Regenerator Location Problem in Flexible Optical Networks

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Abstract

In this study we introduce the regenerator location problem in flexible optical networks (RLP-FON). With a given traffic demand, RLP-FON considers the regenerator location, routing, bandwidth allocation and modulation selection problems jointly to satisfy data transfer demands with the minimum cost regenerator deployment. We propose a novel branch-and-price algorithm for this challenging problem. Using real world network topologies, we conduct extensive numerical experiments to both test the performance of the proposed solution methodology and evaluate the practical benefits of flexible optical networks. In particular, our results show that making routing, bandwidth allocation, modulation selection and regenerator placement decisions in a joint manner, it is possible to obtain drastic capacity enhancements when only a very modest portion of the nodes is endowed with the signal regeneration capability.

1 Introduction

Wider availability Internet access, introduction of the mobile communication devices (smart phones, tablets, etc.) and booming sector of mobile applications have taken Internet age to a new stage (Agrawal 2012). In 2011, global mobile data traffic was eight times the size of the whole Internet in 2000, and is expected to increase 18-fold by 2016 (Index 2012). As the growth of Internet surpasses even the highest estimates, utilized bandwidth of the optical fibers rapidly approaches its theoretical limits (Essiambre et al. 2010, Tomkos et al. 2012). Just worsening the problem, the rigid nature of the current optical networks cannot efficiently use available optical bandwidth to support this increasing traffic. The energy consumption of telecommunications networks is also adversely affected by wasteful resource utilization. Such inefficiencies unnecessarily increase the number of required active network equipments ultimately increasing the total energy consumption of the network. This is an issue of increasing importance since the power consumption of Internet is estimated to reach to the 10% of the worldwide energy consumption in a very short notice (Plan 2007). In the US alone, a 1% saving in the energy consumption of Internet due to the adoption of energy efficient network management strategies is estimated to result in savings of $5 billion per year given that the price of electricity is 17 cents per kWh (Shen and Tucker 2009). The concern over climate change and heavy carbon footprint of energy generation only increases the importance of energy efficiency of telecommunications networks.

Motivated by this urgent practical problem, researchers developed the flexible optical network (FON) architecture that can flexibly choose its transmission parameters according to the varying traffic conditions and significantly increase the resource utilization efficiency (Essiambre et al. 2010, Tomkos et al. 2012).

The major sources of these inefficiencies, remedies offered by FON architecture and the algorithmic challenges raised by the adoption of this novel technology can be summarized as follows.

In the current optical network architecture, the available bandwidth is divided into a set of fixed bandwidth channels each serving a single transmission demand. However, due to the increasing variability of the offered online services, the capacity demands of connections come from a much broader range with granularities of several gigabits per second to 100 Gb/s or more. Due to the granularity mismatch between the widths of these channels and demand sizes, the already drained fiber bandwidth cannot be fully utilized (Jinno et al. 2009). On the other hand, with FON, the optical spectrum is divided into fine bandwidths called *slots* and custom-size bandwidths are created by the contiguous concatenation of those slots. Such custom-size transmission channels can significantly reduce the bandwidth waste and increase the amount of available fiber bandwidth.

The data transfer capacity of backbone networks is not solely dependent on the range of the available bandwidth. Indeed, this capacity is jointly determined by the available bandwidth range of the fiber and the modulation level that induces the amount of data that could be transferred on a fixed bandwidth. Modulation levels with higher bit rates can carry more data on a given bandwidth. But the down side of using higher bit rate modulations is the shorter optical reach which is defined as the maximum distance a signal can traverse before its quality degrades. As a significant source of inefficiency, current optical networks use fixed modulation levels and waste bandwidth by using the same modulation level for both the short and long distance transmissions (Jinno et al. 2010). FON has been designed to increase the data transmission capacity of optical networks by smartly managing routing and modulation level selection in coordination and in particular, by utilizing high bit rate modulation schemas to increase bandwidth efficiency. However, implementing such an approach is quite challenging due to the optical reach limitations. Optical reach is a decreasing function of the bit rate and optical reach limitations can significantly restrict the potential gains of flexible modulation selection. One key technology to extend optical reach and overcome this issue is opto-electro-optical (OEO) regeneration. Processed by an OEO regenerator, the optical signal is rejuvenated and after this renewal it can travel up to its optical reach before it arrives to a new regenerator or its final destination. So, with the expense of more capital investment and operating cost (such as energy and maintenance), it is possible to enhance the optical reach of a signal by employing regenerator equipments. Moreover, FON also allows different modulation levels at each segment of a light-path that connects the source of a demand to its destination possibly passing through several regenerators to maintain a certain level of signal quality. So with this new architecture, it is possible to use regenerators to switch modulation formats on a light-path such that the spectrum allocation is minimum while the signal quality is within the predefined limits.

Since the OEO regenerators are expensive devices to obtain and operate, there is great motivation to design/operate optical networks with few regeneration points. In short, the better exploitation of the opportunities offered by the FON architecture requires the solution of routing, bandwidth allocation, modulation level selection and regenerator location problems jointly. The problem of solving all these problems concurrently is a challenging one. Indeed, researchers indicate that lack of such an efficient algorithm constitutes a major barrier for the adoption of this novel technology (Tomkos et al. 2012). Despite this urgent need, due to the novelty and the high complexity of the problem, there are not so many studies in the literature that address regenerator location problem in flexible optical networks. Considering a static demand structure and assuming fixed routes for each transmission demand Klinkowski (2012b) proposes a heuristic algorithm for jointly solving spectrum allocation and regenerator location problems. Similar to our results, his numerical experiments indicate that a smart placement of regenerators could significantly increase bandwidth efficiency in the network. Relaxing the fixed route assumption, Kahya (2013) presents a sequential solution heuristic approach to solve routing, regenerator location and spectrum allocation problems in flexible optical networks. In this study a fixed modulation level is assumed.

Motivated by the recent developments in virtualized elastic regenerator (VER) technologies that enable efficient regeneration of various bandwidth super-channels Jinno et al. (2015) proposes a heuristic algorithm for calculating the minimal VER placement, routing, and the least congestion resources assignment in a translucent FON based on Nyquist wavelength-division multiplexing (WDM) super-channels. The authors assume a single modulation level and present computational results which indicate significant efficiency gains in the network due to the strategic VER placements. In a recent study, Wang et al. (2014) investigate the impact of modulation conversion in FONs. The authors present an MIP formulation to solve RPP-FON. Since their formulation cannot solve realistic size problems they propose a sequential solution heuristic in which they randomly partition the demand set. Their computational study results show that benefiting from the elastic network structure, proper use of signal regenerators and wavelength converters can significantly decrease the bandwidth requirement depending on the topology of the network. To the best of our knowledge, our study is the first one to present an exact algorithm to solve routing, bandwidth allocation, modulation level selection and regenerator location problems jointly for realistic size problem instances.

Even in the current networks, regenerators are crucial elements as regeneration costs make up a significant portion of a network's set-up and management costs (Yang and Ramamurthy 2005). Motivated by the practical considerations, regenerator location problem (RLP) which tries to find the minimum cost regenerator deployment to facilitate communication between the network nodes has attracted significant research effort in the recent years. Yetginer and Karasan (2003) are the first to introduce the sparse regenerator placement in a static routing environment. Taking the geographical aspect of the RLP into account, Chen et al. (2009) introduce it to the operations research literature, proving its NP-completeness and showing that it can be modeled as a special Steiner arborescence problem. Pointing to the equivalence of the maximum leaf spanning tree problem, the minimum connected dominating set problem and RLP, Sen et al. (2008), Lucena et al. (2010) and more recently Gendron et al. (2012) suggest several exact and heuristic algorithms for RLP. Addressing the network survivability concerns, Yildiz and Karasan (2015) introduce two new facets to the problem. They formulate the RLP as a mixed integer program (MIP) and present an efficient branch-and-cut algorithm which they extend to solve regenerator reliable and node reliable versions of the problem. In none of these studies, fiber capacity constraints are addressed. Pavon-Mariño et al. (2009) explicitly address the fiber bandwidth capacities and study regenerator location problem in a static demand environment. Different than our work, the authors assume single modulation level and do not consider FON architecture. An MIP formulation that contains a large number of variables is presented. Two heuristic algorithms are proposed to solve large problem instances that cannot be solved by the MIP formulation. In a recent study, considering the FON setting and multiple modulation levels, Castro et al. (2012) investigate dynamic demand routing and spectrum allocation problem (RSA). The authors present a high quality heuristic that can solve dynamic RSA problem and propose a spectrum reallocation algorithm to deal with the spectrum fragmentation problem which can significantly limit the available fiber bandwidth. Different than this study, the optical reach constraints are not examined.

In this study we introduce the regenerator location problem in flexible optical networks (RLP-FON). RLP-FON seeks the best routing, modulation level and regenerator location combination that minimizes the regenerator deployment costs while not using more than a predetermined portion of the fiber bandwidth. In other words, promoting the exclusive capabilities of the new FON architecture, RLP-FON finds the minimum amount of network resources needed to satisfy a given set of transmission demands. Since FON architecture is quite new, despite its immense practical importance, this theoretically challenging problem is not well studied in the literature and this study is a first attempt to close this gap.

RLP-FON arises both in the design and network management layers. In the design phase, RLP-FON determines the minimum amount of network equipment (regenerators, router chassis, optic line cards etc.) required to satisfy the targeted demand whereas in the network operation layer RLP-FON can help reduce the operating cost of the network by identifying the network elements that could be put into sleep when the actual demand is less than the maximum supported demand size. The significance of the latter can be better understood considering the fact that optical backbone networks are designed somewhat to support the worst case demand scenarios and peak demand is more than two times larger than the minimum observed on the same day (Rizzelli et al. 2012). Indeed, motivated by such an opportunity, hardware developers intensified their research and development efforts on manufacturing network devices with capabilities to go to sleep mode to save energy.

Path based formulations are very powerful to model problems for which the amount of cost incurred/profit gained or resource depleted depends on the routes chosen. As such, they are widely used in network design and management problems in telecommunications and transportation. Despite their advantages, path based formulations usually suffer from the exponential number of variables with only a fraction of them actually appearing in a feasible solution. Column generation and branch-and-price methods have been successfully applied to those problems to develop efficient algorithms (Parker and Ryan 1993, Barnhart et al. 1994, Park et al. 1996, Barnhart et al. 1998, 2000, Cohn and Barnhart 2003, Degraeve and Jans 2007, Desaulniers 2010).

Within RLP-FON, for each transmission demand a routing problem is solved to find a path that connects source and sink nodes and that respects regeneration constraints. However, with a path based formulation, it is hard to address signal regeneration constraints while generating new columns in a column generation framework. Because of that we define path-segments as the simple paths on which the signal does not get into a regeneration and build the routes by the proper concatenation of those path-segments.

In this study we:

- Introduce RLP-FON problem that adds two new facets to RLP:
  - RLP-FON jointly solves routing, modulation level selection and regenerator location problems.
  - RLP-FON respects the bandwidth capacity limitations of the fiber links.
- Present a path-segment formulation for RLP-FON and develop a branch-and-price algorithm to solve it. To the best of our knowledge this is the first study in which path-segments instead of paths are used as the variables in a column generation framework.
- Conduct extensive numerical experiments on realistic reference network topologies to test the computational performance of the proposed algorithm and to offer managerial insights. In particular, results of these experiments show that a strategic deployment of regenerators on a small portion of nodes can achieve capacity enhancements comparable to the case where all the nodes in the network have regeneration capability.

2 Mathematical Model

In this section we formally define RLP-FON and present the details of the proposed branch-and-price algorithm.

2.1 Problem Definition and Notation

For each connection demand, there is a certain amount of data at the origin coded into optical signals to be carried to the destination in a unit time. This coding is done with one of the technologically available modulation levels. For an optical signal, the chosen modulation level determines the number of bandwidth slots required to transfer this signal on a fiber link and sets the optical reach, i.e., the maximum distance to be covered before a regeneration. Higher modulation levels use optical bandwidth more efficiently (require less bandwidth) but they have shorter optical reach. An optical signal is a light-path, that is a path from the source node to the destination node in the given optical network. When regenerator nodes are visited on this path, the light-path can be viewed as the concatenation of several path-segments where a path-segment is a simple path joining two consecutive regenerators on the light-path or joining a regenerator with the source or the destination node. In other words, except for the one that ends in the destination node, at the end of each path-segment there is a regenerator that restores the signal quality. Regenerators are also capable of re-coding and emitting the incoming signal with a different modulation level. i.e. regenerators have the capability to switch the modulation level of an optical signal. Each fiber link in the network has a certain bandwidth capacity which will be consumed by the light-paths passing through it. Considering all the demands simultaneously, a solution for the RLP-FON needs to respect these bandwidth capacities of fibers. Moreover, the modulation level and the path-segment chosen for a particular demand should be in harmony with respect to optical reach considerations. Thus, the solution of the problem consists of the routing decisions for each demand, location decisions of the regeneration equipment and the modulation level selections to be used for each demand on each one of its path-segments on its light-path. The objective is to find a solution that minimizes the regenerator deployment cost and obeys the link capacity and optical reach constraints.

We now provide some notation for formalism. Let the undirected weighted graph $G = (N, E)$ represent a flexible optical network instance with node set $N = \{1, 2, 3, \dots n\}$ and edge set E. Edge lengths and the total number of slots that exist on each fiber link $e \in E$ are denoted by $l(e) \geq 0$ and $c(e) \in \mathbb{N}$ respectively. The cost for regenerator placement in node $i \in N$ is denoted by h_i. Induced by the edge set E we define the arc set A which contains two arcs $\bar{e} = (i, j)$ and $\underline{e} = (j, i)$ for each edge $e = \{i, j\} \in E$ with $l(\bar{e}) = l(\underline{e}) = l(e)$. We define $M = \{1, 2, \dots, \mu\}$ as the set of modulation levels and assume that the m^{th} component of vector $\Delta = (\Delta^1, \dots, \Delta^\mu)$, is the threshold of regeneration-free communication (optical reach) for the modulation level m. In other words, two nodes with distance at most Δ^m can communicate without any need for signal regeneration using modulation m. We assume without loss of generality that $\Delta^m \geq \Delta^{\bar{m}}, \forall m < \bar{m}$ and $l(e) \leq \Delta^1$ for every $e \in E$ since any edge violating this condition can simply be deleted from G.

Another problem instance parameter is the set of transmission demands $D = \{1, 2, \dots, \delta\}$. For each $d \in D$, we denote $\mathcal{S}(d)$ as the source node, $\mathcal{T}(d)$ as the destination node and define $\psi(d)$ as the data rate demanded by d. The number of slots a demand d requires on any fiber link is a function of the modulation level chosen, say m, and the required data transfer rate of the demand $\psi(d)$. For practical purposes, it is important to note that for the same transfer rate $\psi(d)$, higher modulation levels require less bandwidth (less number of slots) but have more limited optical reach. We define $v(d, m)$ as the number of slots a path-segment of demand d occupies on each fiber optical link it traverses for a chosen modulation level m.

