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Do We Need Models?

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A truth valuational semantics consists of an assignment of truth values to atomic formulas together with a definition of truth keyed to the devices through which formulas are built up from simpler formulas. The definition serves to extend the assignment of truth values from atomic to nonatomic formulas. Each truth valuation V for a language L thereby effects a distribution of truth values over the formulas of L.

It is a central feature of truth valuational semantics that truth values are assigned to atomic formulas directly, not via assignments of objects to individual symbols or sets of n-tuples of objects to n-ary predicates. Further, the truth value of every nonatomic formula evaluated by V is a function of the truth values of simpler formulas. In particular, the truth value of a universal quantification is defined in terms of the truth-values of its instances in a way which does not involve assignment of objects to individual symbols. A truth valuational semantics is nondenotative. It is its nondenotational character which makes a truth valuational semantics of special philosophical interest.

By contrast, a semantics which assigns objects to individual symbols, sets of n-tuples of objects to n-ary predicates, etc., is *denotational*. Standard formalizations of denotational semantics, employing models, also serve to effect distributions of truth values over the formulas of L. Logical terms are defined relative to these distributions. For example,

> A sentence S of L is a logical truth of L just in case S is true relative to each model M of L

i.e., just in case every model of L determines a truth value distribution over the sentences of L on which S is assigned truth. Adapting the just cited definition to truth valuations, we obtain

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> A sentence S of L is a logical truth of L just in case S is true relative to each truth valuation V of L.

These exactly parallel definitions are extensionally equivalent, for a sentence S of L is true relative to each model M of L if and only if it is true relative to each truth valuation V of L.

Next consider the notion of finite consistency. Here the exactly parallel definitions would be as follows:

> For any finite set K of sentences of L, K is consistent just in case there is a model M of L such that each member of K is true relative to M.

> For any finite set K of sentences of L, K is consistent just in case there is some truth valuation V of L such that each member of K is true relative to V.

These definitions also are extensionally equivalent, for there is a model M of L relative to which each member of a finite set K of sentences is true if and only if there is a truth valuation V of L relative to which each member of K is true. A similar situation holds in respect to the consequence relation on finite sets of sentences.

Well-known difficulties arise, however, in connection with infinite sets of sentences, in particular the ω-inconsistent sets.

For expositional simplicity let L be a language with denumerably many individual constants and let the truth valuations for L be based on truth value distributions to atomic sentences. Consider now the set membering the negation NQF of L and each instance Fv/c of the quantification QF which results from dropping the quantifier Q and replacing, for the variable v in Q, each of its free occurrences in F by some individual constant c. There will be no truth valuation V of L relative to which NQF and each Fv/c are true. But there will be models of L relative to which all those sentences are true. Thus, the parallel definitions of general consistency,

> A set K of sentences of L is consistent just in case there is some model M of L relative to which each member of K is true,

> A set K of sentences of L is consistent just in case there is some truth valuation V of L relative to which each member of K is true,

determine different extensions for 'consistent'. The reason is that the truth valuations for L do not provide for all of the truth value distributions provided for by the models for L.

A similar divergence holds for the parallel definitions of general consequence.

Thus, one problem facing the truth valuation approach is that of how to attain the extensions for consistency and consequence fixed for these notions by denotational semantics employing models given that models provide for distributions of truth values unavailable within the truth valuation approach.

The basic line of attack has been to revise the definitions for these terms; i.e., to replace the parallel definitions noted above by nonparallel definitions.

The guiding idea for one revision was perhaps first indicated by Hintikka

([1], p. 33) when he suggested that names could be added as needed to embed consistent sets of sentences in model sets. This yields the notion of a language which results from a given language by the addition of individual constants. Given a truth valuation V of L it will be easy to define a truth valuation V' for a name-extension L' of L. And now the definition of consistency due to Dunn and Belnap ([2], p. 183) is ready at hand:

> A set K of sentences of L is consistent just in case there is a name-extension L' of L such that for some truth valuation V' of L' each member of K is true relative to V'.

An alternative method is developed by Leblanc in [3] and [4]. Let set K' of sentences of L be an alphabetic variant of a set K of sentences of L just in case the sentences in K' result from the sentences in K by some systematic rewriting of individual parameters. Leblanc then defines:

> A set K of sentences of L is consistent just in case there is an alphabetic variant K' of K such that for some truth valuation V of L, each member of K' is true relative to V.

