Notre Dame Journal of Formal Logic
Volume 24, Number 3, July 1983

A Simplification of the Logic of Conditionals

E. J. LOWE

In this paper I aim to show how the conditional connectives in nonmaterial conditional sentences may be defined in terms of truth-functional operators and monadic modal operators ('$\Box$' and '$\Diamond$'), thus reducing the logic of such conditionals to a branch of ordinary monadic modal sentential logic. This is a project which conflicts with the influential existing systems of conditional logic due to Stalnaker [7] and Lewis [3], and where appropriate I shall justify my departures from their theories.

I make no distinction between nonmaterial conditionals on the basis of their being indicative or subjunctive in mood, nor on the basis of whether or not they are counterfactual in force. It is my view that these differences are immaterial to the logical characteristics of such conditionals.[1] In this I differ from Lewis (see [3], pp. 3-4). However, I agree with Lewis in distinguishing between what I call 'strong' and 'weak' nonmaterial conditionality, that is, between the forms '$p \mathrel{\Box\!\!\rightarrow} q$' and '$p \mathrel{\Diamond\!\!\rightarrow} q$' ([3], pp. 1-2).[2] Though, of course, I do not insist as Lewis does that these forms be read only counterfactually as 'If it were the case that p, then it would be the case that q' and 'If it were the case that p, then it might be the case that q', respectively.

The problem (probably insoluble) of trying to find in ordinary language perfectly general and natural *conditional* readings of the forms '$p \mathrel{\Box\!\!\rightarrow} q$' and '$p \mathrel{\Diamond\!\!\rightarrow} q$' which prescind from the distinction between indicative and subjunctive mood may be evaded by reading these forms respectively as '(That) p necessitates (that) q' and '(That) p possibilifies (that) q', though in adopting such readings we must be ready to adjust the strength of these modal expressions according to context, since it is plain that very often we shall be concerned not with *logical* but at most with *physical* necessitation and possibilification. This approach may invite comparisons with Von Wright's [8] system of dyadic

Received June 11, 1982; revised August 25, 1982

modal sentential logic,[3] and, indeed, I shall take it that our problem is effectively one of reducing a certain system of dyadic modal logic to one of the monadic varieties. (Von Wright resists any such attempted reduction.)

My procedure will be as follows. First I shall delineate some of the key logical characteristics of nonmaterial conditionals, subsequently formalising these results by framing a system of dyadic modal sentential logic incorporating these characteristics. Then I shall show how this system may be recovered from one well-known system of monadic modal sentential logic when appropriate definitions of the strong and weak nonmaterial conditional connectives are added to the latter. Some attempt to justify these definitions on independent grounds will be made. At no time, however, shall I argue for my position by appeal to considerations involving 'possible worlds', because I find this notion so fraught with epistemological and ontological difficulties that to explicate conditionals in terms of possible worlds must, in my view, be to explain the obscure by the still more obscure.

First, then, I agree with Lewis ([3], p. 2) in regarding the forms '$p \mathrel{\Box\!\!\rightarrow} q$' and '$p \mathrel{\Diamond\!\!\rightarrow} q$' as interdefinable, as follows:

$$p \mathrel{\Diamond\!\!\rightarrow} q =_{df} \sim(p \mathrel{\Box\!\!\rightarrow} \sim q).$$

Thus, for example, 'If it rains, then we may not get wet'[4] is equivalent to the negation of 'If it rains, then we shall get wet', as is evident from the fact that to assert the former would be a way of contradicting an assertion of the latter. Stalnaker [6] would apparently dispute this. But suppose X and Y are on open moorland with threatening clouds above and X says 'If it rains, then we shall get wet'. Y, knowing unlike X that there is a hut some little way off, may reply 'No, we *may* not—we may reach shelter in time'. To dispute X's claim Y need not be in a position to assert 'If it rains, then we *shan't* get wet'. Again, to take a different example, suppose X were to assert 'If I toss this coin, then it may land tails', believing the coin to be a normal one. Y, knowing that the coin is in fact double-headed, would deny X's claim by replying 'No, it won't—it'll land heads', the implication being that the contradictory of X's assertion is 'If I toss this coin, then it won't land tails'. To receive such a reply X need not have made the stronger assertion 'If I toss this coin, then it *will* land tails'.

