

THE TWO LOGICS: TRADITIONAL AND MODERN

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The objective of this paper is to improve our understanding of the relationship between traditional logic—Aristotle's logic of the assertoric syllogism as it has come to be—and modern logic. Let us join the thin ranks of those who still believe that the hope for such an endeavor resides in locating one or more near analogues of the syllogistic structure within the first-order predicate calculus and then analyzing and accounting for it or for them. The search for an analogue begins with the introduction of a set of rules for translating the statement-forms of traditional logic wherein term-variables occur into certain quantified statement-forms of modern logic wherein predicate-variables occur. We shall say that an *analogue*—a *near* analogue unless shown to be *strict*, i.e., such that it meets any additional standards of similitude we may wish to impose—has been discovered when, with each of the valid inference patterns normally included in the traditional system expressed as a compound statement-form with logical connectives, the first-order predicate translation of each is a valid statement-form.

Using '*Asp*' for '*All S is P*,' '*Esp*' for '*No S is P*,' '*Isp*' for '*Some S is P*,' and '*Osp*' for '*Some S is not P*,' the "traditional laws" we must successfully translate are as follows:

Laws of Immediate Inference

Simple Conversion

$$Esp \equiv Eps$$

$$Isp \equiv Ips$$

Obversion

$$Asp \equiv Es\bar{p}$$

$$Esp \equiv As\bar{p}$$

$$Isp \equiv Os\bar{p}$$

$$Osp \equiv Is\bar{p}$$

Conversion *per accidens*

$$Asp \supset Ips$$

$$Esp \supset Ops$$

(The Laws of Obverted Conversion, Partial Contraposition, Full Contraposition, Partial Inversion, and Full Inversion are derivative from those listed.)

Laws of the Square of Opposition

| | |
|-----------------------|-------------------|
| $Asp \equiv \sim Osp$ | $Isp \vee Osp$ |
| $Esp \equiv \sim Isp$ | $Asp \supset Isp$ |
| $\sim(Asp \cdot Esp)$ | $Esp \supset Osp$ |

Laws of the Syllogism

First Figure

| |
|-------------------------------|
| $(Amp \cdot Asm) \supset Asp$ |
| $(Emp \cdot Asm) \supset Esp$ |
| $(Amp \cdot Ism) \supset Isp$ |
| $(Emp \cdot Ism) \supset Osp$ |
| $(Amp \cdot Asm) \supset Isp$ |
| $(Emp \cdot Asm) \supset Osp$ |

Second Figure

| |
|-------------------------------|
| $(Epm \cdot Asm) \supset Esp$ |
| $(Apm \cdot Esm) \supset Esp$ |
| $(Epm \cdot Ism) \supset Osp$ |
| $(Apm \cdot Osm) \supset Osp$ |
| $(Epm \cdot Asm) \supset Osp$ |
| $(Apm \cdot Esm) \supset Osp$ |

Third Figure

| |
|-------------------------------|
| $(Amp \cdot Ams) \supset Isp$ |
| $(Imp \cdot Ams) \supset Isp$ |
| $(Amp \cdot Ims) \supset Isp$ |
| $(Emp \cdot Ams) \supset Osp$ |
| $(Omp \cdot Ams) \supset Osp$ |
| $(Emp \cdot Ims) \supset Osp$ |

Fourth Figure

| |
|-------------------------------|
| $(Apm \cdot Ams) \supset Isp$ |
| $(Apm \cdot Ems) \supset Esp$ |
| $(Ipm \cdot Ams) \supset Isp$ |
| $(Epm \cdot Ams) \supset Osp$ |
| $(Epm \cdot Ims) \supset Osp$ |
| $(Apm \cdot Ems) \supset Osp$ |

What is to be our position with respect to the commonly acknowledged fact that Professor Strawson has already found an analogue under the conditions for a near analogue just prescribed?* Does not this render further search futile? And this the more so since, seemingly, both Strawson and his critics agree that his analogue is neither logically nor philosophically illuminating? Our response is simply that there is nothing in his demonstration which precludes the possibility of an additional analogue, that the comforting assumption that in all instances there can be at most a single analogue of one formal structure within another is just that, a comforting assumption, and that his analogue is—when made logically precise—instructive to the point that it encourages further discovery.