A directed path is a sequence of arcs $(a_1, a_2, \dots, a_\beta)$ with $a_i = (n_{i-1}, n_i) \in A, \forall i = 1, \dots, \beta$ and $n_i \in N$ for $i = 0, \dots, \beta$. The directed path is non-simple if it repeats nodes and simple otherwise. Our formulation depends on the notion of *path-segments*. A *path-segment* p is a directed simple path with an associated modulation level $m(p)$. Thus, by associating different modulations to the same simple path, it is possible to generate different path-segments. We denote the source and destination nodes of a path-segment p as $s(p)$ and $t(p)$, respectively. For each *path-segment* p we denote $\bar{p}$ as the set of edges $e \in E$ such that p passes through $\bar{e}$ or $\underline{e}$ and define the indicator function $I(e, p)$ that returns 1 if $e \in \bar{p}$ and 0 otherwise. The length of a path-segment $l(p) = \sum_{e \in \bar{p}} l(e)$ is the sum of the lengths of the edges contained in $\bar{p}$. In our formulation we only consider path-segments with total length less than the optical reach of the associated modulation level and call such path-segments as *feasible*. More formally a path-segment p is feasible if $l(p) \leq \Delta^{m(p)}$. We define $\mathcal{P}$ as the set of all those feasible path-segments.

A light-path $P = (p^1 \dots, p^k)$ is an ordered union of path-segments $p^i, i \in 1, \dots, k$ where $t(p^i) = s(p^{i+1})$ forall $i = 1, \dots, k-1$. We call a light-path feasible for a demand $d \in D$ if $s(p^1) = \mathcal{S}(d), t(p^k) = \mathcal{T}(d)$, $l(p^i) \leq \Delta^{m(p^i)}, i \in 1, \dots, k$, and $t(p^i)$ is a regenerator node $\forall i = 1, \dots, k-1$. The regenerator usage cost for each transmission demand is denoted as η.

A solution for RLP-FON is allowed to use only a portion $\alpha \in (0, 1]$ of the available transmission capacity (bandwidth slots) on a link. That is, the number of *slots* available on a link $e \in E$ is given by $\lfloor c(e) \times \alpha \rfloor$. The parameter α actually represents a managerial decision. Due to the quality of service considerations (such as uninterrupted service, accommodating unexpected demands etc.) network management does not want to use all the existing bandwidth of a link and smaller values of α are preferred. However, smaller values for α limit the data transfer capacity of the network and may increase the required number of regenerators. A more detailed discussion about this topic is presented in Section 4. The notation we use throughout this paper is outlined in Table 1.

The formal definition of RLP-FON is as follows.

Definition 1. *Regenerator Location Problem for Flexible Optical Networks (RLP-FON): An RLP-FON instance has associated data $\langle G, D, M, l, c, h, v, \alpha, \eta \rangle$. The aim is to find the minimum cost regenerator deployment and signal routing such that for each $d \in D$ there is a feasible light-path P^d in G such that for all links $e \in E$, $\sum_{d \in D} \sum_{p \in P^d} I(e,p) v(d, m(p)) \leq \lfloor c(e) \times \alpha \rfloor$.*

Table 1: Outline of notation

G	: Input graph representing the optical network
N	: Set of nodes in G; $N = \{1, \dots, n\}$
E	: Set of edges in G
A	: Arc set induced by E
D	: Set of connection demands; $D = \{1, 2, \dots, \delta\}$
M	: Set of modulation levels; $M = \{1, \dots \mu\}$
$\mathcal{P}$	: Set of feasible path-segments
$l(e)$	: Length of an edge $e \in E$
$c(e)$	: Number of slots that exist on each fiber link $e \in E$
$\mathcal{S}(d)$	: Source node of a connection demand $d \in D$
$\mathcal{T}(d)$	: Destination node of a connection demand $d \in D$
$s(p)$	: Source node of a path-segment $p \in \mathcal{P}$
$t(p)$	: Destination node of a path-segment $p \in \mathcal{P}$
$m(p)$	: Modulation level used by path-segment $p \in \mathcal{P}$
$\bar{p}$	: Set of edges a path-segment $p \in \mathcal{P}$ passes through
$I(p, e)$	: Indicator function that returns 1 if $e \in \bar{p}$ and 0 otherwise
h_i	: Cost for regenerator placement in node $i \in N$
η	: Regenerator usage cost
α	: Maximum link utilization ratio
Δ^m	: Optical reach for the modulation level $m \in M$
$v(d, m)$	: Number of slots required by demand $d \in D$ transmitted via modulation level $m \in M$

2.2 RLP-FON Path-Segment Formulation (PS)

In this subsection we present the path-segment formulation PS for RLP-FON and provide the details of the proposed branch-and-price algorithm to solve it.

Recall that each demand d is required to follow a union of path-segments from $\mathcal{S}(d)$ to $\mathcal{T}(d)$ with a regenerator at the end of each used path-segment p for which $t(p) \neq \mathcal{T}(d)$. As such, our path-segment formulation admits a very natural representation of signal regeneration constraints. We define the following decision variables.

$$r_i = \begin{cases} 1, & \text{if node } i \in N \text{ is a regeneration point} \\ 0, & \text{otherwise,} \end{cases}$$

$$x_p^d = \begin{cases} 1, & \text{if demand } d \in D \text{ uses path-segment } p \\ 0, & \text{otherwise,} \end{cases}$$

We name $r_i, i \in N$ as the regeneration variables and $x_p^d, d \in D, p \in \mathcal{P}$ as the arc flow variables. With these decision variables, PS can be stated as follows.

$$\min \sum_{i \in N} h_i r_i + \sum_{\substack{d \in D \\ p \in \mathcal{P} \\ t(p) \neq \mathcal{T}(d)}} \eta x_p^d \tag{1}$$

$$\text{s.t.} \sum_{\substack{p \in \mathcal{P}, \\ s(p)=i}} x_p^d - \sum_{\substack{p \in \mathcal{P}, \\ t(p)=i}} x_p^d = \begin{cases} 1, & \text{if } i = \mathcal{S}(d) \\ \text{-1}, & \text{if } i = \mathcal{T}(d) \\ 0, & \text{otherwise} \end{cases} \qquad \forall i \in N, d \in D, \tag{2}$$

$$\sum_{d \in D} \sum_{\substack{p \in \mathcal{P}: \\ e \in \bar{p}}} v(d, m(p)) x_p^d \leq \lfloor c(e) \times \alpha \rfloor \qquad \forall e \in E, \tag{3}$$

$$\sum_{\substack{p \in \mathcal{P}: \\ t(p)=i}} x_p^d \leq r_i \qquad \forall d \in D, i \in N \setminus \mathcal{T}(d) \tag{4}$$

$$r_i \in \{0, 1\} \qquad \forall i \in N, \tag{5}$$

$$x_p^d \in \{0, 1\} \qquad \forall d \in D, p \in \mathcal{P}. \tag{6}$$

The objective function (1) has two components. The first one represents the fixed cost of setting up a regenerator site on a node. Since some of its constituents can be node dependent, regenerator placement cost h_i can take different values for different nodes $i \in N$. The second piece represents the cost of adding a new regenerator device to a regenerator site. Depending on the features of the practical setting, this cost can also represent the signal regeneration cost incurred at the end of a path-segment. Note that, with this flexible construction, the objective function is quite general accommodating the cost structure of a wide range of practical problems. In the next section we present a more detailed discussion on this topic. Constraints (2) are the flow balance equations that force each demand to be carried from its source to its destination by the concatenation of feasible path-segments. Constraints (3) are the capacity constraints which ensure that the number of slots occupied is not more than the maximum allowed. Constraints (4) enforce regeneration requirements by ensuring regeneration at the end of each feasible path-segment that does not end in the destination node of the associated demand. Constraints (5)-(6) are the domain restrictions. Note that this formulation is equivalent to a flow formulation on a network where different path-segments between pairs of nodes are simply represented by parallel arcs.

Although this formulation considers a static problem where all the connection demands are known/estimated and regenerator locations are chosen accordingly, it is flexible enough to cover a more realistic case where demands emerge in an incremental manner. That is when there are a collection of flows already and we need to solve the problem to accommodate additional flows incurring minimum additional cost. In that case we can simply fix $r_i = 1$ or assume $h_i = 0$ for those regenerators that are in use (already located) to enable existing light-paths. Regarding the edge capacities in the new problem we have two alternatives. If the initial routes have to remain fixed we would reduce the number of available bandwidth slots on the edges which have already been used. On the other hand, if we have the rerouting capability there is no need to update edge capacities and we can solve the problem considering existing and new demands together by simply updating regenerator placement costs as stated above.

2.3 Solution Approach:

In this section we present a novel branch-and-price algorithm to solve PS. During the algorithm, column generation technique is employed to solve the linear relaxation of PS, say PS-LP, and obtain a lower bound for each node of the branch-and-bound tree.

2.3.1 LP Solution (Column Generation):

Pricing Problem: Let RPS be the restricted PS-LP formulation with a fraction of its columns. At every iteration we determine whether there exists a column with negative reduced cost such that including it in the RPS might improve the objective function. If such columns are detected we add them to the RPS and repeat the procedure until there is no column left with a negative reduced cost.

Let π_i^d represent the unrestricted dual variables associated with Constraints (2), and κ_e and γ_i^d be the nonnegative dual variables associated with constraints (3) and (4), respectively. For a path-segment p of demand d the reduced cost $\bar{c}_p^d$ for a fixed modulation level m is given as:

$$\bar{c}_p^d = \begin{cases} \pi_{t(p)}^d - \pi_{s(p)}^d + \sum_{e\in\bar{p}} v(d,m)\kappa_e, & \text{if } t(p) = \mathcal{T}(d) \\ \pi_{t(p)}^d - \pi_{s(p)}^d + \sum_{e\in\bar{p}} v(d,m)\kappa_e + \gamma_{t(p)}^d + \eta, & \text{if } t(p) \neq \mathcal{T}(d) \end{cases} \tag{7}$$

Definition 2. *An ordered node pair $(i,j), \in N \times N$ is called a plausible-pair for demand d if the potential difference:*

$$\pi_{(i,j)}^d = \begin{cases} \pi_j^d - \pi_i^d, & \text{if } j = \mathcal{T}(d) \\ \pi_j^d - \pi_i^d + \gamma_j^d + \eta, & \text{if } j \neq \mathcal{T}(d) \end{cases} \tag{8}$$

is negative. The set of all the plausible-pairs for demand d is denoted by Π^d.

In order to identify columns that price out it is required to pick out plausible-pairs (i,j) for each demand $d \in D$ and check whether there exists a path p of modulation m from node i to j with length $\sum_{e\in\bar{p}} v(d,m)\kappa_e < -\pi_{(i,j)}^d$. If the signal regeneration was not necessary, such a path could be efficiently identified by solving a shortest path problem for each modulation level $m \in M$, over a graph with arc costs equal to $v(d,m)\kappa_e$ for each arc $\bar{e}, \underline{e} \in A$. However, a path-segment p is feasible only if it satisfies signal regeneration constraint $l(p) = \sum_{e\in\bar{p}} l(e) \leq \Delta^{m(p)}$. Thus, the pricing problem actually requires to solve a number of *constrained shortest path problems (CSP)* (Garey and Johnson 1979). Recall that given a directed graph with costs and resources associated with arcs, the CSP problem seeks a minimum cost path from a given source node to a given destination node with a side constraint on the total resource of the path. Our pricing problem can be solved exactly by solving a CSP instance from node i to node j on a directed graph (N, A) where the cost is $v(d,m)\kappa_e$ and the resource is $l(e)$ for each $\bar{e}, \underline{e} \in A$ and the resource limit is Δ^m. We denote this pricing graph as G_m^d.

Since the number of plausible-pairs is $O(n^2)$ and since CSP is NP-Hard (Garey and Johnson 1979), we propose a heuristic method (H_k) to solve the pricing problem and resort to the exact solution of CSP, which employs the solution approach proposed by Santos et al. (2007), if the heuristic method fails to produce a negative reduced cost column.

For each node pair $(i, j) \in N \times N$, paths with short lengths are *good* paths in a sense that they can support higher bit rate modulations and thus use less network resources to transmit data. Therefore, those paths are more likely to be detected as solutions of the pricing problem. Thus, it is a fruitful idea to store some limited number, say k, of those *good path-segments* and at each pricing step check those paths first before resorting to the costly solution of CSP.

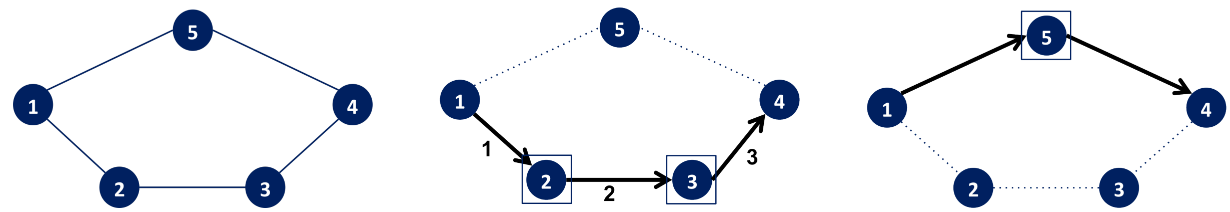
Let $P^k_{(i,j)} = \{p^1_{(i,j)}, p^2_{(i,j)}, \dots, p^k_{(i,j)}\}$ be the set of k-shortest paths from node i to node j in G with non-decreasing order of lengths. For notational simplicity, we also define the cost of the path-segment $p \in P^k_{(i,j)}$ in the pricing graph G^d_m as $l^d_m(p) = \sum_{e \in \bar{p}} v(d, m)\kappa_e$. Initially, we store k-shortest paths for each node pair (i, j) and call Algorithm 1 (H_k) to detect negative reduced cost path-segments.

Algorithm 1: H_k

Input: $\langle G, D, M, P^k_{(i,j)}, \pi, \kappa, \gamma \rangle$
Output: $\langle \Omega \rangle$
begin
  Set $\Omega = \emptyset$
  forall the $d \in D$ **do**
    forall the $(i, j) \in \Pi^d$ **do**
      Set $m = |M|$
      Set $GoToNextPair = false$
      while $m > 0$ *or* $GoToNextPair = false$ **do**
        Set $\sigma = 1$
        while $\sigma \leq k$ *or* $GoToNextPair = false$ **do**
          if $\Delta^m \geq l(p^\sigma_{(i,j)})$ *and* $\pi^d_{(i,j)} + l^d_m(p^\sigma_{(i,j)}) < 0$ **then**
            $\Omega = \Omega \cup \{p^\sigma_{(i,j)}\}$
            $GoToNextPair = true.$
          end
          $\sigma = \sigma + 1$
        end
        $m = m - 1$
      end
    end
  end
end

If H_k for a chosen k returns $\Omega = \emptyset$, then we continue with the exact solution methodology. When solving the pricing problem after the application of H_k there is no need to consider plausible node pairs (i, j) such that $l(p^k_{(i,j)}) > \Delta^m$. Thus, exact solution of the pricing problem requires significantly less computational effort when we first apply H_k. Note that, once we have the solution of the k-shortest path problem, which we solve just once at the very beginning, H_k requires $O(n^2|D|\mu)$ time when seeking for a negative reduced cost column among all plausible pairs, all demands $d \in D$ and modulation levels $m \in M$. So the total time complexity of the heuristic solution of the pricing problem is bounded by $O(kn(|A| + nlogn) + n^2|D|\mu)$.

Determining an Initial Set of Columns: Defining variables as the path-segments instead of light-paths diverts from the widely used path based formulations for which column generation technique has been applied very successfully for a wide range of problems (see Lübbecke and Desrosiers (2005) for a detailed survey). Path-segments as variables necessitate a more careful approach to determine the initial variable pool. In a typical column generation algorithm it is sufficient to have a feasible solution at hand



(a) Network instance (b) Initial columns (bold lines) (c) Optimal solution

Figure 1: A simple example depicting the fact that initial columns matter

to start the procedure. However, in PS-LP, it is not enough to start with an arbitrary feasible solution. Figure 1 depicts a simple problem instance where there is only one level of modulation with the maximum optical reach of 1 unit. This instance contains a single data transfer demand from 1 to 4 for which just one bandwidth slot is enough to carry the signal with the available modulation. Links contain two slots and have lengths of 1 unit. All nodes have unit regenerator placement costs and $\eta = 0$. Figure (1b) shows an initial column pool that consists of three path-segments (numerically depicted). With these columns, one can build a feasible solution that requires two regenerators (at nodes 2 and 3). Figure (1c) depicts the optimal solution with one regenerator at node 5. However, starting with the three path-segments given at (b) there is no way to detect negative reduced cost columns and move to a better feasible solution. Thus, PS-LP is stuck with the initial solution and cannot obtain the optimal solution from here.