Yet another new definition due to Leblanc [3] is as follows:

> A set K of sentences of L is consistent just in case for each finite subset K' of K there is a truth valuation V of L such that each member of K' is true relative to V.

As the authors show, these revised definitions all yield the classical extensions for consistency.

But is it quite necessary to proceed by revising the definitions? Would it not be possible to so revise truth valuations as to yield all of the truth value distributions available in terms of models? That is, might we not so revise the underlying truth valuational semantics that each classical distribution of truth values would be matched by some truth valuational distribution of truth values?

In fact this is not possible. The reason is that whereas the classical denotational semantics does not make the truth values of all of the truth valued formulas a function of the truth values of simpler truth valued formulas, it belongs to the very character of truth valuational semantics to do so. A semantics is truth valuational in character precisely in viewing all truth as a function of some truth. But the truth value distributions classically but not truth valuationally available are just those which involve truth value assignments which are not functions of such assignments to simpler formulas.

To see this point quite clearly consider any first-order language L and model M of L with domain set D and function v assigning a domain element to each individual symbol of L and sets of n-tuples of elements of D to the n-ary predicates of L. Let an α-variant of M be model M' like M except at most in what v' of M' assigns to variable α of L. Similarly, let a variant M' of M be a model like M except at most in what v' of M' assigns to the variables of L. Finally let a satisfaction relation for the formulas of L relative to model M of L be so defined that M satisfies an atomic formula $\psi\tau_1 \ldots \tau_n$ just in case $\langle v(\tau_1), \ldots, v(\tau_n)\rangle \in v(\psi)$; and M satisfies $(\alpha)\phi$ just in case, for each α-variant M' of M, M' satisfies ϕ. Truth and falsity are then defined as follows:

> A formula ϕ of L is true relative to M just in case it is satisfied by each variant M' of M, and is false relative to M just in case it is satisfied by no variant M' of M.

Now, there will be many formulas of L whose truth values are a function of the truth values of simpler formulas of L. But this is not the general case, for there also are formulas of L which are truth valued relative to M whose truth value relative to M is not a function of the truth values relative to M of simpler formulas of L. In general, truth is a function of satisfaction, not of truth.

Now, it is precisely in virtue of this feature that the classical denotational semantics just sketched yields more truth value distributions than are available through a truth valuational semantics. Thus, since a truth valuational semantics is based on the idea that all truth is a function of some truth, no variation on the truth valuational approach will provide for all of the truth value distributions available on the denotational approach.

Nonetheless, the aim of this paper is to provide a nondenotational semantics for first-order languages which will match, one for one, each distribution of truth values available in terms of a denotational semantics.

This will require the development of an *alternative* nondenotational semantics, a nondenotational semantics which is not a truth valuational semantics.

Our basic idea is to define a nondenotative relation parallel to satisfaction. Our term for this nondenotative relation will be *comprehension*. We also define a nondenotative parallel to a model which we call an *atomic system*. We shall define truth and falsity as follows:

> A formula ϕ of L is true relative to an atomic system A of L just in case, for every variant ϕ' of ϕ, A comprehends ϕ' and is false relative to A just in case, for every variant ϕ' of ϕ, ϕ' is not comprehended by A,

where ϕ' is a variant of ϕ just in case ϕ' results from ϕ by rewriting the variables in ϕ.

What we will show is that for every first-order language L (with or without individual constants or operations symbols) and model M of L there is an atomic system A of L such that for any formula ϕ of L, ϕ is true relative to M if and only if ϕ is true relative to A and ϕ is false relative to M if and only if ϕ is false relative to A; and conversely.

Given this result it will not be necessary to revise the definitions for consistency and consequence beyond replacing "true relative to model M" by "true relative to atomic system A".

Atomic systems, comprehension and truth. By a language we shall mean countable set $N \cup P \cup O$, for set N of names, nonempty set P of predicates, and set O of operation symbols. By a term we mean a name, a variable, or $\theta\tau_1, \ldots \tau_n$ where θ is an n-ary operation symbol and each τ_i is a term. Let T be a function from the terms into the variables satisfying these conditions:

$$T(\alpha) = \alpha,$$
$$T(\theta\tau_1 \ldots \tau_n) = T(\theta T(\tau_1) \ldots T(\tau_n)),$$

for each n-ary operation symbol $\theta \in 0$, variable α, and terms $\tau_1, \ldots, \tau_n$. Let A be a set of atomic formulas constructed from language L each of which is an n-ary predicate flanked by n occurrences of variables. Then $\langle A, T\rangle$ is an atomic system for L.