Next, it seems clear that '$p \mathrel{\Box\!\!\rightarrow} q$' entails '$p \mathrel{\Diamond\!\!\rightarrow} q$': thus, 'If it rains, then we shall get wet' obviously entails 'If it rains, then we may get wet'. (Lewis, however, does not concede this unrestrictedly, maintaining that '$p \mathrel{\Diamond\!\!\rightarrow} q$' will be false if '$p \mathrel{\Box\!\!\rightarrow} q$' is 'vacuously' true ([3], pp. 21f). But our intuitions provide no clear guidance here, it seems to me, so that the only relevant considerations are ones of overall systematic simplicity. On these grounds I dismiss Lewis's restriction.)

It seems equally clear that '$p \mathrel{\Box\!\!\rightarrow} q$' entails the *material* conditional '$p \rightarrow q$', since it is incompatible with the negation of the latter, which is equivalent to 'p & $\sim q$'. Thus 'If it rains, then we shall get wet' is incompatible with 'It will rain, but we shan't get wet'. This is in agreement with Lewis ([3], p. 27).

I believe, in opposition to both Lewis and Stalnaker, that the strong nonmaterial conditional connective is *transitive,* that is, that '$p \mathrel{\Box\!\!\rightarrow} q$' and '$q \mathrel{\Box\!\!\rightarrow} r$' together entail '$p \mathrel{\Box\!\!\rightarrow} r$' ([3], pp. 32ff). Thus there seems little doubt that 'If it rains, then we shall get wet' and 'If we get wet, then we shall catch colds' together entail 'If it rains, then we shall catch colds'. Lewis and Stalnaker offer counterexamples, but I shall argue in an appendix to this paper that these are not convincing.

Finally, I consider that the conjunction of '$p \mathrel{\Box\!\!\rightarrow} q$' and '$p \mathrel{\Box\!\!\rightarrow} r$' entails and is entailed by '$p \mathrel{\Box\!\!\rightarrow} (q \,\&\, r)$'. This is exemplified by the apparent equivalence of 'If it rains, then we shall get wet; and if it rains, then the match will be called off' with 'If it rains, then we shall get wet and the match will be called off'.

We are now in a position to formalize the foregoing results within the framework of the following axiomatic system, which I call System *D1* ('*D*' denoting that this is a system of *dyadic* modal logic).

The System D1

Vocabulary

$p, q, r, \ldots$	sentential letters
$\sim, \&, \vee, \rightarrow, \longleftrightarrow$	truth-functional operators
$\Box\!\!\rightarrow, \Diamond\!\!\rightarrow$	strong and weak nonmaterial conditional connectives
(,)	brackets.

Well-formed formulas (wffs)

1. Any sentential letter standing alone is a wff.
2. If A is a wff, then $\sim A$ is a wff.
3. If A and B are wffs, then $(A \,\&\, B)$, $(A \vee B)$, $(A \rightarrow B)$, $(A \longleftrightarrow B)$, $(A \mathrel{\Box\!\!\rightarrow} B)$ and $(A \mathrel{\Diamond\!\!\rightarrow} B)$ are wffs.[5]
4. Nothing else is a wff.

Definitions The truth-functional operators are interdefinable in the usual ways. In addition we adopt

Definition $\Diamond\!\!\rightarrow$ $\quad p \mathrel{\Diamond\!\!\rightarrow} q =_{df} \sim(p \mathrel{\Box\!\!\rightarrow} \sim q)$.