We apply Algorithm 2 to obtain the set of initial variables. Note that the data transfer capacity of the network is maximum when all the nodes have the regeneration capability and each link $e \in E$ uses the most bandwidth efficient (highest bit rate) modulation level m^* such that $l(e) \leq \Delta^{m^*}$. Thus, if the restricted problem with the column set Ω_0 is infeasible, PS-LP is infeasible as well. Moreover, for each (d, a) pair $d \in D, a \in A$, there exists a variable x_p^d in Ω_0 with $p = a$. Thus, values of all the dual variables can be properly calculated once a solution is obtained with the variables in Ω_0. Thus, employing Algorithm 2 we can obtain the initial set of columns in $O(\mu|A|)$ time.

Algorithm 2: Initial variable set generation

```
Output: ⟨Ω0⟩
begin
    Set Ω0 = ∅
    forall the a ∈ A do
        -find the highest bit rate modulation m* such that Δ^{m*} ≥ l(a)
        -build the single arc path-segment p = a with modulation level m(p) = m*
        -Set Ω0 = Ω0 ∪ {x_p^d | d ∈ D}
    end
end
```

2.3.2 IP Solution

Branching Rules: One key step towards developing an effective branch-and-price algorithm is to identify a branching rule which eliminates the fractional solutions but does not disrupt the special structure of the pricing problem. Keeping this in mind we propose the following two branching rules: one for the regeneration variables r_i and one for the arc flow variables x_p^d.

Branching on Regeneration Variables: Encountering a fractional solution we first detect fractional regeneration variables and branch on the variable $0 < r_i < 1$ where $i = argmin_{j \in N}\{|r_j - 0.5|\}$.

Note that in formulation PS arc flow variables x_p^d are tied to the regeneration variables r_i by the constraints (4). Thus, branching decisions on regeneration variables affect a significant number of arc flow variables. Let r_i have a fractional value:

- **Branching-cut-1** $r_i = 0$ **:** In this case the set of arc flow variables $\underline{X_i} = \{x_p^d | d \in D, \mathcal{T}(d) \neq i \text{ and } p \in \mathcal{P}, t(p) = i\}$ are implicitly set to 0. Thus, we must make sure that in the pricing problem any path-segment $x_p^d \in \underline{X_i}$ should not appear as a negative reduced cost column. This can be easily done by setting $\gamma_i^d = \infty \quad \forall d \in D, \mathcal{T}(d) \neq i$. Note that with this modification only the lengths of the arcs in the pricing graph are changed and the structure of the pricing problem is not affected.

- **Branching-cut-2** $r_i = 1$ **:** Implementation of this branching cut is straightforward and does not require any change in the pricing problem.

Branching on Arc Flow Variables: Our branching rule on the arc flow variables is closely related with the one proposed by Barnhart et al. (2000). For a branching rule which is based on the arc flow variables, it is very likely that branching cuts destroy the special structure of the pricing problem. One remedy is to consider original links and base the branching decisions on the usage of an arc in A by a demand $d \in D$.

We derive our branching rule by observing that if an arc flow variable x_p^d has a fractional value, then there must exist a node $i \in N$ such that there are at least two variables $x_{p^1}^d > 0, x_{p^2}^d > 0$ where $s(p^1) = s(p^2) = i$. We call node i as the root node. For the distinct path-segments p^1 and p^2, starting with the *root node* and inspecting one arc at a time we can find two different arcs a^1 and a^2 where $s(a^1) = s(a^2) = \bar{i}$. The node $\bar{i}$ is called the *divergence node*. We denote the set of arcs originating from $\bar{i}$ as $A(\bar{i})$ and let $A(\bar{i}, a^1)$ and $A(\bar{i}, a^2)$ represent a partition of $A(\bar{i})$ where $A(\bar{i}, a^1)$ contains a^1 and $A(\bar{i}, a^2)$ contains a^2. Let $\mathcal{P}(a)$ denote the set of path-segments containing arc $a \in A$. Now consider the following two sets of arc flow variables.

- $X^1 = \{x_p^d | s(p) = i, p \in \mathcal{P}(a) \text{ for some } a \in A(\bar{i}, a^1)\}$
- $X^2 = \{x_p^d | s(p) = i, p \in \mathcal{P}(a) \text{ for some } a \in A(\bar{i}, a^2)\}$

The main idea for the branching rule for the flow variables follows from the observation that in the optimal solution either arc flow variables X^1 or those of X^2 are all set to 0.

- **Branching-cut-1** $\sum_{x_p^d \in X^1} x_p^d = 0$ **:** In this case the set of arc flow variables X^1 are set to 0. Let i be the root node and $\bar{i}$ be the divergence node. In order to force this constraint in the following pricing problems we simply remove arcs $A(\bar{i})$ from the arc set of the constrained shortest path instances $\langle G_m^d, i, j, l_m^d, l, \Delta^m \rangle \quad \forall m \in M, d \in D$ and $j \in N$, where G_m^d is the input graph, i is the origin node, j is the destination node, l_m^d is the cost vector and l is the resource vector. Similarly for H_k we can simply update $l_m^d(\bar{e}) = \infty, \forall \bar{e} \in A(\bar{i})$ when calculating the lengths of the paths $p \in P_{(i,j)}^k \forall j \in N$.

- **Branching-cut-2** $\sum_{x_p^d \in X^2} x_p^d = 0$ **:** The implementation of this branching cut is analogous to the previous one.

2.4 Heuristic Solutions

The bulk of the columns generated in the branch-and-price algorithm is actually obtained during the column generation in the root node. Thus, solving the problem with only those columns can provide a good heuristic for RLP-FON. We call this heuristic as H-Root and apply it to obtain an initial feasible solution to reduce the overall size of the branch-and-bound tree. Obviously a similar procedure can be applied at any given branch-and-bound node (other than the root node). Indeed, during our implementation phase, at the end of some definite intervals we pause the branch-and-price algorithm and try to find an integer solution with the columns generated so far.

3 Insights to Problem Complexity

In this section we investigate theoretical results about the complexity of RLP-FON and its special cases in an attempt to understand the challenges involved. We start by presenting the computational complexity of RLP-FON.

Theorem 1. *RLP-FON is NP-Hard.*

Proof. Chen et al. (2009) prove that Regenerator Location Problem (RLP) is NP-Hard. The result follows from the observation that RLP is a special case of RLP-FON where:

- only a single modulation level m is considered,
- regenerator usage cost η is zero,
- between any node pair $(i, j) \in N \times N$ such that $i < j$, the set D contains a demand $d_{i,j}$ with a fixed transmission rate, i.e., we have $\sigma = v(d, m)$ for each demand $d \in D$,
- all the links $e \in E$ have capacities $c(e) = \frac{n(n-1)}{2}\sigma$.

□

Establishing the computational complexity of RLP-FON, the next question is to explore what makes this problem hard. For most network problems properties of the input graph is an important dimension in an answer to this question. For various well know NP-Hard problems there are polynomial time algorithms to solve them if the input graph has some special structure. However, as we show by the next theorem, this is not the case for RLP-FON which retains its computational challenge even when we consider a tree as an input graph.

Theorem 2. *RLP-FON is NP-Hard even if the input graph is a tree.*

Proof. We provide a polynomial time reduction that transforms an arbitrary 0-1 knapsack instance into an RLP-FON instance on a tree.

Consider a knapsack instance with n items of capacity W with z_i corresponding to the value and w_i corresponding to the weight of item $i \in \{1, \dots, n\}$ respectively. We construct an RLP-FON instance as follows.

- $N = \{s, \bar{s}, \bigcup_{i=1}^{n} t_i, \bigcup_{i=1}^{n} \bar{t}_i\}$,
- $E = \{\{s, \bar{s}\}, \bigcup_{i=1}^{n} \{\bar{s}, \bar{t}_i\}, \bigcup_{i=1}^{n} \{t_i, \bar{t}_i\}\}$,
- $l(e) = 1$ and $c(e) = W \ \forall e \in E$,
- $h_s = h_{\bar{s}} = \infty$, $h_{t_i} = 0$ and $h_{\bar{t}_i} = z_i$ for $i \in \{1, 2, ..., n\}$, and $\eta = 0$,

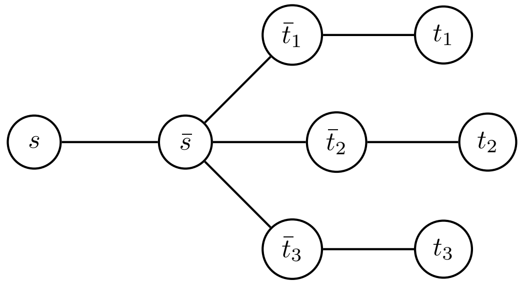


Figure 2: Depiction of a small transformation example with three knapsack items $1, 2$ and 3.

- $M = \{1, 2\}$ with $\Delta^1 = 2$ and $\Delta^2 = 3$,
- $D = \{1, \ldots, n\}$ where $\mathcal{S}(\rangle) = \int, \mathcal{T}(\rangle) = \sqcup_\rangle$ for $i \in \{1, \ldots, n\}$,
- $v(i, 1) = 0$ and $v(i, 2) = w_i, \forall i \in \{1, 2, ..., n\}$.

Figure 2 depicts a small example of building the input graph G for a given knapsack instance.

Note that, for the given RLP-FON instance, we have two choices for each demand $i \in D$. We can either use modulation level 2 and reach to the destination without any regeneration but occupying w_i number of slots on the edge $\{s, \bar{s}\}$ or we can use modulation level 1 and reach to the destination by visiting a regenerator at $\bar{t}_i$ and without consuming any bandwidth on the edge $\{s, \bar{s}\}$. Observing that it is always advantageous to use modulation 2 and our freedom of using this modulation is limited with the W capacity of the bottleneck edge $\{s, \bar{s}\}$, it is easy to see the relation between the given knapsack problem and its RLP-FON transformation. Item i for $i \in \{1, \ldots, n\}$ will be chosen in an optimal solution of the knapsack problem if and only if demand i in RLP-FON uses modulation level 2 in an optimal solution for the RLP-FON and RLP-FON instance will have a solution of cost at most Z if and only if the knapsack instance has a solution with value at least $\sum_{i=1}^{n} z_i - Z$. $\square$

The number of transmission demands is another significant dimension of the problem complexity. As we see with the following two theorems, while there is a polynomial time algorithm to solve RLP-FON when we consider a single transmission demand, RLP-FON is still a challenging problem even if we have only two transmission demands.

Theorem 3. *There is a polynomial time algorithm to solve an RLP-FON instance with* $|D| = 1$.

Proof. Let $\langle G, D, M, l, c, h, v, \alpha, \eta \rangle$ be an RLP-FON instance with $D = \{d\}$. For each modulation level $i \in M$ we denote its *feasible graph* $G_i = (N, E_i)$ as the subgraph of $G = (N, E)$ where $E_i = \{e \in E | v(d, i) \leq \lfloor c(e)\alpha \rfloor\}$. For each feasible graph G_i, consider the closure graph $G_i^c = (N, E_i^c)$. The notion of closure graph is introduced by Chen et al. (2009) and used in Yildiz and Karasan (2015). Namely, $\{i, j\} \in E_i^c$ if and only if the length of the shortest path from i to j in G_i is at most Δ^i. Now we define the united closure graph $G^c = (N, E^c)$ where $E^c = \bigcup_{i=1}^{\mu} E_i^c$.

Note that, we can generate feasibility graphs and solve all pairs shortest path problem on these graphs to obtain their closure graphs in polynomial time. So the generation of the united closure graph can be accomplished in polynomial time. Let each edge in E^c have cost η and node $i \in N$ have cost h_i. Solving a node weighted shortest path problem on G^c from $\mathcal{S}(\lceil)$ to $\mathcal{T}(\lceil)$ gives the optimal solution to the RLP-FON instance. $\square$

Theorem 4. *RLP-FON is NP-Hard even if* $|D| = 2$.

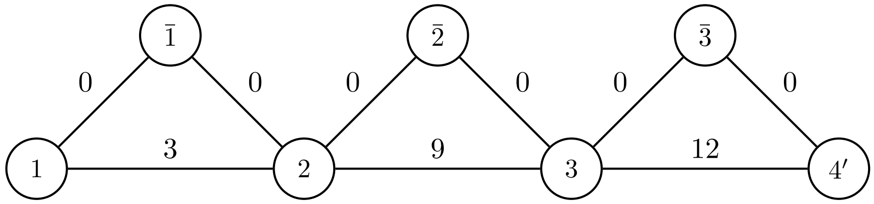


Figure 3: Depiction of a small transformation example for $S = \{3, 9, 12\}$.

Proof. We prove this theorem by showing that a set partitioning problem can be reduced to an RLP-FON instance with two transmission demands. Let $S = \{a_1, \ldots, a_{|S|}\}$ where $\sum_{i=1}^{|S|} a_i$ is even be an arbitrary instance of a set partitioning problem. We construct an RLP-FON instance as follows.

- $N = \{\bigcup_{i=1}^{|S|} \{i, \bar{i}\}, |S| + 1\}$,
- $E = \{\bigcup_{i=1}^{|S|} \{i, \bar{i}\}, \bigcup_{i=1}^{|S|} \{\bar{i}, i + 1\}, \bigcup_{i=1}^{|S|} \{i, i + 1\}\}$,
- $l(\{i, \bar{i}\}) = l(\{\bar{i}, i + 1\} = 0$ for $i \in \{1, \ldots, |S|\}$,
- $l(\{i, i + 1\}) = a_i$ for $i \in \{1, \ldots, |S|\}$,
- $c(e) = 1$ for $e \in E$,
- $h_i = 1$ for $i \in N$, $\eta = 0$, $\alpha = 1$,
- $D = \{1, 2\}$, $\mathcal{S}(\infty) = \mathcal{S}(\in) = \infty$ and $\mathcal{T}(\infty) = \mathcal{T}(\in) = |\mathcal{S}| + \infty$,
- $M = \{1\}$ and $\Delta^1 = \frac{\sum_{i=1}^{|S|} a_i}{2}$,
- $v(1, 1) = v(2, 1) = 1$.

In Figure 3 we present a small example of transforming a set partitioning problem into an RLP-FON instance with the desired properties.

Now observe that any solution to the given RLP-FON instance will construct two edge disjoint light-paths say π_1 and π_2 for each connection demand in D. Let $S_1 = \{a_i \in S : (i, \bar{i}) \in \pi_1\}$ and $S_2 = S \setminus S_1$. Then the given set partitioning instance has a solution if and only if RLP-FON has a zero cost solution. □

3.1 Uncapacitated Edges

Obviously, having capacity limits for the edges increases the difficulty of RLP-FON. Now we investigate the case in which these constraints are relaxed. In many studies motivated by different practical applications, uncapacitated edges are considered for regenerator/relay/refueling station placement (Yetginer and Karasan 2003, Kuby and Lim 2005, Cabral et al. 2007, Pachnicke et al. 2008, Chen et al. 2009, Uster and Kewcharoenwong 2011, Flammini et al. 2011, Kewcharoenwong and Uster 2014, Yildiz and Karasan 2015, Chen et al. 2015, Yildiz et al. 2015). We denote this problem as RLP-FON-U. Note that, once the edge capacities are relaxed, we can find the optimal routing by using only the lowest modulation level that can transmit signals furthest since occupying more bandwidth is not an issue when the capacity limits are not considered for the edges. However, such a simplification does not make the problem an easy one.