An atomic system for a language is our syntactic analogue of a model of that language. The set A represents in syntactic terms assignments of extensions to predicates. T similarly represents assignments of extensions to terms.

Our syntactic analogue of satisfaction is *comprehension*, spelled out as follows. First, formula $\bar{\phi}$ is an α-variant of formula ϕ just in case $\bar{\phi}$ differs at most from ϕ in having bound occurrences of distinct variables where and only where ϕ has bound occurrences of distinct variables and having free occurrences of variable ω where and only where ϕ has free occurrences of variable α. Second, call a formula ψ a T-variant of formula ϕ just in case ψ results from ϕ by replacing one or more occurrences of term τ in ϕ by $T(\tau)$, or conversely. Then formula ϕ is comprehended by atomic system $K = \langle A, T\rangle$ just in case

(i) $\phi \in A$; or
(ii) ϕ is $\neg\psi$ and ψ is not comprehended by K; or
(iii) ϕ is $\psi \vee \chi$ and either ψ or χ is comprehended by K; or
(iv) ϕ is $(\alpha)\psi$ and $\bar{\psi}$ is comprehended by K for each α-variant $\bar{\psi}$ of ψ; or
(v) ϕ is a T-variant of ψ and ψ is comprehended by K.

In the model-theoretic sense $(\alpha)\phi$ is satisfied by model M just in case ϕ is satisfied by all *α-variants* of M; i.e., all models which differ from M at most on their assignments to variable α. An α-variant of formula ϕ is our syntactical representation of an α-variant of a model M.

Model relative truth as developed in terms of satisfaction is defined as follows: A formula ϕ is true relative to a model M just in case it is satisfied by all *variants* of M, i.e., all models which differ from M at most in their assignments to variables. We syntactically represent the notion of a variant of a model M by a *variant of a formula* ϕ, i.e., a formula ϕ' which results from ϕ by simultaneously replacing all occurrences of all the distinct variables in ϕ by distinct variables. Then a formula ϕ is *true relative to atomic system* K just in case K comprehends all variants of ϕ. This is our syntactic analogue of model relative truth.

Note that just as 'satisfaction by all variants of model M' generalizes 'satisfaction by all α-variants of model M', so also 'comprehends all variants of ϕ' generalizes 'comprehends all α-variants of ϕ', since each α-variant of ϕ is also a variant of ϕ, though not conversely.

Example: Let $L = \{0, =, ', +, \cdot\}$. Thus L consists of one name, one predicate, one unary operation symbol, and two binary operation symbols. It is a language suitable for arithmetic. Now consider an atomic system $K = \langle A, T\rangle$ defined for L relative to the standard enumeration of the variables as follows:

$A = \{$'$x = x$', '$y = y$', '$z = z$', $\ldots\}$
$T($'0'$) =$ 'x'
$T(\alpha') = \omega$ iff α is the nth variable and ω is the $n + 1$th variable
$T(\alpha + \omega) = \gamma$ iff α is the nth variable and ω is the mth variable and γ is the $(n - 1) + (m - 1)$th variable

$T(\alpha \cdot \omega) = \gamma$ iff α is the nth variable and ω is the mth variable and γ is the $(n-1)\cdot(m-1)$th variable.

By the standard enumeration of the variables, x is the first variable, y the second, z the third. Here is how to show $0' + 0' = 0''$ is true relative to K: K comprehends $z = z$, which is a T-variant of $y + y = z$, which is a T-variant of $y + y = y'$, which is a T-variant of $x' + x' = x''$ which is a T-variant of $0' + 0' = 0''$. Thus, $0' + 0' = 0''$ is comprehended by K. Since any variant of that formula is that formula, the formula is also true relative to K. Note also that although $y + y = z$ is comprehended by K, not all of its variants are (e.g., K does not comprehend $x + x = y$); thus, $y + y = z$ is not true relative to K.