Axioms

D1.1	$(p \vee p) \rightarrow p$
D1.2	$q \rightarrow (p \vee q)$
D1.3	$(p \vee q) \rightarrow (q \vee p)$
D1.4	$(q \rightarrow r) \rightarrow ((p \vee q) \rightarrow (p \vee r))$
D1.5	$p \mathrel{\Box\!\!\rightarrow} q \rightarrow p \mathrel{\Diamond\!\!\rightarrow} q$
D1.6	$p \mathrel{\Box\!\!\rightarrow} q \rightarrow (p \rightarrow q)$
D1.7	$(p \mathrel{\Box\!\!\rightarrow} q \,\&\, q \mathrel{\Box\!\!\rightarrow} r) \rightarrow p \mathrel{\Box\!\!\rightarrow} r$
D1.8	$(p \mathrel{\Box\!\!\rightarrow} q \,\&\, p \mathrel{\Box\!\!\rightarrow} r) \longleftrightarrow p \mathrel{\Box\!\!\rightarrow} (q \,\&\, r)$.

(Axioms D1.1-D1.4 are the *Principia Mathematica* axioms for nonmodal sentential logic (see [1], p. 17).)

Rules of Inference

MP. Modus Ponens: if A is a thesis and $(A \rightarrow B)$ is a thesis, then B is a thesis.

US. Uniform Substitution: if A is a thesis containing one or more occurrences of a sentential letter B and C is obtainable from A by substituting a wff D for *every* occurrence of B in A, then C is a thesis.

SE. Substitution of Equivalents: if $(A \longleftrightarrow B)$ is a thesis and C is a thesis containing one or more occurrences of A and D is obtainable from C by substituting B for one or more of the occurrences of A in C, then D is a thesis.

Consistency If we reinterpret the system so as to let $A \mathrel{\Box\!\!\rightarrow} B = (A \,\&\, B)$, all the axioms of *D1* turn out to be truth-functional tautologies, while the rules of inference are unaffected and are also rules of inference of nonmodal sentential logic, which is known to be consistent. But this means that no contradiction of the form $A \,\&\, {\sim}A$ can be derivable in *D1*, so that *D1* must be consistent.

A serious drawback of System *D1* is that it makes no provision for the monadic modalities. This may be remedied by defining the monadic modalities in terms of the dyadic modalities (or nonmaterial conditional connectives), as follows:

Definition $\Box$ $\Box p =_{df} t \mathrel{\Box\!\!\rightarrow} p$
Definition $\Diamond$ $\Diamond p =_{df} t \mathrel{\Diamond\!\!\rightarrow} p$,

where 't' represents any tautology, such as '$q \vee {\sim}q$'.[6] Such definitions appear intuitively quite plausible. Thus, the strong nonmaterial conditional 'Whether it rains or it doesn't rain, the match will be called off'[7] is, according to Definition $\Box$, equivalent simply to the categorical sentence 'The match is bound to be called off' (more stiltedly, 'Necessarily, the match will be called off'), and this seems reasonable, for it is hard to see what else anyone could mean by asserting the former.

Given that the monadic modalities are to be added to the system, we shall need also to adopt a further rule of inference which is a standard feature of all the more familiar monadic modal logics, the so-called *rule of necessitation* (see [1], p. 31):

Rule of Necessitation (RN). If A is a thesis, then $\Box A$ is a thesis.

Finally, there is one further axiom involving *both* monadic *and* dyadic modalities which arguably ought to be added to the system:

D2.9 $(p \mathrel{\Box\!\!\rightarrow} q \vee \Box{\sim}p) \longleftrightarrow \Box(p \rightarrow q)$.

The rationale of D2.9 may be better understood if we look at its 'dual' (that is, the equivalent thesis expressed in terms of possibility rather than necessity). This is:

D2.10 $(p \mathrel{\Diamond\!\!\rightarrow} q \,\&\, \Diamond p) \longleftrightarrow \Diamond(p \,\&\, q)$.

What D2.10 asserts is that a conjunction is possible if and only if one of the conjuncts is possible and it possibilifies the other. Thus 'Possibly, it will rain and we shall get wet' is true, according to D2.10, if and only if 'Possibly, it will rain; and if it rains, then we may get wet' is true. This seems intuitively plausible. (The plausibility may be enhanced still further if we put the matter yet another way: what D2.10 implies is that a conjunction is *im*possible if and only if *either* one of the conjuncts is impossible on its own account *or* one of the conjuncts doesn't possibilify the other. It is hard to see on what other grounds a conjunction should be considered impossible.)

