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On Generalizations of a Theorem of Vaught

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Abstract This paper deals with the cylindric algebraic version of Vaught's theorem on the existence of prime models of atomic theories. It is proved that the algebraic version proved by Serény, which states that under certain conditions every isomorphism between two cylindric set algebras (*Cs*'s) is a lower base-isomorphism, extends to generalized cylindric algebras (*Gs*'s) although it does not extend to generalized weak cylindric set algebras (*Gws*'s); indeed, it is not true for weak cylindric set algebras (*Ws*'s).

In this paper we investigate the following question: To which subclasses of cylindric algebras can the algebraic version of Vaught's theorem (Theorem 2.3.4 of Chang and Keisler [6] or Proposition 3 below) concerning the existence of prime models of atomic theories be extended? A version of Vaught's theorem has already been stated for cylindric set algebras in Serény [11], according to which every isomorphism between two cylindric set algebras (*Cs*'s) satisfying certain conditions is a lower base-isomorphism. We prove that this theorem is true not only for *Cs*'s but for generalized cylindric set algebras (*Gs*'s) as well, although it is not true for generalized weak cylindric set algebras (*Gws*'s). Indeed, it is false for weak cylindric set algebras (*Ws*'s).

It is worth adding that we have already proved for *Cs*'s that Serény's theorem requires all the conditions given in its statement (see Biró [2], [3], and [4] and Biró and Shelah [5]).

Our treatment is based on the books *Cylindric Algebras*, Parts I and II, by Henkin, Monk, and Tarski ([7] and [8]), and *Cylindric Set Algebras* by Henkin, Monk, Tarski, Andréka, and Németi [9]. Here we recall only the notions connected with our central concepts. The background of the following definitions is in [8]. Throughout, α is an ordinal. Let U be any set and p any element of ${}^{\alpha}U$. A *cylindric set algebra* of dimension α (Cs_{α}) with base U or a *weak cylindric set algebra* of dimension α (Ws_{α}) with base U determined by p is a Boolean set algebra whose elements have α-sequences as points and whose unit element is the

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set ${}^{\alpha}U$ or ${}^{\alpha}U^{(p)}$, respectively (where ${}^{\alpha}U^{(p)}$ is the set of those sequences in ${}^{\alpha}U$ that differ from p only at finitely many places), enriched by cylindrification operations and diagonal elements. Let $\{U_i : i \in I\}$ be a family of sets. A *generalized cylindric set algebra* or a *generalized weak cylindric set algebra* of dimension α (Gs_α or Gws_α) with set of subbases $\{U_i : i \in I\}$ is a structure defined similarly to those above but having a unit element $\bigcup\{V_i : i \in I\}$, where for each $i \in I$, V_i is the unit element of a Cs_α or Ws_α, respectively, with base U_i and, for $i \neq j$, $V_i \cap V_j = 0$. We note that all the above structures belong to the class Gws_α. The *base* of a Gws_α is the union of its subbases (see Definition 3.1.1 of [8] or I.1.1 of [9]). We note that for a given first-order language a *Cs* corresponds to a model, while a *Gs* corresponds to a set of models (see Section 4.3 of [8]). Let $\mathfrak{A}$ be a Gws_α with base U and unit element V. Let f be a bijection from U onto a set W. We set $\tilde{f}X = \{y \in {}^{\alpha}W : f^{-1} \circ y \in X\}$ for any $X \in A$ and call $\tilde{f}$ the *base-isomorphism* induced by f. Let $V' \subseteq V$. We set $\mathrm{rl}_{V'}X = X \cap V'$ for any $X \in A$. Then $\mathrm{rl}_{V'}$ is the *relativization* (function) of $\mathfrak{A}$ to V'. Let $\mathfrak{C}$ be $\mathfrak{Sb}V'$, the full cylindric-relativized set algebra with unit element V'; i.e., the algebra with universe $\mathrm{Sb}\,V'$ with operations defined in the natural way (see Definition 3.1.2(i) of [8]). Let $\mathfrak{Rl}_{V'}\mathfrak{A} =_{def} \mathfrak{Sg}^{(\mathfrak{C})}\mathrm{rl}^{*}_{V'}A$. That is, $\mathfrak{Rl}_{V'}\mathfrak{A}$ is the algebra generated by the set $\{X \cap V' : X \in A\}$, the *relativization* (algebra) of $\mathfrak{A}$ to V' (see Definition 2.1 in Andréka and Németi [1]). If $\mathrm{rl}^{\mathfrak{A}}_{V'}$ is an isomorphism then it is called an *ext-isomorphism.* If in addition $V' = {}^{\alpha}U'$ for a set $U' \subseteq U$ then $\mathrm{rl}^{\mathfrak{A}}_{V'}$ is called a *strong ext-isomorphism.* A Gws_α $\mathfrak{A}$ is *base-minimal* if it is not strongly ext-isomorphic to any Gws_α except itself. A function g is called a *lower base-isomorphism* if $g = k^{-1} \circ h \circ t$ for some strong ext-isomorphisms k and t and for some base-isomorphism h (see Definition 3.1 of [1]).

