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TOWARDS A LOGIC OF SIGNIFICANCE

PART I: THE SENTENTIAL BASIS

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I tried to show in a previous article[1] that some formal account can be taken of Ryle's theory of categories if we allow that predicates generate three classes of individuals: the class for which they are true, the class for which they are false and the class for which they are absurd (non-significant[2]). In order to deal with these matters formally, however, I there assumed that predicative and relational sentences of the form 'ϕx', 'xRy', *etc.* take on three values: 1 (true), 0 (false) and n (non-significant); but this assumption gives rise to difficulties at the level of interpretation. In particular, it might be thought that absurdity or non-significance is not a third value comparable with the two truth-values, truth and falsity, since there are important semantic differences between them; and even granted that it is, problems arise concerning which particular three-valued logic should be adopted.

I now want to look at these and related difficulties in more detail, though most of what I say in Part I will be confined to the problems as they arise in sentential logic, apart from some brief necessary excursions into predicate logic, and I shall be more concerned with the pre-formal intuitive basis for a significance theory than with detailed formal developments. I hope to discuss the similar but special problems of predicate logic in Part II.

1. *Three values* Consider, first, the question of whether 'non-significance' is a value comparable with the two truth-values.

It might be said that we cannot take an arbitrary grammatical sentence and consider it to be either true, false or non-significant for this amounts to saying that it might be used to make a true statement, a false statement or an absurd statement, and 'absurd statement' is a contradiction in terms. We might perhaps say that it can be used to make a true statement, a false

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statement or an absurd *utterance* but so to say is to admit a difference of semantic level between truth, falsity and absurdity: truth and falsity are characteristic of statements while absurdity is a characteristic of non-statements, not a possible third value of statements. The proper order is first to divide the grammatical sentences into two classes: those which can be used to make statements and are therefore significant, and those which cannot and are therefore non-significant; and then to subdivide the first class into those which are true and those which are false. So if we wish to construct a logic of significance we should first develop a *two-valued* logic of *sentences* whose values are *n* (those which cannot be used to make statements) and *non-n* (those which can) and then a second *two-valued* logic of *statements* (in fact the classical propositional calculus) whose values are 1 (true) and 0 (false). In this way we recognise an essential semantic hierarchy; and though we may define 'non-significant' to mean 'not true and not false' the 'not' so used is not the 'not' of negation; instead the description 'not true and not false' means something like "the words 'true' and 'false' do not apply". So, for example, an expression such as 'Rainbows eat flies' is simply not capable of being true or false; it is non-significant independently of its context of use because it cannot be used significantly in any context.

It is this general view I wish to reject. It rests on the mistaken assumption that significance is a static feature of sentences, rather than a feature of their use, and this in turn derives from an unhealthy interest in sentence types rather than sentence tokens. One might, for example, simply point to the fact that although it is true that if we so use the word 'rainbow' in a given context to refer to visual phenomena in the sky then particular sentence tokens of the type 'Rainbows eat flies' cannot be used to make significant statements, nevertheless we might also use the word in other contexts to refer to trout, and in this case tokens of the same type can be used to make significant statements. Hence, it seems, significance does depend crucially on the context of use, and we may express this by saying that any given token of the type 'Rainbows eat flies' can be used in three possible ways: to make a true statement, a false statement or an absurdity.