The system which results from the addition of Definitions $\Box$ and $\Diamond$, the rule of inference RN, and Axiom D2.9 to System *D1* (and appropriately augmenting the definition of wffs) I shall call System *D2*. The consistency of *D2* may be proved by again reinterpreting it so that $A \mathrel{\Box\!\!\rightarrow} B = (A \,\&\, B)$, in which case D1.1-D2.9 again all turn out to be truth-functional tautologies, and the rules of inference MP, US, and SE remain rules of nonmodal sentential logic and rule RN reduces to the rule: if A is a thesis, then $(t \,\&\, A)$ is a thesis, which is (trivially) a rule of nonmodal sentential logic.

Two particularly important theorems of *D2* are the following:

D2.11 $\Box p \rightarrow p$
D2.12 $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$.

The significance of these is that together they constitute (along with some appropriate nonmodal axioms such as D1.1-D1.4) the axiomatic base of the simplest well-known system of monadic modal sentential logic, the so-called system T (see [1], pp. 22ff). Thus *D2* actually includes this system, since it not only contains all of its axioms either as axioms or theorems and retains the standard interdefinability of '$\Box$' and '$\Diamond$', but also has the same (or equivalent) rules of inference.[8]

We have already noticed that *D2* differs importantly from the systems of Stalnaker and Lewis in allowing that the strong conditional connective '$\Box\!\!\rightarrow$' is transitive (Axiom D1.7). Failure of transitivity for strong nonmaterial conditionals explains, according to Lewis (and Stalnaker's views are in accord with him in this), just one of three important invalid inference-patterns involving such conditionals: besides the 'fallacy of transitivity', there is also, supposedly, the 'fallacy of strengthening the antecedent' and the 'fallacy of contraposition' (see [3], pp. 31ff). It is noteworthy, however, that *D2* does *not* sanction these other inferences unrestrictedly. That is to say, the following formulas are *not* theses of *D2*:

$$p \mathrel{\Box\!\!\rightarrow} q \rightarrow (p \,\&\, r) \mathrel{\Box\!\!\rightarrow} q$$
$$p \mathrel{\Box\!\!\rightarrow} q \rightarrow {\sim}q \mathrel{\Box\!\!\rightarrow} {\sim}p.$$

This is easily proven by observing that under the reinterpretation $A \mathrel{\Box\!\!\rightarrow} B = (A \,\&\, B)$ neither of these formulas turns out to be a truth-functional tautology, unlike the axioms of *D2*, and so neither is derivable from those axioms by the rules of inference of *D2* (these rules being rules of nonmodal sentential logic under the reinterpretation). However, *D2 does* contain the following theorem:

D2.13 $({\sim}\Box q \,\&\, p \mathrel{\Box\!\!\rightarrow} q) \rightarrow ({\sim}q \mathrel{\Box\!\!\rightarrow} {\sim}p)$,

allowing the contraposition of a strong nonmaterial conditional whose consequent does not express a necessity. (And the modal system to be described in a moment also contains a theorem permitting a restricted form of strengthening the antecedent: see M1.10 below.)

I shall now show how, by adding appropriate definitions of the strong and weak nonmaterial conditional connectives to the monadic modal system T, we obtain a system (which I shall call System *M1*) which includes all the theses of System *D2*. This system may be described as follows.

The System M1

Vocabulary

$p, q, r, \ldots$	sentential letters
$\sim, \&, \vee, \rightarrow, \longleftrightarrow$	truth-functional operators
$\Box, \Diamond$	monadic modal operators
$\Box\!\!\rightarrow, \Diamond\!\!\rightarrow$	strong and weak nonmaterial conditional connectives
(,)	brackets

Well-formed formulas (wffs)

1. Any sentential letter standing alone is a wff.
2. If A is a wff, then $\sim A$, $\Box A$, and $\Diamond A$ are wffs.
3. If A and B are wffs, then $(A \& B)$, $(A \vee B)$, $(A \rightarrow B)$, $(A \longleftrightarrow B)$, $(A \Box\!\!\rightarrow B)$, and $(A \Diamond\!\!\rightarrow B)$ are wffs.
4. Nothing else is a wff.