In a little more detail, we will first show that Theorem 1 of [11] on *Cs*'s, which is an algebraic version of Vaught's theorem referred to above, extends to *Gs*'s. Then we will show that this is not true if the reference to *Gs*'s is replaced by a reference to *Gws*'s. That is, if $\mathfrak{A}_0$ and $\mathfrak{A}_1$ are countably generated, regular, locally finite-dimensional, and infinite-dimensional *Gs*'s with atomic neat n-reducts for any finite n, and f is an isomorphism from $\mathfrak{A}_0$ onto $\mathfrak{A}_1$, then f is a lower base-isomorphism, although this is not true for *Gws*'s instead of *Gs*'s.

Throughout, for typographical reasons, items of notation introduced with a symbol in a subscript will sometimes be written with the same symbol in brackets. For example, $Rl_W\mathfrak{A}$ and $\mathfrak{B}_{mi}$ (or $\mathfrak{B}_{m,i}$) have the same denotations as $\mathrm{Rl}(W)\mathfrak{A}$ and $\mathfrak{B}(m,i)$, respectively. Furthermore, in formulas the letter 'm' always denotes either 0 or 1.

First we recall Serény's theorem (Theorem 1 of [11]):

Theorem 1 *Suppose that $\mathfrak{A}_0$ and $\mathfrak{A}_1$ are Cs_α's such that:* (i) $\alpha \geq \omega$, (ii) $\mathfrak{A}_m$ *is locally finite-dimensional,* (iii) *for every $n \in \omega$ $\mathfrak{Nr}_n\mathfrak{A}_m$ is atomic,* (iv) $\mathfrak{A}_m$ *is regular,* (v) $\mathfrak{A}_m$ *is countably generated, and* (vi) $f \in \mathrm{Is}(\mathfrak{A}_0,\mathfrak{A}_1)$. *Then f is a lower base-isomorphism.*

We note that Theorem 1 is a natural generalization of the following theorem on Boolean algebras, which is equivalent to the Axiom of Choice:

Proposition 2 *If $\mathfrak{A}_0$ and $\mathfrak{A}_1$ are atomic Boolean set algebras and* $f \in \mathrm{Is}(\mathfrak{A}_0, \mathfrak{A}_1)$ *then f is a lower base-isomorphism.* (Note that Boolean set algebras can be identified with one-dimensional cylindric *set* algebras.)

Further, we recall Vaught's theorem (the algebraic version of which is Theorem 1):

Proposition 3 (See Chang and Keisler [6] Theorem 2.3.4, or Monk [10] Theorem 27.10) *Suppose that* Λ *is an ordinary first-order language,* Th *is a complete theory over* Λ, *and for every* $n \in \omega$ *the Boolean algebra of the formulas of* Λ *with at most* n *free variables* $v_0, \ldots, v_{n-1}$ *modulo* Th *is atomic. Then* Th *has an (elementary) prime model.*

First we will prove that Theorem 1 holds for Gs's. We start by proving some lemmas:

Lemma 4 *Suppose that* $\mathfrak{A} \in Gs_\alpha^{reg}$ *is such that the set of its subbases is* $\{U_i : i \in I\}$ *with* $U_i \cap U_j = 0$ *for* $i \neq j$. *Suppose also that* $X \in Zd\mathfrak{A}$. *Then there is a* $J \subseteq I$ *such that* $X = \bigcup\{{}^\alpha U_i : i \in J\}$.

Proof: We may assume that $X \neq 0$. Let $f \in X$ and $f \in {}^\alpha U_i$. Let $g \in {}^\alpha U_i$ be arbitrary. Then, by Corollary 3.1.26 of [8], $g \in X$. Consequently, ${}^\alpha U_i \subseteq X$.

