Online Appendix: Short-Sales Constraints and the Diversification Puzzle

Appendix A: Regressions using Simulated Data

In this Appendix, we connect the theoretical predictions in Proposition 1 and Proposition 2 to the empirical tests we perform. While our hypotheses result directly from the comparative statics in the theory, it may not be clear to the reader how our empirical strategy is connected to them. In Table A.1, we use simulated prices to emulate the analyses in Table 4 and Table 5. These prices are created by assuming a uniform distribution for the parameters that measure differences of opinion (α) and lendable supply of shares (λ), assuming that there is one pure play and one conglomerate for each pair of parameters, and performing analyses on portfolio sorts and regressions mimicking our empirical work. Overall, the qualitative results are quite similar between the simulated and the real-world data.

In the model, $\lambda \in [0, 1]$ and $\alpha \geq 0$. In order to generate simulated data, we assume a bivariate uniform distribution of $\{\alpha, \lambda\}$ such that $\lambda = \{0, 0.1, 0.2, ..., 1\}$ and $\alpha = \{0, 0.1, 0.2, ..., 2\}$. This generates $11 \times 21 = 231$ pairs of $\{\alpha, \lambda\}$. For each pair of parameters, we calculate prices using the formulae of Proposition 1 and Proposition 2, for a conglomerate and a pure play. This provides 462 observations, each associated with a price, a value for α (i.e., proxy for differences of opinions), a value for λ (i.e., proxy for the availability of shares for shorting), and a conglomerate indicator variable.

In Panel A of Table A.1, we sort these data into terciles of $\lambda \leq 0.3$, $0.3 < \lambda < 0.7$, and $0.7 \leq \lambda$, and terciles of $\alpha \leq 0.7$, $0.7 < \alpha < 1.4$, and $1.4 \leq \alpha$. In two-by-two sorts, we consider observations with low λ and low α, low λ and high α, high λ and low α, and high λ and high α. This analysis is similar to the standard procedure of examining differences in outcomes across groupings sorted by a particular variable. In each of the four categories, we calculate the average conglomerate discount observed in the simulated data. Panel A replicates the analysis of Table 4, with the results being qualitatively similar to the ones found with Compustat data. On average, conglomerates trade at a discount relative to pure-play firms, with the discount increasing in the level of short-sales constraints and differences of opinion.

In Panel B of Table A.1, we use the 462 simulated observations to estimate five regressions that are similar to those in Table 5. In each case, price is the dependent variable. In column (1), we regress price on a conglomerate indicator variable equal to one for the conglomerate;

zero otherwise. By construction, the coefficient on the indicator variables is expected to be negative: conglomerates are cheaper. In column (2), we add our parameter for differences of opinion, α. As our model shows, higher dispersion raises prices for all firms. In column (3), we add our parameter λ, with a higher λ being associated with weaker constraints on shorting. As the theory predicts, weaker short-sales constraints correlate with lower prices. In column (4), we include both α and λ in the same regression and find the same results. Finally, in column (5), we include the interaction between $\alpha \times \lambda$. As expected, the interaction is positive: differences of opinion and short-sales constraints are complementary in raising prices.

The fact that the signs on α, λ, and the interaction term are consistent with the theory should be of no surprise: the underlying simulated prices come straight from using the model's pricing equations. The purpose of running these regressions with simulated prices is to see how the coefficient on the conglomerate indicator variable changes as controls are added. It is not clear from the theory what will occur. As Table A.1 shows, this coefficient does not change as controls are added. This is consistent with our empirical findings, being an additional piece of evidence that the data are consistent with the theory.

Note that the regressions in Table 5 exhibit one major difference from those using simulated prices in Table A.1. The explanatory variables that proxy for differences of opinion and short-sales constraints in Table 5 are computed from the benchmark of pure-play firms, rather than for the firm itself. In Table A.1 we directly use the proxies for the simulated firms we constructed, either a conglomerate or a pure-play firm. Therefore, we expect the coefficients on α, λ, and the interaction term to switch signs relative to the ones in Table A.1.

Table A.1: Simulated Data: Short-Sales Constraints, Differences of Opinion, and Pricing
This table uses a simulated set of firms that are distributed bivariate uniformly over the parameter space $\{\alpha, \lambda\}$ such that $\lambda = 0, 0.1, 0.2, ..., 1$ and $\alpha = 0, 0.1, 0.2, ..., 2$. This generates $11 \times 21 = 231$ pairs of $\{\alpha, \lambda\}$. For each parameter pair, we assume a single conglomerate and a single pure play, and we calculate their prices using the formulae of Proposition 1 and Proposition 2, respectively. This provides 462 observations, each associated with a price, a value for α (which is dispersion of beliefs), a value for λ (which is the availability of shares for shorting), and a conglomerate dummy. Panel A displays the conglomerate discount sorted on the lendable supply of shares (λ) and differences of opinion (α) using the following procedure: we sort the simulated data into terciles of $\lambda \leq 0.3$, $0.3 < \lambda < 0.7$, and $0.7 \leq \lambda$, and terciles of $\alpha \leq 0.7$, $0.7 < \alpha < 1.4$, and $1.4 \leq \alpha$. Then, we compute the conglomerate discount based on the equation in Proposition (2) for each of the highest and lowest combinations of each variable. *Joint (High – Low)* computes the difference between two distinct groups of firms: (i) *High(λ & α)* are firms in the top tercile of λ and then, within this tercile, those in the top tercile of α; (ii) *Low(λ & α)* are firms in the lowest tercile of λ and bottom tercile of α. Panel B estimates regressions of prices as a function of: (i) *D(Conglomerate)*, an indicator variable equal to one if the firm is a conglomerate; (ii) the lendable supply of shares (λ) of the firm; and (iii) the differences of opinion (α). Standard errors are reported in brackets.

