Notre Dame Journal of Formal Logic
Volume XIX, Number 2, April 1978
NDJFAM

ANALYTICA PRIORA I, 38 AND REDUPLICATION

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Although many commentators have summarized chapter 38 of *Analytica Priora I* as if it was perfectly clear to them, I have not found their explanations satisfactory enough. In fact, I think Aristotle's text needs badly some sort of clarification that makes it meaningful to modern logicians. In this note I wish to propose one such reconstruction.

The understanding of chapter 38 requires first some analysis of the so-called "reduplicative" phrases that occur in it. In Greek and Latin there are many such phrases, but for simplicity I will restrict myself to just one of them: *qua*. Also for simplicity I will standardize reduplicative sentences as follows: "S est P qua M" where P and M are predicates, S can be a singular term or a predicate (this is why I leave the Latin copula "est", in order to cover both predication proper and subordination of predicates). For the purposes of this note, however, it is enough to consider S as a predicate and, further, to take P as one of those predicates that "belong to all S's", not just to some S. There are historical reasons suggesting the following reconstruction of our standard reduplicative sentence:

(1) $$\bigwedge x . x \,\varepsilon\, S \rightarrow x \,\varepsilon\, P . \wedge \bigwedge x . x \,\varepsilon\, P \rightarrow x \,\varepsilon\, M.$$[1]

Chapter 38 is neatly divided into two parts. In the first part Aristotle considers people who want to prove "S est P qua M". Clearly, the question is: how to construct premises that yield *this* conclusion? *Somewhere* in the premises the 'qua M' must show up: but where? There seem to be the following two possibilities (with B as middle term):

(2)	B est P qua M	(3)	B est P
	S est B		S est B qua M

*The 'ε' in, for example, $x \,\varepsilon\, P$, means that the predicate P is predicated of the object x. Dots are used instead of parentheses.

Received August 11, 1977

which become for us:

(2′) $\bigwedge x . x \varepsilon B \rightarrow x \varepsilon P . \wedge \bigwedge x . x \varepsilon P \rightarrow x \varepsilon M .$
$\bigwedge x . x \varepsilon S \rightarrow x \varepsilon B .$

(3′) $\bigwedge x . x \varepsilon B \rightarrow x \varepsilon P .$
$\bigwedge x . x \varepsilon S \rightarrow x \varepsilon B . \wedge \bigwedge x . x \varepsilon B \rightarrow x \varepsilon M .$

We observe that (2′) and not (3′) is the couple of premises yielding the desired conclusion, or rather the modern translation of the desired conclusion: our (1) above. Thus we make sense of Aristotle's doctrine in the first part of chapter 38: "the reduplication should be attached to the major term".

In the second part of chapter 38 Aristotle has in mind people who want to prove "S est P qua M" as a conclusion of a syllogism and who *already* have a proof of "S est P", for example the following:

(4)
$$\begin{array}{l} B \text{ est } P \\ \underline{S \text{ est } B} \\ S \text{ est } P \end{array}$$
or (4′)
$$\begin{array}{l} \bigwedge x . x \varepsilon B \rightarrow x \varepsilon P . \\ \underline{\bigwedge x . x \varepsilon S \rightarrow x \varepsilon B .} \\ \bigwedge x . x \varepsilon S \rightarrow x \varepsilon P . \end{array}$$

By the first part of chapter 38 we should write:

(5)
$$\begin{array}{l} B \text{ est } P \text{ qua } M \\ \underline{S \text{ est } B \qquad\quad} \\ S \text{ est } P \text{ qua } M \end{array}$$

but this is not accurate enough. Consider our translation of (5):

(5′)
$$\begin{array}{l} \bigwedge x . x \varepsilon B \rightarrow x \varepsilon P . \wedge \bigwedge x . x \varepsilon P \rightarrow x \varepsilon M . \\ \underline{\bigwedge x . x \varepsilon S \rightarrow x \varepsilon B . \qquad\qquad\qquad\qquad\qquad} \\ \bigwedge x . x \varepsilon S \rightarrow x \varepsilon P . \wedge \bigwedge x . x \varepsilon P \rightarrow x \varepsilon M . \end{array}$$

and observe that the first premise implies $\bigwedge x . x \varepsilon B \rightarrow x \varepsilon M .$, which means that the middle B that did well in (4) now in (5) may fail to secure or to preserve truth in the premises . . . in case there are B's that are not M. Hence we must take a middle term *magis contractum* (to use Monlorius' phrase), more restricted than B, in fact any class not larger than $B \cap M$. Such is the main point of chapter 38.

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