Our first task is to correct Strawson's analogue. The corrected version is free of a certain characteristic that marks the original, a characteristic which works against understanding the relationship of the two logics. Our second task is to introduce another analogue. The final task for us is, with the aid of these materials, to develop a promising account of that relationship, a relationship between a term logic and a predicate logic.

1 Strawson's set of translation rules (*tr* 1) is exhibited by

*P. F. Strawson, *Introduction to Logical Theory*, Methuen, London (1952), pp. 163-73.

$$\begin{aligned}
Asp & \sim(\exists x)(Sx \cdot \sim Px) \cdot (\exists x)(Sx) \cdot (\exists x)(\sim Px) \\
Esp & \sim(\exists x)(Sx \cdot Px) \cdot (\exists x)(Sx) \cdot (\exists x)(Px) \\
Isb & (\exists x)(Sx \cdot Px) \vee \sim(\exists x)(Sx) \vee \sim(\exists x)(Px) \\
Osb & (\exists x)(Sx \cdot \sim Px) \vee \sim(\exists x)(Sx) \vee \sim(\exists x)(\sim Px).
\end{aligned}$$

The set of translation rules (*tr 1'*) leading to a logically reduced version of the same analogue is exhibited by

$$\begin{aligned}
Asp & \sim(\exists x)(Sx \cdot \sim Px) \cdot (\exists x)(Sx \cdot Px) \cdot (\exists x)(\sim Sx \cdot \sim Px) \\
Esp & \sim(\exists x)(Sx \cdot Px) \cdot (\exists x)(Sx \cdot \sim Px) \cdot (\exists x)(\sim Sx \cdot Px) \\
Isb & (\exists x)(Sx \cdot Px) \vee \sim(\exists x)(Sx \cdot \sim Px) \vee \sim(\exists x)(\sim Sx \cdot Px) \\
Osb & (\exists x)(Sx \cdot \sim Px) \vee \sim(\exists x)(Sx \cdot Px) \vee \sim(\exists x)(\sim Sx \cdot \sim Px).
\end{aligned}$$

It is readily shown that the respective expressions in these two sets are logically equivalent. Consider '*Asp*' by *tr 1* and by *tr 1'* for example: Since ' $\sim(\exists x)(Sx \cdot \sim Px) \cdot (\exists x)(Sx) \cdot (\exists x)(\sim Px)$,' i.e., '*Asp*' by *tr 1*, is equivalent to ' $\sim(\exists x)(Sx \cdot \sim Px) \cdot [(\exists x)(Sx \cdot \sim Px) \vee (\exists x)(Sx \cdot Px)] \cdot [(\exists x)(Sx \cdot \sim Px) \vee (\exists x)(\sim Sx \cdot \sim Px)]$,' since the latter may be viewed as a substitution-instance of ' $\sim p \cdot (p \vee q) \cdot (p \vee r)$,' and since this in turn is equivalent to the simpler ' $\sim p \cdot q \cdot r$,' therefore, the comparable substitution-instance of ' $\sim p \cdot q \cdot r$,' namely, ' $\sim(\exists x)(Sx \cdot \sim Px) \cdot (\exists x)(Sx \cdot Px) \cdot (\exists x)(\sim Sx \cdot \sim Px)$,' i.e., '*Asp*' by *tr 1'*. Or again, '*Isb*' by *tr 1* and by *tr 1'*: Since ' $(\exists x)(Sx \cdot Px) \vee \sim(\exists x)(Sx) \vee \sim(\exists x)Px$,' i.e., '*Isb*' by *tr 1*, is equivalent to ' $(\exists x)(Sx \cdot Px) \vee [(\exists x)(Sx \cdot Px) \cdot \sim(\exists x)(Sx \cdot \sim Px)] \vee [(\exists x)(Sx \cdot Px) \cdot \sim(\exists x)(\sim Sx \cdot \sim Px)]$,' since the latter may be viewed as a substitution-instance of ' $p \vee (\sim p \cdot \sim q) \vee (\sim p \cdot \sim r)$,' and since this in turn is equivalent to the simpler ' $p \vee \sim q \vee \sim r$,' therefore, the comparable substitution-instance of ' $p \vee \sim q \vee \sim r$,' namely, ' $(\exists x)(Sx \cdot Px) \vee \sim(\exists x)(Sx \cdot \sim Px) \vee \sim(\exists x)(\sim Sx \cdot \sim Px)$,' i.e., '*Isb*' by *tr 1'*.