Panel A: Simulated Conglomerate Discount

	α	
λ	Low	High
Low	5.36%	64.46%
High	0.63%	16.95%
Joint(High – Low):	63.83%***	

Panel B: Simulated Regressions

Variables	(1)	(2)	(3)	(4)	(5)
D(Conglomerate)	-0.227***	-0.227***	-0.227***	-0.227***	-0.227***
	[0.021]	[0.019]	[0.019]	[0.017]	[0.015]
α		0.165***		0.165***	0.366***
		[0.015]		[0.014]	[0.023]
λ			-0.276***	-0.276***	0.126***
			[0.030]	[0.026]	[0.046]
$\alpha * \lambda$					-0.402***
					[0.039]
Intercept	0.727***	0.562***	0.865***	0.700***	0.499***
	[0.015]	[0.020]	[0.020]	[0.022]	[0.028]

*** p-value<0.01, ** p-value<0.05, * p-value<0.10

Appendix B: Proofs and Model Extensions

This section contains three additions to the model that do not warrant inclusion in the body of the manuscript but are worth developing nonetheless. Appendix B.1 develops a micro-foundation for the linear demand curves associated with each investor type. Appendix B.2 allows for disagreement among investors not only about demand for each stock, but also about demand for the overall market. Because the base model is a special case of this extension, proofs of the relevant propositions in Appendix B.2 are also proofs for propositions in the main text. Appendix B.3 allows for disagreement over synergies. This section shows that, when such disagreement exceeds disagreement over pure-play shares, the conglomerate discount can become a premium.

B.1 A Micro-foundation of the Base Model

We present here an example of a micro-founded model whose implications are consistent with the reduced-form model developed in the body of the manuscript. Suppose that stock $j \in \{A, B\}$ has a dividend that investor $i \in \{1, 2\}$ dogmatically believes to have distribution $D_i^j = x_i^j + \varepsilon^j$. Both agree that $\varepsilon^j \sim N(0, \sigma^2)$ and that the errors are uncorrelated across stocks A and B. Investor 1 has beliefs $x_1^A = x + \alpha$ and $x_1^B = x - \alpha$. Similarly, investor 2 has beliefs $x_2^A = x - \alpha$ and $x_2^B = x + \alpha$. If investors have mean-variance preferences for each asset over the size of the dividend net of the purchase price, with risk aversion parameter γ. Investors of type 1 demand shares of stock A according to

$$\begin{aligned} X_1^A &= \arg\max_X E\left[X\left(D_1^A - p^A\right)\right] - \frac{\gamma}{2} var\left(X D_1^A\right) \\ &= \arg\max_X X\left(x + \alpha - p^A\right) - \frac{\gamma}{2} X^2 \sigma^2, \end{aligned}$$

so the type 1 demand for stock A is

$$X_1^A = \frac{x + \alpha - p^A}{\gamma \sigma^2}.$$

Assuming that $\gamma = 1/\sigma^2$ and $x = 1$, demand from type 1 investors is

$$\begin{aligned} D_1^A &= 1 + \alpha - p^A \\ D_1^B &= 1 - \alpha - p^B, \end{aligned}$$

while demand of investor type 2 is the reverse

$$\begin{aligned} D_2^A &= 1 - \alpha - p^A \\ D_2^B &= 1 + \alpha - p^B. \end{aligned}$$

Note that we could also solve the problem in which investors have mean-variance preferences over their total dividend from both shares.

B.2 Market-level and Stock-level Disagreement

In this section, we relax our restrictions on investor demand functions so that there is disagreement about both the quality of each stock and the appeal of shares generally. This will allow one type of investor to prefer to short both pure-play stocks and the merged firm, if she is pessimistic enough. The qualitative results established in the previous section continue to hold, so these results are not conditional upon the conglomerate firm's shares being easy to borrow. This is comforting, because some investors always short conglomerate shares in practice. We also find additional results connecting overall market pessimism and optimism to the diversification discount.

Let demand of investor type 1 be

$$\begin{aligned} D_1^A &= 2 - \kappa + \alpha - p^A \\ D_1^B &= 2 - \kappa - \alpha - p^B, \end{aligned}$$

while the demand of investor type 2 is

$$\begin{aligned} D_2^A &= \kappa - \alpha - p^A \\ D_2^B &= \kappa + \alpha - p^B. \end{aligned}$$

The level of disagreement about which stock is better continues to be parameterized by α, but the share of aggregate demand is allowed to vary between the two investors. We call α the *stock-level disagreement*, and note that stock-level disagreement increases in α. We call κ the *market-level disagreement*, and note that market-level disagreement increases as κ moves away from unity in either direction.

The lowest level of market-level disagreement is found when $\kappa = 1$ and the two types of investors share identical demand intercepts. As κ moves away from 1 in either direction, there is increasing divergence in the overall level of stock holdings. Just as with α, by including κ in both demand functions, we hold constant the overall level of demand for shares. Adjusting κ simply adjusts the split of aggregate demand between the two types of investor.

Proposition 5. *There are three regions of disagreement that generate differing diversification discounts.*

1. *If there is low stock-level and market-level disagreement, such that* $1/2+\alpha \leq \kappa \leq 3/2-\alpha$, *then there is no diversification discount.*
2. *If there is high market-level disagreement relative to stock-level disagreement, such that* $\kappa \leq 1/2-\alpha$ *or* $\kappa \geq 3/2+\alpha$, *then there is no diversification discount.*
3. *If there is low market-level disagreement relative to stock-level disagreement, such that none of the four preceding inequalities holds, then there is a diversification discount.*

Proof. First, consider stock A:

1. If both investors choose a long position, then the price is given by $D_1^A + D_2^A = N^A$, which implies $2-\kappa+\alpha-p^A+\kappa-\alpha-p^A = 1$, which implies $p^A = 1/2$, as before. At this price, demand by investor 2 is $D_2^A = \kappa-\alpha-p^A = \kappa-\alpha-1/2$, and demand by investor type 1 is $D_1^A = 2-\kappa+\alpha-p^A = 3/2-\kappa+\alpha$. Both are positive if and only if $1/2+\alpha \leq \kappa \leq 3/2+\alpha$.
2. If investor type 2 chooses a short position and investor type 1 chooses a long position, then type 2 can only short λ times her demand, so the price is given by $D_1^A+\lambda D_2^A = N^A$, which implies $2-\kappa+\alpha-p^A+\lambda(\kappa-\alpha-p^A) = 1$, which implies $p^A = \frac{1-\kappa+\alpha+\lambda(\kappa-\alpha)}{1+\lambda}$. At this price, demand by investor 2 is $D_2^A = \kappa-\alpha-p^A = \frac{2x-2\alpha-1}{1+\lambda}$, which is negative if and only if $\kappa < 1/2+\alpha$.
3. If investor type 2 chooses a long position and investor type 1 chooses a short position, then type 1 can only short λ times her demand, so the price is given by $\lambda D_1^A+D_2^A = N^A$, which implies $\lambda\left(2-\kappa+\alpha-p^A\right)+\kappa-\alpha-p^A = 1$ which implies $p^A = \frac{2\lambda-\lambda(\kappa-\alpha)-(1-\kappa+\alpha)}{1+\lambda}$. At this price, demand by investor 1 is $D_1^A = 2-\kappa+\alpha-p^A = \frac{3-2\kappa+2\alpha}{1+\lambda}$, which is negative if and only if $\kappa > 3/2+\alpha$.

