funded c_i, $\rho_1 < p_i \leq \rho_1 + \rho_2$, the principal only implements g_1 and g_2. Therefore, we can express the principal's revenues associated with implementing a Stage 2 project with probability p_i as

$$\gamma(p_i) = \begin{cases} p_i g_1 & \text{for } p_i \leq \rho_1 \\ \rho_1 g_1 + (p_i - \rho_1) g_2 & \text{for } \rho_1 < p_i \leq \rho_1 + \rho_2 \\ \text{M} & \text{M} \\ \displaystyle\sum_{j=1}^{k-1} \rho_j g_j + \left(p_i - \sum_{j=1}^{k-1} \rho_j \right) g_h & \text{for } \displaystyle\sum_{j=1}^{k-1} \rho_j < p_i \leq \sum_{j=1}^{k} \rho_j \end{cases}.$$

Define the principal's utility as a function of the implementation probability vector $P = (p_1, \text{K}, p_h) \in [0,1]^h$ and budget vector $B = (b_1, \text{K}, b_h)$ as $U^h(P,B) = \dfrac{1}{n} \displaystyle\sum_{i=1}^{h} (\gamma(p_i) - c_i - b_i)$ or as $U^h(P) = U^h(P, B^*(h))$ where $B^*(h)$ denotes the optimal budgets obtained in Proposition 1, with

$$b_i^* = c_i + \delta \sum_{q=i+1}^{h} \frac{p_q^*}{p_i^*}.$$

We label the least profitable (in terms of the continuation value) Stage 2 project that the principal implements with positive probability after funding a manager with reported Stage 1 cost c_i, as g_{w_i}; i.e., $p_{i,j} > 0 \Leftrightarrow j \leq w_i$.

Part (i)

We first show that over-investment relative to First-Best in Stage 1 is never optimal.

Define h^{FB} as the optimal Stage 1 hurdle in the First-Best model that funds all managers with cost less than or equal to $c_{h^{FB}}$. Fixing $h = h^{FB} + 1$ in the Commitment model and optimizing over P yields the solution: $P^*(h^{FB} + 1) = \{p_1^*(h^{FB} + 1), p_2^*(h^{FB} + 1), ..., p_{h^{FB}+1}^*(h^{FB} + 1)\}$. The principal's expected utility is thus:

$$U^{h^{FB}+1}\left(P^*\left(h^{FB}+1\right)\right)=\frac{1}{n}\sum_{i=1}^{h^{FB}+1}\left(\gamma\left(p_i^*\left(h^{FB}+1\right)\right)-c_i-\frac{\delta}{p_i^*\left(h^{FB}+1\right)}\sum_{s=i+1}^{h^{FB}+1}p_s^*\left(h^{FB}+1\right)\right).$$

Now, consider an alternative allocation policy where the principal does not fund the manager with re-ported Stage 1 cost $c_{h^{FB}+1}$, but retains the all other aspects of the $P^*\left(h^{FB}+1\right)$ policy. For this alternative policy the principal's expected utility is given by (note the h^{FB} superscript on U)

$$U^{h^{FB}}\left(P^*\left(h^{FB}+1\right)\right)=\frac{1}{n}\sum_{i=1}^{h^{FB}}\left(\gamma\left(p_i^*\left(h^{FB}+1\right)\right)-c_i-\frac{\delta}{p_i^*\left(h^{FB}+1\right)}\sum_{s=i+1}^{h^{FB}}p_s^*\left(h^{FB}+1\right)\right).$$

The marginal benefit attributed to the decreased number of funded projects in the Commitment model is given by:[17]

$$
\begin{aligned}
U^{h^{FB}}-U^{h^{FB}+1} &=\frac{1}{n}\sum_{i=1}^{h^{FB}}\left(\gamma\left(p_i^*\right)-c_i-\frac{\delta}{p_i^*}\sum_{s=i+1}^{h^{FB}}p_s^*\right)-\frac{1}{n}\sum_{i=1}^{h^{FB}+1}\left(\gamma\left(p_i^*\right)-c_i-\frac{\delta}{p_i^*}\sum_{s=i+1}^{h^{FB}+1}p_s^*\right) \\
&=-\frac{1}{n}\left(\gamma\left(p_{h^{FB}+1}^*\right)-c_{h^{FB}+1}-\sum_{s=i+1}^{h^{FB}}\frac{\delta p_{h^{FB}+1}^*}{p_i^*}\right)
\end{aligned}
$$