Indeed RLP is a special case of RLP-FON-U in which all nodes are required to communicate with each other and the regenerator usage cost η is assumed to be zero. Thus RLP-FON-U is also NP-Hard.

Depending on the application area, one of the two costs: regenerator placement or usage costs, would dominate the overall cost. Now we fist consider the case where the regenerator placement costs are negligible and show that there exists a polynomial time algorithm to solve this special case.

Theorem 5. *There exists a polynomial time algorithm to solve an instance $\langle G, D, l, h, \eta \rangle$ of RLP-FON-U where $h_i = 0$ for each $i \in N$.*

Proof. By solving an all pairs shortest path problem on G and considering the reach limit Δ^1 we can easily generate a closure graph with edge lengths all equal to regeneration cost η. Then solving the RLP-FON-U instance entails finding the shortest path from $\mathcal{S}(d)$ to $\mathcal{T}(d)$ on this closure graph for each demand $d \in D$. □

The second case we investigate is the one for which the regenerator placement costs dominate the total cost. Unlike the previous case, this time RLP-FON-U remains an NP-Hard problem. So we consider special network topologies, i.e., path and ring networks, for which we can present interesting characterizations for the optimal solutions and attain polynomial time algorithms.

Considering a special case of RLP-FON-U by assuming unit costs for the regenerator placement, unit edge lengths and fixed routes for OD pairs, Flammini et al. (2011) show that there exist polynomial time algorithms to solve path (line) and ring network topologies. Now we show that these results can be extended even when unit cost and fixed route assumptions are relaxed.

Although path networks are seldom in real world applications, investigating properties of the optimal solutions for these networks can be quite useful to develop solution algorithms for general networks. Consider an RLP-FON-U instance $\langle G, D, l, h, \eta \rangle$ where the input graph $G = (N, E)$ is a path (line). Assume without loss of generality that the nodes are labelled from 1 to n such that $\{i, j\} \in E$ if and only if $|i - j| = 1$. For $i \leq j$, let $[i, j] = \{k \in N | k \leq j \text{ and } i \leq k\}$ be the set of nodes lying in the interval from i through j in this ordering. Since G is undirected, without loss of generality we may assume that $\mathcal{S}(d) < \mathcal{T}(d)$ with respect to this ordering for each $d \in D$.

Lemma 1. *Consider an RLP-FON-U instance $\langle G, D, l, h, \eta \rangle$ where $\eta = 0$ and $D = \{1, 2\}$. If $[\mathcal{S}(2), \mathcal{T}(2)] \subset [\mathcal{S}(1), \mathcal{T}(1)]$, then solving the problem for only $D = \{1\}$ provides the optimal solution for $D = \{1, 2\}$.*

Proof. It is clear that a subset of the regenerator locations enabling a feasible light-path from $\mathcal{S}(1)$ to $\mathcal{T}(1)$ with Δ^1 reach limitation will enable a feasible light-path from $\mathcal{S}(2)$ to $\mathcal{T}(2)$ as well. □

Lemma 2. *Consider an RLP-FON-U instance $\langle G, D, l, h, \eta \rangle$ where $\eta = 0$ and $D = \{1, 2\}$. If $[\mathcal{S}(1), \mathcal{T}(1)] \cap [\mathcal{S}(2), \mathcal{T}(2)] = \emptyset$, then it is possible to attain an optimal solution by solving the single demand subproblems individually.*

Proof. Let R_1 and R_2 be the regenerator locations when solving the given RLP-FON-U instance with $D = \{1\}$ and $D = \{2\}$, respectively. It is clear that $R_1 \cup R_2$ will be an optimal solution for $D = \{1, 2\}$. □

Lemma 3. *Consider an RLP-FON-U instance $\langle G, D, l, h, \eta \rangle$ where $\eta = 0$ and $D = \{1, 2\}$. Assume $[\mathcal{S}(1), \mathcal{T}(1)] \cap [\mathcal{S}(2), \mathcal{T}(2)] \neq \emptyset$, $[\mathcal{S}(1), \mathcal{T}(1)] \not\subset [\mathcal{S}(2), \mathcal{T}(2)]$, $[\mathcal{S}(2), \mathcal{T}(2)] \not\subset [\mathcal{S}(1), \mathcal{T}(1)]$ and $\mathcal{S}(1) < \mathcal{S}(2)$. Then one of the following must hold.*

- *Solving the single demand subproblems individually will provide an optimal solution for the original problem,*

- *Solving the single demand subproblem from $\mathcal{S}(1)$ to $\mathcal{T}(2)$ will provide an optimal solution for the original problem.*

Proof. Let R be an optimal solution of the RLP-FON-U instance. There are two cases to consider.

- **Case-1:** There are two feasible light-paths connecting $\mathcal{S}(1)$ to $\mathcal{T}(1)$ and $\mathcal{S}(2)$ to $\mathcal{T}(2)$ visiting nonintersecting sets of regenerators. In this case it is clear that the first claim holds.

- **Case-2:** Every pair of light-paths connecting $\mathcal{S}(1)$ to $\mathcal{T}(1)$ and $\mathcal{S}(2)$ to $\mathcal{T}(2)$ in an optimal solution share at least one regenerator. Let r be such a shared regenerator. It is clear that nodes $\mathcal{S}(1), \mathcal{S}(2), \mathcal{T}(1)$ and $\mathcal{T}(2)$ all have feasible light-paths to connect to r. Let π_1 and π_2 be the light-paths through which nodes $\mathcal{S}(1)$ and $\mathcal{T}(2)$ connect to r. Then the light-path $\pi = (\pi_1, \pi_2)$ is a feasible one that connects $\mathcal{S}(1)$ to $\mathcal{T}(2)$ as desired. By Lemma 1, the regenerator locations on π provide feasible light-paths for both demands.

□

Theorem 6. *There is a polynomial time algorithm to solve an RLP-FON-U instance $\langle G, D, l, h, \eta \rangle$, if $\eta = 0$ and the input graph G is a path.*

Proof. Without loss of generality assume that the demand set cannot be partitioned into nonoverlapping intervals, otherwise one can solve the problem by solving the disjoint intervals independently as suggested by Lemma 2. In a similar fashion, assume that $[\mathcal{S}(d), \mathcal{T}(d)] \not\subset [\mathcal{S}(d'), \mathcal{T}(d')]$ for distinct $d, d' \in D$ since otherwise demand d can be ignored without loss of generality using Lemma 1. Assume the demands are ordered such that $\mathcal{S}(1) < \mathcal{S}(2) < \ldots < \mathcal{S}(\delta)$.

Let $Z(i), i = 1, 2, \ldots, \delta$ be the optimal solution value of the RLP-FON-U instance $\langle G_i, D_i, l, h, \eta \rangle$ where $G_i = [\mathcal{S}(\delta - i + 1), \mathcal{T}(\delta)]$ and $D_i = D \setminus \bigcup_{j=1}^{\delta-i} \{j\}$, i.e., $Z(i)$ is the optimal solution of the problem considering only the last i demands in D. In particular $Z(\delta)$ is the optimal solution value for $\langle G, D, l, h, \eta \rangle$. We also define $\overline{Z}(i)$ as the optimal solution value of the RLP-FON-U instance considering the single OD pair $(\mathcal{S}(1), \mathcal{T}(i))$. Finally, let $\mathcal{T}(i^*) \in \{\mathcal{T}(1), \mathcal{T}(2), \ldots, \mathcal{T}(n)\}$ be the largest index node that $\mathcal{S}(1)$ would have a feasible light-path in an optimal solution of $\langle G, D, l, h, \eta \rangle$. Now observe that for $\langle G, D, l, h, \eta \rangle$ adding an artificial demand $d^* = (\mathcal{S}(1), \mathcal{T}(i^*))$ to D does not change the optimal solution and at the presence of d^* we can disregard demands $i \leq i^*$ since they are all dominated by it. Moreover, $\mathcal{S}(1)$ cannot reach nodes $\mathcal{T}(i), i > i^*$ in an optimal solution implies that, none of the demands $i > i^*$ uses a regenerator that could be reached by $\mathcal{S}(1)$ via a feasible light-path since otherwise $\mathcal{S}(1)$ could reach $\mathcal{T}(i)$ first reaching the shared regenerator. So we have:

$$Z(\delta) = \begin{cases} \overline{Z}(i^*) + Z(\delta - i^*), & \text{if } i^* < \delta \\ \overline{Z}(\delta), & \text{if } i^* = \delta \end{cases} \tag{9}$$

Since we have only δ possible values for i^* and $\overline{Z}(1), Z(1)$ can both be calculated in polynomial time by Theorem 3, a dynamic programming algorithm can find $Z(\delta)$ in polynomial time by recursively calculating $Z(i)$ for each $i < \delta$.

□

In telecommunications, ring networks are pervasive as cost effective and easy to implement solutions to protect network traffic against edge and node failures (Vachani et al. 1996). As minimal cycles, they have interesting properties from theoretical perspective as well. So we pay a special attention to these networks and present two important results regarding the RLP-FON-U.

Let $G = (N, E)$ represent a ring. For a pair of nodes a and b on this ring, let the interval $[a, b]$ depict the path starting from a and traversing the ring in a clockwise manner to reach node b. Let $\langle G, D, l, h, \eta \rangle$ be an RLP-FON-U instance where $\eta = 0$, the input graph G is a ring and R is the set of regenerator locations in an optimal solution. Consider the graph $T = (R, \overline{E})$ where $\overline{E} = \{\{i, j\}$: i and j are two consecutive nodes of R on G with clockwise distance at most $\Delta^1\}$.

Proposition 1. *The graph T is a forest.*

Proof. Assume to the contrary that $\overline{E}$ includes a cycle. Let $r \in R$ be a regenerator node. Let r_1 and r_2 be the neighbors of r in the clockwise and counter-clockwise directions on the ring. Any node i in N which communicates through r will communicate through either r_1 or r_2, thus removal of r will not spoil feasibility which contradicts the optimality of R.

□

Theorem 7. *There is a polynomial time algorithm to solve an RLP-FON-U instance $\langle G, D, l, h, \eta \rangle$ when the input graph G is a ring and η is zero.*

Proof. For the sake of simplicity and without loss of generality we assume that there are at least four regenerator locations in any optimal solution. Note that if this is not the case one can find an optimal solution by a simple enumeration since the feasibility of a given regenerator set can be checked in polynomial time. We also assume that there is no OD pair in D with shortest path distance less than or equal to Δ^1 since it could simply be omitted from D without loss of generality. Let Y^* be the value of the optimal solution for the problem instance $\langle G, D, l, h, \eta \rangle$ and $\mathcal{R}$ be the collection of sets of feasible regenerator locations for this instance.

Let $i, j \in N$ be such that the length of the interval $[i, j]$ denoted as $l_{[i,j]}$ is larger than Δ^1. We name nodes i and j as detachment and attachment nodes, respectively. Considering the demand set $D_{[i,j]} = \{(i, j)\}$ with the single OD pair (i, j) and the path graph $G_{[i,j]}$ derived from G by removing all the edges on $[i, j]$ we obtain the reduced problem instance $\langle G_{[i,j]}, D_{[i,j]}, l, h, \eta \rangle$ for which we denote the optimal regenerator locations as $R_{[i,j]}$. We define:

$$\overline{Y}_{[i,j]} = \begin{cases} \sum_{i \in R_{[i,j]}} h_i, & \text{if } R_{[i,j]} \in \mathcal{R} \\ \infty & \text{otherwise} \end{cases} \tag{10}$$

Note that if T is a tree, then $Y^* = min_{i,j \in N} \overline{Y}_{[i,j]}$.

If T is a forest with more than one disconnected trees, then there exist two sets of detachment-attachment nodes $(i, j), (l, m)$ that we can visit i, j, l, m when traversing G in the clockwise direction. Let $\langle G_{[i,j]}, D^1_{[i,j][l,m]}, l, h, \eta \rangle$ and $\langle G_{[i,j]}, D^2_{[i,j][l,m]}, l, h, \eta \rangle$ be two RLP-FON-U instances where the demand set $D^k_{[i,j][l,m]}, k = 1, 2$ initially set to D is updated as follows. For an OD pair $(s, t), s \in [i, j]$, if (s, t) can reach each other using i, j, l or m as a regenerator we remove it from $D^k_{[i,j][l,m]}, k = 1, 2$. If this is not the case we proceed as follows.

- If s can reach only one of the regenerators, say i, we replace (s, t) with (i, t) in $D^k_{[i,j][l,m]}, k = 1, 2$
- If s can reach both i and j,
  - if $t \in [m, i]$ we replace the OD pair (s, t) with (i, t) in $D^k_{[i,j][l,m]}, k = 1, 2$ (similarly we replace it with (j, t) if $t \in [j, l]$).
  - If $t \notin [m, i]$ and:

* it can reach only one of the regenerators, say m, we replace (s,t) with (i,m) in $D^k_{[i,j][l,m]}, k = 1,2$
* it can reach both regenerators l and m we replace (s,t) with (i,m) and (j,l) in $D^1_{[i,j][l,m]}$ and $D^2_{[i,j][l,m]}$, respectively.

Now let $R^k_{[i,j][l,m]}$ be the optimal regenerator locations for the problem instances $\langle G_{[i,j]}, D^k_{[i,j][l,m]}, l, h, \eta\rangle, k = 1,2$, respectively. We define:

$$\overline{Y}^k_{[i,j][l,m]} = \begin{cases} \sum_{i\in R^k_{[i,j][l,m]}} h_i, & \text{if } R^k_{[i,j][l,m]} \in \mathcal{R} \\ \infty & \text{otherwise} \end{cases} \tag{11}$$

Let $\overline{Y}_{[i,j][l,m]} = min_{k=1,2}\overline{Y}^k_{[i,j][l,m]}$. Then it is clear that if T is a forest with more than one tree then we have $Y^* = min\{\overline{Y}_{[i,j][l,m]} | i,j,l,m \in N, l_{[i,j]} > \Delta^1 \text{ and } l_{[l,m]} > \Delta^1\}$.

By Proposition 1, it must be the case that T is a single tree or a forest with more than one disconnected trees. We have shown that for each case we can enumerate a polynomially bounded number of RLP-FON-U instances on path networks, $O(|N|^2)$ for the single tree case and $O(|N|^4)$ for the forest case, and find the optimal solution. Hence the result follows by by Theorem 6. □

3.2 Computational Performance of the Branch-and-Price Algorithm for Special Problem Instances

In this subsection we investigate the computational performance of our branch-and-price algorithm on some special problem instances. In the first part we present some problem instances for which we can put a theoretical performance guarantee for our pricing algorithm and in the second part we propose a useful optimality cut to improve the PS formulation when regenerator usage costs are assumed to be negligible.

3.2.1 Ring Networks

In many practical settings ring networks are quite common in telecommunications. For these networks our H_k heuristic can solve the pricing problem exactly when $k = 2$. This is simply due to the fact that in these networks there could be at most two different simple paths that connect any two nodes. Since the computational complexity of our heuristic solution approach is $O(kn(|A|+nlogn)+n^2|D|\mu)$, we can solve the pricing problem in polynomial time. Obviously, for any network where the number of feasible path-segments between any two nodes is bounded by some positive integer K (such as path and tree networks), H_K can solve the pricing problem exactly in polynomial time. Moreover, since we solve the K-shortest path problem just once in the beginning, such a computational efficiency can boost the performance of the branch-and-price algorithm significantly.