Next, consider stock B:

1. If both investors choose a long position, then the price is given by $D_1^B + D_2^B = N^B$, which implies $p^B = 1/2$, as before. At this price, demand by investor 2 is $D_2^B = \kappa + \alpha - p^B = \kappa + \alpha - 1/2$, and demand by investor type 1 is $D_1^B = 2 - \kappa - \alpha - p^B = 3/2 - \kappa - \alpha$. Both are positive if and only if $1/2 - \alpha \leq \kappa \leq 3/2 - \alpha$.

2. If investor type 2 chooses a short position and investor type 1 chooses a long position, then type 2 can only short λ times her demand, so the price is given by $D_1^B + \lambda D_2^B = N^B$, which implies $2 - \kappa - \alpha - p^B + \lambda(\kappa + \alpha - p^B) = 1$, which implies $p^B = \frac{1-\kappa-\alpha+\lambda(\kappa+\alpha)}{1+\lambda}$. At this price, demand by investor 2 is $D_2^B = \kappa + \alpha - p^B = \frac{2\kappa+2\alpha-1}{1+\lambda}$, which is negative if and only if $\kappa < 1/2 - \alpha$.

3. If investor type 2 chooses a long position and investor type 1 chooses a short position, then type 1 can only short λ times her demand, so the price is given by $\lambda D_1^B + D_2^B = N^B$, which implies $\lambda \left(2 - \kappa - \alpha - p^B\right) + \kappa + \alpha - p^B = 1$ which implies $p^B = \frac{2\lambda - \lambda(\kappa+\alpha)-(1-\kappa-\alpha)}{1+\lambda}$. At this price, demand by investor 1 is $D_1^B = 2 - \kappa + \alpha - p^B = \frac{3-2\kappa-2\alpha}{1+\lambda}$, which is negative if and only if $\kappa > 3/2 - \alpha$.

Finally, consider the merged firm. Demand is given by

$$\begin{aligned} D_1^M &= D_1^A + D_1^B = 2 - \kappa + \alpha - p^M + 2 - \kappa - \alpha - p^M = 4 - 2\kappa - 2p^M \\ D_2^M &= D_2^A + D_2^B = \kappa + \alpha - p^M + \kappa - \alpha - p^M = 2\kappa - 2p^M. \end{aligned}$$

1. If both investors choose a long position, then demand is given by $D_1^M + D_2^M = N^M$, which implies $4 - 2\kappa - 2p^M + 2\kappa - 2p^M = 2$, which implies $p^M = 1/2$. This requires $D_1^M, D_2^M \geq 0$, which requires $1/2 \leq \kappa \leq 3/2$.

2. If investor type 1 chooses a long position and type 2 chooses a short position, then the price is given by $D_1^M + \lambda D_2^M = N^M$, which implies $p^M = \frac{1-(1-\lambda)\kappa}{1+\lambda}$. At this price, demand by investor type 2 is $D_2^M = 2\kappa - 2p^M = \frac{4\kappa-2}{1+\lambda}$, which is negative if and only if $\kappa < 1/2$.

3. If investor type 2 chooses a long position and investor type 1 chooses a short position, then type 1 can only short λ times her demand, so the price is given by $\lambda D_1^M + D_2^M = N^M$, which implies $\lambda \left(4 - 2\kappa - 2p^M\right) + 2\kappa - 2p^M = 2$, which implies $p^M = \frac{2\lambda - \lambda\kappa - (1-\kappa)}{1+\lambda}$. At this price, demand by investor 1 is $D_1^M = 4 - 2\kappa - 2p^M = \frac{6-4\kappa}{1+\lambda}$, which is negative if and only if $\kappa > 3/2$.

Recall that the discount is defined to be $\delta = 1 - \frac{p^M}{(p^A+p^B)/2}$. For simplicity, and without loss of generality, focus on the case of $\kappa < 1$: all results will be symmetric for $\kappa > 1$ with the substitution $y = 2 - \kappa$. We need to calculate the diversification discount in six cases.

1. If $\kappa < 1/2$ and:

   (a) $\kappa \leq 1/2 - \alpha$, then investors of type 2 choose to short both stocks A and B, as well as the merged firm, while investors of type 1 choose a long position on all three. Then the diversification discount is $\delta = 1 - \frac{p^M}{(p^A+p^B)/2} = 1 - \frac{\frac{1-(1-\lambda)\kappa}{1+\lambda}}{\left(\frac{1-\kappa+\alpha+\lambda(\kappa-\alpha)}{1+\lambda} + \frac{1-\kappa-\alpha+\lambda(\kappa+\alpha)}{1+\lambda}\right)/2} = 0$.

(b) $1/2-\alpha<\kappa\leq 3/2-\alpha$, then investors of type 2 choose to short stock A and the merged firm, but choose a long position on stock B. Investors of type 1 choose a long position on all three. Then the diversification discount is $\delta=1-\frac{p^M}{(p^A+p^B)/2}=\frac{\alpha+\kappa-1/2}{3/2-\kappa+\alpha+\lambda\kappa-\alpha\lambda+\lambda/2}(1-\lambda)>0$.

(c) $3/2-\alpha<\kappa$, then investors of type 2 choose to short stock A and the merged firm, and investors of type 1 choose to short stock B. Then the diversification discount is $\delta=1-\frac{p^M}{(p^A+p^B)/2}=\frac{\alpha+\kappa-1}{\alpha-\alpha\lambda+\lambda}(1-\lambda)>0$.

