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TWO SETS OF PERFECT SYLLOGISMS

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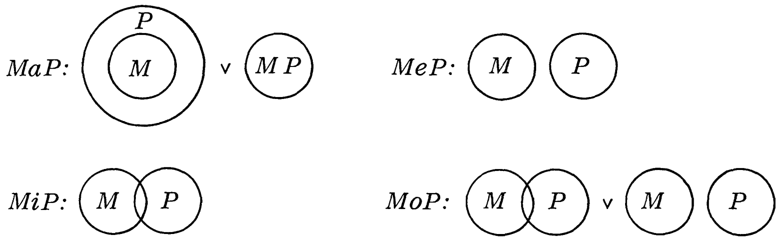
Aristotle divides all syllogisms into perfect and imperfect.

> I call that a perfect syllogism which needs nothing other than what has been stated to make the necessity evident; a syllogism is imperfect, if it needs either one or more components which are necessary by the terms set down, but have not been stated by the premises. *Cf.* [1].

Since Aristotle the categorical syllogisms have been divided into these two groups. Examination of the 256 possible categorical syllogisms reveals that there are not one but two sets of perfect syllogisms.

When the Venn diagram is adapted to the concepts of the new mathematics, *cf.* [2], two sets of perfect syllogisms can be defined. In this examination the Venn diagram with its three overlapping circles was replaced by a diagram in which three or more discs may or may not overlap.

Figure One



In the disc diagram (Figure One) *'All M are P'* or *'MaP'* is indicated by a disc M lying inside a disc P or by a disc M identical with a disc P. *'No M are P'* or *'MeP'* is indicated by a disc M that has no area in common with a disc P. *'Some M are P'* or *'MiP'* is indicated by a disc M that overlaps a disc P but is not totally coincident with it. *'Some M are not P'* or *'MoP'* is indicated by either a disc M overlapping a disc P or by a disc M that fails to overlap disc P and has no area in common with it.

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In this examination all 256 possible categorical syllogisms were diagrammed, and two sets of perfect syllogisms could be discerned. The first of these sets of perfect syllogisms is the set of valid syllogisms:

Fig. 1	Fig. 2	Fig. 3	Fig. 4
AAA	*AEE*	*AII*	*AEE*
AII	*AOO*	*EIO*	*EIO*
EAE	*EAE*	*IAI*	*IAI*
EIO	*EIO*	*OAO*	*AAI*
AAI	*AEO*	*AAI*	*AEO*
EAO	*EAO*	*EAO*	*EAO*

The second set of perfect syllogisms appears to include:

Fig. 1	Fig. 2	Fig. 3	Fig. 4
AAE	*AEA*	*AAE*	*AAE*
AIE	*AEI*	*EAA*	*AEA*
EAA	*AOA*	*EIA*	*AEI*
EAI	*EAA*	*IAE*	*EAA*
EIA	*EAI*	*OAA*	*EIA*
AAO	*EIA*	*AIE*	*IAE*

I have called these syllogisms:

Candace	*Alberta*	*Candace*	*Candace*
Camille	*Amelia*	*Beata*	*Alberta*
Beata	*Carlotta*	*Celia*	*Amelia*
Melanie	*Beata*	*Diane*	*Beata*
Celia	*Melanie*	*Roxana*	*Celia*
Sara Jo	*Celia*	*Camille*	*Diane*

It will be noticed that the second set of perfect syllogisms is the first set with the conclusion stated as the contradictory to the conclusion. The first set of perfect syllogisms might be called the Perfect-True syllogisms, and the second set of perfect syllogisms might be called the Perfect-False syllogisms. The Perfect-True Syllogisms, in which it is impossible for the premises to be true and the conclusion to be false, appear to be coincident with and identical to the valid syllogisms.

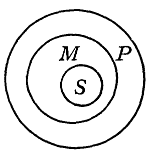
What has been called the Imperfect Syllogisms has also been called the invalid syllogisms. But what has been called the Imperfect Syllogisms appears to be two disjoint sets of syllogisms. The one set is the set of Perfect-False syllogisms and the other is the Imperfect Syllogisms. A deductive argument is said to be valid in the categorical syllogisms when and only when it is impossible for the premises to be true and the conclusion to be false. In logic a deductive argument has been said to be invalid when it is possible for the premises to be true and the conclusion to be either true or false. However, a set of categorical syllogisms appears to exist wherein it is impossible for the premises to be true and the conclusion to be true.