For another example, $T(\alpha \cdot \omega') = T((\alpha \cdot \omega) + \alpha) = \gamma$ for any variables α, ω and some variable γ. So, since K comprehends $\gamma = \gamma$ for any variable γ, K comprehends $(x)(y)((x \cdot y') = (x \cdot y) + x)$ and all variants of this formula. Thus this formula is true relative to K.

By similar reasoning it can be shown that the usual axioms of arithmetic are true relative to K. It can also be shown that K has this version of ω-completeness: If ϕ^{α}/τ is true relative to K, for all closed terms τ, $(\alpha)\phi$ is also true relative to K. Thus K is equivalent to a standard model of arithmetic in the sense that a formula of L is true relative to the model just in case it is true relative to K.

Theorem *Every atomic system for a language L is equivalent to a model for L.*

Our strategy is to prove this plus its converse: every model for L is equivalent to an atomic system for L, where a model for a language L and an atomic system for L are *equivalent* just in case the set of formulas true in the model is the set of formulas true relative to the atomic system.

Though entrenched, the notion of truth in a model is characterized in diverse ways, so it will not hurt to spell out our characterization. First, a model for a language L is a pair $\langle D, v\rangle$ for some nonempty set D and function v defined for L plus the variables. In particular, $v(\beta) \in D$ and $v(\alpha) \in D$ for each name β and variable α; v assigns the usual things to the predicates and operation symbols in L as well as all terms constructed from L. In addition,

$\psi\tau_1, \ldots, \tau_n$ is satisfied by $\langle D, v\rangle$ iff $\langle v(\tau_1), \ldots, v(\tau_n)\rangle \in v(\psi)$, for each atomic formula $\psi\tau_1 \ldots \tau_n$ of L

$\neg\phi$ is satisfied by $\langle D, v\rangle$ iff ϕ is not satisfied by $\langle D, v\rangle$, for each negation $\neg\phi$ of L (similarly for disjunctions $\phi \vee \psi$)

$(\alpha)\phi$ is satisfied by $\langle D, v\rangle$ iff ϕ is satisfied by $\langle D, v'\rangle$, for each function v' which differs from v at most in what it assigns to α ('each $v' =_{\alpha} v$' for short).

A formula ϕ is *true* (false) in a model M just in case ϕ is satisfied by all (no) variants of M.

A model of L is *complete* just in case, for each quantification formula $(\alpha)\phi$ of L, the model satisfies $(\alpha)\phi$ iff it satisfies every α-variant of ϕ.

Let $K = \langle A, T\rangle$ be any atomic system for L. Then an equivalent model is constructed as follows:

D is the set of variables
$v(\alpha) = \alpha$, for all variables α
$v(\beta) = T(\beta)$, for each name $\beta \in L$
$v(\theta) = \{\langle\langle \alpha_1, \ldots, \alpha_n\rangle, \alpha\rangle\colon T(\theta\alpha_1 \ldots \alpha_n) = \alpha\}$ for each n-ary operation symbol θ and variables $\alpha, \alpha_1, \ldots, \alpha_n$
$v(\psi) = \{\langle v(\alpha_1), \ldots, v(\alpha_n)\rangle\colon \psi\alpha_1 \ldots \alpha_n$ is comprehended by $K\}$, for each n-ary predicate $\psi \in L$ and variables $\alpha_1, \ldots, \alpha_n$.

Now $\langle D, v\rangle$ is complete. This follows from the fact that v maps the set of terms *onto* $D =$ the set of variables. Using the fact that $\langle D, v\rangle$ is complete, an induction on the number of occurrences of logical signs (= connectives and quantifiers) in a formula ϕ establishes that $\langle D, v\rangle$ satisfies ϕ just in case K comprehends ϕ. Thus K is equivalent to $\langle D, v\rangle$.

Theorem *Every model is equivalent to an atomic system.*

This is the main thing we seek to show.

First we shall say a model $\langle D, v\rangle$ is a *falsification* model just in case, for each quantification formula $(\alpha)\phi$ of L, if the model fails to satisfy $(\alpha)\phi$ then, for some variable ω free in ϕ^α/ω wherever α is free in ϕ ('ω free for α in ϕ', for short), the model fails to satisfy ϕ^α/ω. $\langle D, v\rangle$ will be called *normal* just in case for each term τ there is a variable α such that $v(\tau) = v(\alpha)$. Finally, two models of a language will be said to be *equivalent* just in case the same formulas of the language are true in each model.