2. If $1/2\leq\kappa\leq 1$ and:

(a) $\kappa<1/2+\alpha$, then investors of type 2 choose to short stock A and choose a long position on stock B and the merged firm, while investors of type 1 choose a long position on all three. Then the diversification discount is $\delta=1-\frac{p^M}{(p^A+p^B)/2}=\frac{1/2+\alpha-\kappa}{1-(\kappa-\alpha)(1-\lambda)+1/2+\lambda/2}(1-\lambda)>0$.

(b) $1/2+\alpha\leq\kappa\leq 3/2-\alpha$, then no investors ever prefer to short sell a stock, and the diversification discount is trivially $\delta=1-\frac{p^M}{(p^A+p^B)/2}=1-\frac{1/2}{(1/2+1/2)/2}=0$.

(c) $3/2-\alpha\leq\kappa$, then investors of type 2 choose to short stock A and take a long position on stock B, while investors of type 1 choose to short stock B and take a long position on stock A. No investors choose to short in the merged firm. Then the diversification discount is $\delta=1-\frac{p^M}{(p^A+p^B)/2}=\frac{2\alpha-1}{2\alpha-2\lambda\alpha+2\lambda}(1-\lambda)>0$.

□

Corollary 5. *If parameters are such that the diversification discount exists, then it increases in α and decreases in λ. Short interest is higher for the pure plays than for the merged firm.*

Proof. Calculate the derivative of δ with respect to α and λ in each of the four cases above in which $\delta>0$.

1. If $\kappa<1/2$ and $1/2-\alpha<\kappa\leq 3/2-\alpha$, then $\delta=\frac{(\kappa+\alpha-1/2)(1-\lambda)}{2-(\kappa-\alpha-1/2)(1-\lambda)}$, so:

$$sign\left(\frac{d\delta}{d\alpha}\right)=sign\left(\frac{\begin{array}{c}(1-\lambda)\left(3/2-\kappa+\alpha+\lambda x-\alpha\lambda+\lambda/2\right)\\ -\left(\alpha+\kappa-1/2\right)(1-\lambda)^2\end{array}}{\left(3/2+(\alpha-\kappa)(1-\lambda)+\lambda/2\right)^2}\right).$$

The denominator is positive because it is a square. Thus:

$$\begin{aligned}sign\left(\frac{d\delta}{d\alpha}\right) &= sign\left(\left(3/2-\kappa+\alpha+\lambda\kappa-\alpha\lambda+\lambda/2\right)-\left(\alpha+\kappa-1/2\right)(1-\lambda)\right)\\ &= sign\left(2(1-\kappa)+2\lambda\kappa\right)\\ &> 0.\end{aligned}$$

Therefore, δ is increasing in α in this region. Taking a derivative with respect to λ yields

$$sign\left(\frac{d\delta}{d\lambda}\right) = sign\left(\frac{\begin{array}{c}-(\kappa+\alpha-1/2)\left(2-(\kappa-\alpha-1/2)(1-\lambda)\right)\\ -(\kappa-\alpha-1/2)(\kappa+\alpha-1/2)(1-\lambda)\end{array}}{\left(2-(\kappa-\alpha-1/2)(1-\lambda)\right)^2}\right)$$

The denominator is positive because it is a square. Thus:

$$sign\left(\frac{d\delta}{d\lambda}\right) = sign(1/2-\kappa-\alpha).$$

Because $1/2-\alpha<\kappa$, $1/2-\alpha-\kappa<0$, so $\frac{d\delta}{d\lambda}<0$.

2. If $\kappa<1/2$ and $3/2-\alpha<\kappa$, then $\delta=\frac{\alpha+\kappa-1}{\alpha-\alpha\lambda+\lambda}(1-\lambda)$, so:

$$\begin{aligned} sign\left(\frac{d\delta}{d\alpha}\right) &= sign\left(\frac{(1-\lambda)(\alpha-\alpha\lambda+\lambda)-(\alpha+\kappa-1)(1-\lambda)^2}{(\alpha(1-\lambda)+\lambda)^2}\right)\\ &= sign\left(1-\kappa(1-\lambda)\right)\\ &> 0. \end{aligned}$$

Taking a derivative with respect to λ yields

$$sign\left(\frac{d\delta}{d\lambda}\right) = sign\left(\frac{-(\alpha+\kappa-1)(\alpha(1-\lambda)+\lambda)-(1-\alpha)(\alpha+\kappa-1)(1-\lambda)}{(\alpha(1-\lambda)+\lambda)^2}\right)$$

The denominator is positive because it is a square. Thus:

$$sign\left(\frac{d\delta}{d\lambda}\right) = sign\left(1-\alpha-\kappa\right).$$

Because $3/2-\alpha<\kappa$, $1-\alpha-\kappa<3/2-\alpha-\kappa<0$, so $\frac{d\delta}{d\lambda}<0$.

3. If $1/2\le\kappa\le1$ and $\kappa<1/2+\alpha$, then $\delta=\frac{1/2+\alpha-\kappa}{1-(\kappa-\alpha)(1-\lambda)+1/2+\lambda/2}(1-\lambda)$, so:

$$sign\left(\frac{d\delta}{d\alpha}\right) = sign\left(\frac{\begin{array}{c}(1-\lambda)\left(1-(\kappa-\alpha)(1-\lambda)+1/2+\lambda/2\right)\\ -(1/2+\alpha-\kappa)(1-\lambda)^2\end{array}}{\left(1-(\kappa-\alpha)(1-\lambda)+1/2+\lambda/2\right)^2}\right).$$

The denominator is positive because it is a square. Thus:

$$\begin{aligned} sign\left(\frac{d\delta}{d\alpha}\right) &= sign\left(\begin{array}{c}(1-(\kappa-\alpha)+1/2+\lambda/2)\\ -(1/2+\alpha-\kappa)(1-\lambda)\end{array}\right)\\ &= sign\left(1+\lambda-(\kappa-a)\lambda\right)\\ &> 0, \end{aligned}$$

where this last inequality follows because $\kappa-a<1/2$. Taking a derivative with respect to λ yields

$$sign\left(\frac{d\delta}{d\lambda}\right) = sign\left(\frac{\begin{array}{c}-\left(\alpha-\kappa+1/2\right)\left(2+(\alpha-\kappa-1/2)(1-\lambda)\right)\\ +\left(\alpha-\kappa+1/2\right)(1-\lambda)(\alpha-\kappa-1/2)\end{array}}{\left(2+(\alpha-\kappa-1/2)(1-\lambda)\right)^2}\right).$$

The denominator is positive because it is a square. Thus:

$$sign\left(\frac{d\delta}{d\lambda}\right) = sign\left(-\left(1/2+\alpha-\kappa\right)\right).$$

Because $\kappa < 1/2+\alpha$, $1/2+\alpha-\kappa > 0$ so $\frac{d\delta}{d\lambda} < 0$.

