九州大学学術情報リポジトリ
Kyushu University Institutional Repository

A new approach to the Pareto stable matching problem

Kamiyama, Naoyuki
Institute of Mathematics for Industry, Kyushu University

https://hdl.handle.net/2324/24491

出版情報：MI Preprint Series. 2012-8, 2012-07-30. 九州大学大学院数理学研究院
バージョン：
権利関係：

MI Preprint Series

Kyushu University
The Global COE Program
Math-for-Industry Education & Research Hub

A new approach to the Pareto stable matching problem

Naoyuki Kamiyama

MI 2012-8

(Received July 30, 2012)

Faculty of Mathematics
Kyushu University
Fukuoka, JAPAN

A New Approach to the Pareto Stable Matching Problem

Naoyuki Kamiyama[*][†]

Abstract

In two-sided matching markets, the concept of stability proposed by Gale and Shapley (1962) is one of the most important solution concepts. In this paper, we consider a problem related to the stability of a matching in a two-sided matching market with indifferences (i.e., ties). The introduction of ties into preference lists dramatically changes the properties of stable matchings. For example, stable matchings need not have the same size. Furthermore, it is known that stability do not guarantee Pareto efficiency that is also one of the most important solution concepts in two-sided matching markets. This fact naturally leads to the concept of Pareto stability, i.e., both stable and Pareto efficient. Erdil and Ergin (2006, 2008) proved that there always exists a Pareto stable matching in a one-to-one/many-to-one matching market with indifferences and gave a polynomial-time algorithm for finding it. Furthermore, Chen (2012) proved that there always exists a Pareto stable matching in a many-to-many matching market with indifferences and gave a polynomial-time algorithm for finding it. In this paper, we propose a new approach to the problem of finding a Pareto stable matching in a many-to-many matching market with indifferences. Our algorithm is an alternative proof of the existence of a Pareto stable matching in a many-to-many matching market with indifferences.

1 Introduction

Since the seminal work of Gale and Shapley [7], two-sided matching markets have been extensively studied in both Economics and Computer Science [8, 21, 15]. In this model, there exist two groups of agents and each agent has a preference ranking over members of the other group. The goal is to find a matching between these two groups with some specified properties. In two-sided matching markets, the concept of stability proposed by Gale and Shapley [7] is one of the most important solution concepts.

In this paper, we consider a problem related to the stability of a matching in a two-sided matching market with indifferences (i.e., ties). The introduction of ties into preference lists dramatically changes the properties of stable matchings [12]. For example, stable matchings need not have the same size. The problem of finding a maximum-size stable matching is $\mathcal{NP}$-hard [11, 16], and several approximation algorithms have been proposed [13, 17]. Moreover, it is known that stability do not guarantee Pareto efficiency, where Pareto efficiency means that there exists no other matching improving some agent without hurting everyone else (for its definition, see Section 2). Since the concept of Pareto efficiency is also one of the most important solution concepts in two-sided matching markets, this fact has received much attention in Economics [1, 2, 5, 6, 23] and naturally leads to the concept of Pareto stability [23], i.e., both stable and Pareto efficient. Erdil and Ergin [5, 6] proved that there always exists a Pareto stable matching in a one-to-one/many-to-one matching market with indifferences and gave a

[*]Institute of Mathematics for Industry, Kyushu University.

[†]kamiyama@imi.kyushu-u.ac.jp

polynomial-time algorithm for finding it. Furthermore, Chen [3] proved that there always exists a Pareto stable matching in a many-to-many matching market with indifferences and gave a polynomial-time algorithm for finding it.

In this paper, we propose a new approach to the problem of finding a Pareto stable matching in a many-to-many matching market with indifferences. Compared with the algorithms proposed in [5, 6, 3], our algorithm is not based on the characterization of a Pareto stable matching using a Pareto improvement cycle/chain (see Section 2). In our algorithm, we iteratively compute a rank-maximal matching [10, 14, 18, 9] that is another solution concept in matching markets. Our algorithm is an alternative proof of the existence of a Pareto stable matching in a many-to-many matching market with indifferences.

The rest of this paper is organized as follows. In Section 2, we formally define our problem and review previous results. In Section 3, we propose our algorithm. The next three sections are preliminaries to the proof of the correctness of our algorithm. In Section 4, we consider a relationship between Pareto efficiency and rank-maximality. In Section 5, we give a reduction from the rank-maximal matching problem to the maximum-cost matching problem. In Section 6, we introduce an auxiliary directed graph. In Section 7, we prove the correctness of our algorithm.

2 Problem formulation

Let $\mathbb{N}$ (resp., $\mathbb{Z}_+$) be the set of positive integers (reps., non-negative integers).

Throughout this paper, we are given a finite simple undirected bipartite graph G with a vertex set V and an edge set E. Define $n := |V|$ and $m := |E|$. Assume that a vertex set V is partitioned into two disjoint subsets P and Q such that every edge of E connects a vertex of P and one of Q. For each edge e that connects a vertex v of P and a vertex w of Q, we write $e(v, w)$. For each vertex v of V and each subset F of E, we denote by $F(v)$ the set of edges of E that are incident to v. Furthermore, we are given a capacity function $c\colon V \to \mathbb{N}$, and we are given a preference ranking function $\pi_v\colon E(v) \to \mathbb{N}$ for each vertex v of V. If $\pi_v(e) > \pi_v(f)$, then a vertex v strictly prefers an edge e to an edge f. If $\pi_v(e) = \pi_v(f)$, then v is indifferent between e and f. We call a bipartite graph G with a capacity function c and preference ranking functions π_v $(v \in V)$ a **many-to-many matching market with indifferences**.

A subset A of E is said to be **feasible** with respect to a vertex v of V, if $|A(v)| \leq c(v)$. A subset A of E is called an **assignment**, if A is feasible with respect to every vertex v of V. Assume that we are given an assignment A. For each edge e of $E \setminus A$, an endpoint v of e is said to be **free** with respect to A, if

1. $A \cup \{e\}$ is feasible with respect to v, and/or
2. there exists an edge f of $A(v)$ such that $\pi_v(e) > \pi_v(f)$.

An edge e of $E \setminus A$ is called a **blocking edge** with respect to A, if both endpoints of e are free with respect to A. An assignment A is said to be **stable**, if there exists no blocking edge with respect to A.

Now we define the concept of Pareto efficiency. For this, we have to define the preference of a vertex v of V over subsets of $E(v)$. Let F be a subset of $E(v)$ consisting of p edges $e_1, e_2, \ldots, e_p$ such that $\pi_v(e_1) \geq \pi_v(e_2) \geq \cdots \geq \pi_v(e_p)$. Furthermore, let F' be a subset of $E(v)$ consisting of q edges $f_1, f_2, \ldots, f_q$ such that $\pi_v(f_1) \geq \pi_v(f_2) \geq \cdots \geq \pi_v(f_q)$. We say that a vertex v **prefers** F to F' (denoted by $F \succeq_v F'$), if $p \geq q$ and $\pi_v(e_i) \geq \pi_v(f_i)$ for every positive integer i with $i \leq q$. We say that a vertex v **strictly prefers** F to F' (denoted by $F \succ_v F'$), if

1. v prefers F to F' (i.e., $F \succeq_v F'$), and

2. $p > q$ and/or $\pi_v(e_s) > \pi_v(f_s)$ for some positive integer s with $s \leq q$.

This preference is said to be responsive in Economics [21].

We say that an assignment A **dominates** an assignment B, if $A(v) \geq_v B(v)$ for every vertex v of V and there exists a vertex w of V such that $A(w) >_w B(w)$. An assignment A is said to be **Pareto efficient**, if there exists no assignment B that dominates A. An assignment is said to be **Pareto stable**, if it is stable and Pareto efficient.