3.2.2 Networks with Equal Edge Lengths

Optical or electrical signals do not lose their quality just traveling long distances, they also deteriorate when they pass through a switch, router or any other network device represented as nodes. So especially when the distances between the network nodes are not large, the main concern becomes the number of nodes a signal visits instead of the total distance it travels. Considering this situation there are a widestream of RLP literature which consider the number of hops as the reach constraint instead of the distance travelled (Ramamurthy et al. 1999, Huang et al. 2005, Cardillo et al. 2006, Zsigmond et al. 2007, He et al. 2007, Pachnicke et al. 2008, Manousakis et al. 2009). Note that considering such hop constraints is equivalent to having equal edge lengths in the input graph and using the reach limit as usual. For this case solving the pricing problem can be accomplished in polynomial time by iterating the Bellman-Ford shortest path algorithm on the pricing graph as many times as the allowed number of hops.

3.2.3 Strengthening the PS Formulation:

In this part, we present a logical cut to tighten the LP lower bound and improve the strength of the branching cuts specifically for r variables when the regenerator usage cost η is assumed to be zero.

Definition 3. *A node $i \in N$ is called an internal node of path-segment p if it is visited by p and it is neither the source nor the destination node of p. The set of path-segments that contain a node i as an internal node is denoted by $\mathcal{P}(i)$.*

Proposition 2. *Let (x, r) be a feasible solution of an RLP-FON instance. If there exists a node $i \in N$ for which $r_i = 1$ and i is an internal node of a path-segment p such that $x_p^d = 1$ for some $d \in D$, then there exists an alternative feasible solution $(\bar{x}, r)$ which satisfies the following conditions.*

- *i is not an internal node of any path-segment p that satisfies $\bar{x}_p^d = 1$ for some $d \in D$,*
- *for each arc $\bar{e} \in A$ the number of slots occupied by the solution $(\bar{x}, r)$ is less than or equal to that of (x, r).*

Proof. Let (x, r) be a feasible solution of an RLP-FON instance and assume $r_i = 1$ and i is an internal node of a path-segment $p = (p_1, p_2)$ where $t(p_1) = s(p_2) = i$. Let $\bar{D} = \{d \in D | x_p^d = 1\}$. Assume $\bar{D}$ is not empty.

Since $l(p) > l(p^1)$ and $l(p) > l(p^2)$ we can choose $m(p^1) \geq m(p)$ and $m(p^2) \geq m(p)$. Now we modify x by setting $x_p^d = 0$ and $x_{p^1}^d = x_{p^2}^d = 1 \quad \forall d \in \bar{D}$ and obtain the vector $\bar{x}$. By our assumption we have $r_i = 1$ and $r_{t(p)} = 1$. Thus, replacing x with $\bar{x}$ does not necessitate a change in the number and locations of the regenerators. The same will be true if $t(p) = \mathcal{T}(d)$ holds as well. Moreover, $m(p^1) \geq m(p)$ and $m(p^2) \geq m(p)$ implies that p^1 and p^2 utilize optical bandwidth more efficiently and the amount of bandwidth slots used by $(\bar{x}, r)$ is less than or equal to the one used by (x, r). Since i was arbitrary and this procedure can be repeated as many times as needed the result follows. $\square$

By Proposition 2, the following is an optimality cut for PS.

$$\sum_{\substack{d \in D, \\ p \in \mathcal{P}(i)}} x_p^d \leq K(1 - r_i) \qquad \forall i \in N, \tag{12}$$

where K is a large number.

The proposed cut forces that if a node $i \in N$ is chosen as a regeneration node (i.e. $r_i = 1$), none of the path-segments utilized by a positive x variable should contain i as an internal node. The modified formulation containing (12) is denoted $\overline{PS}$. Note that, denoting the highest capacity edge as e^*, choosing $K = \lfloor c(e^*) \times \alpha \rfloor$ is sufficient to assure the validity of the cut, since each path-segment x_p^d occupies at least 1 bandwidth slot.

In a branch-and-price framework, adding cuts to the model requires special attention since maintaining the structure of the pricing problem is crucial to retain tractability of the solution approach. In our case, we can easily modify the reduced cost calculations and preserve the special structure of the pricing problem as follows.

Let $\theta_i, i \in N$ be the dual variables associated with the constraints (12) in $\overline{PS}$ and p^o denote the set of internal nodes of a path-segment p. With the following modifications, solution of the pricing problem for $\overline{PS}$ follows exactly the same steps explained above.

- The reduced cost calculation (7) is changed as:

$$\bar{c}_p^d = \begin{cases} \pi^d_{t(p)} - \pi^d_{s(p)} + \sum_{e\in\bar{p}} v(d,m)\kappa_e + \sum_{i\in p^o} \theta_i, & \text{if } t(p) = \mathcal{T}(d) \\ \pi^d_{t(p)} - \pi^d_{s(p)} + \sum_{e\in\bar{p}} v(d,m)\kappa_e + \sum_{i\in p^o} \theta_i + \gamma^d_{t(p)}, & \text{if } t(p) \neq \mathcal{T}(d) \end{cases} \tag{13}$$

- The length function for the pricing graph $l^d_m(e)$ modified as:

$$l^d_m(\bar{e}) = \begin{cases} v(d,m)\kappa_e + \theta_i, & \text{if } t(\bar{e}) \in p^o \\ v(d,m)\kappa_e & \text{o.w.} \end{cases} \tag{14}$$

The main function of the cut (12) is not to raise the LP bound in the root node, it helps to speed up the solution procedure by reinforcing the strength of the branching cuts that remove the fractional solutions for the regeneration variables r. This is because when a variable r_i is set to its upper bound 1, variables $x^p_d, \quad \forall d \in D$ and $p \in \mathcal{P}(i)$ are all forced to 0 with the presence of the constraints (12). As such, we need to modify our branching rule for the Branching-cut-2 as follows.

- **Branching-cut-2** $r_i = 1$ **:** In this case the set of arc flow variables $\overline{X_i} = \{x^d_p | d \in D, p \in \mathcal{P}$ and $i \in p^o\}$ are implicitly set to 0. In order to make sure that any path-segment $x^d_p \in \overline{X_i}$ would not appear as a solution of the pricing problem we can simply remove the node i from all the pricing graphs where i is neither the source nor the sink node. Similarly for H_k we can update $l^d_m(\bar{e}) = \infty, \forall \bar{e} \in \{a \in A | s(a) = i, \mathcal{T}(d) \neq i$ and $\mathcal{S}(d) \neq i\}$ and implement the algorithm without any change.

4 Numerical Experiments

Extensive numerical experiments are conducted in order to both test the performance of the proposed solution methodology and derive insights from the instances closely representing the real world problems. We implemented the branch-and-price algorithm using Java under Linux and CPLEX 12.4. All experiments are done on an AMD Opteron(tm) Processor 6282 SE machine with 2GB RAM.

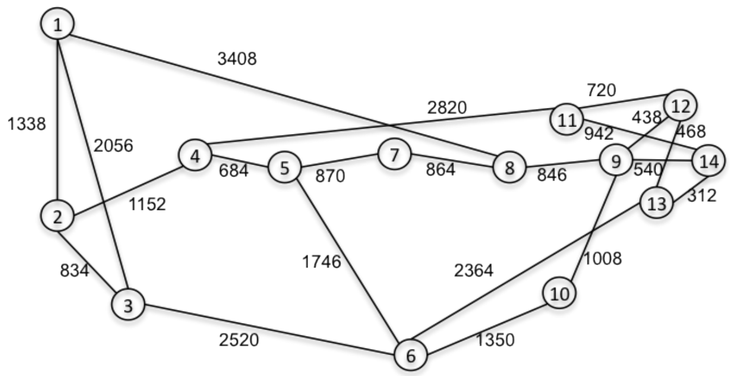
4.1 Network and Traffic

For our experiments we studied two well known network topologies from the literature: NSF-US network (Figure 2(a)) and COST-266 Pan European network (Figure 2(b)) (Hulsermann et al. 2004). Table 2 presents the topological parameters of these networks. In this table, for each optical network, we denote the minimum (min), maximum (max) and average (mean) values for the node degrees and physical edge lengths in kilometers.

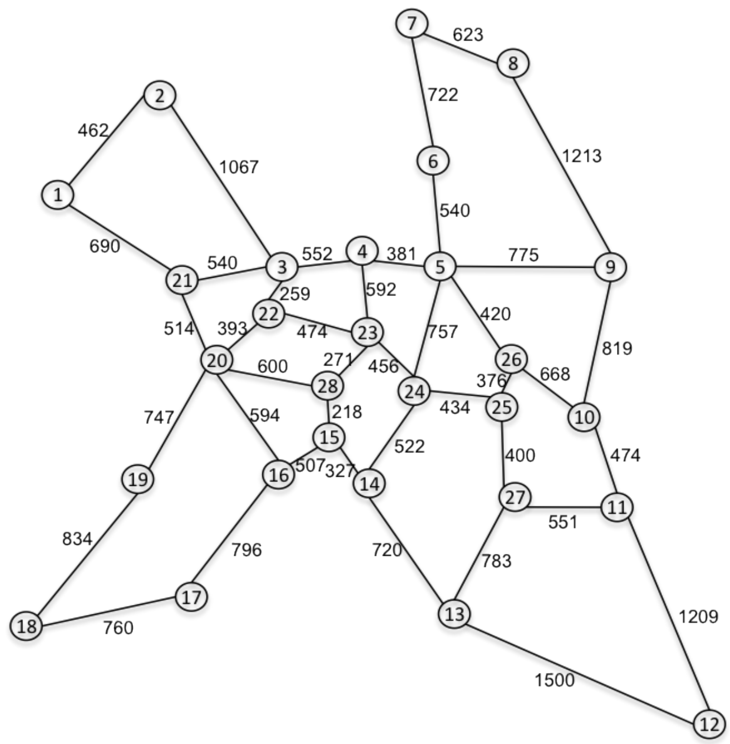
Table 2: Topological parameters

			Node Degree			Edge Length (km)		
Network	#nodes	#edges	min	max	mean	min	max	mean
NSF	14	21	2	4	3	312	3408	1299.1
COST-266	28	41	2	5	2.9	218	1500	625.4

For both network topologies we studied problem instances with 75,100,125 and 150 number of transmission demands. For each connection, the demanded data transfer rate (DTR) is assumed to be a uniform random variable that attains values 10, 40, 100 and 400 Gbps. For each demand we randomly



(a) NSF US network



(b) COST-266 Pan European network

Figure 4: Network topologies; link lengths in km. (Hulsermann et al. 2004)

choose an origin-destination (O-D) pair among all possible node pairings. For the O-D pair selection we investigate two cases: *uniform distribution (U)* and *traffic density distribution (TD)*. In the first case, all pairs have equal probability whereas in the second case the probability of a pair is assumed to be proportional to its IP traffic volume as reported in Hulsermann et al. (2004).

Abiding by the general approach in the RLP literature (Kim and Seo 2001, Yetginer and Karasan 2003, Pachnicke et al. 2008, Chen et al. 2009, Kewcharoenwong and Uster 2014, Duarte et al. 2014, Yildiz and Karasan 2015, Chen et al. 2015), we assumed $\eta = 0$ and considered unit cost for the regenerator placement. There are mainly two practical reasons for such an approach. The first one is the high set up and maintenance costs of regeneration sites for OEO regenerators (Yang and Ramamurthy 2005, Wang et al. 2014). Thus, placement and operation of such a regenerator site is much more costly when compared with the addition of an extra regenerator slot in an existing site. Secondly, once a regenerator site is deployed on a node, it is very costly to relocate it. Thus, as a strategic level problem, RLP is more concerned about the locations of the regenerator sites rather than the number of regenerator devices in each site. Moreover, from the algorithmic perspective as we have shown in the previous section, the problem gets harder under the cost structure when the regenerator placement costs are dominant.

4.2 Fiber and Modulation Level Parameters

Cables are assumed to be non-zero dispersion-shifted fiber (NZDSF) and four modulation formats are considered: BPSK, QPSK, 8-QAM and 16-QAM. The number of frequency slots per fiber is 360 (Klinkowski 2012a) and Table 3 shows the optical reaches (Δ) (Bosco et al. 2011) and the number of slots required by each modulation format (MF) (Klinkowski 2012a).

We generated problem instances with 6 different α values from the set $C = \{\alpha_{min}, 0.2, 0.4, 0.6, 0.8, 1\}$ where the value α_{min} represents the smallest α value for which the problem is feasible with the given set of parameters. Since the problem has no solution for $\alpha < \alpha_{min}$, for each problem setting we studied $\alpha \in C$ such that $\alpha \geq \alpha_{min}$.

There are two main motivations to generate problem instances with the minimum bandwidth allocation (i.e. $\alpha = \alpha_{min}$). From the algorithmic point of view, as our numerical results will attest to, problem instances with the limited arc capacities are harder to solve and those hard problem instances are required for a comprehensive performance test of algorithmic efficiency. On the other hand, from the managerial perspective, α_{min} constitutes an upper bound on the spectrum efficiency in a network. Thus, it is interesting to solve problem instances with the minimum α values to see the trade-off between the number of regenerators and bandwidth utilization efficiency. Indeed, finding the α_{min} value for each problem setting is an optimization problem in its own right. Thus, we solve the following mixed integer program for finding the minimum α value for which the problem stays feasible when deploying regenerators at each node of the network.

$$\min \ \alpha_{min} \tag{15}$$

$$\text{s.t.} \sum_{\substack{\bar{e} \in A: \\ s(\bar{e})=i}} x^d_{\bar{e}} - \sum_{\substack{\bar{e} \in A: \\ t(\bar{e})=i}} x^d_{\bar{e}} = \begin{cases} 1, & \text{if } i = \mathcal{S}(d) \\ \text{-1}, & \text{if } i = \mathcal{T}(d) \\ 0, & \text{otherwise} \end{cases} \qquad \forall i \in N, d \in D, \tag{16}$$

$$\sum_{d \in D} v(d, m^*_{\bar{e}})(x^d_{\bar{e}} + x^d_{\underline{e}}) \leq c(e) \times \alpha_{min} \qquad \forall e \in E, \tag{17}$$

$$x^d_{\bar{e}} \in \{0, 1\} \qquad \forall d \in D, \bar{e} \in A \tag{18}$$

The above formulation assumes that each node in the network has the regeneration capacity. The objective is to minimize the α_{min} value for the given problem instance. Constraints (16) are the flow balance

Table 3: Modulation level parameters

Modulation Format (m)	Δ^m(km)	DTR(Gbps)			
		10	40	100	400
BPSK (1)	2880	2	4	8	32
QPSK (2)	1080	2	2	4	16
8-QAM (3)	630	2	2	4	12
16-QAM (4)	270	2	2	2	8

equations that force each demand to be carried from its source to its destination. The decision variable $x^d_{\bar{e}}$ attains the value 1 if the route for the demand $d \in D$ includes the arc $\bar{e} \in A$ and 0 otherwise. Constraints (17) are the capacity constraints which ensure that the total number of slots occupied is not more than the maximum allowed. For each arc $\bar{e} \in A$, the best modulation $m^*_{\bar{e}}$ is defined as the highest modulation level which has an optical reach larger than or equal to the length of the arc. We have $m^*_{\bar{e}} = m^*_{\underline{e}}$ since $l(\bar{e}) = l(\underline{e}) = l(e)$. Also note that $m^*_{\bar{e}}$ is indeed the most spectrum efficient feasible modulation since higher modulation levels can transmit more data with less number of bandwidth slots. Constraints (18) are the variable restrictions.