4. If $1/2 \leq \kappa < 1$ and $3/2-\alpha \leq \kappa$, then $\delta = \frac{2\alpha-1}{2\alpha-2\lambda\alpha+2\lambda}(1-\lambda)$, so:

$$\begin{aligned} sign\left(\frac{d\delta}{d\alpha}\right) &= sign\left(\frac{2(1-\lambda)\left(2\alpha-2\lambda\alpha+2\lambda\right)-2(2\alpha-1)(1-\lambda)^2}{\left(2\alpha(1-\lambda)+2\lambda\right)^2}\right) \\ &= sign\left(1+\lambda\right) \\ &> 0. \end{aligned}$$

Taking a derivative with respect to λ yields

$$sign\left(\frac{d\delta}{d\lambda}\right) = sign\left(\frac{-\left(2\alpha-1\right)\left(2\alpha(1-\lambda)+2\lambda\right)-2(1-\alpha)\left(2\alpha-1\right)\left(1-\lambda\right)}{\left(2\alpha(1-\lambda)+2\lambda\right)^2}\right).$$

The denominator is positive because it is a square. Thus:

$$sign\left(\frac{d\delta}{d\lambda}\right) = sign\left(-(2\alpha-1)\right).$$

Because $3/2-\alpha \leq \kappa$, and $\kappa < 1$, it must be the case that $\alpha > 1/2$, so $2\alpha-1 > 0$ and $\frac{d\delta}{d\lambda} < 0$.

$\square$

In the following pages, we derive implicit equations for equilibrium prices in the stock and equity lending markets. These derivations follow similar lines to the proof of Proposition 5. The difference is that there are two prices and two markets. In all cases, $p^i = 1/2 + \gamma p_s^i$ for $i \in \{A, B, M\}$. We must calculate functions for p^A, p^B, and p^M that describe equilibrium in the equity lending market, and then substitute the appropriate equation above to get expressions for p^i and p_s^i for $i \in \{A, B, M\}$. In each case, we state the equation defining p^i solely in terms of parameters. Substituting that value into the equation $p^i = 1/2 + \gamma p_s^i$ yields an equation defining p_s^i solely in terms of parameters. For the sake of brevity, we do not state those equations, but they are easily derived.

First, consider stock A:

1. If $\kappa < 1/2+\alpha$, then investors of type 2 will choose to be short. Then equilibrium in the equity lending market requires $\lambda p_s^A D_1^A = -D_2^A$, which implies $\lambda p_s^A \left(2-\kappa+\alpha-p^A+\gamma p_s^A\right) = -\left(\kappa-\alpha-p^A+\gamma p_s^A\right)$, which can be written

$$\begin{aligned} p^A &= \frac{\lambda\gamma\left(p_s^A\right)^2+(2\lambda-\kappa\lambda+\alpha\lambda+\gamma)p_s^A+(\kappa-\alpha)}{1+\lambda p_s^A} \\ &= \frac{\lambda\gamma\left(\frac{p^A-1/2}{\gamma}\right)^2+(2\lambda-\kappa\lambda+\alpha\lambda+\gamma)\frac{p^A-1/2}{\gamma}+(\kappa-\alpha)}{1+\lambda\frac{p^A-1/2}{\gamma}}. \end{aligned}$$

2. If $1/2+\alpha<\kappa<3/2+\alpha$, then no investors choose to be short and prices are given by $p^A=1/2$, $p_s^A=0$.

3. If $\kappa > 3/2+\alpha$, then investors of type 1 will choose a short position. Then equilibrium in the equity lending market requires $\lambda p_s^A D_2^A = -D_1^A$, which implies $\lambda p_s^A \left(\kappa-\alpha-p^A+\gamma p_s^A\right) = -\left(2-\kappa+\alpha-p^A+\gamma p_s^A\right)$, which can be written

$$\begin{aligned} p^A &= \frac{\lambda\gamma\left(p_s^A\right)^2+\left(\kappa\lambda-\alpha\lambda+\gamma\right)p_s^A+(2-\kappa+\alpha)}{1+\lambda p_s^A} \\ &= \frac{\lambda\gamma\left(\frac{p^A-1/2}{\gamma}\right)^2+\left(\kappa\lambda-\alpha\lambda+\gamma\right)\frac{p^A-1/2}{\gamma}+(2-\kappa+\alpha)}{1+\lambda\frac{p^A-1/2}{\gamma}}. \end{aligned}$$

Next, consider stock B:

1. If $\kappa < 1/2-\alpha$, then investors of type 2 will choose a short position. Then equilibrium in the equity lending market requires $\lambda p_s^A D_1^A = -D_2^A$, which implies $\lambda p_s^B \left(2-\kappa-\alpha-p^A+\gamma p_s^A\right) = -\left(\kappa+\alpha-p^A+\gamma p_s^A\right)$, which can be written

$$\begin{aligned} p^B &= \frac{\lambda\gamma\left(p_s^B\right)^2+\left(2\lambda-\kappa\lambda-\alpha\lambda+\gamma\right)p_s^B+\kappa-\alpha}{1+\lambda p_s^B} \\ &= \frac{\lambda\gamma\left(\frac{p^B-1/2}{\gamma}\right)^2+\left(2\lambda-\kappa\lambda-\alpha\lambda+\gamma\right)\frac{p^B-1/2}{\gamma}+\kappa-\alpha}{1+\lambda\frac{p^B-1/2}{\gamma}}. \end{aligned}$$

2. If $1/2-\alpha \leq \kappa \leq 3/2-\alpha$, then no investors will choose a short position and prices are given by $p^B = 1/2$, $p_s^B = 0$.