Lemma 5 *Suppose that* $\mathfrak{A} \in Gs_\alpha^{reg} \cap Lf_\alpha$ *and* U *is a subbase of* $\mathfrak{A}$. *Set* $V = {}^\alpha U$ *and* $\mathfrak{B} = \mathfrak{Rl}_V\mathfrak{A}$. *Then* $\mathfrak{B} \in Cs_\alpha^{reg} \cap Lf_\alpha$.[1]

Proof: Trivially, for every $X \in A$

$$\Delta^{(\mathfrak{B})}(X \cap V) \subseteq \Delta^{(\mathfrak{A})}X.$$

Hence, $\mathfrak{B}$ is also locally finite-dimensional. Let $X \in A$. We will show that $X \cap V$ is regular in $\mathfrak{B}$. Set $\Delta = \Delta^{(\mathfrak{A})}X$, $\Sigma = \Delta^{(\mathfrak{B})}(X \cap V)$, and $\Theta = \Delta \sim \Sigma$. Let $f \in X$ and $g \in {}^\alpha U$ be such that $\Sigma \upharpoonright g \subseteq f$. We have to show that $g \in X$ (see Corollary 3.1.23 of [8]). We define the function g' on α as follows: For $\kappa \in \alpha$ let

$$g'(\kappa) = \begin{cases} f(\kappa), \text{ if } \kappa \in \Theta \\ g(\kappa), \text{ otherwise.} \end{cases}$$

We have $\Delta \upharpoonright g' \subseteq f$ so, by the regularity of X, $g' \in X$. Hence, $g \in c_{(\Theta)}^{(\mathfrak{B})}(X \cap V) = X \cap V$ since $\Theta \cap \Delta^{(\mathfrak{B})}(X \cap V) = 0$.

Now we are ready to state the generalization of Theorem 1 for Gs_α's.

Theorem 6 *Assume that* $\mathfrak{A}_0$ *and* $\mathfrak{A}_1$ *are* Gs_α*'s and* f *is an isomorphism from* $\mathfrak{A}_0$ *onto* $\mathfrak{A}_1$ *such that conditions* (i) *through* (v) *of* Theorem 1 *are satisfied for* $m < 2$. *Then* f *is a lower base-isomorphism.*[2]

Proof: Recall that, throughout, in formulas the letter m stands for either 0 or 1.

(1) Let the unit element of $\mathfrak{A}_m$ *be* $\bigcup\{{}^\alpha U_{mi} : i \in I_m\}$ with:
(1a) $U_{mi} \neq 0$, and
(1b) $U_{mi} \cap U_{mj} = 0$ for $i \neq j \in I_m$.

Set $V_{mi} = {}^\alpha U_{mi}$ and $\mathfrak{B}_{mi} = \mathfrak{Rl}_{V(m,i)}\mathfrak{A}_m$. By Lemma 4 there exists a Boolean set algebra $\mathfrak{C}_m$ with base I_m such that

(2) $Zd\mathfrak{A}_m = \{\bigcup\{V_{mi} : i \in T\} : T \in C_m\}$.

Claim 6.1 *Assume that $X \in At\mathfrak{Zd}\mathfrak{A}_m$, so that, by* (2), $X = \bigcup\{V_{mi} : i \in J_X\}$ *for some $J_X \subseteq I_m$. Further, let $Y \in A_m$ be such that $0 \neq Y \subseteq X$ and choose $k \in J_X$. Then $rl^{\mathfrak{A}(m)}_{V(m,k)} Y \neq 0$.*

Proof: Set $\mathfrak{D} = \mathfrak{Rl}_X \mathfrak{A}_m$. Then, by Theorem 3.2.76 of [8]

(3) $rl^{\mathfrak{D}}_{V(m,k)} \in \mathrm{Hom}(\mathfrak{D}, \mathfrak{B}_{mk})$.

So, by (3) $X \supseteq c^{(\mathfrak{A}(m))}_{(\Delta Y)} Y \in Zd\mathfrak{A}_m$. As X is an atom of $\mathfrak{Zd}\mathfrak{A}_m$ the inclusion in the last sentence implies $c^{\mathfrak{A}(m)}_{(\Delta Y)} Y = X$. As $\mathfrak{B}_{mk} = \mathfrak{Rl}_{V(m,k)} \mathfrak{A}_m$ it follows, by (3), that $c^{(\mathfrak{B}(m,k))}_{(\Delta Y)} rl^{\mathfrak{A}(m)}_{V(m,k)} Y = X \cap V_{mk} = V_{mk}$. Thus, by (1a), $Y \cap V_{mk} \neq 0$, proving Claim 6.1.