4.3 Implementation Details

Before presenting the results of the numerical experiments we briefly state the implementation details of the proposed algorithm.

For each problem instance we run H-Root to find an initial feasible solution and repeat this procedure at every 75 branch-and-bound nodes to improve the current solution at hand. As we present in the following section, our numerical experiments show that H-Root can produce very high quality solutions and boosts the performance of the branch-and-price algorithm drastically.

For the exact solution of the pricing problem we employed the state-of-the art algorithm proposed by Santos et al. (2007) (Alg 3 by their notation). As a subroutine, Alg 3 requires to solve several k-shortest path problems and the authors use the algorithm presented in de Azevedo et al. (1994) for this task. Although very efficient, this algorithm can produce non simple paths and performs rather poorly in our pricing graph instances that can include arcs with zero length. Therefore, different than Santos et al. (2007), we implemented Yen's loop-less k-shortest path algorithm (Yen (1971)) as a subroutine in Alg 3.

Our preliminary results have shown that for all the problem instances we have studied, employing heuristic H_k has been very useful to reduce solution times. For some problem instances (mostly for those with α_{min} in COST-266 network) we were not able to find the optimal solution within the time limit of 1 hour unless we apply H_k. Since the 4^{th} and 15^{th} shortest loop-less paths between any two nodes of the NSF and COST-266 networks, respectively, has lengths more than the optical reach of the BPSK modulation (2880 km), the pricing problem is solved exactly by the heuristics H_4 and H_{15}. Hence, as the solution of the pricing problem we use H_4 and H_{15} for NSF and COST-266 networks, respectively.

We conduct a best bound search to explore the branch-and-bound tree. Although this strategy cannot explore as many nodes as the depth first search strategy, our computational studies showed that it can converge much faster by exploiting the high quality heuristic solution of H_k that can prune a significant part of the search tree.

4.4 Performance of the Branch-and-Price Algorithm

In this subsection we investigate the performances of the proposed solution methodology and discuss the effects of the various problem parameters on the difficulty of the resulting instances.

Our experimental design has 480 problem instances in total (2 networks, 4 demand sizes, 2 demand distributions, 6 α choices, and 5 random seeds). However, for some instances $\alpha = 0.2, 0.4$ and even $\alpha = 0.6$ are larger than α_{min} and hence there is no feasible solution. Tables 4,5,6 and 7 report the solutions of the branch-and-price algorithm for the total of 316 problem instances for which there exists a feasible solution. In these tables, $|D|$ is the number of connection demands, Seed is the key used to generate random numbers for the relevant problem instances, $R.LP$ is the solution of the linear relaxation, HS is the number regenerators found by the heuristic solution H-Root, $HS.RT$ is the run time (in seconds) for H-Root, NR is the solution found by the branch-and-price algorithm, $NR.LP$ is the proven lower bound for the number of regenerators, RT is the solution time (in seconds) for the branch-and-price algorithm, $\#BB$ is the number of branch-and-bound nodes explored by the branch-and-price algorithm, $\#Col.G.$ is the total number of columns generated during the branch-and-price algorithm, %SW. shows the percentage of demands that have gone through at least one modulation swap (at a regenerator node) on its path and finally *Reg.Loc.* reports the nodes that are chosen to be a regenerator point by the branch-and-price algorithm.

As we can see from the results, 303 out of 316 problems were solved optimally and for the remaining 13 the optimality gap was reduced to just 1 regenerator within the given time limit of **3600** seconds. For 12 out of the 13 unsolved problem instances, the bandwidth utilization level is equal to the minimum (i.e. $\alpha = \alpha_{min}$).

Table 8 depicts a summary of the results in Tables 4,5,6 and 7. Cells occupied with "-" are the cases where the related problem has no feasible solution. It is easy to read from Table 8 that the smaller α values (limited link capacities) makes the RLP-FON instances harder to solve. This is due to the fact that the short supply of bandwidth slots entails high percentage of the flow variables to assume fractional values in the LP solution of the problem and the branch-and-bound tree grows significantly.

The results also show that problem instances with the NSF network are much easier to solve than those of COST-266. This is in part due to the higher number of nodes and edges in the latter. However, the substantial difference in the solution times points to a more significant effect in play. In NSF network the mean edge length is more than two times larger than that of the COST-266 network and consequently the optical reach constraints are more binding. As a result, for each demand, the number of alternative paths is rather limited for the NSF network compared to the COST-266 network. Moreover, the topology is quite different between the two networks. In the NSF network most of the nodes are in the periphery. On the other hand, the COST-266 network has a much crowded core which contains more than half of the nodes. Such a composition significantly increases the number of alternative routings for each demand and makes problem instances challenging.

Not surprisingly, problem instances with higher number of demands are harder to solve. What is interesting is the higher solution times for the problem instances with the *traffic density* demand distribution. A possible explanation for this result could be the higher concentration of connection demands on some specific node pairs which exacerbates the problem of bandwidth capacity limitations.

Our numerical experiments show that H-Root can produce high quality solutions. Among the 316 problem instances, H-Root could find the optimal solution in 214 (67.72%). For the 65 instances out of the remaining 102, H-Root could find a solution with an optimality gap of just 1 regenerator. Interestingly, for some instances, H-Root would return a higher number of regenerators when α increases (e.g. Table 4, $|D| = 75$, $Seed = 1$) . The reason for such a result is the lack of some critical path-segments that are not generated in the root node when α is larger and LP relaxation of the problem has to be solved with

Table 4: Results for COST-266 network with uniform demand distribution

$\|D\|$	Seed	α	R.LP	HS	H.RT	NR	NR.LB	RT	#BB	#Col.G.	% SW.	Reg.Loc.
75	1	0.23	3.00	8	37	8	7	**3600**	410	21765	53.3	5 6 9 14 15 20 23 24
		0.40	1.50	2	14	2	2	19	0	9819	21.3	4 14
		0.60	1.50	2	4	2	2	7	0	4102	24.0	5 17
		0.80	1.50	2	3	2	2	6	0	4242	26.7	5 17
		1.00	1.50	5	4	2	2	12	6	5253	21.3	4 27
	2	0.27	2.52	5	38	5	5	223	29	18845	49.3	5 6 20 23 25
		0.40	1.50	2	35	2	2	43	0	14167	33.3	5 23
		0.60	1.50	2	6	2	2	9	0	4674	40.0	4 5
		0.80	1.50	2	4	2	2	7	0	4385	52.0	4 5
		1.00	1.50	2	3	2	2	6	0	4070	21.3	5 17
	3	0.13	2.62	6	33	6	6	353	73	18049	49.3	3 5 14 15 20 24
		0.20	1.50	3	21	2	2	168	75	15224	21.3	3 24
		0.40	1.50	2	3	2	2	6	0	3846	21.3	10 20
		0.60	1.50	4	4	2	2	12	7	6028	10.7	3 13
		0.80	1.50	4	4	2	2	12	7	5983	32.0	4 5
		1.00	1.50	2	1	2	2	5	0	5334	4.0	9 16
	4	0.16	6.04	12	52	12	11	**3600**	3777	9754	57.3	3 4 5 6 10 13 14 15 16 20 24 27
		0.20	2.70	6	48	6	6	868	113	22633	52.0	5 14 16 22 24 26
		0.40	1.50	2	5	2	2	8	0	5265	41.3	4 5
		0.60	1.50	2	3	2	2	6	0	4617	8.0	4 13
		0.80	1.50	2	2	2	2	5	0	4027	37.3	5 14
		1.00	1.50	2	2	2	2	4	0	4363	1.3	9 14
	5	0.29	1.50	2	23	2	2	34	0	14597	20.0	23 24
		0.40	1.50	2	10	2	2	13	0	6153	9.3	14 26
		0.60	1.50	2	3	2	2	6	0	4857	6.7	9 20
		0.80	1.50	4	4	2	2	13	6	6258	37.3	23 28
		1.00	1.50	2	4	2	2	7	0	4341	30.7	5 17
100	1	0.26	5.73	11	204	10	10	414	75	14752	53.0	3 4 5 6 9 14 15 20 23 24
		0.40	1.50	3	55	3	2	168	10	22279	32.0	5 22 24
		0.60	1.50	2	4	2	2	10	0	6127	16.0	13 23
		0.80	1.50	3	6	2	2	30	14	7591	15.0	10 14
		1.00	1.50	2	9	2	2	14	0	6410	26.0	5 19
	2	0.33	3.66	6	66	6	6	363	26	22108	40.0	5 6 9 20 23 24
		0.40	1.63	3	134	3	3	224	4	22460	34.0	5 23 25
		0.60	1.50	7	14	2	2	99	75	13323	23.0	5 20
		0.80	1.50	7	11	2	2	28	5	8586	24.0	4 25
		1.00	1.50	8	9	2	2	29	9	8213	20.0	11 20
	3	0.19	4.20	8	54	8	7	**3600**	1590	21501	55.0	4 5 6 14 15 20 23 24
		0.20	3.01	7	44	6	6	482	75	25693	46.0	4 5 6 20 23 25
		0.40	1.50	2	17	2	2	27	0	13523	15.0	10 23
		0.60	1.50	4	8	2	2	25	6	8278	14.0	3 9
		0.80	1.50	7	7	2	2	22	5	9245	24.0	14 24
		1.00	1.50	4	7	2	2	20	5	8211	18.0	3 14
	4	0.21	6.76	12	69	12	11	**3600**	1701	20007	56.0	4 5 6 9 13 14 15 16 20 22 24 25
		0.40	1.50	2	29	2	2	44	0	17230	14.0	4 15
		0.60	1.50	2	7	2	2	12	0	6200	22.0	14 24
		0.80	1.50	3	5	2	2	22	6	7544	14.0	6 15
		1.00	1.50	3	8	2	2	21	4	7423	33.0	23 24
	5	0.32	2.07	5	58	4	4	956	75	32616	48.0	5 20 23 24
		0.40	1.50	3	60	2	2	356	75	23494	26.0	23 24
		0.60	1.50	2	5	2	2	11	0	6182	13.0	9 23
		0.80	1.50	2	5	2	2	10	0	6455	11.0	1 24
		1.00	1.50	5	7	2	2	29	11	8414	25.0	10 23
125	1	0.32	4.63	9	66	9	9	191	26	18262	57.6	4 5 6 9 14 15 20 22 24
		0.40	1.78	4	87	4	4	687	28	37169	44.0	5 15 23 24
		0.60	1.50	3	36	2	2	193	75	22814	16.0	3 24
		0.80	1.50	4	11	2	2	88	75	12872	18.4	23 24
		1.00	1.50	6	11	2	2	49	15	13058	34.4	5 20
	2	0.40	2.98	6	216	6	5	**3600**	551	35617	48.0	3 5 6 15 23 24
		0.60	1.50	3	151	2	2	435	75	29420	28.0	3 5
		0.80	1.50	8	22	2	2	146	75	16685	9.6	11 14
		1.00	1.50	2	7	2	2	15	0	8626	26.4	5 15
	3	0.23	4.25	9	44	8	8	589	75	32706	54.4	4 5 6 7 14 20 23 25
		0.40	1.50	2	43	2	2	65	0	21391	17.6	14 24
		0.60	1.50	3	15	2	2	54	19	11774	16.8	4 14
		0.80	1.50	7	10	2	2	36	7	11457	23.2	23 24
		1.00	1.50	8	11	2	2	27	4	9321	17.6	4 27
	4	0.29	5.68	11	58	10	10	352	75	20767	55.2	4 5 6 13 14 15 16 22 24 26
		0.40	1.79	4	107	4	4	814	33	35450	44.0	3 5 14 24
		0.60	1.50	2	34	2	2	45	0	15497	19.2	14 23
		0.80	1.50	5	23	2	2	60	19	10327	18.4	24 28
		1.00	1.50	5	8	2	2	29	9	9626	26.4	5 28
	5	0.37	2.65	6	86	6	6	1809	146	43467	52.0	5 14 19 20 23 25
		0.40	2.13	4	80	4	4	362	11	30128	47.2	5 20 23 24
		0.60	1.50	3	63	2	2	329	75	26272	28.8	23 24
		0.80	1.50	2	5	2	2	13	0	7728	24.0	5 23
		1.00	1.50	4	15	2	2	45	7	10982	44.0	4 5
150	1	0.36	4.50	8	62	8	8	205	21	16932	54.0	4 5 6 14 15 20 23 24
		0.40	2.35	5	69	5	5	254	18	30980	45.3	3 5 14 15 24
		0.60	1.50	2	63	2	2	90	0	24709	32.7	4 5
		0.80	1.50	3	42	2	2	185	75	17660	12.0	6 14
		1.00	1.50	4	11	2	2	136	75	14450	6.0	17 24
	2	0.49	2.92	6	156	5	5	1800	75	48989	32.7	5 6 9 23 24
		0.60	1.61	3	163	3	3	361	5	40331	22.7	4 6 24
		0.80	1.50	2	40	2	2	58	0	16952	22.7	3 5
		1.00	1.50	2	10	2	2	23	0	10800	6.7	4 27
	3	0.31	3.58	6	257	6	6	572	33	32811	34.7	3 5 6 9 15 24
		0.40	1.54	3	95	2	2	542	75	34671	28.0	5 23
		0.60	1.50	2	49	2	2	66	0	20486	19.3	20 24
		0.80	1.50	8	20	2	2	156	75	16714	6.0	3 13
		1.00	1.50	6	24	2	2	105	36	16419	20.0	3 10
	4	0.36	5.41	9	199	9	9	283	13	15145	48.0	3 4 5 6 13 14 15 20 24
		0.40	2.94	6	303	6	6	706	46	36044	50.7	4 5 13 15 20 24
		0.60	1.50	3	112	2	2	338	75	32311	26.7	5 23
		0.80	1.50	3	23	2	2	115	62	17032	35.3	5 23
		1.00	1.50	2	24	2	2	35	0	12711	26.7	3 5
	5	0.38	3.47	7	56	7	7	601	64	42469	54.0	4 5 13 14 16 20 23
		0.40	2.99	6	80	6	6	849	62	44329	42.0	5 14 16 20 23 24
		0.60	1.50	3	157	2	2	564	75	37924	25.3	24 28
		0.80	1.50	5	28	2	2	167	75	18406	16.0	3 28
		1.00	1.50	6	25	2	2	162	75	16697	25.3	23 28