2.1 Known results

Here we review the known algorithms for finding a Pareto stable matching. We first review the algorithms proposed by Erdil and Ergin [5, 6]. Their algorithms are based on the characterization of a Pareto stable matching using a **Pareto improvement cycle/chain** (for the formal definitions, see [5, 6, 3]). Roughly speaking, we can improve some agent without hurting everyone else along a Pareto improvement cycle/chain. It is known [5, 6, 3] that an assignment A is Pareto efficient if and only if there exists no Pareto improvement cycle/chain with respect to A. Furthermore, it is known [5, 6] that in a one-to-one/many-to-one matching markets (i.e., $c(v) = 1$ for every vertex v of V, or $c(v) = 1$ for every vertex v of P), the stability is preserved even if we improve an assignment along a Pareto improvement cycle/chain. So, we can obtain the following algorithm. Starting from any stable assignment, keep improving an assignment by eliminating Pareto improvement cycles/chains until none remain. However, it is also known [3] that this algorithm does not work in a many-to-many matching market. This is because improving an assignment along a Pareto improvement cycle/chain need not preserve the stability in a many-to-many matching market (see Example 1 in [3]). For overcoming this difficulty, Chen [3] proposed an algorithm based on an alternative to Gale-Shapley algorithm presented by Roth and Vande Vate [20]. In this algorithm, men "arrive" one by one, and a Pareto improvement chain is carefully eliminated. Although the author does not explicitly evaluate the time complexity of this algorithm, if we naively evaluate its time complexity, then it becomes $O(n^6m^2\Delta)$, where Δ denotes the time required to checking and finding a Pareto improvement cycle. We should remark that Chen and Ghosh [4] also considered Pareto stable solutions in many-to-many matching markets. However, in the paper [4], every pair of agents can transact any number of units, which is very different from the present paper in which at most one unit can be assigned.

3 Algorithm

For each vertex v of V, let $\pi_{v,1} > \pi_{v,2} > \cdots$ be the distinct values of $\pi_v(e)$ ($e \in E(v)$). For each vertex v of V and each edge e of $E(v)$, let $r_v(e)$ be the positive integer i such that $\pi_{v,i} = \pi_v(e)$. For each subset F of E and each positive integer i, we denote by $\lambda_i(F)$ and $\gamma_i(F)$ the numbers of edges $e(v, w)$ of F such that $r_v(e) = i$ and $r_w(e) = i$, respectively.

3.1 Rank-maximal matchings

For each subsets A, B of E, we write $A \succeq B$, if one of the following conditions is satisfied.

(R1) For every positive integer i, $\lambda_i(A) = \lambda_i(B)$ and $\gamma_i(A) = \gamma_i(B)$.

(R2) There exists a positive integer s such that $\gamma_s(A) > \gamma_s(B)$ and $\gamma_i(A) = \gamma_i(B)$ for every positive integer i with $i < s$.

(R3) For every positive integer i, $\gamma_i(A) = \gamma_i(B)$. Moreover, there exists a positive integer s such that $\lambda_s(A) > \lambda_s(B)$ and $\lambda_i(A) = \lambda_i(B)$ for every positive integer i with $i < s$.

If (R2) or (R3) holds, then we write $A \succ B$.

Assume that we are a function $\alpha\colon P \times \mathbb{N} \to \mathbb{Z}_+$. A subset A of E is said to be **α-eligible**, if

1. $\lambda_i(A(v)) \le \alpha(v,i)$ for every vertex v of P and every positive integer i, and
2. $|A(v)| \le c(v)$ for every vertex v of Q.

Assume that we are given a subset F of E. An α-eligible subset A of F is called a **rank-maximal α-matching** on F, if $A \succeq B$ for every α-eligible subset B of F.

3.2 Description of the algorithm

For each vertex v of P and each subset F of E, define

$$\partial_F(v) := \begin{cases} \min\Big\{ i \in \mathbb{N} \;\Big|\; \displaystyle\sum_{j=1}^{i} \lambda_j(F(v)) \ge c(v) \Big\} & \text{if } |F(v)| \ge c(v) \\ \max\{ i \in \mathbb{N} \mid \lambda_i(F(v)) \ne 0 \} & \text{if } |F(v)| < c(v). \end{cases}$$

For each subset F of E, define a function $\omega_F\colon P \times \mathbb{N} \to \mathbb{Z}_+$ as follows. If $|F(v)| < c(v)$, then define $\omega_F(v,i) := m+1$ for each vertex v of P and each positive integer i. If $|F(v)| \ge c(v)$, then define

$$\omega_F(v,i) := \begin{cases} m+1 & \text{if } i < \partial_F(v) \\ c(v) - \displaystyle\sum_{j=1}^{i-1} \lambda_j(F(v)) & \text{if } i = \partial_F(v) \\ 0 & \text{if } i > \partial_F(v) \end{cases}$$

for each vertex v of P and each positive integer i.

Assume that A, F are subsets of E with $A \subseteq F$. For each vertex v of P and each positive integer i, define

$$\kappa_{v,i}(A;F) := \begin{cases} \{ e \in F(v) \setminus A(v) \mid r_v(e) = i \} & \text{if } \lambda_i(A(v)) < \omega_F(v,i) \\ \emptyset & \text{if } \lambda_i(A(v)) \ge \omega_F(v,i). \end{cases}$$

Then, define

$$\kappa(A;F) := \bigcup_{v \in P} \bigcup_{i=1}^{\partial_F(v)} \kappa_{v,i}(A;F).$$

We are now ready to propose our algorithm, called ParetoStable.

Step1: Set $R_0 := \emptyset$ and $t := 0$

Step2: Repeat the following (2-a) to (2-c) until the algorithm halts.

- **(2-a)** Update $t := t+1$, and set $F_t := E \setminus R_{t-1}$.
- **(2-b)** Find a rank-maximal ω_{F_t}-matching A_t on F_t.
- **(2-c)** If $\kappa(A_t;F_t) = \emptyset$, then output A_t and halt. Otherwise, set $R_t := R_{t-1} \cup \kappa(A_t;F_t)$.

Since $R_0 \subsetneq R_1 \subsetneq \cdots$ holds, this algorithm halts in finite time. In the sequel, the algorithm halts when $t = T$. Notice that T is at most m.

4 Pareto efficiency and rank-maximality

In this section, we consider a relationship between Pareto efficiency and rank-maximality. For each subset F of E and each positive integer i, define

$$\Lambda_i(F) := \sum_{j=1}^{i} \lambda_j(F), \qquad \Gamma_i(F) := \sum_{j=1}^{i} \gamma_j(F).$$

Lemma 1. *If an assignment A dominates an assignment B, then*

(D1) $\Lambda_i(A(v)) \geq \Lambda_i(B(v))$ *for every vertex v of P and every positive integer i, and*

(D2) $\Gamma_i(A(v)) \geq \Gamma_i(B(v))$ *for every vertex v of Q and every positive integer i.*

Furthermore,

(E1) $\Lambda_s(A(w)) > \Lambda_s(B(w))$ *for some vertex w of P and some positive integer s, and/or*

(E2) $\Gamma_s(A(w)) > \Gamma_s(B(w))$ *for some vertex w of Q and some positive integer s.*

Proof. We first prove that (D1) holds. Assume that v is a vertex of P and i is a positive integer. Moreover, assume that $A(v)$ consists of p edges $e_1, e_2, \ldots, e_p$ such that $\pi_v(e_1) \geq \pi_v(e_2) \geq \cdots \geq \pi_v(e_p)$ and $B(v)$ consists of of q edges $f_1, f_2, \ldots, f_q$ such that $\pi_v(f_1) \geq \pi_v(f_2) \geq \cdots \geq \pi_v(f_q)$. Let j be the maximum positive integer such that $j \leq q$ and $\pi_v(f_j) \geq \pi_{v,i}$. Since A dominates B, $\pi_v(e_{j'}) \geq \pi_{v,i}$ for every positive integer j' with $j' \leq j$. This implies that $\Lambda_i(A(v)) \geq \Lambda_i(B(v))$. Similarly, we can prove that (D2) holds.