Since the optimal First-Best hurdle (Stage 1) cost c_{FB} is defined as the largest c_i such that $\left(\max_p\gamma(p)\right)-c_i\geq 0$, it follows that $\gamma\left(p_{h^{FB}+1}^*\right)-c_{h^{FB}+1}<0$, hence the differ-ence $U^{h^{FB}}\left(P^*\left(h^{FB}+1\right)\right)-U^{h^{FB}+1}\left(P^*\left(h^{FB}+1\right)\right)$ is positive and the principal never over-invests in Stage 1 relative to First-Best.

Next we show the possibility of under-investment.

Assume that under First-Best h^{FB} managers receive Stage 1 funding. Now consider the principal's solu-tion to the Commitment model assuming that the same h^{FB} managers receive funding in Stage 1: $P^*\left(h^{FB}\right)=\left(p_1^*,p_2^*,\text{K},p_{h^{FB}}^*\right)\in[0,1]^{h^{FB}}$. Let $P^{FB}(h^{FB})=\left(p_1^{FB},\text{K},p_{h^{FB}}^{FB}\right)\in[0,1]^{h^{FB}}$ denote the First-Best solu-tion and $U_{FB}^{h^{FB}}\left(P^{FB}(h^{FB})\right)$ the principal's expected utility under First-Best employing h^{FB} and $P^{FB}(h^{FB})$. By definition, $U_{FB}^{h^{FB}-1}\left(P^{FB}(h^{FB})\right)-U_{FB}^{h^{FB}}\left(P^{FB}(h^{FB})\right)<0$. Comparing the FOC for the First-Best and Com-mitment models shows that $p_{h^{FB}}^*\left(h^{FB}\right)\leq p_{h^{FB}}^{FB}$, hence $\gamma\left(p_{h^{FB}}^*\left(h^{FB}\right)\right)-c_{h^{FB}}\leq\gamma\left(p_{h^{FB}}^{FB}\right)-c_{h^{FB}}$, implying: $U_{FB}^{h^{FB}-1}\left(P^{FB}(FB)\right)-U_{FB}^{h^{FB}}\left(P^{FB}\left(h^{FB}\right)\right)\leq U^{h^{FB}-1}\left(P^*(h^{FB})\right)-U^{h^{FB}}\left(P^*(h^{FB})\right)$. Thus, the cost of under-

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[17] We omit the $\left(h^{FB}+1\right)$ notation below as all implementation probabilities below are from $P^*\left(h^{FB}+1\right)$.

investment at Stage 1 is always greater in the First-Best setting relative to that in the Commitment setting, which implies that the principal weakly under-invests at Stage 1 in the model with Commitment.

Part (ii)

We next prove that Stage 1 under-investment is mitigated with the Commitment model relative to the No-Commitment model. In the Commitment case, the principal solves: $\frac{1}{n}\max_{P,B} U^h(P,B)$ subject to (IR^C) and (TT^C). Let $\{P^*(h),B^*(h)\}$ denote the principal's solution to the Commitment model with h managers receiving Stage 1 funding. Similarly, the No-Commitment (NC) solution can be specified as $\{P^{NC}(h),B^{NC}(h)\}$ with $B^{NC}(h)=(c_h,\text{K},c_h)$. The NC model assumes that the principal cannot commit to a Stage 2 implementation rule and therefore acts sequentially rationally; consequently, $p_i^{NC}(h)=p^{NC}(h)$ for all $i\leq h$. The total expected profit in the NC model given a Stage 1 cost hurdle, c_h, is therefore given by $U^h\left(P^{NC}(h),B^{NC}(h)\right)=\frac{h}{n}\left(\gamma\left(p^{NC}(h)\right)-c_h\right)$. To establish the result, we will show that the incremental profit of funding a manager with cost c_{h+1} in the Commitment model is weakly greater than the incremental profit obtained in the NC setting. That is, we will show that:

$$U^{h+1}\left(P^*(h+1),B^*(h+1)\right)-U^h\left(P^*(h),B^*(h)\right)\geq U^{h+1}\left(P^{NC}(h+1),B^{NC}(h+1)\right)-U^h\left(P^{NC}(h),B^{NC}(h)\right).$$

The proof is by construction. To do so, we begin with the solution $\left(P^*(h),B^*(h)\right)$, and use it to generate a feasible $\left(P(h+1),B(h+1)\right)$ contract to the $\max_{P(h+1),B(h+1)} U^{h+1}\left(P(h+1),B(h+1)\right)$ problem, such that

$$U^{h+1}\left(P(h+1),B(h+1)\right)-U^h\left(P^*(h),B^*(h)\right)\geq U^{h+1}\left(P^{NC}(h+1),B^{NC}(h+1)\right)-U^h\left(P^{NC}(h),B^{NC}(h)\right).$$ The approach is sufficient, because $U^{h+1}(P^*(h+1),B^*(h+1))\geq U^{h+1}\left(P(h+1),B(h+1)\right)$ by definition.

First note that adding an additional manager in the NC model yields incremental profit to the principal of

$$U^{h+1}\left(P^{NC}(h+1),B^{NC}(h+1)\right)-U^h\left(P^{NC}(h),B^{NC}(h)\right)=\frac{1}{n}\left(\gamma\left(p^{NC}\right)-c_{h+1}-h\delta\right),$$

as all managers with cost $c_i<c_{h+1}$ now earn an additional rent of δ dollars.

Let s denote the smallest index, such that $p^{NC}\geq p_s^*$ (the existence of s is guaranteed by Proposition 4). Define the proposed probability vector $P(h+1)$ as $P(h+1)=\left(p_1^*,\text{K},p_{s-1}^*,p^{NC},p_s^*,\text{K},p_h^*\right)$ and the proposed budget vector $B(h+1)$ as $B(h+1)=\left(c_1+\delta\sum_{q=2}^{h+1}\frac{p_q}{p_1},\text{K},c_h+\delta\frac{p_{h+1}}{p_h},c_{h+1}\right)$. The proposed probability vector for $h+1$ managers begins with the original solution for h managers, and changes the implementation probability for all managers with cost $c_i\geq c_s$. For $c_i=c_s$, the proposed implementation probability is p^{NC}

($p_s = p^{NC}$) and for $c_i > c_s$, the proposed implementation probability is the same probability as for manager c_{i-1} in the original solution ($p_i = p_{i-1}^*$ for $i > s$). Therefore one interpretation for the proposed probability vector is that the additional manager was "inserted" between managers c_{s-1} and c_s in the original solution vector. For example, if $p^{NC} = 0.7$, $h = 4$ and $P^*(h) = \{.9,.8,.5,.4\}$, then $s = 3$, in which case the proposed probability vector is given by $P(h+1) = \{.9,.8,.7,.5,.4\}$. Alternatively, if $p^{NC} = .95$, then $s = 1$, in which case the proposed probability vector is given by $P(h+1) = \{.95,.9,.8,.5,.4\}$. Note that our proposed probability vector increases the overall likelihood of project implementation by $\frac{1}{n}p^{NC}$ over the prior solution, $P^*(h)$, which is the same as in the NC setting. Because all Stage 1 cost observations are equally likely, the expected incremental revenue only varies with the overall likelihood of project implementation; therefore the incremental revenue associated with $P(h+1)$ is identical to that incurred in the NC setting: $\frac{1}{n}\gamma(p^{NC})$.[18] The proposed budget vector, $B(h+1)$, was chosen according to the optimal budget allocation outlined in Proposition 1, therefore $\{P(h+1), B(h+1)\}$ satisfies the (TT^C) and (IR^C) constraints for all managers.