We also have

(4) $\mathfrak{Zd}\mathfrak{A}_m$ is atomic, and
(5) $\mathfrak{A}_m$ is strongly ext-isomorphic to a Gs_α $\mathfrak{A}'_m$ such that each atom of $\mathfrak{Zd}\mathfrak{A}'_m$ has the form ${}^\alpha U$ for some subbase U of $\mathfrak{A}_m$.

In fact, (4) is a particular case of condition (iii) of Theorem 1. For (5), use (2) to select $J_X \subseteq I_m$ such that $X = \bigcup\{V_{mi} : i \in J_X\}$ and choose $i_X \in J_X$ for every $X \in At\mathfrak{Zd}\mathfrak{A}_m$. Further, set $V'_m = \bigcup\{V_{i(X)} : X \in At\mathfrak{Zd}\mathfrak{A}_m\}$ and $U'_m = \bigcup\{U_{i(X)} : X \in At\mathfrak{Zd}\mathfrak{A}_m\}$. Then:

(6) $rl^{\mathfrak{A}(m)}_{V'(m)}$ is a strong ext-isomorphism.

In fact, by Theorem 3.1.77 of [8], $rl^{\mathfrak{A}(m)}_{V'(m)}$ is a homomorphism. Suppose that $Y \in A_m$ is such that $rl^{\mathfrak{A}(m)}_{V'(m)} Y = 0$. Then, for every $X \in At\mathfrak{Zd}\mathfrak{A}_m$, $Y \cap X \in A_m$ and $Y \cap V_{i(X)} = 0$. Thus, by Claim 6.1, for every $X \in At\mathfrak{Zd}\mathfrak{A}_m$ we have $Y \cap X = 0$. Hence, by (4), $Y = 0$. Thus $rl^{\mathfrak{A}(m)}_{V'(m)}$ is an ext-isomorphism. By (1b), $rl^{\mathfrak{A}(m)}_{V'(m)} = rl^{\mathfrak{A}(m)}({}^\alpha U'_m)$, so it is strong. The last three statements prove (6), which proves (5).

Since the composition of two strong ext-isomorphisms is a strong ext-isomorphism, by (5) we may assume that:

(7) Every atom of $\mathfrak{Zd}\mathfrak{A}_m$ has the form ${}^\alpha U$ for some subbase U of $\mathfrak{A}_m$.

Now, let W_m be the set-theoretical union of the atoms of $\mathfrak{Zd}\mathfrak{A}_m$. It can be easily seen that $rl^{\mathfrak{A}(m)}_{W(m)}$ is a strong ext-isomorphism. (This can be proved by an argument similar to that used in the proof of (6).) By (4) and (7), if U is a subbase of $\mathfrak{Rl}_{W(m)} \mathfrak{A}_m$ then ${}^\alpha U \in Zd\mathfrak{Rl}_{W(m)} \mathfrak{A}_m$. By the last two statements, similarly to assumption (7), we may assume:

(8) For every subbase U of $\mathfrak{A}_m$ we have ${}^\alpha U \in Zd\mathfrak{A}_m$.

By (v), (vi), and (8):

(9) There exists a bijection η from I_0 onto I_1 such that, for every $i \in I_0$, $fV_{0i} = V_{1,\eta i}$.

Throughout, we suppose that η satisfies condition (9). Let $i \in I_0$. Then it is easy to prove by (vi), (8), and (9) that $f_i =_{def} Rl^{\mathfrak{A}(0)}_{V(0,i)} \upharpoonright f$ is an isomorphism from B_{0i} onto $B_{1,\eta i}$. Trivially, B_{0i} and $B_{1,\eta i}$ satisfy condition (i) of Theorem 1. By (3) $rl^{\mathfrak{A}(m)}_{V(m,i)}$ is a homomorphism, so $\mathfrak{B}_{0i}$ and $\mathfrak{B}_{1,\eta i}$ also satisfy conditions (ii), (iii), and (v); while, by Lemma 5, they satisfy condition (iv). Hence, by Theorem 1, f_i is a lower base-isomorphism from B_{0i} onto $B_{1,\eta i}$. By (9) this means that:

(10) For $i \in I_m$ there is a $U''_{mi} \subseteq U_{mi}$ and a mapping ζ_i from U''_{0i} onto $U''_{1,\eta i}$ such that

(10a) ζ_i is a bijection (setting $V''_{mi} = {}^\alpha U''_{mi}$)

(10b) For every $i \in I_m$, $rl^{\mathfrak{B}(m,i)}_{V''(m,i)}$ is a strong ext-isomorphism, and

(10c) For every $i \in I_0$, $f_i = (rl^{\mathfrak{B}(1,\eta i)}_{V''(1,\eta i)})^{-1} \circ \zeta_i \circ rl^{\mathfrak{B}(0,i)}_{V''(0,i)}$.

Set $U^*_m = \bigcup\{U''_{mi} : i \in I_m\}$, $V^*_m = \bigcup\{V''_{mi} : i \in I_m\}$, and $\delta = \bigcup\{\zeta_i : i \in I_0\}$. By (1b) and (10a):

(11) δ is a bijection from U^*_0 onto U^*_1.

By (1b), (9), and (10c) we have the following derivation:

$$
\begin{aligned}
(12)\quad f &= \bigcup\{f_i : i \in I_0\} \\
&= \bigcup\{(rl^{\mathfrak{B}(1,\eta i)}_{V''(1,\eta i)})^{-1} \circ \tilde{\zeta}_i \circ rl^{\mathfrak{B}(0,i)}_{V''(0,i)} : i \in I_0\} \\
&= \bigcup\{(rl^{\mathfrak{A}(1)}_{V''(1,\eta i)})^{-1} \circ \tilde{\zeta}_i \circ rl^{\mathfrak{A}(0)}_{V''(0,i)} : i \in I_0\} \\
&= (rl^{\mathfrak{A}(1)}_{V^*(1)})^{-1} \circ \tilde{\delta} \circ rl^{\mathfrak{A}(0)}_{V^*(0)} \\
&= (rl^{\mathfrak{A}(1)}({}^\alpha U^*_1))^{-1} \circ \tilde{\delta} \circ rl^{\mathfrak{A}(0)}({}^\alpha U_0).
\end{aligned}
$$

By (1b), (11), and (12), f is a lower base-isomorphism.

We have now proved that Theorem 1 extends to Gs_α's. Now we prove that it is false for the much larger class Gws_α:

Theorem 7 *For every infinite ordinal α there exist Ws_α's $\mathfrak{A}_0$ and $\mathfrak{A}_1$ such that they satisfy conditions* (i) *through* (v) *of Theorem 1 and these two conditions:*

(vii) $\mathfrak{A}_0 \cong \mathfrak{A}_1$,

(viii) $\mathfrak{A}_0$ *and* $\mathfrak{A}_1$ *are not lower base-isomorphic.*

Proof: To prove Theorem 7 it is obviously sufficient to construct base-minimal Ws_α's $\mathfrak{A}_0$ and $\mathfrak{A}_1$ such that they satisfy conditions (vii) and (i) through (v) and are not base-isomorphic.

We assume that $\alpha = \omega \cdot 2$. For other infinite ordinals the construction is similar. Set $p_0 = \alpha \times \{0\}$, the α-sequence with range $\{0\}$, and $p_1 = \omega \times \{0\} \cup (\alpha \sim \omega) \times \{1\}$, the α-sequence whose values are 0 at finite numbers and 1 otherwise. For $m < 2$ let $\mathfrak{A}_m$ be the minimal subalgebra $\mathfrak{Mn}\ \mathfrak{Sb}({}^\alpha 2^{(p(m))})$ of the full α-dimensional weak cylindric set algebra with base 2 determined by p_m. By 2.1.16, 3.1.26, and 3.1.49 of [7] and [8] it can easily be proved that $\mathfrak{A}_0$ and $\mathfrak{A}_1$ are Ws_α's that satisfy conditions (i), (ii), (iv), (v), and (vii) of Theorem 1. It can also easily be proved that for each $n \in \omega$ and for $m < 2$ the set of atoms of $\mathfrak{Nr}_n \mathfrak{A}_m$ is the set $\{\Pi\{d_\Sigma : \Sigma \text{ is an equivalence class of } E\} \cdot \bar{d}(\alpha \times \alpha \sim E) : E \text{ is an equivalence relation on } \alpha\}$ (see Definitions 1.8.1, 1.9.1, and 2.6.28 of [7]).