Next we prove that (E1) and/or (E2) holds. Since A dominates B, there exists a vertex w of V such that $A(w) >_w B(w)$. Now we prove that if $w \in P$, then (E1) holds. Assume that $A(w)$ consists of p edges $e_1, e_2, \ldots, e_p$ such that $\pi_w(e_1) \geq \pi_w(e_2) \geq \cdots \geq \pi_w(e_p)$ and $B(w)$ consists of of q edges $f_1, f_2, \ldots, f_q$ such that $\pi_w(f_1) \geq \pi_w(f_2) \geq \cdots \geq \pi_w(f_q)$. We first consider the case of $p > q$. Let s be a positive integer such that $\pi_{w,s} = \pi_w(e_{q+1})$. Then,

$$\Lambda_s(A(w)) \geq q + 1 > q = |B(w)| \geq \Lambda_s(B(w)).$$

Next we consider the case where there exists a positive integer j such that $j \leq q$ and $\pi_w(e_j) > \pi_w(f_j)$. Let s be the positive integer with $\pi_{w,s} = \pi_w(e_j)$. Since $\pi_w(f_j) < \pi_w(e_j) = \pi_{w,s}$, we have

$$\Lambda_s(A(w)) \geq j > j - 1 \geq \Lambda_s(B(w)).$$

Similarly, we can prove that if $w \in Q$, then (E2) holds. □

Lemma 2. *If an assignment A dominates an assignment B, then $A \succ B$.*

Proof. We first assume that (E2) of Lemma 1 holds. Let s be the minimum positive integer i such that there exists a vertex w of Q with $\Gamma_i(A(w)) > \Gamma_i(B(w))$. For every vertex v of Q and every positive integer i with $i < s$, $\Gamma_i(A(v)) = \Gamma_i(B(v))$ follows from (D2). By this and (D2),

$$\begin{aligned} \gamma_i(A(v)) &= \gamma_i(B(v)), && \forall v \in Q, \ \ \forall i \in \mathbb{N} \text{ with } i < s \\ \gamma_s(A(v)) &= \gamma_s(B(v)), && \forall v \in Q \text{ with } \Gamma_s(A(v)) = \Gamma_s(B(v)) \\ \gamma_s(A(v)) &> \gamma_s(B(v)), && \forall v \in Q \text{ with } \Gamma_s(A(v)) > \Gamma_s(B(v)). \end{aligned}$$

This implies that $\gamma_s(A) > \gamma_s(B)$ and $\gamma_i(A) = \gamma_i(B)$ for every positive integer i with $i < s$, which implies that (R2) holds.

Next we consider the case where (E2) does not hold, but (E1) holds. In this case, by (D2), $\gamma_i(A(v)) = \gamma_i(B(v))$ for every vertex v of Q and every positive integer i, i.e., $\gamma_i(A) = \gamma_i(B)$ for every positive integer i. Moreover, we can prove that there exists a positive integer s such that $\lambda_s(A) > \lambda_s(B)$ and $\lambda_i(A) = \lambda_i(B)$ for every positive integer i with $i < s$ in the same way as the first case. This implies that (R3) holds. $\square$

5 Maximum-cost matchings

In this section, we consider the reduction from finding a rank-maximal matching to finding a maximum-cost matching defined as follows. This reduction is a standard technique in the study of rank-maximal matchings [10, 14, 18, 9].

For each vertex v of V, let $\phi(v)$ be the number of the distinct values of $\pi_v(e)$ ($e \in E(v)$). Define $\phi_P := \max_{v \in P} \phi(v)$ and $\phi_Q := \max_{v \in Q} \phi(v)$. For each edge $e(v, w)$ of E, define

$$k_P(e) := m^{\phi_P - r_v(e)}, \qquad k_Q(e) := m^{\phi_Q - r_w(e) + \phi_P}, \qquad k(e) := k_P(e) + k_Q(e).$$

For each subset F of E, define

$$k_P(F) := \sum_{e \in F} k_P(e), \qquad k_Q(F) := \sum_{e \in F} k_Q(e), \qquad k(F) := k_P(F) + k_Q(F).$$

Assume that we are given a subset F of E and a function $\alpha \colon P \times \mathbb{N} \to \mathbb{Z}_+$. An α-eligible subset A of F is called a **maximum-cost α-matching** on F, if $k(A) \geq k(B)$ for every α-eligible subset B of F.

Lemma 3. *For each subsets A, F of E with $A \subseteq F$ and each function $\alpha \colon P \times \mathbb{N} \to \mathbb{Z}_+$, A is a rank-maximal α-matching on F if and only if A is a maximum-cost α-matching on F.*

Proof. ($\Rightarrow$) Assume that A, B are α-eligible subsets of F with $A \succeq B$. We prove that $k(A) \geq k(B)$. If $A = \emptyset$, then $B = \emptyset$, i.e., $k(A) = k(B)$. So, we can assume that $A \neq \emptyset$. If (R1) holds, then $k_P(A) = k_P(B)$ and $k_Q(A) = k_Q(B)$, i.e., $k(A) = k(B)$.

Assume that (R2) holds. In this case, the size of B is clearly at most $m - 1$. So, since $A \neq \emptyset$ and $1 \leq k_P(e) \leq m^{\phi_P - 1}$ for every edge e of E,

$$k_P(A) - k_P(B) \geq 1 - (m-1) \cdot m^{\phi_P - 1} = 1 + m^{\phi_P - 1} - m^{\phi_P}$$

Let s be a positive integer such that $\gamma_s(A) > \gamma_s(B)$ and $\gamma_i(A) = \gamma_i(B)$ for every positive integer i with $i < s$. Let B' be the set of edges $e(v, w)$ of B such that $r_w(e) \leq s$. It is not difficult to see that $k_Q(A) - k_Q(B') \geq m^{\phi_Q - s + \phi_P}$. If $s = \phi_Q$, then $B = B'$. So,

$$k(A) - k(B) = k_P(A) - k_P(B) + k_Q(A) - k_Q(B') \geq 1 + m^{\phi_P - 1} > 0.$$

If $s < \phi_Q$, then

$$k_Q(B \setminus B') \leq (m-1) \cdot m^{\phi_Q - (s+1) + \phi_P} = m^{\phi_Q - s + \phi_P} - m^{\phi_Q - s + \phi_P - 1}.$$

By this and $\phi_Q - s + \phi_P - 1 \geq \phi_P$,

$$\begin{aligned} k(A) - k(B) &= k_P(A) - k_P(B) + k_Q(A) - k_Q(B') - k_Q(B \setminus B') \\ &\geq 1 + m^{\phi_P - 1} - m^{\phi_P} + m^{\phi_Q - s + \phi_P - 1} \geq 1 + m^{\phi_P - 1} > 0. \end{aligned}$$

Assume that (R3) holds. In this case, $k_Q(A) = k_Q(B)$. Let s be a positive integer such that $\lambda_s(A) > \lambda_s(B)$ and $\lambda_i(A) = \lambda_i(B)$ for every positive integer i with $i < s$. Let B' be the set of edges $e(v, w)$ of B such that $r_v(e) \leq s$. It is not difficult to see that $k_P(A) - k_P(B') \geq m^{\phi_P - s}$. If $s = \phi_P$, then $B = B'$. So, $k(A) - k(B) = k_P(A) - k_P(B') \geq 1$. If $s < \phi_P$, then

$$k_P(B \setminus B') \leq (m-1) \cdot m^{\phi_P - (s+1)} = m^{\phi_P - s} - m^{\phi_P - s - 1}.$$

By this,

$$k(A) - k(B) = k_P(A) - k_P(B') - k_P(B \setminus B') \geq m^{\phi_P - s - 1} > 0$$

($\Leftarrow$) Assume that A, B are α-eligible subsets of F with $k(A) \geq k(B)$. If $B \succ A$, i.e., (R2) or (R3) holds, then $k(B) > k(A)$ follows from the above discussion. This contradicts $k(A) \geq k(B)$. So, $A \succeq B$ holds. This completes the proof. $\square$

6 Auxiliary Directed Graph

For each vertex v of P and each positive integer i with $i \leq \phi(v)$, let $v(i)$ be a new vertex. Define

$$U_P := \{v(1), v(2), \ldots, v(\phi(v)) \mid v \in P\}, \qquad U := U_P \cup Q.$$

For each edge $e(v, w)$, define $v(e) := v(r_v(e))$. For each edge $e(v, w)$ of E, let e^+ be an arc from $v(e)$ to w, and let e^- be an arc from w to $v(e)$. For each subset F of E, define $F^+ := \{e^+ \mid e \in F\}$ and $F^- := \{e^- \mid e \in F\}$. For each vertex u of U_P (resp., Q) and each subset L of E^+, let $L(u)$ be the set of arcs of L leaving (resp., entering) u. For each edge e of E, define $l(e^+) := k(e)$ and $l(e^-) := -k(e)$. Define $l(L) := \sum_{a \in L} l(a)$ for each subset L of $E^+ \cup E^-$.