3. If $\kappa > 3/2-\alpha$, then investors of type 1 will choose a short position. Then equilibrium in the equity lending market requires $\lambda p_s^A D_2^A = -D_1^A$, which implies $\lambda p_s^A \left(\kappa+\alpha-p^A+\gamma p_s^A\right) = -\left(2-\kappa-\alpha-p^A+\gamma p_s^A\right)$, which can be written

$$
\begin{aligned}
p^B &= \frac{\lambda\gamma\left(p_s^B\right)^2+(\kappa\lambda+\alpha\lambda+\gamma)\,p_s^B+(2-\kappa+\alpha)}{1+\lambda p_s^B} \\
&= \frac{\lambda\gamma\left(\frac{p^B-1/2}{\gamma}\right)^2+(\kappa\lambda+\alpha\lambda+\gamma)\,\frac{p^B-1/2}{\gamma}+(2-\kappa+\alpha)}{1+\lambda\frac{p^B-1/2}{\gamma}}.
\end{aligned}
$$

Finally, consider the merged firm. Demand is given by

$$
\begin{aligned}
D_1^M &= D_1^A+D_1^B = 2-\kappa+\alpha-p^M+\gamma p_s^M+2-\kappa-\alpha-p^M+\gamma p_s^M = 4-2\kappa-2p^M+2\gamma p_s^M \\
D_2^M &= D_2^A+D_2^B = \kappa+\alpha-p^M+\gamma p_s^M+\kappa-\alpha-p^M+\gamma p_s^M = 2\kappa-2p^M+2\gamma p_s^M.
\end{aligned}
$$

1. If $\kappa < 1/2$, then investors of type 2 will choose a short position. Then equilibrium in the equity lending market requires $\lambda p_s^M D_1^M = -D_2^M$, which implies $\lambda p_s^M \left(4-2\kappa-2p^M+2\gamma p_s^M\right) = -\left(2\kappa-2p^M+2\gamma p_s^M\right)$, which can be written

$$
\begin{aligned}
p^M &= \frac{\lambda\gamma\left(p_s^M\right)^2+(2\lambda-\kappa\lambda+\gamma)\,p_s^M+\kappa}{1+\lambda p_s^A} \\
&= \frac{\lambda\gamma\left(\frac{p^M-1/2}{\gamma}\right)^2+(2\lambda-\kappa\lambda+\gamma)\,\frac{p^M-1/2}{\gamma}+\kappa}{1+\lambda\frac{p^M-1/2}{\gamma}}.
\end{aligned}
$$

2. If $1/2 \leq \kappa \leq 3/2$, then no investors will choose a short position and prices are given by $p^M = 1/2$, $p_s^M = 0$.

3. If $\kappa > 3/2$, then investors of type 1 will choose a short position. Then equilibrium in the equity lending market requires $\lambda p_s^M D_2^M = -D_1^M$, which implies $\lambda p_s^M \left(2\kappa-2p^M+2\gamma p_s^M\right) =$

$-\left(4-2\kappa-2p^M+2\gamma p_s^M\right)$, which can be written

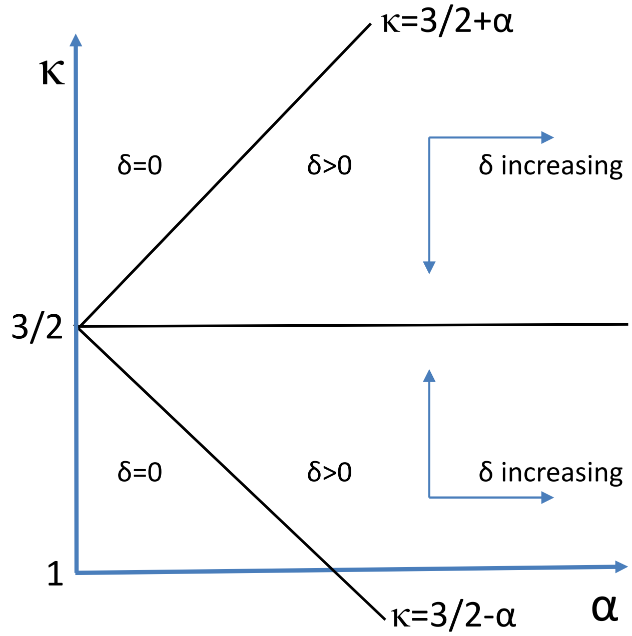
$$\begin{aligned} p^M &= \frac{\lambda\gamma\left(p_s^M\right)^2+(\kappa\lambda+\gamma)\,p_s^M+(2-\kappa)}{1+\lambda p_s^A} \\ &= \frac{\lambda\gamma\left(\frac{p^M-1/2}{\gamma}\right)^2+(\kappa\lambda+\gamma)\,\frac{p^M-1/2}{\gamma}+(2-\kappa)}{1+\lambda\frac{p^M-1/2}{\gamma}}. \end{aligned}$$

Figure B.2 shows these regions graphically, with attention restricted to the case of $\kappa \geq 1$. This is without loss of generality, as the figure is symmetric about $\kappa = 1$. Between $\kappa = 1$ and $\kappa = 3/2$, both investors hold a long position in the merged firm, whereas for $\kappa > 3/2$, investors of type 1 hold a short position in the merged firm. If both investors hold a long position in the merged firm, then a diversification discount will arise so long as there is sufficient stock-level disagreement for investors of type 1 to hold a short position in one of the pure-play stocks, which happens if $\kappa > 3/2 - \alpha$. Recalling that $\alpha = 1/2$ was the threshold for the diversification discount arising in the base case where there is market-level agreement, we see that the discount can arise for $\alpha < 1/2$ once market-level disagreement is introduced. In the neighborhood in which market-level disagreement is low, allowing market-level disagreement makes the diversification discount more prevalent.

There is a flip side, however, as the level of market-level disagreement becomes significant. In this case, represented as $\kappa > 3/2$, investors of type 1 will hold a short position in the merged entity. The diversification discount arises if type 1 investors also would choose a short position in only one of the pure plays, but it does not arise if type 1 investors would choose a short position in both pure plays.