We outline the proof of our theorem by means of a series of lemmas.

Lemma 1 *Every model of a language is equivalent to a falsification model of that language which is also normal.*

Proof: Let $\langle D, v\rangle$ be a model of L and let R well order D. Let $\chi_1, \chi_2, \ldots$ enumerate the quantification formulas of L. We construct a series of sets $S_0, S_1, \ldots$ of variables as follows: S_0 is the null set; for $i > 0$, $S_i = S_{i-1} \cup \{\gamma_1, \ldots, \gamma_n\} \cup \{\omega, \omega_1, \ldots, \omega_m\}$ where $\gamma_1, \ldots, \gamma_n$ are all the variables free in $\chi_i = (\alpha)\phi$ but not in S_{i-1}, ω is the first variable free for α in ϕ which is not in $S_{i-1} \cup \{\gamma_1, \ldots, \gamma_n\}$ and, where m is the number of terms in χ_i, $\omega_1, \ldots, \omega_m$ are the first m variables not in $S_{i-1} \cup \{\gamma_1, \ldots, \gamma_n, \omega\}$. We next construct a series of functions $v_0, v_1, \ldots$ which agree with v on L. v_0 is the subfunction of v defined on L, i.e., on the names, predicates and operation symbols of L. For each $i > 0$, v_i is defined on S_i in three stages. First, for each $\alpha \in S_{i-1}$, $v_i(\alpha) = v_{i-1}(\alpha)$; and for each γ_k, $v_i(\gamma_k) = v(\gamma_k)$. Second, for each ω_k, $v_i(\omega_k) = v_i(\tau)$ where τ is the kth occurring term in χ_i. Third, $v_i(\omega) = v(\omega)$ if $\langle D, v'\rangle$ satisfies $(\alpha)\phi$ where v' differs from v at most in assigning to the free variables of $\chi_i = (\alpha)\phi$ what v_i assigns to those variables; otherwise $v_i(\omega)$ is the R-least element of $\{d\colon d \in D$ and for some $v^* =_\alpha v'$, $\langle D, v^*\rangle$ does not satisfy ϕ and $v^*(\alpha) = d\}$.

Let $\bar{v}$ be the union of the $v_0, v_1 \ldots$. Then first observe that for each variable α and functions v_i and v_j defined for α, $v_i(\alpha) = v_j(\alpha)$. Thus $\bar{v}$ also is a function. Since each v_i agrees with v on L, so does $\bar{v}$. Finally, note that each variable is assigned an element of D by some v_i. Thus $\langle D, \bar{v}\rangle$ is a model for L. Now $\langle D, \bar{v}\rangle$ satisfies the lemma because, first, since $\bar{v}$ agrees with v on L,

$\langle D, \bar{v}\rangle$ is equivalent to $\langle D, v\rangle$, and, second, by the ω-assignments, $\langle D, \bar{v}\rangle$ is a falsification model for L which is also normal.

Lemma 2 *Every model of a language L is equivalent to a complete model of that language which is also normal.*

Proof: Every falsification model is complete. This fact plus Lemma 1 yields Lemma 2.

Lemma 3 *Let $\langle D, v\rangle$ be a model of L which is complete and normal. Let $A = \{\psi\alpha_1 \ldots \alpha_n$: $\langle D, v\rangle$ satisfies $\psi\alpha_1 \ldots \alpha_n\}$ for n-ary predicate $\psi \in L$ and variables $\alpha_1, \ldots, \alpha_n$. Let T satisfy these conditions: $T(\beta) = \alpha$ iff $v(\beta) = v(\alpha)$, for name β and variable α (that the model is normal guarantees that this condition can be satisfied); $T(\theta\alpha_1 \ldots \alpha_n) = \alpha$ iff $v(\theta\alpha_1 \ldots \alpha_n) = \alpha$, for variables α, $\alpha_1, \ldots, \alpha_n$ and n-ary operation symbol θ. Then $\langle A, T\rangle$ is an atomic system equivalent to $\langle D, v\rangle$.*

Proof: First, since $\langle D, v\rangle$ is normal it follows by the construction of T that $T(\tau) = \alpha$ iff $v(\tau) = v(\alpha)$, for term τ and variable α. This fact yields the result that $\langle D, v\rangle$ satisfies ϕ just in case $\langle A, T\rangle$ comprehends ϕ if ϕ is an atomic formula. Then an inductive proof which relies upon $\langle D, v\rangle$ being complete establishes the same for any formula ϕ. This establishes the equivalence of $\langle A, T\rangle$ and $\langle D, v\rangle$.