Assume that we are given a function $\alpha\colon P \times \mathbb{N} \to \mathbb{Z}_+$. A subset K of E^+ is said to be **α-good**, if $|K(v(i))| \leq \alpha(v, i)$ for every vertex $v(i)$ of U_P and $|K(v)| \leq c(v)$ for every vertex v of Q. Assume that L is a subset of E^+. An α-good subset K of L is called a **maximum-length α-matching** on L, if $l(K) \geq l(M)$ for every α-good subset M of L. For each subsets A, F of E with $A \subseteq F$ and each function $\alpha\colon P \times \mathbb{N} \to \mathbb{Z}_+$, it is easy to see that A is a maximum-cost α-matching on F if and only if A^+ is a maximum-length α-matching on F^+. So, it follows from Lemma 3 that A is a rank-maximal α-matching on F if and only if A^+ is a maximum-length α-matching on F^+.

Let A, F be subsets of E with $A \subseteq F$. Define $F^+/A := (F^+ \setminus A^+) \cup A^-$. Let $D(F)/A$ be a directed graph with a vertex set U and an arc set F^+/A. Assume that S is a directed path from a vertex u of U to a vertex w of U on $D(F)/A$. (In this paper, a directed path may pass through the same vertex more than once, but we do not allow that a directed path passes through the same arc more than once. Furthermore, a single vertex is regarded as a directed path with no arc.) Define the length $l(S)$ of a directed path S by the sum of the lengths of arcs contained in S. A directed path S is called a circuit, if $u = w$ and S contains at least one arc. Assume that S is not a circuit. We call S a **semi-augmenting path**, if (i) $w \in U_P$, or (ii) $w \in Q$ and $|A^+(w)| < c(w)$. We call a semi-augmenting path S an **α-augmenting path**, if (i) $u \in Q$, or (ii) $u \in U_P$ and $|A^+(u)| < \alpha(v, i)$, where $u = v(i)$ for a vertex v of P and a positive integer i. It is known [22, Theorem 12.1] that A^+ is a maximum-length α-matching on F^+ if and only if there exist no circuit C and α-augmenting path S on $D(F)/A$ such that $l(C) > 0$ and $l(S) > 0$.

For each positive integer t with $t \leq T$, define $D_t := D(F_t)/A_t$. For each vertex v of Q and each positive integer t with $t \leq T$, let $d_t(v)$ be the maximum-length of a semi-augmenting path from v on D_t. Notice that since $c(v) > 0$ for every vertex v of Q, there always exists a semi-augmenting path from each vertex v of Q on D_t. Furthermore, since A_t^+ is a maximum-cost ω_{F_t}-matching on F_t^+, there exists no circuit C on D_t with $l(C) > 0$. Thus, there exists a semi-augmenting path S passing through each vertex at most once such that $l(S) = d_t(v)$.

Lemma 4. *For each vertex v of Q and each positive integer t with $2 \leq t \leq T$, $d_t(v) \leq d_{t-1}(v)$.*

Proof. Assume that v is a vertex of Q and t is positive integer with $2 \leq t \leq T$. Since

$$A_{t-1} \subseteq F_{t-1} \qquad A_{t-1} \cap \kappa(A_{t-1}; F_{t-1}) = \emptyset \qquad F_t = F_{t-1} \setminus \kappa(A_{t-1}; F_{t-1}),$$

A_{t-1} is a subset of F_t. Define $D' := D(F_t)/A_{t-1}$. Let $d'(v)$ be the maximum-length of a semi-augmenting path from v on D'. Since D' can be obtained from D_{t-1} by removing some arcs, we have $d'(v) \leq d_{t-1}(v)$. So, it suffices to prove that $d_t(v) \leq d'(v)$.

It is not difficult to see that D_t can be obtained from D' by reversing the orientation of all arcs of

$$\{e^+ \mid e \in A_t \setminus A_{t-1}\} \cup \{e^- \mid e \in A_{t-1} \setminus A_t\}. \tag{1}$$

It is known [22, the proof of Theorem 12.1] that (1) can be decomposed into a collection $\mathcal{L}$ of arc-disjoint circuits and ω_{F_t}-augmenting paths on D' such that (i) $l(L) \geq 0$ for every member L of $\mathcal{L}$, and (ii) for each vertex u of U_P, there exist ω_{F_t}-augmenting paths from u, or there exist ω_{F_t}-augmenting paths to u, or there exists no ω_{F_t}-augmenting path from/to u (i.e., exactly one of these three cases holds).

Assume that $d'(v) < d_t(v)$. Let S be a semi-augmenting path from v to v' on D_t such that $l(S) = d_t(v)$. Without loss of generality, we can assume that S passes through each vertex at most once. Let S^a be the set of arcs of S contained in D'. Notice that an arc of S^a is not contained in a member of $\mathcal{L}$. Let S^b be the set of arcs a of S with $a \notin S^a$, i.e., the arc obtained by reversing the orientation of a is contained in D'. Let S^p be the set of arcs of D' obtained by reversing the orientation of arcs of S^b. Notice that every arc of S^p are passed through by exactly one member of $\mathcal{L}$.

We prove that there exists a directed path K from v on D' satisfying the following.

- **(P1)** K is a directed path to v' or a vertex u of U such that there exists an ω_{F_t}-augmenting path of $\mathcal{L}$ to u.
- **(P2)** Let v_{co} be the vertex of S that K passes through last. Let K' (resp., S') is a subpath of K (resp., K) from v to v_{co}. Then, we have $l(K') \geq l(S')$.

Assume that there exists a directed path K from v on D' satisfying (P1) and (P2). If K is a directed path to v', then $v_{\mathrm{co}} = v'$, i.e., $K' = K$ and $S' = S$. If (i) $v' \in U_P$ or (ii) $v' \in Q$ and $|A^+_{t-1}(v')| < c(v')$, then K is a semi-augmenting path from v on D'. So, $d'(v) \geq l(K) \geq l(S) = d_t(v)$, which contradicts $d'(v) < d_t(v)$. Since S is a semi-augmenting path to v' on D_t, if $v' \in Q$, then $|A^+_t(v')| < c(v')$. So, if $v' \in Q$ and $|A^+_{t-1}(v')| = c(v')$, then there exists an ω_{F_t}-augmenting path L of $\mathcal{L}$ from v'. Let KL be the directed path obtained by combining K and L. Since $l(L) \geq 0$ and KL is a semi-augmenting path from v on D',

$$d'(v) \geq l(KL) \geq l(K) \geq l(S) \geq d_t(v).$$

This contradicts $d'(v) < d_t(v)$. Assume that K is a directed path to a vertex u such that there exists an ω_{F_t}-augmenting path of $\mathcal{L}$ to u. In this case, K is a semi-augmenting path from v on D'. Let K'' and S'' be subpaths of K and S obtained by removing K' and S', respectively. Then, $l(K'') < l(S'')$ follows from $l(K) \leq d'(v) < d_t(v) = l(S)$. Since there exists an ω_{F_t}-augmenting path of $\mathcal{L}$ to u, if $u \in U_P$, then $|A^+_t(u)| < \omega_{F_t}(w, i)$, where $u = w(i)$ for a vertex w of P and a positive integer i. Furthermore, there exists a directed path K^r on D_t obtained by reversing the orientation of arcs of K''. So, a directed path S^* obtained by combining K^r and S'' is an ω_{F_t}-augmenting path on D_t such that $l(S^*) = l(S'') - l(L'') > 0$. This contradicts the fact that A^+_t is a maximum cost ω_{F_t}-matching on F^+_t. This completes the proof.