We can learn from the materials of these demonstrations how it is that an expression which appears to be in its simplest logical form can nevertheless have an adscititious componential characteristic. Let us regard ' $(\exists x)(Sx \cdot Px) \vee [(\exists x)(Sx \cdot \sim Px) \cdot \sim(\exists x)(Sx \cdot \sim Px)] \vee [(\exists x)(Sx \cdot Px) \cdot \sim(\exists x)(\sim Sx \cdot \sim Px)]$,' the intermediary statement-form in the proof that '*Isb*' by *tr 1* and by *tr 1'* are equivalent, as an initial assertion. It is manifest (1) that this expression is constituted by three disjuncts which are truth-functionally dependent (to wit, if the first disjunct is *true*, then the latter disjuncts can only be *false*), and (2) that this expression is not in its logically simplest form. Until (2) is rectified, we must withhold judgment about whether or not the general componential characteristic of truth-functional dependency—of which (1) is a specific variety—is inherent, i.e., is such that it obtains for every member in the set of expressions logically equivalent to the initial one.

Two modes of simplification suggest themselves. By regarding our initial expression as a ' $(\exists x)(Sx \cdot Px)$ '/'*p*', ' $\sim(\exists x)(Sx \cdot Px) \cdot \sim(\exists x)(Sx \cdot \sim Px)$ '/'*q*', ' $\sim(\exists x)(Sx \cdot Px) \cdot \sim(\exists x)(\sim Sx \cdot \sim Px)$ '/'*r*' substitution-instance of the admittedly logically simple ' $p \vee q \vee r$ ' and by drawing on our knowledge of quantification, we can move to ' $(\exists x)(Sx \cdot Px) \vee \sim(\exists x)(Sx) \vee \sim(\exists x)(Px)$,' i.e., '*Isb*' by *tr 1*, as a ' $(\exists x)(Sx \cdot Px)$ '/'*p*', ' $\sim(\exists x)(Sx)$ '/'*q*', ' $\sim(\exists x)(Px)$ '/'*r*' substitution-instance of the self-same ' $p \vee q \vee r$.' But this is surely no more

than a typographical simplification; no *logical* reduction has occurred. Thus, the evident fact that this statement-form, no less than the initial one, possesses the general componential characteristic of truth-functional dependency fails to establish that that characteristic is inherent.

The second mode of simplification is, of course, that which leads to $(\exists x)(Sx \cdot Px) \vee \sim(\exists x)(Sx \cdot \sim Px) \vee \sim(\exists x)(\sim Sx \cdot Px)$, i.e., '*Is_p*' by *tr 1*', and it is displayed in the latter portion of the equivalency proof. Here the initial expression is viewed as a substitution-instance of the fully articulated ' $p \vee (\sim p \vee \sim q) \vee (\sim p \vee \sim r)$ ' rather than of the under articulated ' $p \vee q \vee r$ '; here, as it were, the initial expression is put up for logical reduction rather than offered as reduced even though prolix. The move to ' $p \vee \sim q \vee \sim r$ ' and then, through substitutions as previously settled upon, to '*Is_p*' by *tr 1*', presents us with our desideratum, i.e., with the simplest logical form of the initial statement-form. This being so, the inspection of '*Is_p*' by *tr 1*' for the general componential characteristic of truth-functional dependency becomes decisive: It does not have this characteristic.