Hence, condition (iii) is also satisfied. On the other hand $\mathfrak{A}_0$ and $\mathfrak{A}_1$ are base-minimal since they are infinite-dimensional Ws's with finite bases (see the remark following Definition 2.4.61 of [7]). But $\mathfrak{A}_0$ and $\mathfrak{A}_1$ are not base-isomorphic because for every element q of the unit element of A_1 there are, for all $i \in \alpha$, infinitely many $j \in \alpha$ such that $q_i \neq q_j$, yet this is not true for $\mathfrak{A}_0$. Thus (viii) is also satisfied.

Theorem 7 proves that Theorem 1 does not extend to Gws_α's, and actually is not true for Ws_α's.

Now we note that Vaught's theorem (Proposition 3) can easily be generalized for noncomplete theories using Theorem 6 and Section 4.3 of [8]. That is, we have the following model-theoretical theorem (Corollary 8), where the terminology of pages 44–46 of [7] is used. Thus Corollary 8 has a cylindric algebraic proof. A theory Th over a language Λ is called atomic iff for all $n \in \omega$, the Boolean algebra of the formulas having at most n free variables among $v_0, \ldots, v_{n-1}$, modulo Th, is atomic (see page 93 in Section 2.3 in Chang and Keisler [6], and footnote 2 on page 169 of [7], or Definition 27.5 of Monk [10]).

Corollary 8 *Assume that* Λ *is an ordinary first-order language,* Th *is a theory over* Λ *and* $\mathfrak{M}$ *and* $\mathfrak{N}$ *are classes of models of* Λ *such that:*

(ix) Λ *has countably many nonlogical constants,*
(x) $\Theta\rho\mathfrak{M} = \Theta\rho\mathfrak{N} = \mathrm{Th}$, *and*
(xi) Th *is an atomic theory.*

Then there is some $\mathfrak{M}' \subseteq \mathfrak{M}$ *such that* $\Theta\rho\mathfrak{M}' = \mathrm{Th}$, *and for every* $\mathfrak{M} \in \mathfrak{M}'$ *there is an* $\mathfrak{N} \in \mathfrak{N}$ *and a model* $\mathfrak{K}$ *of* Th *such that* $\mathfrak{K}$ *can be elementarily embedded to both* $\mathfrak{M}$ *and* $\mathfrak{N}$.

We note that if Th is a complete theory then the conclusion of the corollary is a straightforward consequence of Vaught's theorem (Proposition 3). However, in some countable languages there exist atomic and noncomplete theories. Using Lemma 13.11 of Monk [10] it can be seen that an example of such a theory is the pure theory of equality (which has no nonlogical constants and axioms). From Corollary 8 the following corollary easily follows:

Corollary 9 *If* $\mathfrak{M}$ *and* $\mathfrak{N}$ *are classes of models of a first-order language of power* ω *such that* $\Theta\rho\mathfrak{M} = \Theta\rho\mathfrak{N}$ *and this common theory* Th *is atomic then there is an* $\mathfrak{M}' \subseteq \mathfrak{M}$ *such that* $\Theta\rho\mathfrak{M}' = \mathrm{Th}$ *also holds; and for every* $\mathfrak{M} \in \mathfrak{M}'$ *there is an* $\mathfrak{N} \in \mathfrak{N}$ *such that* $\mathfrak{M} \equiv \mathfrak{N}$, *i.e.* $\mathfrak{M}$ *and* $\mathfrak{N}$ *are elementarily equivalent.*

Remark By closely analyzing the proof of Theorem 5 we can conclude that Corollary 9 can also be proved easily by a simple model-theoretical argument; furthermore, in that argument it is enough to assume, instead of Th being atomic (as was stated in the hypotheses of Corollary 9), that the Boolean algebra of the sentences of our language, modulo Th, is atomic. Then $\mathfrak{M}'$ can be chosen to be the class of those models in $\mathfrak{M}$ that satisfy some 0-atomic formulas over Th, i.e., some atoms of the last-mentioned Boolean algebra of sentences. Further, Corollary 8 follows from Corollary 9 and Vaught's theorem (Proposition 3).

NOTES

1. As R. J. Thompson pointed out, the regularity of $\mathfrak{B}$ can be proved without assuming that $\mathfrak{A}$ is *Lf*, using Lemma 3.1.42 of [8].

2. A negative solution for some particular cases of Problems 3.6 and 4 of Andréka and Németi [1] follows from this theorem.

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