What remains is to prove that there exists a directed path K on D' satisfying (P1) and (P2). Such a path can be constructed by the following operation. Starting from a vertex v on D', we go along arcs of S^a until we meet the head of an arc of S^p (see Figure 1(a)). Recall that there exists a unique member L of $\mathcal{L}$ that passes through this arc (see Figure 1(b)). So, we go along L until we meet the tail of an arc a of L with $a \in S^p$ (see Figure 1(c)). Then, we again go along arcs of S^a until we meet the head of an arc of S^p (see Figure 1(d)). During this operation, if we meet v' or a vertex u of U such that there exists an ω_{F_t}-augmenting path of $\mathcal{L}$ to u, this operation halts. By repeating this operation (see Figure 1(e)-(f)), our path eventually meet v' or a vertex u of U such that there exists an ω_{F_t}-augmenting path of $\mathcal{L}$ to u. Notice that it is not difficult to see that this operation halts in finite time. (We can prove this by contradiction. Assuming that $\hat{S}$ is the first subpath of S that our path passes through more than once. However, since members of $\mathcal{L}$ are arc-disjoint, there must exist another subpath of S that our path passes through more than once before passing through $\hat{S}$ twice.)

Let K be the directed path obtained by the above operation. Now we prove that (P2) holds. Assume that the form of K' is $(S_0, K_1, S_1, \ldots, K_h, S_h)$, where $S_0, S_1, \ldots, S_h$ are subpaths of S such that arcs of S_i are contained in S^a for every non-negative integer i with $i \leq h$ and $K_1, K_2, \ldots, K_h$ are subpaths of some member of $\mathcal{L}$. For each positive integer i with $i \leq h$, let p_i and q_i be vertices of S such that K_i is a directed path from p_i to q_i. Define $q_0 := v$ and $p_{h+1} := v_{\rm co}$. Notice that $p_i \neq q_i$ for every positive integer i with $i \leq h$. Furthermore, for each non-negative integer i with $i \leq h$, S_i is a subpath of S from q_i to p_{i+1}. For each non-negative integer i with $i \leq h$, let $S_{\leq i}$ be a directed path whose form is $(S_0, K_1, S_1, \ldots, K_i, S_i)$. Notice that $S_{\leq h} = K'$. For each positive integer i with $i \leq h$, let $K_{\leq i}$ be a directed path obtained by combining $S_{\leq i-1}$ and K_i. For each non-negative integer i with $i \leq h$, let S_i° be a subpath of S from v to p_{i+1}. Notice that $S_h^\circ = S'$. For each positive integer i with $i \leq h$, let $S_i^\bullet$ be a subpath of S from v to q_i. Since $S_{\leq 0} = S_0^\circ$, we have $l(S_{\leq 0}) = l(S_0^\circ)$. Now we prove that $l(K_{\leq i}) \geq l(S_i^\bullet)$ and $l(S_{\leq i}) \geq l(S_i^\circ)$ for every positive integer i with $i \leq h$, which implies that $l(K') \geq l(S')$. Assume that $l(S_{\leq i-1}) \geq l(S_{i-1}^\circ)$ for a positive integer i with $i \leq h$. We first prove that $l(K_{\leq i}) \geq l(S_i^\bullet)$. Assume that S passes through p_i before q_i (see Figure 1(c)). Let $S_{\rm sub}$ be a subpath of S from p_i to q_i. Since A_t^+ is a maximum-cost ω_{F_t}-matching on F_t^+ (i.e., there exists no circuit C on D_t with $l(C) > 0$), we have $l(S_{\rm sub}) - l(K_i) \leq 0$, i.e., $l(K_i) \geq l(S_{\rm sub})$. So,

$$l(K_{\leq i}) = l(S_{\leq i-1}) + l(K_i) \geq l(S_{i-1}^\circ) + l(S_{\rm sub}) = l(S_i^\bullet).$$

Assume that S passes through q_i before p_i (see Figure 1(e)). Let $S_{\rm sub}^r$ be a subpath of S from q_i to p_i. Then, $l(S_{\rm sub}^r) \geq -l(K_i)$ follows from $l(S) = d_t(v)$. So,

$$l(K_{\leq i}) = l(S_{\leq i-1}) + l(K_i) \geq l(S_{i-1}^\circ) - l(S_{\rm sub}^r) = l(S_i^\bullet).$$

Since $S_{\leq i}$ (resp., S_i°) is a directed path obtained by combining $K_{\leq i}$ (resp., $S_i^\bullet$) and S_i, $l(S_{\leq i}) \geq l(S_i^\circ)$ follows from $l(K_{\leq i}) \geq l(S_i^\bullet)$. This completes the proof. □

Lemma 5. *If $e(v, w)$ is an edge of R_{T-1}, then $l(e^+) + d_T(w) \leq 0$.*

Proof. Let $e(v, w)$ be an edge of R_{T-1}. Assume that $r_v(e) = i$ and $e \in \kappa(A_t; F_t)$ for a positive integer t with $t < T$. This implies that $\lambda_i(A_t(v)) < \omega_{F_t}(v, i)$, i.e., $|A_t^+(v(i))| < \omega_{F_t}(v, i)$. Since A_t^+ is a maximum-length ω_{F_t}-matching on F_t^+, there exists no circuit C and ω_{F_t}-augmenting path S on D_t with $l(C) > 0$ and $l(S) > 0$, which implies that $l(e^+) + d_t(w) \leq 0$. Since $d_T(w) \leq d_t(w)$ follows from Lemma 4, we have $l(e^+) + d_T(w) \leq 0$. □

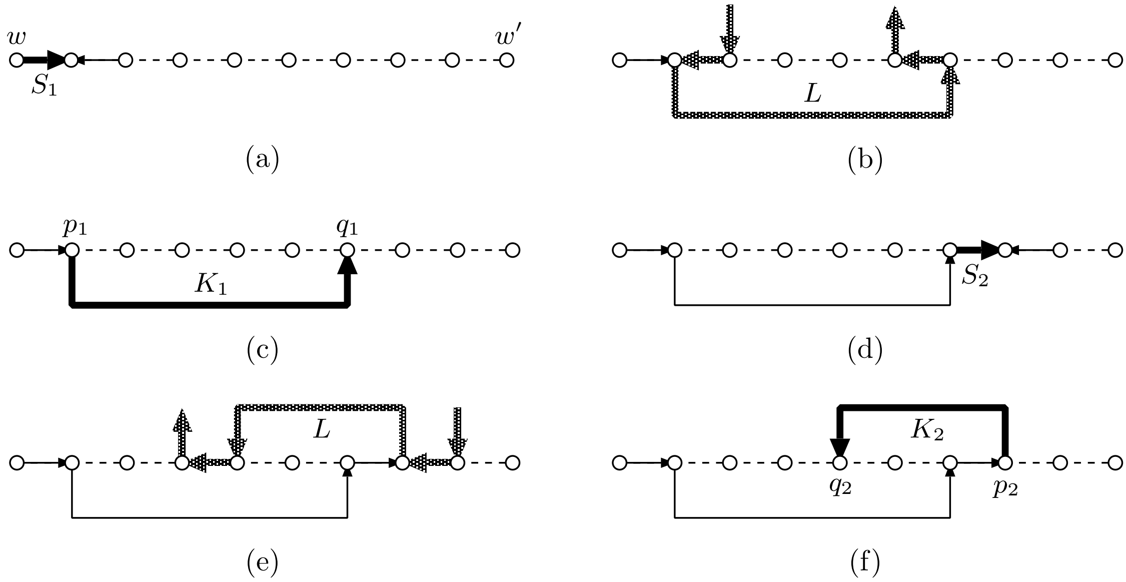

Figure 1: Construction of a directed path K.

7 Correctness

We are now ready to prove the correctness of the algorithm ParetoStable. For simplicity. define $A := A_T$, $F := F_T$, $d_T(\cdot) := d(\cdot)$ and $R := R_{T-1}$. Since A is ω_F-eligible, A is an assignment. Next we prove that A is stable. For this, we need the following lemma.