Perhaps it is possible to say that a deductive argument in the categorical syllogisms may be said to be valid when and only when it is impossible for the premises to be true and the conclusion to be false, and a deductive argument in the categorical syllogisms may be said to be invalid when and only when it is impossible for the premises to be true and the conclusion to be true, and a deductive argument in the categorical syllogisms may be said to be neither valid nor invalid when it is possible for the premises to be true and the conclusion to be either true or false. These syllogisms that are neither valid nor invalid might be called the indefinite syllogisms. They are the Imperfect Syllogisms. There may indeed be three disjoint sets of categorical syllogisms:

(1) the valid and perfect-true syllogisms
(2) the invalid and perfect-false syllogisms
(3) the indefinite and imperfect syllogisms.

Perhaps these three disjoint sets may best be illustrated with an example of each. A valid and perfect-true syllogism:

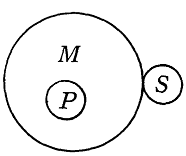
AAA Figure 1 (Barbara)



MaP	All Monocotyledons are Plants	T
SaM	*All Silk-grasses are Monocotyledons*	T
SaP	All Silk-grasses are Plants	T

An invalid and perfect-false syllogism:

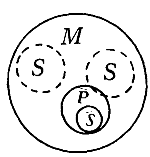
AEA Figure 2 (Alberta)



PaM	All Opossums are Marsupials	T
SeM	*No Snakes are Marsupials*	T
SaP	All Snakes are Opossums	F

An example of an indefinite and imperfect syllogism:

AAA Figure 2



PaM	All princes are male human beings	T
SaM	*All Sheikhs are male human beings*	T
SaP	All Sheikhs are Princes	T U F

The valid and perfect-true syllogisms appear to number twenty-four. The invalid and perfect-false syllogisms appear to number twenty-four. The indefinite and imperfect syllogisms appear to number 208 among the categorical syllogisms.

The Rules of the Syllogism by which a syllogism's validity may be tested are (*cf.* [3]):

(1) Every valid syllogism has the middle term distributed at least once.
(2) No term in the conclusion may be distributed unless also distributed in the premises.
(3) No valid syllogism has two negative premises.
(4) In a valid syllogism the conclusion may be negative if and only if one or the other premise is negative.

The Rules of the Syllogism by which a syllogism's invalidity may be tested are

(1) Every invalid syllogism has the middle term distributed at least once.
(2) In an invalid syllogism no term is distributed (D) in the conclusion *and* undistributed (U) in the premises.
(3) An invalid syllogism always has one and only one negative statement and exactly two affirmative statements.
(4) One premise is negative only if the conclusion is affirmative; the conclusion is negative only when both premises are affirmative.

A Structural Illustration of the Perfect-False and Invalid Syllogisms

DU	MaP	DU	PaM	DU	MaP	DU	PaM
DU	*SaM*	DD	*SeM*	DU	*MaS*	DU	*MaS*
DD	SeP	DU	SaP	DD	SeP	DD	SeP
DU	MaP	DU	PaM	DD	MeP	DU	PaM
UU	*SiM*	DD	*SeM*	DU	*MaS*	DD	*MeS*
DD	SeP	UU	SiP	DU	SaP	DU	SaP
DD	MeP	DU	PaM	DD	MeP	DU	PaM
DU	*SaM*	UD	*SoM*	UU	*MiS*	DD	*MeS*
DU	SaP	DU	SaP	DU	SaP	UU	SiP
DD	MeP	DD	PeM	UU	MiP	DD	PeM
DU	*SaM*	DU	*SaM*	DU	*MaS*	DU	*MaS*
UU	SiP	DU	SaP	DD	SeP	DU	SaP
DD	MeP	DD	PeM	UD	MoP	DD	PeM
UU	*SiM*	DU	*SaM*	DU	*MaS*	UU	*MiS*
DU	SaP	UU	SiP	DU	SaP	DU	SaP
DU	MaP	DD	PeM	DU	MaP	UU	PiM
DU	*SaM*	UU	*SiM*	UU	*MiS*	DU	*MaS*
UD	SoP	DU	SaP	DD	SeP	DD	SeP

In summary it may be said that there are three disjoint sets among the categorical syllogisms and that these three disjoint sets are the valid and perfect-true syllogisms, the invalid and perfect-false syllogisms, and the indefinite and imperfect syllogisms. The recognition of three disjoint sets makes it possible to say that a syllogism is valid, invalid or indefinite.

Perhaps what has appeared to be a little white and a great deal of black can be seen to be a little white, a little black, and a great deal of gray.

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REFERENCES

[1] Aristotle, *Prior Analytics* and Jan Łucasiewicz, *Aristotle's Syllogistic*, Clarendon Press, Oxford (1957).

[2] Wilson, Jack D., *Elementary Mathematics, a Modern Approach*, McGraw-Hill, New York (1967).

[3] Barker, Stephan F., *The Elements of Logic*, McGraw-Hill, New York (1967).

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