The failure to eliminate adscititious characteristics from statement-forms proposed as basic spawns logical puzzles (resolvable, to be sure) and philosophical misunderstandings (correctable, we may hope). A ready illustration—and one suited to our purposes—of the former is as follows: As seems proper for equivalent expressions, '*Es_p*' by *tr 1* and by *tr 1*' can be represented by one and the same diagram (Figure 1); yet, diagrammed disjunctively, equivalent expressions '*Is_p*' by *tr 1* (Figure 2) and by *tr 1*' (Figure 3) differ notably. Furthermore, as judged by their diagrams, is not '*Is_p*' by *tr 1*' rather than '*Is_p*' by *tr 1*' the genuine contradictory of '*Es_p*' by *tr 1*'?

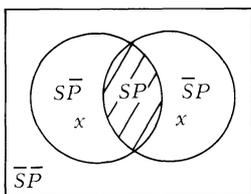


Figure 1.

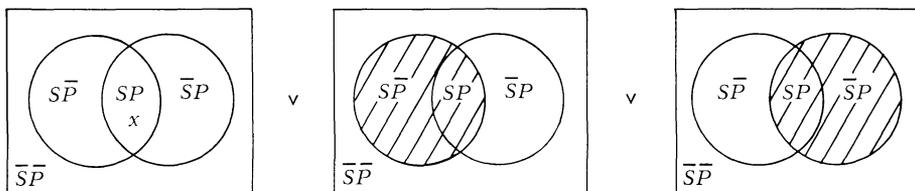


Figure 2.

3 Quite obviously a key feature both of Strawson's corrected analogue and of ours is that, when regarded in and of themselves, the '*Asp*' and '*Esp*' translations therein must be false in empty or very small nonempty domains of discourse and the '*Is p* ' and '*Osp*' translations must be true ($D < 2$ for Strawson's and $D < 3$ for ours). Thus, speaking casually, we may say, for example, that the celebrated problem of how to deal with those inferences from "all" to "some" and "no" to "some are not" is solved from the beginning. In contrast, translations in accordance with the most familiar set of all translation rules (*tr* 0), those exhibited by

$$\begin{aligned} Asp & (x)(Sx \supset Px) \\ Esp & (x)(Sx \supset \sim Px) \\ Isp & (\exists x)(Sx \cdot Px) \\ Osp & (\exists x)(Sx \cdot \sim Px), \end{aligned}$$

invite us to discredit traditional logic for underwriting such inferences, or, alternatively, to impugn modern logic for its failure to sustain them; and, if neither of these reactions, to exaggerate the incommensurateness of the two logics, or to undertake bold adventures in the philosophy of logic.

But what, in more detail, is the relationship between a statement-form of traditional logic with term-variables and its translation by either *tr* 1' or *tr* 2, a statement-form with predicate-variables? The rough answer is that the latter is the first-order predicate calculus (without identity) reflection of the former (adapted to statement connectives) within a context wherein the concept of validity as defined for that calculus (and not for traditional logic) is controlling. We can be more precise than this however. For one thing, when we invoke the fundamental but normally neglected distinction between those statement-forms of the first-order predicate calculus which are valid *only* in whatsoever nonempty domain and those which are valid *not only* in whatsoever nonempty domain *but also* in whatsoever empty domain, we note that all of the statement-forms comprising the two analogues are of the latter sort.

For another thing—and of greater immediate importance—neither analogue fully reflects the term characteristics of traditional logic. We must get at this issue. The basic difference between the terms of traditional logic and the predicates of modern logic comes to this: The characteristic of denoting at least one individual, i.e., of having application, is intrinsic to terms but not to predicates. Furthermore, for terms, no expression ' ϕ ' can be accounted to be a term unless both it and its complement ' $\sim\phi$ ' have application, i.e., ' ϕ ' is a term if and only if ' $\sim\phi$ ' is a term.