Lemma 6. *If $e(v, w)$ is an edge of R, then $|A(w)| = c(w)$ and $\pi_w(e) \leq \pi_w(f)$ for every edge f of $A(w)$.*

Proof. If $|A(w)| < c(w)$, then $d(w) \geq 0$. By this, $l(e^+) + d(w) > 0$, which contradicts Lemma 5. Assume that there exists an edge f of $A(w)$ with $\pi_w(e) > \pi_w(f)$. In the same way of the proof of Lemma 3, we can prove that $l(e^+) > l(f^+)$. Since f^- is a semi-augmenting path from w on D_T, we have $d(w) \geq l(f^-) = -l(f^+)$. So, $l(e^+) + d(w) > 0$, which contradicts Lemma 5. □

Lemma 7. *An assignment A is stable.*

Proof. Let $e(v, w)$ be an edge of $E \setminus A$. If $|A(v)| = c(v)$ and $\pi_v(e) \leq \pi_v(f)$ for every edge f of $A(v)$, then e is not a blocking edge with respect to A. So, we can assume that $|A(v)| < c(v)$ and/or there exists an edge f of $A(v)$ such that $\pi_v(e) > \pi_v(f)$. If $e \in R$, then it follows from Lemma 6 that e is not a blocking edge with respect to A. So, it suffices to prove that $e \in R$. Assume that $e \notin R$, i.e., $e \in F$. If $|F(v)| < c(v)$, then $\omega_F(v, i) > m$ for every positive integer i. So, $e \in \kappa(A; F)$, which contradicts $\kappa(A; F) = \emptyset$. So, we can assume that $|F(v)| \geq c(v)$. If $|A(v)| < c(v)$, then $\kappa(A; F) \neq \emptyset$, which contradicts $\kappa(A; F) = \emptyset$. If there exists an edge f of $A(v)$ such that $\pi_v(e) > \pi_v(f)$, then $r_v(e) < \partial_F(v)$. Hence, $e \in \kappa(A; F)$ follows from $\omega_F(v, r_v(e)) > m$, which contradicts $\kappa(A; F) = \emptyset$. This completes the proof. □

Finally we prove that A is Pareto efficient. For this, we need the following lemma.

Lemma 8. *An assignment A is a rank-maximal ω_F-matching on E.*

Proof. It suffices to prove that there exist no circuit C and ω_F-augmenting path S on $D(E)/A$ such that $l(C) > 0$ and $l(S) > 0$. Assume that C is a circuit and S is an ω_F-augmenting path on $D(E)/A$. Since A^+ is a maximum-length ω_F-matching on F^+, if every arcs of C and S are contained in F^+/A, we have $l(C) \leq 0$ and $l(S) \leq 0$. So, we consider the case where C and S contain an arc e^+ for some edge e of $E \backslash F$. For every edge $e(v, w)$ of $E \backslash F$ such that $|F(v)| \geq c(v)$ and $r_v(e) > \partial_F(v)$, since $\omega_F(v, r_v(e)) = 0$ and $A^+(v(e)) = \emptyset$, e^+ can not be contains in C and S. So, if an edge $e(v, w)$ of $E \setminus F$ with $|F(v)| \geq c(v)$ is contained in S or C, then we can assume that $r_v(e) \leq \partial_F(v)$, i.e., $e \in R$. Notice that every edge $e(v, w)$ of $E \setminus F$ with $|F(v)| < c(v)$ is contained in R.

Assume that the form of C is $(e_1^+, S_1, e_2^+, S_2, \ldots, e_h^+, S_h)$, where $e_1, e_2, \ldots, e_h$ are edges of R and $S_1, S_2, \ldots, S_h$ are semi-augmenting paths on D_T. By Lemma 5, $l(e_i^+) + l(S_i) \leq 0$ for every positive integer i with $i \leq h$. This implies that $l(C) \leq 0$.

Assume that the form of S is $(S_0, e_1^+, S_1, e_2^+, S_2, \ldots, e_h^+, S_h)$, where $e_1, e_2, \ldots, e_h$ are edge of R and $S_0, S_1, \ldots, S_h$ are semi-augmenting paths on D_T. Notice that S_0 is an ω_F-augmenting path on D_T. So, $l(S_0) \leq 0$. Moreover, by Lemma 5, $l(e_i^+) + l(S_i) \leq 0$ for every positive integer i with $i \leq h$. This implies that $l(S) \leq 0$. □

Lemma 9. *An assignment A is Pareto efficient.*

Proof. Assume that an assignment B dominates A. Since $B \succ A$ follows from Lemma 2, if B is ω_F-eligible, then this contradicts Lemma 8. So, we can assume that B is not ω_F-eligible. In this case, there exist a vertex w of P and a positive integer s with $|B(w)| > \omega_F(w, s)$. From the definition of ω_F, we can see that $|F(w)| \geq c(w)$ and $s \geq \partial_F(w)$. Notice that $|A(w)| = c(w)$ by $|F(w)| \geq c(w)$ and $\kappa(A; F) = \emptyset$. Since B is assignment, $|B(w)| \leq c(w)$, i.e., $|B(w)| \leq |A(w)|$. If $s = \partial_F(w)$, then $\lambda_s(A(w)) \leq \omega_F(v, s) < \lambda_s(B(w))$. So, since $|B(w)| \leq |A(w)|$ and $r_v(e) \leq s$ for every edge e of $A(w)$, we have $\Lambda_{s-1}(A(w)) > \Lambda_{s-1}(B(w))$, which contradicts Lemma 1. If $s > \partial_F(w)$, then there exists an edge f of $B(w)$ with $r_w(f) > \partial_F(w)$. For every edge e of $A(w)$, $\pi_w(e) > \pi_w(f)$ follows from $r_w(e) \leq \partial_F(w)$, which contradicts the fact that B dominates A. □

Theorem 10. *There always exists a Pareto stable assignment in a many-to-many matching market with indifferences.*

Proof. The theorem immediately follows form Lemmas 7 and 9. □

7.1 Time complexity

Here we consider the time complexity of the algorithm ParetoStable. Since the number of iterations of Step 2 is at most m. So, the algorithm ParetoStable can find a Pareto stable assignment in $O(m \cdot \mathsf{MC})$ time, where MC represents the time required to find a rank-maximal ω_F-matching on F, i.e., a maximum-length ω_F-matching on F^+ for one subset F of E. Notice that due to the definition of ω_F, when we find a maximum-length ω_F-matching on F^+, we need only two vertices instead of vertices $v(1), v(2), \ldots, v(\phi(v))$. We can contract vertices $v(1), \ldots, v(\partial_F(v) - 1)$ into one vertex, and we can delete vertices $v(\partial_F(v) + 1), \ldots, v(\phi(v))$. So, if we use the algorithm proposed in [19], then MC becomes $O(m^3 \log n + m^2 n \log^2 n)$, where we assume that arithmetic on numbers $O(m^{2m})$ takes $O(m)$ time. Thus, the time complexity of the algorithm ParetoStable becomes $O(m^4 \log n + m^3 n \log^2 n)$ time.

References

[1] A. Abdulkadiroglu, P. Pathak, and A. E. Roth. Strategy-proofness versus efficiency in matching with indifferences: Redesigning the NYC high school match. *American Economic Review*, 99(5):1954–1978, 2009.

[2] A. Abdulkadiroglu and T. Sonmez. School choice: A mechanism design approach. *American Economic Review*, 93(3):729–747, 2003.

[3] N. Chen. On computing pareto stable assignments. In *STACS'12*, pages 384–395, 2012.

[4] N. Chen and A. Ghosh. A market clearing solution for social lending. In *IJCAI'11*, pages 152–157, 2011.

[5] A. Erdil and H. Ergin. Two-sided matching with indifferences. Working paper, 2006.

[6] A. Erdil and H. Ergin. What's the matter with tie-breaking? Improving efficiency in school choice. *American Economic Review*, 98(3):669–689, 2008.

[7] D. Gale and L. S. Shapley. College admissions and the stability of marriage. *The American Mathematical Monthly*, 69(1):9–15, 1962.

[8] D. Gusfield and R. W. Irving. *The Stable Marriage Problem: Structure and Algorithm.* MIT Press, 1989.

[9] C-C. Huang and T. Kavitha. Weight-maximal matchings. In *MATCH-UP'12*, pages 87–98, 2012.

[10] R. W. Irving, T. Kavitha, K. Mehlhorn, D. Michail, and K. E. Paluch. Rank-maximal matchings. *ACM Transactions on Algorithms*, 2(4):602–610, 2006.

[11] K. Iwama, D. Manlove, S. Miyazaki, and Y. Morita. Stable marriage with incomplete lists and ties. In *ICALP'99*, pages 443–452, 1999.