Some intuition is in order. If type 1 investors, for example, are so pessimistic that they choose to be short in both pure plays and in the merged entity, then the overpricing is the same for all three. The reason is that our model is linear and there are two types of investor. If type 1 investors are short everything, then their aggregate short demand is the same for the merged firm as for the two pure-plays. In a non-linear model, this fact, and the associated conclusion that pure plays and conglomerates will be equally overpriced, would not necessarily hold. Which type of firm would be more overpriced with a market-wide pessimist would depend on the details of the model.

Figure B. 2. Diversification Discount Existence



The diversification discount will be zero if investors generally agree on the overall attractiveness of the two stocks, such that α is low and κ is close to unity. As stock-level disagreement, α, increases, the diversification discount will appear and increase. As market-level disagreement increases via κ approaching 3/2, the diversification discount appears and increases. However, for $\kappa > 3/2 + \alpha$, market-level disagreement is sufficient to make one type of investor sell short both pure-play firms, as well as the combined firm. In this case, all stocks become overpriced and the diversification discount disappears. This intuition is identical when $\kappa < 1$. The graph is a mirror image across the $\kappa = 1$ axis.

We close this section with a discussion of the insights gained by allowing for both market-level and stock-level disagreement among investors. First, there is an element of realism that makes the model more accurate. Investors of differing levels of optimism and pessimism concerning a pair of stocks can also differ in their general outlook on the market. Further, it is true in practice that, typically, conglomerates are shorted. It is more realistic to advance a model that allows for these options.

Second, additional novel results arise from allowing an additional form of disagreement. Most notably, market-level disagreement has a non-monotonic effect on pricing and the diversification discount. Very low levels of market-level disagreement, if paired with low levels of stock-level disagreement, yield consistent pricing between pure plays and conglomerates. However, as market-level disagreement rises, the diversification discount comes into being. As market-level disagreement keeps rising, however, it overwhelms stock-level disagreement and the discount associated with it.

Third, this additional realism does not affect the qualitative results from the baseline model. Most important, the diversification discount is always weakly positive, and, as shown in Corollary 5, is larger when stock-level disagreement is higher.[1]

B.3 Disagreement over Synergies

Until now, we have not considered synergies in valuing the merged firm. Trivially, if all investors agree on the size, positive or negative, of the synergies, then the conglomerate will simply be more or less valuable due to those synergies. This is not particularly interesting. More interesting, given our arguments above, is what happens when investors disagree about the level of synergies. We show in this section that if disagreement about synergies is less than the disagreement within each pure-play, our preceding results continue to hold: there will by a conglomerate discount if the disagreement within pure-plays is sufficiently large. If, however, disagreement over synergies is greater than disagreement within pure-plays, then our result flips: a conglomerate premium will arise if the disagreement over synergies is sufficiently high.

Suppose that there are four types of investor. Within both type 1 and type 2 investors are optimists and pessimists about possible synergies. A fraction y of type 1 investors are optimists and a fraction $(1-y)$ of type 2 investors are optimists. y measures the association between being optimistic about synergies and being optimistic about type A shares. For simplicity, in order to maintain the assumption that with no constraints the merged firm is worth the same as the separate firms, we assume that type o investors have demands $D^M_{1,o} = y(1+s-p^M)$ and $D^M_{2,o} = (1-y)(1+s-p^M)$. Type p investors have demands $D^M_{1,p} = (1-y)(1-s-p^M)$ and $D^M_{2,p} = (1-y)(1-s-p^M)$. We can see that if we add the optimistic and pessimistic demand within type 1 (type 2) investors, the aggregate demand for that group accords with the prior equations. We can also see that total demand for the merged firm is equally split between optimists and pessimists, regardless of y. That is, it is irrelevant whether optimism regarding a merger's synergies correlates with optimism about firm A or firm B.

Proposition 6. *The price of the merged firm is $p^M = 1/2$ if disagreement over synergies is*

[1] It may be the case that models with more than two assets and a more sophisticated correlation structure would invalidate this conclusion. Without a more sophisticated model, it is difficult to say how robust these conclusions may be.

low, and $p^M = \frac{\lambda + s(1-\lambda)}{1+\lambda} > 1/2$ *if disagreement over synergies is high.*

Proof. If short sales constraints do not bind, then aggregate demand is $D^M_{1,o} + D^M_{2,o} + D^M_{1,p} + D^M_{2,p} = 4 - 4p^M$. Setting equal to the supply we get $4 - 4p^M = 2$ so $p^M = 1/2$. If short sales constraints bind, then long demand is $2 + 2s - 2p^M$ and short demand is $\lambda \left(2 - 2s - 2p^M\right)$. Setting equal to supply yields

$$p^M = \frac{\lambda + s(1-\lambda)}{1+\lambda}.$$

□

The familiar expressions for the possible prices of the merged firm immediately yield the key result of this section.

Proposition 7. *There is a conglomerate discount if disagreement within pure plays is greater than disagreement over synergies, i.e., if* $\alpha > s$*. There is a conglomerate premium if* $\alpha < s$*. If* $\alpha = s$*, then the pure plays and the conglomerate are equally valued.*

Proof.

$$\begin{aligned} p^A - p^M &= p^B - p^M \\ &= \frac{\lambda + \alpha(1-\lambda)}{1+\lambda} - \frac{\lambda + s(1-\lambda)}{1+\lambda} \\ &= (\alpha - s)(1-\lambda). \end{aligned}$$

□

These results suggest that disagreement over synergies can flip our main result on its head: if disagreement over synergies is sufficiently high, then short-sale constraints can combine with disagreement to generate a conglomerate premium.

Appendix C: Robustness Tests

In Appendix C we show additional robustness tests.

Table C.1 uses institutional ownership concentration as an additional proxy for short sales constraints as proposed by Prado et al. (2016).

Table C.2 includes *Cash Holdings* and *Tangibility* as additional control variable to Table 5 in the main text.