Definitions The truth-functional operators are interdefinable in the usual ways, as are the monadic modal operators. In addition we adopt

Definition $\Box\!\!\rightarrow$ $p \Box\!\!\rightarrow q =_{df} (\Box(p \rightarrow q) \& (\Box\sim p \rightarrow \Box q))$
Definition $\Diamond\!\!\rightarrow$ $p \Diamond\!\!\rightarrow q =_{df} \sim(p \Box\!\!\rightarrow \sim q)$.

Axioms M1.1-M1.4 are isomorphous with D1.1-D1.4. We also have:

M1.5 $\Box p \rightarrow p$
M1.6 $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$.

(These, of course, are isomorphous with D2.11 and D2.12.)

Rules of Inference
MP, US, SE and RN (see above).

Consistency The consistency of *M1* may be proved in the same way as that of *T* (see [1], p. 41).

It is a straightforward, if rather tedious, task to prove that D1.5-D2.9 are all derivable as theorems in *M1*. Not only this, but *M1* also contains the following theorems derivable in *D2* with the aid of Definitions $\Box$ and $\Diamond$:

M1.7 $\Box p \longleftrightarrow t \Box\!\!\rightarrow p$
M1.8 $\Diamond p \longleftrightarrow t \Diamond\!\!\rightarrow p$.

Thus it is clear that *M1* contains *all* the theses of *D2*. But *D2* does *not*, conversely, contain all the theses of *M1* (although it does contain all those of *T*), since the following theorem, derivable in *M1* with the aid of Definition $\Box\!\!\rightarrow$, is *not* derivable in *D2*:

M1.9 $(p \Box\!\!\rightarrow q) \longleftrightarrow (\Box(p \rightarrow q) \& (\Box\sim p \rightarrow \Box q))$.

(The nontheoremhood of M1.9 in *D2* may be established by again using the reinterpretation $A \mathrel{\Box\!\!\rightarrow} B = (A \,\&\, B)$, under which M1.9 does not turn out to be a truth-functional tautology.) Thus *M1* is more comprehensive than *D2*.

I remarked earlier that *M1* contains a theorem permitting a restricted form of strengthening the antecedent for strong nonmaterial conditionals. This theorem is

M1.10 $(p \mathrel{\Box\!\!\rightarrow} q \,\&\, p \mathrel{\Diamond\!\!\rightarrow} r) \rightarrow (p \,\&\, r) \mathrel{\Box\!\!\rightarrow} q.$

Thus, according to M1.10, strengthening the antecedent is allowable provided that what is added to the antecedent is possibilified by (or possible relative to) what was there before.

It remains only to justify the adoption of Definition $\mathrel{\Box\!\!\rightarrow}$ (since Definition $\mathrel{\Diamond\!\!\rightarrow}$ has already been defended). One argument in favour of Definition $\mathrel{\Box\!\!\rightarrow}$ is quite simply that its addition to *T* enables us to recover *D2*, which has already been argued to provide an adequate logic of nonmaterial conditionals (or at least an appreciable fragment of such a logic). The virtue of such a definition is that it renders conditional logic just a branch of ordinary monadic modal logic, which is clearly preferable to leaving the two domains unrelated. That the reduction should be from dyadic to monadic modal logic, rather than *vice versa* (as in *D2* itself), is dictated by the greater simplicity of the latter type of system and the arguably more primitive status of its modal concepts.

It may be noted, further, that no *other* definition of strong nonmaterial conditionality by means of the resources available in the system *T* will do as well as Definition $\mathrel{\Box\!\!\rightarrow}$. This may be seen from the following considerations. First, it is clear from Axiom D2.9 that '$p \mathrel{\Box\!\!\rightarrow} q$' entails '$\Box(p \rightarrow q)$'. However, it would not do simply to define the former as the latter (and here I agree with Lewis ([3], pp. 4ff) even though I do not approve of all his arguments), since on this definition Axiom D1.5 will not turn out to be valid. Hence '$p \mathrel{\Box\!\!\rightarrow} q$' must be (*pace* Lewis) *stronger* than (entail but not be entailed by) '$\Box(p \rightarrow q)$'. Now secondly we know that the following is a theorem of *D2*:

D2.14 $(\Box(p \rightarrow q) \,\&\, {\sim}\Box{\sim}p) \rightarrow p \mathrel{\Box\!\!\rightarrow} q.$

(D2.14 is a truth-functional consequence of Axiom D2.9.) But if '$p \mathrel{\Box\!\!\rightarrow} q$' were defined simply as '$(\Box(p \rightarrow q) \,\&\, {\sim}\Box{\sim}p)$', although all the axioms D1.1-D2.9 of *D2* would turn out valid as required, we should also have as a theorem the formula

$$p \mathrel{\Box\!\!\rightarrow} q \rightarrow \Diamond p,$$

which is clearly undesirable since it would rule out the possibility of there being true strong nonmaterial conditionals with impossible antecedents. Thus we see that the *definiens* of '$p \mathrel{\Box\!\!\rightarrow} q$' must be *weaker* than '$(\Box(p \rightarrow q) \,\&\, {\sim}\Box{\sim}p)$' and *stronger* than '$\Box(p \rightarrow q)$'. It appears that the only definition meeting this requirement and enabling us to recover all the theses of *D2* when added to the system *T* is Definition $\mathrel{\Box\!\!\rightarrow}$.

Apart from the foregoing argument, I would urge that Definition $\mathrel{\Box\!\!\rightarrow}$ derives some rather more direct support from linguistic intuition. There are, I suggest, basically two kinds of case in which we are prepared to assert a strong nonmaterial conditional: (1) when we consider that the conjunction of the antecedent with the negation of the consequent is, in some sense, *impossible*, but not merely in virtue of either the antecedent or the negation of the consequent being impossible on their own account, and (2) when we consider that the consequent is, in some sense, *necessary* (or inevitable) on its own account, so that the antecedent is effectively redundant. An example under case (1) would be 'If the bough breaks, the cradle will fall'; an example under case (2) would be '(Even) if you live till you're eighty, you won't learn patience'.[9] In case (1) we are prepared to assert something of the form '$\sim\Diamond(p \;\&\; \sim q) \;\&\; \Diamond p \;\&\; \Diamond\sim q$'; in case (2) we are prepared to assert something of the form '$\Box q$'. But what these cases have in common is that they each commit us to being prepared to assert something of the form '$\Box(p \to q) \;\&\; (\Box\sim p \to \Box q)$', i.e., the proposed *definiens* of '$p \;\Box\!\!\rightarrow q$' according to Definition $\Box\!\!\rightarrow$. (In fact, it is easily proved that in the system T '$\Box(p \to q) \;\&\; (\Box\sim p \to \Box q)$' is equivalent to the disjunction of '$\sim\Diamond(p \;\&\; \sim q) \;\&\; \Diamond p \;\&\; \Diamond\sim q$' and '$\Box q$'.) Thus, I would claim, Definition $\Box\!\!\rightarrow$ serves to explain why we are prepared to assert something of the form '$p \;\Box\!\!\rightarrow q$' in each type of case, despite their considerable differences.

Appendix: On the alleged fallacy of transitivity Stalnaker gives the following supposed counterexample to transitivity[10]:

(1) If J. Edgar Hoover had been born a Russian, then he would today be a communist.
(2) If J. Edgar Hoover were today a communist, then he would be a traitor.
(3) Therefore, if J. Edgar Hoover had been born a Russian, then he would be a traitor.