[12] K. Iwama and S. Miyazaki. Stable marriage with ties and incomplete lists. In *Encyclopedia of Algorithms*. Springer, 2008.

[13] K. Iwama, S. Miyazaki, and N. Yamauchi. A 1.875-approximation algorithm for the stable marriage problem. In *SODA'07*, pages 288–297, 2007.

[14] T. Kavitha and C. D. Shah. Efficient algorithms for weighted rank-maximal matchings and related problems. In *ISAAC'06*, pages 153–162, 2006.

[15] D. E. Knuth. *Stable marriage and its relation to other combinatorial problems : an introduction to the mathematical analysis of algorithms*. Number 10 in CRM proceedings & lecture notes. American Mathematical Society, 1997.

[16] D. F. Manlove, R. W. Irving, K. Iwama, S. Miyazaki, and Y. Morita. Hard variants of stable marriage. *Theoretical Computer Science*, 276(1-2):261–279, 2002.

[17] E. McDermid. A 3/2-approximation algorithm for general stable marriage. In *ICALP'09*, pages 689–700, 2009.

[18] D. Michail. Reducing rank-maximal to maximum weight matching. *Theoretical Computer Science*, 389(1–2):125–132, 2007.

[19] J. B. Orlin. A faster strongly polynomial minimum cost flow algorithm. *Operations Research*, 41(2):338–350, 1993.

[20] A. Roth and J. Vande Vate. Random paths to stability in two-sided matching. *Econometrica*, 58(6):1475–80, 1990.

[21] A. E. Roth and M. A. O. Sotomayor. *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*, volume 18 of *Economic Society Monographs*. Cambridge University Press, 1990.

[22] A. Schrijver. *Combinatorial Optimization: Polyhedra and Efficiency*. Springer, 2003.

[23] M. Sotomayor. The Pareto-stability concept is a natural solution concept for discrete matching markets with indifferences. *International Journal of Game Theory*, 40:631–644, 2011.

List of MI Preprint Series, Kyushu University

The Global COE Program
Math-for-Industry Education & Research Hub

MI

MI2008-1 Takahiro ITO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI
Abstract collision systems simulated by cellular automata

MI2008-2 Eiji ONODERA
The intial value problem for a third-order dispersive flow into compact almost Hermitian manifolds

MI2008-3 Hiroaki KIDO
On isosceles sets in the 4-dimensional Euclidean space

MI2008-4 Hirofumi NOTSU
Numerical computations of cavity flow problems by a pressure stabilized characteristic-curve finite element scheme

MI2008-5 Yoshiyasu OZEKI
Torsion points of abelian varieties with values in nfinite extensions over a p-adic field

MI2008-6 Yoshiyuki TOMIYAMA
Lifting Galois representations over arbitrary number fields

MI2008-7 Takehiro HIROTSU & Setsuo TANIGUCHI
The random walk model revisited

MI2008-8 Silvia GANDY, Masaaki KANNO, Hirokazu ANAI & Kazuhiro YOKOYAMA
Optimizing a particular real root of a polynomial by a special cylindrical algebraic decomposition

MI2008-9 Kazufumi KIMOTO, Sho MATSUMOTO & Masato WAKAYAMA
Alpha-determinant cyclic modules and Jacobi polynomials

MI2008-10 Sangyeol LEE & Hiroki MASUDA
Jarque-Bera Normality Test for the Driving Lévy Process of a Discretely Observed Univariate SDE

MI2008-11 Hiroyuki CHIHARA & Eiji ONODERA
A third order dispersive flow for closed curves into almost Hermitian manifolds

MI2008-12 Takehiko KINOSHITA, Kouji HASHIMOTO and Mitsuhiro T. NAKAO
On the L^2 a priori error estimates to the finite element solution of elliptic problems with singular adjoint operator

MI2008-13 Jacques FARAUT and Masato WAKAYAMA
Hermitian symmetric spaces of tube type and multivariate Meixner-Pollaczek polynomials

MI2008-14 Takashi NAKAMURA
Riemann zeta-values, Euler polynomials and the best constant of Sobolev inequality

MI2008-15 Takashi NAKAMURA
Some topics related to Hurwitz-Lerch zeta functions

MI2009-1 Yasuhide FUKUMOTO
Global time evolution of viscous vortex rings

MI2009-2 Hidetoshi MATSUI & Sadanori KONISHI
Regularized functional regression modeling for functional response and predictors

MI2009-3 Hidetoshi MATSUI & Sadanori KONISHI
Variable selection for functional regression model via the L_1 regularization

MI2009-4 Shuichi KAWANO & Sadanori KONISHI
Nonlinear logistic discrimination via regularized Gaussian basis expansions

MI2009-5 Toshiro HIRANOUCHI & Yuichiro TAGUCHII
Flat modules and Groebner bases over truncated discrete valuation rings

MI2009-6 Kenji KAJIWARA & Yasuhiro OHTA
Bilinearization and Casorati determinant solutions to non-autonomous 1+1 dimensional discrete soliton equations

MI2009-7 Yoshiyuki KAGEI
Asymptotic behavior of solutions of the compressible Navier-Stokes equation around the plane Couette flow

MI2009-8 Shohei TATEISHI, Hidetoshi MATSUI & Sadanori KONISHI
Nonlinear regression modeling via the lasso-type regularization

MI2009-9 Takeshi TAKAISHI & Masato KIMURA
Phase field model for mode III crack growth in two dimensional elasticity

MI2009-10 Shingo SAITO
Generalisation of Mack's formula for claims reserving with arbitrary exponents for the variance assumption

MI2009-11 Kenji KAJIWARA, Masanobu KANEKO, Atsushi NOBE & Teruhisa TSUDA
Ultradiscretization of a solvable two-dimensional chaotic map associated with the Hesse cubic curve

MI2009-12 Tetsu MASUDA
Hypergeometric τ-functions of the q-Painlevé system of type $E_8^{(1)}$

MI2009-13 Hidenao IWANE, Hitoshi YANAMI, Hirokazu ANAI & Kazuhiro YOKOYAMA
A Practical Implementation of a Symbolic-Numeric Cylindrical Algebraic Decomposition for Quantifier Elimination

MI2009-14 Yasunori MAEKAWA
On Gaussian decay estimates of solutions to some linear elliptic equations and its applications

MI2009-15 Yuya ISHIHARA & Yoshiyuki KAGEI
Large time behavior of the semigroup on L^p spaces associated with the linearized compressible Navier-Stokes equation in a cylindrical domain

MI2009-16 Chikashi ARITA, Atsuo KUNIBA, Kazumitsu SAKAI & Tsuyoshi SAWABE
Spectrum in multi-species asymmetric simple exclusion process on a ring

MI2009-17 Masato WAKAYAMA & Keitaro YAMAMOTO
Non-linear algebraic differential equations satisfied by certain family of elliptic functions

MI2009-18 Me Me NAING & Yasuhide FUKUMOTO
Local Instability of an Elliptical Flow Subjected to a Coriolis Force

MI2009-19 Mitsunori KAYANO & Sadanori KONISHI
Sparse functional principal component analysis via regularized basis expansions and its application

MI2009-20 Shuichi KAWANO & Sadanori KONISHI
Semi-supervised logistic discrimination via regularized Gaussian basis expansions

MI2009-21 Hiroshi YOSHIDA, Yoshihiro MIWA & Masanobu KANEKO
Elliptic curves and Fibonacci numbers arising from Lindenmayer system with symbolic computations

MI2009-22 Eiji ONODERA
A remark on the global existence of a third order dispersive flow into locally Hermitian symmetric spaces

MI2009-23 Stjepan LUGOMER & Yasuhide FUKUMOTO
Generation of ribbons, helicoids and complex scherk surface in laser-matter Interactions

MI2009-24 Yu KAWAKAMI
Recent progress in value distribution of the hyperbolic Gauss map

MI2009-25 Takehiko KINOSHITA & Mitsuhiro T. NAKAO
On very accurate enclosure of the optimal constant in the a priori error estimates for H_0^2-projection

MI2009-26 Manabu YOSHIDA
Ramification of local fields and Fontaine's property (Pm)