Special difficulties of this sort can however be met, and the view that significance is a feature of sentence types rather than the use of tokens retained if we presuppose a standard sense for the words employed. If we take it that the standard use of 'rainbow' is to refer to visual phenomena, then no token of the type 'Rainbows eat flies' can be used significantly and we might reasonably say the type itself, that is the sentence considered independently of its use in a context, or perhaps, more correctly, with respect to its use in a standard context, is non-significant. And this is not an unreasonable assumption to make. For if we are to do logic at all, we must abstract from particular contexts and drop out the odd non-standard cases, otherwise we shall find it impossible to develop general laws applicable in every context. And in any case, words do have a standard sense. By and large we know what people mean when they say something or write it down; it is a general presupposition of rational discourse that there is a standard sense for every word and, correspondingly, paradigmatic contexts in terms of which significance is judged. If it were simply accepted that any word can be used to mean anything at any time just because some words are ambiguous (for that is all that is shown by pointing to the possible use of 'rainbow' to refer to trout) there could be no language. We all know that 'cardinal' may be used to refer to numbers and prelates and hence that 'Cardinals are thoughtful' *might* be non-significant, but we are nevertheless entitled to presume significance until it is disproved, or as one might say, to assume a standard context until it is shown that this assumption is false. Thus we ought really to regard 'rainbow' in 'Rainbows are trout' and 'Rainbows are visual phenomena' as two separate words and, if necessary, in order to get the logic going, distinguish them as 'rainbow$_1$' and 'rainbow$_2$'; in this way we may properly direct our attention to types rather than tokens for then 'Rainbows$_1$ eat flies' and 'Rainbows$_2$ eat flies' are two different types such that any token of the first is significant while any token of the second is not.

But though we accept this, it is only a partial account. We most commonly do presuppose a standard context and judge significance in terms of it, but the significance of many simple sentences cannot be determined independently of their use even though we presuppose a standard sense for the words employed.

Suppose somebody says 'Mary is happy'. Just because the word 'Mary' is used to name people, cows, ships and cylcones, so we cannot, even given the standard sense of 'happy', determine the significance of the sentence independently of its use. Of course, if the context is introduced into the utterance itself, or is part of the surrounding verbal context (the before and after sentences) then there is no difficulty. If somebody says 'Mary, my wife, is happy' the use of the proper name, hence the context, becomes irrelevant to the significance of the statement made. A second predicate 'wife' with a (presumed) standard sense is introduced, and if one knows the language one knows that 'wife' and 'happy' are a significant pair. But the context may simply be the concomitant physical circumstances in which the utterance is made rather than some further verbal qualification, and in this case we can only judge its significance given its actual use. Hence if we are to consider such sentences as 'Mary is happy' in a vacuum, *i.e.* as cases of '*Px*' on the pages of a logic book, then we have to recognise that tokens of the same type may be used to make a true statement, a false statement or an absurd utterance, depending on the context, for on the pages of a logic book the context is not known. But to say this is to say that any expression of the form '*Px*' has three possible, and comparable, values.

There are possible answers to this. We might make the assumption that even proper names have a standard sense; that, for example, we all take 'Mary' to name a woman unless it is made clear that this presumption is shown to be wrong. But even if this is true, and it seems doubtful, it cannot be of use here. For in logic, formal logic anyway, we do not deal with actual proper names but only with variables which take them, any of them, as values; and hence we cannot know in advance what sense is operating. Alternatively we might make the assumption that every item in the universe could be given a different proper name, and we might suppose this done. This would be to treat ordinary proper names as if they were ambiguous general words and to distinguish between, say, 'Mary_1', 'Mary_2', and so on. Given this, there could never by any doubt about which object is being named and hence no doubt about which of 'Mary_1, Mary_2, . . . , is happy' are significant. They would all be different sentences, some true, some false and some non-significant. But quite apart from the drift towards a logically perfect language which this assumption suggests, it is again unhelpful. For it still only solves the problem at the level of interpretation. Given that 'Mary_1 is happy' is significant and 'Mary_2 is happy' is non-significant, it remains true that '*Px*' takes on three possible values, depending on the value chosen for '*x*'. If we choose 'Mary_1' then '*Px*' is significant, if we choose 'Mary_2', not.