Here premises (1) and (2) seem quite probably true, whereas the conclusion seems manifestly false, apparently vindicating Stalnaker's and Lewis's view. However, it is arguable that the plausibility of (1) and (2) rests upon a tacit assumption that the antecedent of each contains a *suppressed clause* which is different in each case: which, if true, would imply that the only sort of fallacy involved in the inference from (1) and (2) to (3) is a fallacy of equivocation. Thus (1) appears to mean something like 'If J. Edgar Hoover had been born a Russian *and had remained a Russian citizen,* then he would today be a communist' (clearly, if he had *not* remained a Russian citizen it is highly likely that he would have become a political exile and hence quite probably *non*-communist today). On the other hand, (2) appears to mean something like 'If J. Edgar Hoover were today a communist *and were still an American citizen with high government office,* then he would be a traitor' (clearly, if he were *not* still an American citizen and had resigned his office, neither of which circumstances need be treasonable, he might well have become a resident citizen of some communist state, where he would certainly be no traitor on account of being a communist). Even if my suggestions as to the precise formulation of these suppressed clauses are disputed, that *some* such clauses must be presumed to be involved seems to me undeniable, because without them (1) and (2) haven't the slightest plausibility. This is because being born a Russian, as such, has no 'necessary connection' with being a communist, nor has being a communist (even for one who has held high government office in America) with being a traitor. (I would have no quarrel with (1) and (2), of course, if the word 'would' in each were changed for 'might'.) The lesson is that examples drawn from ordinary discourse (where much is left unsaid) should be used with extreme caution.

NOTES

1. I have defended this view in [4] and [5]. See also [2], pp. 59ff, for another defence. Those who are not convinced may regard my theory as primarily a theory of indicative conditionals (all my examples are chosen with this in view).
2. Lewis does not use the terms 'strong' and 'weak' in these senses.
3. I shall not, however, discuss these comparisons in the present paper.
4. That this is a nonmaterial conditional is evident from the fact that its truth would not be guaranteed by the falsehood of its antecedent: this provides a general test of nonmaterial conditionality, both 'strong' and 'weak'.
5. Where no confusion results, brackets may be omitted to improve ease of reading. Where this happens, the convention I follow is that the nonmaterial conditional connectives have narrowest possible scope and outermost brackets are omitted; e.g., '$((p \mathrel{\Box\!\!\rightarrow} q) \rightarrow (p \rightarrow q))$' may be simplified to '$p \mathrel{\Box\!\!\rightarrow} q \rightarrow (p \rightarrow q)$'.
6. See [8], pp. 89f. Note that these definitions render '$\Box$' and '$\Diamond$' interdefinable in the standard way, namely, equating '$\Diamond p$' with '$\sim\Box\sim p$'.
7. The absence of an 'If . . ., then ---' construction here does not disguise the underlying logical form.
8. However, *D2* does *not* include any of the well-known higher systems of monadic modal logic, such as *S4* and *S5* (see [1], pp. 43ff).
9. From this example it will be clear that I regard 'even' in such a context as having only pragmatic significance, serving to emphasise the redundancy of the antecedent.
10. See [7]; also cited by Lewis [3], p. 33. Lewis [3] offers another supposed counterexample which can, I believe, be objected to in much the same way that I object to Stalnaker's. It will be observed that the conditionals used in these examples are *counterfactuals.* Those who are not convinced by my contention that counterfactual and indicative conditionals have the same logic and are accordingly invited to regard my theory as a theory of *indicative* conditionals (see Note 1 above), will not therefore see these examples as offering even a *prima facie* threat to my theory.

REFERENCES

[1] Cresswell, M. J. and G. E. Hughes, *An Introduction to Modal Logic,* Methuen and Company Limited, London, 1968.

[2] Ellis, B., *Rational Belief Systems,* Basil Blackwell Limited, Oxford, 1979.

[3] Lewis, D., *Counterfactuals,* Cambridge University Press, Cambridge, 1973.

[4] Lowe, E. J., "Indicative and counterfactual conditionals," *Analysis,* vol. 39, no. 3 (1979), pp. 139-141.

[5] Lowe, E. J., "Reply to Davis," *Analysis,* vol. 40, no. 4 (1980), pp. 187-190.

[6] Stalnaker, R. C., "A defense of conditional excluded middle," in *Ifs*, ed., W. L. Harper et al., D. Reidel Publishing Company, Dordrecht, Holland, 1981.

[7] Stalnaker, R. C., "A theory of conditionals," in *Studies in Logical Theory,* ed., N. Rescher, Basil Blackwell Limited, Oxford, 1968.

[8] Von Wright, G. H., "A new system of modal logic" in his *Logical Studies,* Routledge and Kegan Paul, London, 1957.

Department of Philosophy
University of Durham
Durham, England