With Strawson's corrected analogue in mind, let us illustrate the term-predicate distinction in such a way that it illuminates the relationship between the two logics. Presuming a domain consisting of three individuals, $D = \{a, b, c\}$, employing the power set of $\{a, b, c\}$ to display the entire range of denotative possibilities for expressions, and observing the law of the excluded middle, the following schema indicates a set of assignments for expressions whereby each may be viewed as a simple, but not

necessarily applicatively unique, member of an applicatively complete set of term expressions of degree 1 (term expressions being regarded as n-ary relations):

$$\{a, b, c\} \text{ or } \vee, \{a, b\}, \{a, c\}, \{b, c\}, \{a\}, \{b\}, \{c\}, \wedge .$$

$$A_1, A_2 \quad B \quad C \quad \sim C \quad \sim B \quad \sim A_1, \sim A_2$$

But with the foregoing altered only to the extent that we achieve simple, not necessarily denotatively unique predicate expressions of degree 1 which as a set are denotatively complete, we have:

$$\{a, b, c\} \text{ or } \vee, \{a, b\}, \{a, c\}, \{b, c\}, \{a\}, \{b\}, \{c\}, \wedge .$$

$$D \quad A_1, A_2 \quad B \quad C \quad \sim C \quad \sim B \quad \sim A_1, \sim A_2 \quad \sim D$$

The difference between Strawson's corrected analogue and ours has its origin in an issue which is in part scholarly. It is the issue of whether or not any two simple terms in a genuine argument of the syllogistic can have identical applications. *Tr 2* provides an analogue of traditional logic conceived of with the constraint that all simple terms over which term-variables range are applicatively unique, i.e., that all expressively distinct terms are applicatively distinct, while *tr 1'* provides an analogue of traditional logic conceived of without that constraint. The schema for our analogue is

$$\{a, b, c\} \text{ or } \vee, \{a, b\}, \{a, c\}, \{b, c\}, \{a\}, \{b\}, \{c\}, \wedge$$

$$A \quad B \quad C \quad \sim C \quad \sim B \quad \sim A$$

where the problem of the expressively distinct but applicatively indistinct 'A₁' and 'A₂' of the earlier schemata is presumed resolved at some extra-logical level of inquiry.

It is our general view then that, although it is open for simple predicate expressions of degree 1 to denote an empty set, \wedge , or a universal set, \vee , it is not open for simple term expressions of degree 1 to do so. This means that predicate-variables in, say, 'Asp' by *tr 1'* or by *tr 2* range over expressions with greater denotative possibilities than those over which the term-variables in 'Asp' range. Our search for analogues has in effect been a search for sets of valid statement-forms in modern quantificational logic in which this irreducible denotative difference between the values of predicate- and term-variables has, in other logical ways, been compensated for.

It is reasonable—perhaps only as measured against our hopes—to require that the statement-forms of a strict analogue do, *in their own right*, exactly reflect the truth circumstances of the statement-forms of the original. Our account denies that the statement-forms in either Strawson's corrected analogue or ours do this. They fail because, when regarded *qua* items in the first-order predicate calculus, they are open for occurrences in contexts wherein (a) whatsoever nonempty domain of discourse ($D = 1$ for example) may be imposed, and (b) predicate-variables are instantiated by expressions failing to meet the applicative characteristics of terms.

Whether reflected on singly or jointly, (a) and (b) identically restrict

the maximum claim that we can make for the two analogues. The following is typical: The truth of 'Asp' by *tr 2* is both a necessary and sufficient condition for the truth of 'Asp,' but the falsity of 'Asp' by *tr 2* is only a necessary condition for the falsity of 'Asp.' Again, with respect to 'Isb' by *tr 2* and 'Isb,' the falsity of the former is necessary and sufficient for the falsity of the latter, but the truth of the former is only necessary for the truth of the latter.

We cannot claim the success of having reduced traditional logic to modern logic, of having shown that the old is no more than a proper part of the new. The analogues are near, but they are not strict.

4 Strawson consigns his own analogue to the class of logical curios. Imposing the test that "the constants 'all,' 'some,' and 'no' of . . . [a] . . . system should faithfully reflect the typical logical behavior of these words in ordinary speech," he judges that the *tr 1* system or analogue fails conspicuously. He is so confident about this, and so ready to embark on another type of account, that he relies on a single, casually presented, example to convince us: "It is quite unplausible to suggest that if someone says [(1)] 'Some students of English will get a First this year', it is a sufficient condition of his having made a true statement, that [(2)] no one at all should get a First."