To complete the proof, we demonstrate that the incremental cost of funding an additional manager in the NC setting, $\frac{1}{n}(c_{h+1} + \delta h)$, provides an upper-bound for the incremental cost incurred in our proposed solution to the Commitment problem. By replacing the h manager optimal contract $\{P^*(h), B^*(h)\}$ with the proposed $h+1$ manager contract in the Commitment setting, $\{P(h+1), B(h+1)\}$, the principal incurs an incremental cost of:

$$\frac{1}{n}\sum_{i=1}^{h+1} b_i - \frac{1}{n}\sum_{i=1}^{h} b_i^* = \frac{1}{n}\sum_{i=1}^{s-1}(b_i - b_i^*) + \frac{1}{n}\sum_{i=s}^{h}(b_{i+1} - b_i^*) + \frac{b_s}{n}. \tag{A.1}$$

As the budgets always provide a manager with cost c_i with his true cost c_i plus slack, the above difference is given by the difference in slack for the pre-existing h managers, plus the cost of the newly funded manager, $\frac{1}{n}c_{h+1}$. Therefore we can rewrite the incremental cost, (A.1), as:

$$\frac{1}{n}c_{h+1} + \frac{\delta}{n}\sum_{i=1}^{s-1}\left(\sum_{q=i+1}^{h+1}\frac{p_q}{p_i} - \sum_{q=i+1}^{h}\frac{p_q^*}{p_i^*}\right) + \frac{\delta}{n}\sum_{i=s}^{h}\left(\sum_{q=i+2}^{h+1}\frac{p_q}{p_i} - \sum_{q=i+1}^{h}\frac{p_q^*}{p_i^*}\right) + \frac{1}{n}\delta\sum_{q=s+1}^{h+1}\frac{p_q}{p^{NC}}. \tag{A.2}$$

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[18] Our assumption of discrete, rather than continuous, costs simplifies the search for, s, the smallest index, such that $p^{NC} \geq p_s^*$. Further, our assumption of uniformly distributed costs simplifies the construction of an $h+1$ probability vector, $P(h+1)$, which attains the same incremental revenues as the NC model.

The fact that the proposed probability vector, $P(h+1)$, uses the same implementation probabilities as $P^*(h)$, (p_i^*), for managers with cost $c_i < c_s$, uses $p_s = p^{NC}$, and $p_i = p_{i-1}^*$ for managers with cost $c_i > c_s$ allows us to express the incremental cost, (A.2), as:

$$\frac{1}{n}c_{h+1} + \frac{\delta}{n}\sum_{i=1}^{s-1}\left(\sum_{q=i+1}^{s-1}\frac{p_q^*}{p_i^*} + \frac{p^{NC}}{p_i} + \sum_{q=s}^{h}\frac{p_q^*}{p_i^*} - \sum_{q=i+1}^{h}\frac{p_q^*}{p_i^*}\right) + \frac{\delta}{n}\sum_{i=s}^{h}\left(\sum_{q=i+1}^{h}\frac{p_q^*}{p_i^*} - \sum_{q=i+1}^{h}\frac{p_q^*}{p_i^*}\right) + \frac{\delta}{n}\sum_{q=s+1}^{h+1}\frac{p_q}{p^{NC}}$$

$$= \frac{1}{n}c_{h+1} + \frac{\delta}{n}\sum_{i=1}^{s-1}\frac{p^{NC}}{p_i} + \frac{\delta}{n}\sum_{q=s+1}^{h+1}\frac{p_q}{p^{NC}}.$$

By construction, $p_i > p^{NC}$ for $i < s$, and $p_i < p^{NC}$ for $i > s$, therefore we have derived an upper bound for the incremental cost associated with $\{P(h+1), B(h+1)\}$ in the Commitment model, namely $\frac{1}{n}c_{h+1} + \delta\frac{h}{n}$, which is equal to the incremental cost in the NC model.