Table 5: Results for COST-266 network with traffic density demand distribution

$\|D\|$	Seed	α	R.LP	HS	H.RT	NR	NR.LB	RT	#BB	#Col.G.	% SW.	Reg.Loc.
75	1	0.18	5.17	10	33	10	10	218	42	14669	50.7	1 3 5 6 7 14 15 20 21 22 24
		0.20	3.26	6	41	6	6	189	18	17946	52.0	3 5 6 20 23 24
		0.40	1.50	2	10	2	2	16	0	11138	16.0	9 28
		0.60	1.50	2	3	2	2	7	0	6072	28.0	23 26
		0.80	1.50	3	4	2	2	18	8	6596	20.0	16 24
		1.00	1.50	3	2	2	2	12	5	6711	18.7	4 14
	2	0.14	5.64	12	21	11	10	**3600**	4528	9385	49.3	3 4 5 14 15 16 20 21 23 24 26
		0.20	1.76	4	47	4	3	**3600**	1088	22945	42.7	3 5 22 24
		0.40	1.50	2	3	2	2	6	0	4202	16.0	17 24
		0.60	1.50	2	2	2	2	6	0	4725	0.0	9 21
		0.80	1.50	2	2	2	2	5	0	4087	21.3	24 28
		1.00	1.50	2	2	2	2	6	0	4192	17.3	4 14
	3	0.15	3.57	9	40	8	8	262	75	14120	53.3	3 4 5 14 15 20 22 24
		0.20	1.70	4	33	4	4	237	33	20547	38.7	3 5 22 24
		0.40	1.00	1	2	1	1	6	0	5976	12.0	3
		0.60	1.00	1	2	1	1	5	0	4601	22.7	3
		0.80	1.00	1	2	1	1	5	0	4811	20.0	3
		1.00	1.00	1	2	1	1	5	0	4023	21.3	3
	4	0.28	3.07	6	36	6	6	143	20	13670	38.7	3 5 14 16 20 23
		0.40	1.54	3	45	2	2	184	75	17244	30.7	22 24
		0.60	1.50	3	8	2	2	69	75	6583	9.3	10 14
		0.80	1.50	2	3	2	2	6	0	3628	16.0	1 24
		1.00	1.50	3	4	2	2	11	5	5574	20.0	4 25
	5	0.16	8.49	15	23	14	13	**3600**	3490	7400	58.7	3 4 5 6 13 14 15 20 21 22 23 24 26 27
		0.20	3.33	7	35	6	6	450	75	15863	38.7	3 5 9 20 23 24
		0.40	1.50	2	7	2	2	11	0	8213	16.0	3 9
		0.60	1.50	2	2	2	2	5	0	3331	21.3	3 13
		0.80	1.50	2	2	2	2	5	0	3437	21.3	19 24
		1.00	1.50	2	2	2	2	5	0	3895	13.3	19 26
100	1	0.21	6.11	12	39	12	12	105	20	9965	57.0	3 4 5 6 7 14 15 16 20 21 22 24
		0.40	1.50	2	33	2	2	52	0	19702	16.0	22 24
		0.60	1.50	4	10	2	2	34	10	10007	42.0	5 23
		0.80	1.50	3	4	2	2	23	8	10600	44.0	4 5
		1.00	1.50	3	7	2	2	24	6	9637	29.0	5 15
	2	0.22	5.16	10	25	10	10	174	40	13698	44.0	3 4 5 14 15 16 17 20 23 24
		0.40	1.50	3	31	2	2	236	75	19887	23.0	23 24
		0.60	1.50	5	8	2	2	67	75	8456	21.0	23 24
		0.80	1.50	2	3	2	2	8	0	4645	11.0	9 14
		1.00	1.50	3	4	2	2	17	6	6538	21.0	14 23
	3	0.20	4.39	9	31	9	8	**3600**	1483	23023	53.0	3 4 14 15 16 20 22 24 27
		0.40	1.50	2	15	2	2	23	0	12525	23.0	23 24
		0.60	1.50	3	9	2	2	24	7	7847	23.0	14 24
		0.80	1.50	2	3	2	2	7	0	4612	25.0	3 13
		1.00	1.50	3	6	2	2	15	3	7442	42.0	23 24
	4	0.33	3.37	7	45	6	6	390	75	27118	41.0	3 5 14 16 20 23
		0.40	2.13	4	50	4	4	328	21	28533	39.0	3 5 20 23
		0.60	1.50	2	19	2	2	25	0	10958	24.0	14 24
		0.80	1.50	2	5	2	2	11	0	5317	20.0	3 13
		1.00	1.50	2	2	2	2	7	0	5044	25.0	3 9
	5	0.22	6.86	12	31	12	11	**3600**	1125	16181	61.0	3 4 5 6 13 14 15 20 21 22 24 27
		0.40	1.50	2	34	2	2	48	0	15599	20.0	3 24
		0.60	1.50	2	8	2	2	14	0	9783	11.0	16 24
		0.80	1.50	6	5	2	2	16	5	6097	11.0	4 27
		1.00	1.50	2	2	2	2	7	0	4747	26.0	23 28
125	1	0.26	6.26	13	55	12	12	187	75	13689	55.2	1 3 4 5 6 7 14 15 20 21 22 24
		0.40	1.74	3	70	3	3	115	1	25145	37.6	3 22 24
		0.60	1.50	2	37	2	2	48	0	15076	6.4	9 16
		0.80	1.50	2	9	2	2	18	0	8843	20.0	3 9
		1.00	1.50	4	7	2	2	49	23	12239	12.8	9 23
	2	0.26	5.28	9	33	9	9	334	54	18718	43.2	3 5 14 15 16 19 20 23 24
		0.40	1.76	4	107	3	3	1056	75	37491	27.2	3 5 16
		0.60	1.50	3	17	2	2	138	75	13172	21.6	14 24
		0.80	1.50	2	10	2	2	19	0	6969	30.4	14 24
		1.00	1.50	5	8	2	2	26	6	8032	20.8	5 23
	3	0.26	4.10	8	51	8	8	913	110	32527	44.0	3 4 15 16 20 21 22 24
		0.40	1.50	3	87	2	2	502	75	27471	21.6	3 5
		0.60	1.50	2	12	2	2	18	0	7837	25.6	3 24
		0.80	1.50	4	8	2	2	32	8	11077	35.2	3 5
		1.00	1.50	4	7	2	2	30	8	10870	22.4	5 23
	4	0.41	3.05	6	59	6	6	253	26	29247	38.4	3 5 14 16 20 23
		0.60	1.50	2	76	2	2	113	0	20730	24.8	22 24
		0.80	1.50	2	25	2	2	36	0	16851	22.4	3 15
		1.00	1.50	2	9	2	2	18	0	7270	21.6	20 24
	5	0.28	6.98	13	41	12	11	**3600**	75	18344	54.4	3 4 5 13 14 15 20 21 24 26 27 28
		0.40	1.61	3	72	3	3	155	3	22477	32.0	3 5 24
		0.60	1.50	2	15	2	2	34	0	20330	4.8	9 14
		0.80	1.50	6	9	2	2	117	75	15128	13.6	3 13
		1.00	1.50	2	6	2	2	12	0	6320	17.6	23 24
150	1	0.31	4.95	9	70	9	9	284	27	25216	52.0	1 3 5 6 15 20 21 22 24
		0.40	2.21	4	332	4	3	**3600**	779	44496	44.7	3 5 20 24
		0.60	1.50	3	53	2	2	284	75	30497	26.7	24 28
		0.80	1.50	2	12	2	2	26	0	9671	3.3	9 14
		1.00	1.50	3	12	2	2	46	9	15357	10.0	4 11
	2	0.33	5.27	10	53	10	10	378	53	26104	46.7	3 5 14 15 16 17 19 20 21 23
		0.40	3.20	6	92	6	6	804	35	45207	36.7	3 15 16 17 20 24
		0.60	1.50	3	110	2	2	323	75	26560	24.7	3 15
		0.80	1.50	3	22	2	2	113	75	15019	14.0	4 15
		1.00	1.50	5	17	2	2	79	35	12884	40.0	4 5
	3	0.33	4.66	9	198	9	9	1233	139	40488	32.0	3 4 14 16 17 20 21 23 24
		0.40	2.83	5	121	4	4	934	75	44488	32.7	3 20 24 28
		0.60	1.50	3	44	2	2	190	75	25936	8.7	10 15
		0.80	1.50	3	18	2	2	140	75	15966	5.3	11 28
		1.00	1.50	3	13	2	2	80	31	15993	30.7	3 28
	4	0.49	3.23	7	106	7	7	480	50	35523	46.0	3 4 5 14 16 20 23
		0.60	1.88	4	128	4	4	859	24	45629	32.7	15 20 23 24
		0.80	1.50	3	95	2	2	444	75	35013	28.0	22 24
		1.00	1.50	2	33	2	2	47	0	21249	14.7	24 28
	5	0.34	6.60	11	244	11	10	**3600**	75	11607	52.0	3 4 5 6 14 15 16 20 21 22 24
		0.40	3.22	5	222	5	5	295	6	20825	38.7	3 5 20 23 24
		0.60	1.50	2	77	2	2	98	0	23414	18.7	24 28
		0.80	1.50	2	19	2	2	31	0	15521	6.7	9 28
		1.00	1.50	4	23	2	2	147	75	13260	11.3	1 24

Table 6: Results for NSF network with uniform demand distribution

$\|D\|$	Seed	α	R.LP	HS	H.RT	NR	NR.LB	RT	#BB	#Col.G.	% SW.	Reg.Loc.
75	1	0.48	2.11	3	1	3	3	3	0	901	5.3	3 4 6
		0.60	2.00	3	1	3	3	3	1	718	8.0	3 4 6
		0.80	2.00	2	1	2	2	2	0	673	2.7	4 6
		1.00	2.00	2	0	2	2	1	0	619	18.7	4 11
	2	0.44	2.19	4	1	4	4	10	9	1635	13.3	3 4 6 7
		0.60	2.00	3	1	3	3	3	2	675	14.7	3 4 6
		0.80	2.00	3	1	3	3	4	3	998	21.3	3 4 11
		1.00	2.00	2	0	2	2	1	0	487	21.3	4 11
	3	0.42	2.29	4	2	4	4	6	3	1323	12.0	3 4 6 11
		0.60	2.00	3	1	3	3	3	1	844	5.3	3 4 6
		0.80	2.00	2	1	2	2	2	0	724	2.7	4 6
		1.00	2.00	2	0	2	2	2	0	614	22.7	4 11
	4	0.41	2.17	3	1	3	3	3	0	824	10.7	3 4 6
		0.60	2.00	2	1	2	2	2	0	601	2.7	4 6
		0.80	2.00	2	0	2	2	1	0	536	18.7	4 11
		1.00	2.00	2	0	2	2	2	0	574	20.0	4 11
	5	0.51	2.19	4	1	4	4	5	3	1054	9.3	3 4 6 11
		0.60	2.01	4	1	3	3	7	22	948	14.7	3 6 7
		0.80	2.00	3	1	3	3	2	1	813	2.7	3 4 6
		1.00	2.00	2	1	2	2	2	0	631	1.3	4 6
100	1	0.68	2.20	4	4	4	4	10	2	1541	12.0	3 4 6 11
		0.80	2.02	3	2	3	3	5	0	1310	9.0	3 4 6
		1.00	2.00	3	2	3	3	5	1	1066	15.0	4 6 11
	2	0.62	2.17	3	2	3	3	5	0	1340	10.0	3 4 6
		0.80	2.00	4	2	3	3	9	12	1235	17.0	3 4 6
		1.00	2.00	3	1	3	3	4	2	917	16.0	3 4 6
	3	0.67	2.15	3	2	3	3	5	0	1313	7.0	3 4 6
		0.80	2.00	3	1	3	3	4	1	1057	5.0	3 4 6
		1.00	2.00	3	1	3	3	4	1	1041	4.0	4 6 11
	4	0.44	2.37	4	3	4	4	9	3	1603	17.0	3 4 6 11
		0.60	2.00	3	1	3	3	6	2	1213	9.0	4 6 11
		0.80	2.00	2	1	2	2	3	0	845	3.0	4 6
		1.00	2.00	2	1	2	2	2	0	801	20.0	4 11
	5	0.62	2.17	3	2	3	3	4	0	1243	8.0	3 4 6
		0.80	2.00	4	1	3	3	11	10	1326	7.0	3 4 6
		1.00	2.00	3	1	3	3	3	1	922	6.0	3 4 6
125	1	0.84	2.22	4	4	4	4	14	4	2106	11.2	3 4 6 11
		1.00	2.04	3	2	3	3	7	0	1804	5.6	3 4 6
	2	0.86	2.11	3	2	3	3	6	0	1544	11.2	3 4 6
		1.00	2.00	3	2	3	3	7	1	1195	10.4	3 4 6
	3	0.77	2.12	3	3	3	3	8	0	1843	8.8	3 4 6
		0.80	2.08	3	2	3	3	6	0	1737	11.2	3 4 6
		1.00	2.00	3	2	3	3	5	1	1340	10.4	3 4 6
	4	0.65	2.41	4	6	4	4	14	2	1933	12.8	3 4 6 11
		0.80	2.13	3	3	3	3	8	0	1855	9.6	3 4 6
		1.00	2.00	3	1	3	3	6	2	1176	10.4	3 4 6
	5	0.76	2.18	4	4	4	4	10	3	1877	18.4	3 4 6 11
		0.80	2.13	3	2	3	3	6	0	1318	8.0	3 4 6
		1.00	2.00	3	2	3	3	6	1	1183	5.6	3 4 6
150	1	0.96	2.25	4	6	4	4	14	4	2262	12.7	3 4 6 11
		1.00	2.20	4	11	4	4	16	2	1954	14.0	3 4 6 11
	2	1.04	2.12	4	4	4	4	12	4	2140	14.7	3 4 6 11
	3	0.94	2.14	3	3	3	3	8	0	1980	6.7	3 4 6
		1.00	2.07	3	2	3	3	7	0	1656	11.3	3 4 6
	4	0.79	2.45	4	5	4	4	14	2	2056	14.7	3 4 6 11
		0.80	2.42	4	5	4	4	13	2	1914	12.7	3 4 6 11
		1.00	2.13	3	4	3	3	9	0	2209	9.3	3 4 6
	5	0.93	2.15	3	4	3	3	8	0	1888	6.7	3 4 6
		1.00	2.07	3	3	3	3	7	0	1524	8.0	3 4 6