It is possible that Strawson is calling attention to the following: Statement (1) by *tr 1* becomes, with obvious symbolism, $(\exists x)(Sx \cdot Fx) \vee \sim(\exists x)(Sx) \vee \sim(\exists x)(Fx)$. Referring to the three disjuncts by 'A,' 'B,' and 'C' respectively, it appears not only that the truth of C is a sufficient condition for the truth of (1) by *tr 1* but also that its truth is a sufficient condition for the falsity of A. If this is the criticism Strawson intends, then he is the first victim of his own logical imprecision (compare Figures 2 and 3). Statement (1) by *tr 1'* is $(\exists x)(Sx \cdot Fx) \vee \sim(\exists x)(Sx \cdot \sim Fx) \vee \sim(\exists x)(\sim Sx \cdot Fx)$. With 'A,' 'B,' and 'C' serving comparably, the truth of C is a sufficient condition for the truth of (1) by *tr 1'* but it is *not* a sufficient condition for the falsity of A.

It is possible, however, that the criticism Strawson intends is not tied to the disjuncts in so direct a fashion. In any event, it need not be: We can agree to replace (1) by *tr 1* with (1) by *tr 1'* while continuing to insist that (2)—his instrument of criticism—is effectively $\sim(\exists x)(Fx)$, or better, $\sim(\exists x)(Sx \cdot Fx) \cdot \sim(\exists x)(\sim Sx \cdot Fx)$, by whatever legerdemain of translation. This done, we are confronted with the "unplausible" circumstance that (2) so translated can be a sufficient condition for the truth of C, and so of (1) by *tr 1'*, and also for the falsity of A.

This criticism connects with (b) of our assessment of the *tr 1'* and *tr 2* analogues as near rather than strict. That assessment derives from the fact that the requisite term conditions for the occurrence of statements in the traditional logic may or may not be met in contexts wherein, despite a sufficiency of individuals, quantified statements of modern logic conforming to the analogues can occur. The response to Strawson's criticism can be as simple as this: While *tr 1'* (or *tr 2*) affords us a near analogue it does

not afford us a strict one. The example quite properly calls attention to the latter point, but it does not challenge the former one.

Let us bring this paper, this effort to somewhat improve our understanding of the relationship between the two logics, to a close by offering two additional brief responses to Strawson's criticism. Each will be less fair to Strawson than the foregoing one, but each warns us all that the two known analogues, *qua* near but not strict analogues, are not thereby disqualified for service.

First. Strawson invites us to fill the blanks in a 'Some ___ is ___' statement-form with denotative expressions only one of which has the applicative characteristics of a term. The resulting statement is a pseudo-rather than a genuine statement for traditional logic. Nevertheless, we are encouraged to translate this statement into modern logic through the use of translation rules previously introduced for the express purpose of translating statements that are genuine for traditional logic. It is therefore not at all clear why we are to conclude that he has leveled a criticism at his own or anyone else's proposal for translating traditional logic into modern logic.

Second. In order to resist Strawson's criticism, we have merely to insist that the statement (2) "no one at all will get a First" be subjected to the same set of translation rules as is the statement (1) "Some students of English will get Firsts this year" for whose truth it is alleged to be a sufficient condition. So insisting, Strawson's specific criticism collapses since, as inspection reveals, (2) whether by *tr* 1, *tr* 1', or *tr* 2 is a necessarily false statement, e.g., (2) by *tr* 1' is

$$\begin{aligned} & \sim(\exists x)(Sx \cdot Fx) \cdot (\exists x)(Sx \cdot \sim Fx) \cdot (\exists x)(\sim Sx \cdot Fx) \cdot \sim(\exists x)(\sim Sx \cdot Fx) \cdot (\exists x) \\ & (\sim Sx \cdot \sim Fx) \cdot (\exists x)(Sx \cdot Fx), \end{aligned}$$

a self-contradiction.

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