MI2009-27 Yu KAWAKAMI
Value distribution of the hyperbolic Gauss maps for flat fronts in hyperbolic three-space

MI2009-28 Masahisa TABATA
Numerical simulation of fluid movement in an hourglass by an energy-stable finite element scheme

MI2009-29 Yoshiyuki KAGEI & Yasunori MAEKAWA
Asymptotic behaviors of solutions to evolution equations in the presence of translation and scaling invariance

MI2009-30 Yoshiyuki KAGEI & Yasunori MAEKAWA
On asymptotic behaviors of solutions to parabolic systems modelling chemotaxis

MI2009-31 Masato WAKAYAMA & Yoshinori YAMASAKI
Hecke's zeros and higher depth determinants

MI2009-32 Olivier PIRONNEAU & Masahisa TABATA
Stability and convergence of a Galerkin-characteristics finite element scheme of lumped mass type

MI2009-33 Chikashi ARITA
Queueing process with excluded-volume effect

MI2009-34 Kenji KAJIWARA, Nobutaka NAKAZONO & Teruhisa TSUDA
Projective reduction of the discrete Painlevé system of type$(A_2 + A_1)^{(1)}$

MI2009-35 Yosuke MIZUYAMA, Takamasa SHINDE, Masahisa TABATA & Daisuke TAGAMI
Finite element computation for scattering problems of micro-hologram using DtN map

MI2009-36 Reiichiro KAWAI & Hiroki MASUDA
Exact simulation of finite variation tempered stable Ornstein-Uhlenbeck processes

MI2009-37 Hiroki MASUDA
On statistical aspects in calibrating a geometric skewed stable asset price model

MI2010-1 Hiroki MASUDA
Approximate self-weighted LAD estimation of discretely observed ergodic Ornstein-Uhlenbeck processes

MI2010-2 Reiichiro KAWAI & Hiroki MASUDA
Infinite variation tempered stable Ornstein-Uhlenbeck processes with discrete observations

MI2010-3 Kei HIROSE, Shuichi KAWANO, Daisuke MIIKE & Sadanori KONISHI
Hyper-parameter selection in Bayesian structural equation models

MI2010-4 Nobuyuki IKEDA & Setsuo TANIGUCHI
The Itô-Nisio theorem, quadratic Wiener functionals, and 1-solitons

MI2010-5 Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling and detecting change point via the relevance vector machine

MI2010-6 Shuichi KAWANO, Toshihiro MISUMI & Sadanori KONISHI
Semi-supervised logistic discrimination via graph-based regularization

MI2010-7 Teruhisa TSUDA
UC hierarchy and monodromy preserving deformation

MI2010-8 Takahiro ITO
Abstract collision systems on groups

MI2010-25 Toshimitsu TAKAESU
On the Essential Self-Adjointness of Anti-Commutative Operators

MI2010-26 Reiichiro KAWAI & Hiroki MASUDA
On the local asymptotic behavior of the likelihood function for Meixner Lévy processes under high-frequency sampling

MI2010-27 Chikashi ARITA & Daichi YANAGISAWA
Exclusive Queueing Process with Discrete Time

MI2010-28 Jun-ichi INOGUCHI, Kenji KAJIWARA, Nozomu MATSUURA & Yasuhiro OHTA
Motion and Bäcklund transformations of discrete plane curves

MI2010-29 Takanori YASUDA, Masaya YASUDA, Takeshi SHIMOYAMA & Jun KOGURE
On the Number of the Pairing-friendly Curves

MI2010-30 Chikashi ARITA & Kohei MOTEGI
Spin-spin correlation functions of the q-VBS state of an integer spin model

MI2010-31 Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling and spike detection via Gaussian basis expansions

MI2010-32 Nobutaka NAKAZONO
Hypergeometric τ functions of the q-Painlevé systems of type $(A_2 + A_1)^{(1)}$

MI2010-33 Yoshiyuki KAGEI
Global existence of solutions to the compressible Navier-Stokes equation around parallel flows

MI2010-34 Nobushige KUROKAWA, Masato WAKAYAMA & Yoshinori YAMASAKI
Milnor-Selberg zeta functions and zeta regularizations

MI2010-35 Kissani PERERA & Yoshihiro MIZOGUCHI
Laplacian energy of directed graphs and minimizing maximum outdegree algorithms

MI2010-36 Takanori YASUDA
CAP representations of inner forms of $Sp(4)$ with respect to Klingen parabolic subgroup

MI2010-37 Chikashi ARITA & Andreas SCHADSCHNEIDER
Dynamical analysis of the exclusive queueing process

MI2011-1 Yasuhide FUKUMOTO& Alexander B. SAMOKHIN
Singular electromagnetic modes in an anisotropic medium

MI2011-2 Hiroki KONDO, Shingo SAITO & Setsuo TANIGUCHI
Asymptotic tail dependence of the normal copula

MI2011-3 Takehiro HIROTSU, Hiroki KONDO, Shingo SAITO, Takuya SATO, Tatsushi TANAKA & Setsuo TANIGUCHI
Anderson-Darling test and the Malliavin calculus

MI2011-4 Hiroshi INOUE, Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling via Compressed Sensing

MI2011-5 Hiroshi INOUE
Implications in Compressed Sensing and the Restricted Isometry Property

MI2011-6 Daeju KIM & Sadanori KONISHI
Predictive information criterion for nonlinear regression model based on basis expansion methods

MI2011-7 Shohei TATEISHI, Chiaki KINJYO & Sadanori KONISHI
Group variable selection via relevance vector machine

MI2011-8 Jan BREZINA & Yoshiyuki KAGEI
Decay properties of solutions to the linearized compressible Navier-Stokes equation around time-periodic parallel flow
Group variable selection via relevance vector machine

MI2011-9 Chikashi ARITA, Arvind AYYER, Kirone MALLICK & Sylvain PROLHAC
Recursive structures in the multispecies TASEP

MI2011-10 Kazunori YASUTAKE
On projective space bundle with nef normalized tautological line bundle

MI2011-11 Hisashi ANDO, Mike HAY, Kenji KAJIWARA & Tetsu MASUDA
An explicit formula for the discrete power function associated with circle patterns of Schramm type

MI2011-12 Yoshiyuki KAGEI
Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a parallel flow

MI2011-13 Vladimír CHALUPECKÝ & Adrian MUNTEAN
Semi-discrete finite difference multiscale scheme for a concrete corrosion model: approximation estimates and convergence

MI2011-14 Jun-ichi INOGUCHI, Kenji KAJIWARA, Nozomu MATSUURA & Yasuhiro OHTA
Explicit solutions to the semi-discrete modified KdV equation and motion of discrete plane curves

MI2011-15 Hiroshi INOUE
A generalization of restricted isometry property and applications to compressed sensing

MI2011-16 Yu KAWAKAMI
A ramification theorem for the ratio of canonical forms of flat surfaces in hyperbolic three-space

MI2011-17 Naoyuki KAMIYAMA
Matroid intersection with priority constraints

MI2012-1 Kazufumi KIMOTO & Masato WAKAYAMA
Spectrum of non-commutative harmonic oscillators and residual modular forms

MI2012-2 Hiroki MASUDA
Mighty convergence of the Gaussian quasi-likelihood random fields for ergodic Levy driven SDE observed at high frequency

MI2012-3 Hiroshi INOUE
A Weak RIP of theory of compressed sensing and LASSO

MI2012-4 Yasuhide FUKUMOTO & Youich MIE
Hamiltonian bifurcation theory for a rotating flow subject to elliptic straining field

MI2012-5 Yu KAWAKAMI
On the maximal number of exceptional values of Gauss maps for various classes of surfaces

MI2012-6 Marcio GAMEIRO, Yasuaki HIRAOKA, Shunsuke IZUMI, Miroslav KRAMAR, Konstantin MISCHAIKOW & Vidit NANDA
Topological Measurement of Protein Compressibility via Persistence Diagrams

MI2012-7 Nobutaka NAKAZONO & Seiji NISHIOKA
Solutions to a q-analog of Painlevé III equation of type $D_7^{(1)}$

MI2012-8 Naoyuki KAMIYAMA
A new approach to the Pareto stable matching problem