Proposition 3:

(i weak over-investment for 1) To prove part (i), note that if $p_1 > 0$, then

$$\frac{\partial U^h(P^*)}{\partial p_1} = \frac{1}{n}\left(\gamma'(p_1^*) + \frac{\delta}{(p_1^*)^2}\sum_{t=2}^{h}p_t^*\right) \geq 0,$$ which implies $\gamma'(p_1^*)$ can be negative, even though at the

efficient investment level we would have $\gamma'(p_1^{FB}) \geq 0$. Because $\gamma(\cdot)$ is concave, and $\frac{\partial^2 U^h(P^*)}{\partial p_i \partial p_j} > 0$ for

$i \neq j$, the principal weakly over-invests in the Commitment setting relative to the First-Best setting. If $g_k \geq 0$, over-investment is impossible therefore this is a mute point. Since the probabilities, p_i^*, are ordered, if over-investment takes place for the manager with the lowest reported cost, then the principal may also over-invest for managers with greater reported costs.

To prove **(ii under-invest for h)**; note that if $p_h > 0$, then the FOC with respect to p_h is:

$$\frac{\partial U^h(P^*)}{\partial p_h} = \frac{1}{n}\left(\gamma'(p_h^*) - \delta\sum_{t=1}^{h-1}\frac{1}{p_t^*}\right) \geq 0,$$ which implies $\gamma'(p_h^*) \geq \delta\sum_{t=1}^{h-1}\frac{1}{p_t^*} > 0$; i.e., weak under-

investment relative to First-Best when the manager reports sufficiently large cost realizations.

To prove part **(iii on average under)**: the FOC with respect to p_i for $p_i > 0$

is: $\frac{\partial U^h(P^*)}{\partial p_i} = \frac{1}{n}\left(\gamma'(p_i^*) - \delta\sum_{s=1}^{i-1}\frac{1}{p_s^*} + \delta\sum_{t=i+1}^{h}\frac{p_t^*}{(p_i^*)^2}\right) \geq 0.$ Summing over all FOC, substituting $\gamma'(p_i^*) = g_{w_i}$

and multiplying by n, yields: $\sum_{i=1}^{h}g_{w_i} - \delta\sum_{i=1}^{h}\left(\frac{(h-i)}{p_i^*} - \sum_{t=i+1}^{h}\frac{p_t^*}{(p_i^*)^2}\right) \geq 0.$ Since p_i^* is weakly decreasing in i,

the term in brackets is non-negative, therefore $\sum_{i=1}^{h} g_{w_i} \geq 0$, and hence on average, there is weak under-

investment at Stage 2; i.e. $\sum_{i=1}^{h} \dfrac{g_{w_i}}{h} \geq 0$.

Proposition 4:

It suffices to show that the principal's objective function, $U^h(P)$, is super-modular and exhibits increas-

ing-differences for the result to hold. A sufficient condition is for all cross-partials, $\dfrac{\partial^2 U^h(P)}{\partial p_i \partial p_j}$ with

$i \neq j$ and $\dfrac{\partial^2 U^h(P)}{\partial p_i \partial g_j}$ to carry the same sign (Sundaram 2009). Because $\partial^2 U^h \big/ \partial p_i \partial p_j = \delta \big/ p_j^2$ when $j < i$

and $\partial^2 U^h \big/ \partial p_i \partial p_j = \delta \big/ p_i^2$ when $j > i$, we conclude that U^h is super-modular over $X^h \subseteq \mathrm{R}^h$. Further,

because $\partial^2 U^h \big/ \partial p_i \partial g_j = 1 > 0$, the function U^h is super-modular over the sub-

lattice $\left\{ (p_1, p_2, \mathrm{K}, p_h) : p_{i+1} < p_i, 0 < p_i \leq 1 \right\} \times \left\{ (g_1, g_2, \mathrm{K}, g_k) : g_i < g_{i+1} \right\}$, as was to be shown.

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