Table 7: Results for NSF network with traffic density demand distribution

$\|D\|$	Seed	α	R.LP	HS	H.RT	NR	NR.LB	RT	#BB	#Col.G.	% SW.	Reg.Loc.
75	1	0.43	2.25	3	1	3	3	2	0	661	2.7	3 4 6
		0.60	2.00	3	1	3	3	2	1	616	1.3	3 4 6
		0.80	2.00	2	1	2	2	2	0	546	0.0	4 6
		1.00	2.00	2	1	2	2	2	0	555	13.3	2 7
	2	0.38	2.34	5	2	5	5	7	5	1227	32.0	3 4 6 9 11
		0.60	2.00	3	1	3	3	2	1	899	2.7	3 4 6
		0.80	2.00	3	1	3	3	3	3	972	6.7	3 4 6
		1.00	2.00	2	0	2	2	2	0	650	30.7	4 11
	3	0.48	2.09	3	1	3	3	2	0	713	6.7	3 4 6
		0.60	2.00	3	1	3	3	2	1	739	10.7	3 4 6
		0.80	2.00	3	1	3	3	2	2	687	8.0	3 6 7
		1.00	2.00	2	0	2	2	2	0	571	22.7	4 11
	4	0.71	2.08	3	1	3	3	2	0	749	9.3	3 4 6
		0.80	2.00	3	1	3	3	2	1	759	6.7	3 4 6
		1.00	2.00	3	1	3	3	2	1	719	9.3	3 4 6
	5	0.67	2.19	4	2	4	4	6	4	1138	9.3	3 4 6 11
		0.80	2.00	3	1	3	3	3	1	846	8.0	3 4 6
		1.00	2.00	3	1	3	3	3	1	785	4.0	3 4 6
100	1	0.52	2.19	4	2	3	3	18	53	1524	14.0	3 5 6
		0.60	2.04	3	1	3	3	3	0	904	4.0	3 4 6
		0.80	2.00	2	1	2	2	2	0	697	0.0	4 6
		1.00	2.00	2	1	2	2	2	0	722	2.0	4 6
	2	0.63	2.25	4	3	4	4	9	3	1298	18.0	3 4 6 11
		0.80	2.00	3	1	3	3	4	0	1251	5.0	3 4 6
		1.00	2.00	3	1	3	3	4	1	1045	18.0	4 6 11
	3	0.72	2.00	3	1	3	3	4	1	951	9.0	3 4 6
		0.80	2.00	3	1	3	3	3	1	916	10.0	3 4 6
		1.00	2.00	3	1	3	3	3	1	925	7.0	3 6 7
	4	0.79	2.10	4	1	4	3	5	2	1029	7.0	3 4 6 7
		0.80	2.08	3	1	3	3	4	0	1001	7.0	3 4 6
		1.00	2.00	3	1	3	3	4	1	953	7.0	3 4 6
	5	0.75	2.22	3	2	3	3	6	0	1199	7.0	3 4 6
		0.80	2.14	3	2	3	3	4	0	1112	8.0	3 4 6
		1.00	2.00	3	1	3	3	4	1	1117	5.0	3 4 6
125	1	0.62	2.28	4	3	4	4	10	4	1527	7.2	3 4 6 11
		0.80	2.00	3	2	3	3	7	1	1546	16.8	3 5 6
		1.00	2.00	3	1	3	3	4	1	948	1.6	4 6 11
	2	0.77	2.22	4	3	4	4	10	3	1711	16.0	3 4 6 11
		0.80	2.18	4	4	4	4	11	3	1798	4.8	3 4 6 11
		1.00	2.00	3	2	3	3	5	1	1390	3.2	3 4 6
	3	0.89	2.04	3	2	3	3	5	0	1392	8.0	3 4 6
		1.00	2.00	3	2	3	3	5	1	1237	9.6	3 4 6
	4	0.98	2.15	4	2	4	4	9	3	1494	8.0	3 4 6 7
		1.00	2.13	4	2	4	4	11	4	1786	8.8	3 4 6 7
	5	0.93	2.17	4	2	3	3	26	75	2282	8.0	3 4 6
		1.00	2.08	3	3	3	3	8	0	1397	5.6	3 4 6
150	1	0.73	2.28	4	4	4	4	9	2	1695	7.3	3 4 6 11
		0.80	2.19	4	3	3	3	16	50	1974	14.0	3 5 6
		1.00	2.00	3	2	3	3	7	1	1428	4.7	3 4 6
	2	0.88	2.24	4	4	4	4	9	3	1774	17.3	3 4 6 11
		1.00	2.11	4	6	4	4	11	3	1957	5.3	3 4 6 11
	3	1.01	2.07	3	3	3	3	7	0	1569	6.0	3 4 6
	4	1.25	2.14	4	6	4	4	12	3	1888	13.3	3 4 6 7
	5	1.09	2.21	3	4	3	3	8	0	1816	5.3	3 4 6

a more restricted set of columns.

4.5 Managerial Insights

The relation between the number of regenerators and the spectral efficiency is an interesting one for the network management who wants to deploy/operate minimum number of regenerators to lower the capital investment and operational costs (such as energy and maintenance costs) but also wants to achieve higher bandwidth utilization efficiency to be able to satisfy more demand and build resilience against the failures in the network components. As detailed below, our results show that with FON architecture, a smart deployment of a rather limited number of regenerators can achieve high levels of bandwidth efficiency. These promising results indicate that FON architecture can provide significant cost reductions and capacity enhancements at the same time.

Table 8: Exact solution results for COST-266 and NSF networks

			Average Regenerator Number						Average Run Time(seconds)						Average #BB nodes						Average #Col Generated					
Network	Demand	$\|D\|$	α_{min}	0.2	0.4	0.6	0.8	1.0	α_{min}	0.2	0.4	0.6	0.8	1.0	α_{min}	0.2	0.4	0.6	0.8	1.0	α_{min}	0.2	0.4	0.6	0.8	1.0
COST-266	TD	75	9.8	5.0	1.8	1.8	1.8	1.8	1564.4	1119.1	44.6	18.3	7.7	7.8	1631.0	303.5	15.0	15.0	1.6	2.0	11848.8	19325.3	9354.6	5062.4	4511.8	4879.0
		100	9.8	-	2.4	2.0	2.0	2.0	1573.9	-	137.3	33.0	13.0	14.0	548.6	-	19.2	18.4	2.6	3.0	17997.0	-	19249.2	9410.2	6254.2	6681.6
		125	9.4	-	2.8	2.0	2.0	2.0	1057.4	-	456.8	70.2	44.3	27.1	68.0	-	38.5	15.0	16.6	7.4	22505.0	-	28146.0	15429.0	11773.6	8946.2
		150	9.2	-	4.8	2.4	2.0	2.0	1195.3	-	1408.2	350.7	150.9	80.1	68.8	-	223.8	49.8	45.0	30.0	27787.6	-	38754.0	30407.2	18238.0	15748.6
	U	75	6.6	4.0	2.0	2.0	2.0	2.0	1561.9	518.2	17.7	8.1	8.5	6.8	857.8	94.0	0.0	1.4	2.6	1.2	16602.0	18928.5	7850.0	4855.6	4979.0	4672.2
		100	8.0	6.0	2.4	2.0	2.0	2.0	1786.7	482.1	163.6	31.3	22.3	22.6	693.4	75.0	17.8	16.2	6.0	5.8	22196.8	25693.0	19797.2	8022.0	7884.2	7734.2
		125	7.8	-	3.5	2.0	2.0	2.0	1308.3	-	482.2	211.3	68.7	32.9	174.6	-	18.0	48.8	35.2	7.0	30163.8	-	31034.5	21155.4	11813.8	10322.6
		150	7.0	-	4.8	2.2	2.0	2.0	692.1	-	587.9	283.8	136.3	92.0	41.2	-	50.3	31.0	57.4	37.2	31269.2	-	36506.0	31152.2	17352.8	14215.4
NSF	TD	75	3.6	-	-	3.0	2.8	2.4	4.1	-	-	2.1	2.5	2.1	1.8	-	-	1.0	1.4	0.4	897.6	-	-	751.3	762.0	656.0
		100	3.4	-	-	3.0	2.8	2.8	8.1	-	-	3.4	3.6	3.4	11.8	-	-	0.0	0.2	0.8	1200.2	-	-	904.0	995.4	952.4
		125	3.6	-	-	-	3.5	3.2	11.9	-	-	-	8.7	6.7	17.0	-	-	-	2.0	1.4	1681.2	-	-	-	1672.0	1351.6
		150	3.6	-	-	-	3.0	3.5	8.9	-	-	-	15.8	8.9	1.6	-	-	-	50.0	2.0	1748.4	-	-	-	1974.0	1692.5
	U	75	3.6	-	-	2.8	2.4	2.0	5.4	-	-	3.6	2.3	1.6	3.0	-	-	5.2	0.8	0.0	1147.4	-	-	757.2	748.8	585.0
		100	3.4	-	-	3.0	2.8	2.8	6.6	-	-	6.2	6.3	3.7	1.0	-	-	2.0	4.6	1.0	1408.0	-	-	1213.0	1154.6	949.4
		125	3.6	-	-	-	3.0	3.0	10.6	-	-	-	6.7	6.3	1.8	-	-	-	0.0	1.0	1860.6	-	-	-	1636.7	1339.6
		150	3.6	-	-	-	4.0	3.3	11.2	-	-	-	13.1	9.8	2.0	-	-	-	2.0	0.5	2065.2	-	-	-	1914.0	1835.8

Table 9 shows the lower bounds for the optimal solution values of the 27 problem instances with COST-266 network and traffic density distribution of 75,100,125 and 150 connection demands generated by random number seed 2. Cells depicted with "-" are the cases where the problem has no feasible solution and for each row, the first cell with a numeric value is the case where the percentage of allocated bandwidth is the minimum (i.e $\alpha = \alpha_{min}$). In order not to leave any dents in Table 9 we generate and solve additional problem instances, using different α values than we have in Table 5.

A quick look at the table reveals that certain combinations of regenerator deployment and bandwidth utilization (α) levels could be very attractive for the network management. For example, the first row ($|D| = 75$) of the table shows that with just 3 regenerators (deploying regenerators at 11% of nodes), it is possible to satisfy all the demand by using only 20% of the available bandwidth in each link. Note that for this case the lowest possible utilization level is 0.14 which requires 10 regenerators. Moreover, Tables 4-7 together show that regardless of the network and demand distribution differences, the same

Table 9: Number of required regenerators for COST-266 network with traffic density demand distribution.

	Percentage of bandwidth allocated (α)								
$\|D\|$	0.14	0.20	0.22	0.26	0.33	0.40	0.60	0.80	1.00
75	10	3	3	2	2	2	2	2	2
100	-	-	10	6	3	2	2	2	2
125	-	-	-	9	5	3	2	2	2
150	-	-	-	-	10	6	2	2	2

Table 10: Percentage of nodes appearing as regeneration points in the optimal solutions for the problem instances with COST-266 network and traffic density demand distribution.

		Node Number																											
$\|D\|$	α	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
75	α_{min}	20	0	100	60	100	40	20	0	0	0	0	0	20	100	80	40	0	0	0	100	60	60	60	60	0	20	0	0
	0.20	0	0	100	0	100	25	0	0	25	0	0	0	0	0	0	0	0	0	0	50	0	50	50	100	0	0	0	0
	0.40	0	0	40	0	0	0	0	0	40	0	0	0	0	0	0	0	20	0	0	0	0	20	0	40	0	0	0	20
	0.60	0	0	40	0	0	0	0	0	20	20	0	0	20	20	0	0	0	0	0	0	20	0	20	0	0	20	0	0
	0.80	20	0	20	0	0	0	0	0	0	0	0	0	0	0	0	20	0	0	20	0	0	0	0	80	0	0	0	20
	1.00	0	0	20	60	0	0	0	0	0	0	0	0	0	40	0	0	0	0	20	0	0	0	0	0	20	20	0	0
100	α_{min}	0	0	100	80	80	40	20	0	0	0	0	0	20	100	80	80	20	0	0	100	40	60	40	60	0	0	20	0
	0.40	0	0	40	0	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	20	0	20	60	80	0	0	0	0
	0.60	0	0	0	0	20	0	0	0	0	0	0	0	0	40	0	20	0	0	0	0	0	0	40	80	0	0	0	0
	0.80	0	0	40	40	20	0	0	0	20	0	0	0	40	20	0	0	0	0	0	0	0	0	0	0	0	0	20	0
	1.00	0	0	20	0	20	0	0	0	20	0	0	0	0	20	20	0	0	0	0	0	0	0	60	20	0	0	0	20
125	α_{min}	20	0	100	60	80	20	20	0	0	0	0	0	20	80	80	60	0	0	20	100	60	40	40	60	0	20	20	0
	0.40	0	0	100	0	75	0	0	0	0	0	0	0	0	0	0	25	0	0	0	0	0	25	0	50	0	0	0	0
	0.60	0	0	20	0	0	0	0	0	40	0	0	0	0	40	0	20	0	0	0	0	0	20	0	60	0	0	0	0
	0.80	0	0	80	0	20	0	0	0	20	0	0	0	20	20	20	0	0	0	0	0	0	0	0	20	0	0	0	0
	1.00	0	0	0	0	40	0	0	0	20	0	0	0	0	0	0	0	0	0	0	20	0	0	80	40	0	0	0	0
150	α_{min}	20	0	100	60	80	40	0	0	0	0	0	0	0	80	60	80	40	0	20	100	80	40	60	60	0	0	0	0
	0.40	0	0	100	0	50	0	0	0	0	0	0	0	0	0	25	25	25	0	0	100	0	0	25	100	0	0	0	25
	0.60	0	0	20	0	0	0	0	0	0	20	0	0	0	0	60	0	0	0	0	20	0	0	20	60	0	0	0	40
	0.80	0	0	0	20	0	0	0	0	40	0	20	0	0	20	20	0	0	0	0	0	0	20	0	20	0	0	0	40
	1.00	20	0	20	40	20	0	0	0	0	0	20	0	0	0	0	0	0	0	0	0	0	0	0	40	0	0	0	40

conclusion stays valid. Our results also show that some nodes are more likely to appear as regeneration nodes (in the optimal solutions regenerators are placed/activated on those nodes). In Table 10, each row depicts the results of 5 different RLP-FON instances with the COST-266 network and traffic density demand distribution. For each node, the percentage of solutions for which that node appears in the set of regenerator nodes is given in the table. For example, looking at the first row we can see that nodes 3,5,14 and 20 have been chosen as the regenerator nodes at all of the 5 problem instances solved with the minimum bandwidth allocation ($\alpha = \alpha_{min}$), whereas, nodes 1,13 and 26 were selected just once. Looking at the table, one can see that the optimal locations of the regenerators do not change drastically with the fluctuations in the demand. From Tables 6 and 7 we can see the same conclusion is also true for the NSF network for which the various combinations of the nodes 3,4,6 and 11 constitute approximately the 89.3% of the optimal solutions. Thus, the solutions obtained by the proposed algorithm can be considered as somewhat robust solutions. This is a desired property for the network management especially when it is hard to accurately estimate the demand at the time of planning.

Another interesting data in the given tables is the percentage of modulation swaps. We call it a modulation swap if a demand uses more than one modulation level (by going through modulation conversion in a regenerator node) on its light-path. Our results show that, in general, more strict bandwidth limitations necessitate more modulation swaps to satisfy connection demand with less network resources. For example, in the first line of Table 4, 53.3% of the demands have gone through at least one modulation swap on their determined light-paths when the allowed bandwidth is at its minimum ($\alpha = \alpha_{min}$). This number reduces to 21.3% when all the bandwidth is allowed to be used. A very similar trend is clearly visible in other tables as well. Thus, we can see that, at least according to the results of our numerical experiments, FON architecture's novel capability of multi modulation transmission appears to be very useful in increasing resource utilization efficiency for the optical networks.

5 Conclusions

This study revisits the regenerator location problem from the flexible optical network architecture perspective and introduces this problem to the operations research (OR) literature. Since the concept of FON architecture is quite new (due to the recent maturation of the enabling hardware technologies), this practically significant and theoretically interesting problem is not well studied in the literature. One of the purposes of this study is to draw the attention of the OR researchers to this gap in the literature and promote new studies in this promising area of research.

For the considered problem, we developed a path-segment based formulation. Our solution methodology introduces a new perspective to general path based formulations. In particular, our novel path-segment formulation makes it easy to include some special constraints on the paths which are otherwise harder to incorporate in a plain path based formulation.

We propose an efficient branch-and-price algorithm to solve the problem. We conducted extensive numerical experiments to test the performance of the proposed algorithm. As explained above, RLP-FON requires to solve regenerator location, routing, spectrum allocation and modulation selection problems jointly. The performance of the proposed algorithm is comparable to the state-of-the-art heuristic algorithms that solve these problems sequentially. From the practical point of view, these numerical studies provide significant managerial insights about this urgent problem. In particular, our findings show that with the FON architecture, it is possible to enhance network capacity and reduce the capital and operational costs of the optical network.

Although the practical motivation for RLP-FON comes from the telecommunications applications, this theoretically interesting problem and its simple extensions can actually appear in various application settings. In its current form, arc costs are not considered in the RLP-FON formulation (they are simply assumed to be 0). But this is only because of the practical setting of the problem where arc costs are either negligible or do not scale up to the regenerator deployment costs. From the mathematical point of view, adding arc costs does not disturb the main structure of the proposed algorithm at all. Note that adding arc costs to the objective function or considering new constraints on some resource usage at arcs can be included in the pricing problem by just adding a new term in the calculation of arc lengths of the pricing graph. Thus, our formulation and solution methodology can be easily adapted to the rather general multi commodity, multi modal flow problems and as such we leave it for a future study to investigate the application of the proposed solution methodology in different contexts than the optical networks.

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