From the three lemmas it follows that every model is equivalent to an atomic system.

Here is the main idea of the argument. Let M be any model of L. Then there is a model M' which agrees with M on L and which is complete and normal. Since the models agree on L, a formula is true relative to one model just in case it is true relative to the other. That is, M and M' are equivalent. Since M' is complete and normal, an atomic system K may be constructed which comprehends just the formulas M' satisfies. Hence a formula is true relative to M just in case it is true relative to K. That is, M and K are equivalent.

Applications: Our main purpose has been to show that our nondenotative, system relative theory of truth matches any classical model-theoretic distribution of truth values over first-order languages. We have shown just this, having shown

(1) Every atomic system is equivalent to a model.
(2) Every model is equivalent to an atomic system.

A corollary is that our proposed nondenotative theory of truth yields the standard extensions of logical notions (consequence, consistency, etc.) in a perfectly straightforward way, simply by replacing, in the standard definitions, occurrences of 'model' by occurrences of 'atomic system'.

Further, the semantical properties of theories are explicable in terms of system-relative truth. Consider a language interpreted by model M, and let X be a theory couched in the language. Then X is complete just in case only the theory's theorems are true relative to M. By (2) there is an atomic system K equivalent to M. Thus X is complete by model theoretic criteria just in case only X's theorems are true relative to K.

Next, consider ω-incompleteness. Let M be a model verifying the usual axioms of arithmetic but not the Gödel sentence; in that case M verifies $\neg(\alpha)\phi$. As Gödel showed, for each numeral n, ϕ^{α}/n is provable from the axioms. Thus M verifies each of these as well as $\neg(\alpha)\phi$. Thus M is a nonstandard model of arithmetic. By (2) there is an equivalent atomic system K. Then each sentence $\phi^{\alpha}\backslash n$ is true relative to K even though $(\alpha)\phi$ is not. More generally, ω-incompleteness can be interpreted in nondenotative terms: there are languages with denumerably many closed terms such that, for some universal quantification $(\alpha)\phi$, there is an atomic system K such that ϕ^{α}/τ is true relative to K, for all closed terms τ, while $(\alpha)\phi$ is not true relative to K.

As a final example consider the following reasoning based on Gödel's incompleteness result. Suppose that M verifies the usual axioms, and that each element of the domain is named by some numeral, so that M is a standard model for arithmetic. Let $(\alpha)\phi$ be the Gödel sentence. Since M verifies each axiom, M verifies ϕ^{α}/n for each numeral n. Since M verifies each of these and since each domain element is named, M verifies $(\alpha)\phi$. If $(\alpha)\phi$ is added to the axioms, the Gödel proof may be reapplied. Thus, there is no consistent decidable extension of the axioms from which everything true in M is provable. Given (2) this reasoning can be replicated in terms of system-relative truth: Let Δ be any axiom set suitable for arithmetic. Let K be an atomic system equivalent to the standard model M. Then each element of Δ is true relative to K. So is $(\alpha)\phi$. If that sentence is added to Δ, the Gödel proof may be reapplied. Thus, there is no consistent decidable extension Δ' of Δ such that the set of sentences provable from Δ' is the set of sentences true relative to K.

In all these cases, classical conceptions and arguments can be directly carried out in the framework of our nondenotative semantics. This seems to us a strong reason in favor of the view that in respect to the logic of first-order languages denotative notions are semantically dispensable.

REFERENCES

[1] Hintikka, J., "Two papers on symbolic logic," *Acta Philosophica Fennica*, vol. 8 (1955), pp. 11–51.

[2] Dunn, J. M. and N. D. Belnap, Jr., "The substitution interpretation of the quantifiers," *Noûs*, vol. 2 (1968), pp. 177–185.

[3] Leblanc, H., "A simplified account of validity and implication for quantificational logic," *The Journal of Symbolic Logic*, vol. 33 (1968), pp. 231–235.

[4] Leblanc, H., *Truth-Value Semantics*, North-Holland, New York, 1976.

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