Table C.1: *Concentration of Institutional Ownership* as proxy for SS Constraints in Table 5
This table displays regressions of a measure of excess firm value as a function of differences of opinion and short-sales constraints using a propensity-score matched sample from January 2006 through December 2015. The dependent variable is the logarithm of the ratio between a firm's *EV/Sales* divided by the *EV/Sales* benchmark computed from the average of pure plays operating in the same 2-digit SIC code. For conglomerates, we use the sales-weighted average of the pure-play firms operating in each of the conglomerate's reported segments as in Berger & Ofek (1995), where *Imp(X)* is the imputed mean value of X. *D(Conglomerate)* is an indicator variable equal to one if the firm reports data for more than one segment on Compustat, and zero otherwise. *Dispersion* is the standard deviation of analysts' annual earnings forecasts divided by the absolute value of the mean forecast in IBES, while *HHI IO* is given by the Hirschman-Herfindahl index (*HHI*) based on institutional ownership reported on the 13f files. We use values on the reporting date of the earnings announcement. The covariates used on the first stage to create the matched sample are: total assets (*Ln(Assets)*), institutional ownership (*Total IO*), Amihud's (2002) *ILLIQ*, log of 1 + analyst coverage, *Leverage*, *CAPEX*, *EBIT/Sales*, and *Stock Volatility*. All regressions include firm- and year-fixed effects. Standard errors clustered at the firm level are reported in brackets.

SS Variable:	HHI IO	
Variables	(1)	(2)
D(Conglomerate)	-0.231***	-0.230***
	[0.049]	[0.049]
Imp(Dispersion)	0.218***	0.513***
	[0.050]	[0.174]
Imp(HHI IO)	-0.999***	-0.693*
	[0.374]	[0.410]
*Imp(Dispersion)*Imp(HHI IO)*		-2.015*
		[1.063]
Ln(Assets)	0.078**	0.078**
	[0.037]	[0.037]
Total IO	0.647***	0.645***
	[0.103]	[0.103]
ILLIQ	-0.001*	-0.001*
	[0.000]	[0.000]
Ln(1+Analyst)	0.034*	0.034*
	[0.020]	[0.020]
Leverage	0.213*	0.212*
	[0.121]	[0.121]
CAPEX	0.442	0.44
	[0.286]	[0.286]
Stock Volatility	0.013	0.012
	[0.050]	[0.050]
EBIT/Sales	-0.157***	-0.157***
	[0.026]	[0.026]
Obs.	13,752	13,752
Firms	3,208	3,208
Adj. R^2	0.06	0.06

*** p-value<0.01, ** p-value<0.05, * p-value<0.10

Table C.2: Adding *Cash Holdings* and *Tangibility* as controls in Table 5

This table displays regressions of a measure of excess firm value as a function of differences of opinion and short-sales constraints using a propensity-score matched sample from January 2006 through December 2015. The dependent variable is the logarithm of the ratio between a firm's *EV/Sales* divided by the *EV/Sales* benchmark computed from the average of pure plays operating in the same 2-digit SIC code. For conglomerates, we use the sales-weighted average of the pure-play firms operating in each of the conglomerate's reported segments as in Berger & Ofek (1995), where *Imp(X)* is the imputed mean value of X. *D(Conglomerate)* is an indicator variable equal to one if the firm reports data for more than one segment on Compustat, and zero otherwise. *Dispersion* is the standard deviation of analysts' annual earnings forecasts divided by the absolute value of the mean forecast in IBES, while *SS* is one of the following measures of short-sales constraints: *Supply* is lendable supply as a fraction of market capitalization, *Fee Score* is a measure of daily borrowing costs computed by Markit going from 0 (cheapest) to 5 (most expensive), and *Fee Risk* is the standard deviation of loan fees in the previous 12 months. We use values on the reporting date of the earnings announcement. The covariates used on the first stage to create the matched sample are: total assets (*Ln(Assets)*), institutional ownership (*Total IO*), Amihud's (2002) *ILLIQ*, log of 1 + analyst coverage, *Leverage*, *CAPEX*, *EBIT/Sales*, and *Stock Volatility*. All regressions include firm- and year-fixed effects. Standard errors clustered at the firm level are reported in brackets.

SS Variable:	None	Supply	Fee Score	Fee Risk
Variables	(1)	(2)	(3)	(4)
D(Conglomerate)	-0.237***	-0.224***	-0.234***	-0.229***
	[0.050]	[0.048]	[0.049]	[0.049]
Imp(Dispersion)		-1.295***	0.530***	1.477***
		[0.250]	[0.098]	[0.264]
Imp(SS Constraints)		3.238***	-0.158***	0.023
		[0.882]	[0.041]	[0.018]
*Imp(Dispersion)*Imp(SS Constraints)*		9.276***	-0.259***	-0.209***
		[1.503]	[0.084]	[0.040]
Ln(Assets)	0.068*	0.078**	0.079**	0.068*
	[0.037]	[0.037]	[0.037]	[0.037]
Total IO	0.616***	0.579***	0.628***	0.607***
	[0.102]	[0.102]	[0.103]	[0.103]
ILLIQ	-0.000	-0.001*	-0.001*	-0.000
	[0.000]	[0.000]	[0.000]	[0.000]
Ln(1+Analyst)	0.031	0.033*	0.028	0.031
	[0.020]	[0.020]	[0.019]	[0.020]
Leverage	0.275**	0.260**	0.270**	0.275**
	[0.120]	[0.118]	[0.119]	[0.120]
CAPEX	0.635**	0.607**	0.584*	0.674**
	[0.301]	[0.293]	[0.300]	[0.299]
Stock Volatility	0.014	0.015	0.022	0.016
	[0.050]	[0.050]	[0.050]	[0.050]
EBIT/Sales	-0.154***	-0.157***	-0.159***	-0.156***
	[0.026]	[0.026]	[0.027]	[0.026]
Cash Holdings	0.004***	0.004***	0.004***	0.004***
	[0.001]	[0.001]	[0.001]	[0.001]
Tangibility	-0.236	-0.244	-0.211	-0.249
	[0.207]	[0.205]	[0.206]	[0.207]
Obs.	13,758	13,745	13,745	13,745
Firms	3,208	3,207	3,207	3,207
Adj. R^2	0.06	0.07	0.07	0.06

*** p-value<0.01, ** p-value<0.05, * p-value<0.10