Finally, it might be said that since it is a general presumption of rational discourse that sentences are significant until it is shown otherwise, a sentence such as 'Mary is happy' is significant because the word 'happy' forces the correct (significant) naming use for 'Mary'. That is, we take it that 'Mary is happy' is significant and hence that 'Mary' names a woman and not a cyclone. And this is true, we do; but what this amounts to saying is that predicates "carry" a significance range with them: to be significant '. . . is happy' has to be completed by the name of a person (perhaps an animal). In order to express this formally, however, we have first to accept that an expression such as '*Px*' may be either true, false or non-significant depending on the value given to the variable, and then go on to say that a condition for it to be significant is that the value be chosen from a certain restricted range. In fact, '*Px*' is significant if, and only if, x belongs to the class $\pi \cup \bar{\pi}$, where π is the class of objects for which '*P*' is true and $\bar{\pi}$ the class for which it is false. Thus,

$$x \; \varepsilon \; \pi \cup \bar{\pi} \equiv SPx$$ [3]

In this way, we may express general criteria of significance which hold independently of the context of use, in fact criteria which determine the paradigm context for the standard sense of a general word. So we relate context-dependent significance criteria to context-independent criteria. But we can only do this if we allow '*Px*' to take on three comparable values. Far from destroying the basis of the logic, therefore, this proposed "way out" simply re-affirms it.

I take it, then, that the descriptions 'true statement', 'false statement' and 'absurdity' indicate three possible indicative uses of any given sentence token, and because they are all uses so they are on the same semantic level and may be appropriately represented in a formal logical theory by recognising three comparable values 'true', 'false' and 'non-significant' for the sentential variables. This means that we may develop a sentential logic of significance as an interpreted three-valued calculus if we introduce variables '*a*', '*b*', '*c*', *etc.* which range over sentence tokens whose particular use is not known (apart from the fact that it is assumed to be indicative) because the context is not known. Thus the variables range over sentence tokens with an undetermined context. We could of course make it clear that the context is undetermined by introducing further variables which range over contexts and then consider complexes of the sort '(a, m)' or 'a_m' which represent 'the indicative use of a in context m'[4]. Such refinement is unnecessary however if we are not interested in relations between contexts; it becomes essential if we wish to develop general laws which relate different contexts.

I turn now to the question of which particular three-valued logic is most appropriate in terms of such intuitive criteria as we have for dealing with absurdities.

2. *Which logic?* The problems associated with the choice of a three-valued logic were to some extent bypassed in *PRC* by introducing the operators, '*T*' (it is true that), '*F*' (it is false that) and '*S*' (it is significant (to say) that) and considering only such relations as hold between expressions of the sort '*Ta*', '*Fa*', '*Sb*', *etc.*, where '*a*' and '*b*' are three-valued sentential variables. For if we make the assumption:

I To say of any sentence that it is true, that it is false or that it is non-significant, is to make a significant statement; and in particular, to say of a non-significant sentence that it is true, or that it is false, is to make a false statement,

then the expressions '*Ta*', '*Fa*', '*Sa*' and others definable in terms of these are themselves two-valued, true-or-false, even though they contain a three-valued part. For in terms of this assumption we may define the operators by the following significance tables:

a	Ta	Fa	Sa
1	1	0	1
0	0	1	1
n	0	0	0

and we may describe them as (3:2) mapping operators since they map three values onto two. Hence by restricting what we say, in effect, to statements about[5] three-valued sentences, we may develop a logic in which all the laws are themselves two-valued.

I shall adopt criterion I throughout. Given certain plausible assumptions it seems to follow as a consequence of the thesis that 'true', 'false' and 'non-significant' are three comparable values. For in terms of this thesis, 'not significant' means 'not true and not false' (as opposed to 'not capable of being true or false'). And if we now take it that it is always significant to say of a non-significant sentence that it is non-significant, then the description 'not true and not false' is significant of absurdities. But if this is significant (*i.e.* true or false) of absurdities so, too, it would seem, are its component parts 'not true' and 'not false', and hence, so are the descriptions 'true' and 'false'. It has to be recognised, however, that in arguing thus, assumptions are made about the significance criteria of conjunctions in terms of their component parts, and of negations. These are discussed quite generally below. Nor do I wish to deny that there is a sense of 'not true and not false' which means 'not capable of being true or false', and in terms of which some expressions of the form 'X is true' are themselves non-significant because the word 'true' is being misapplied. If 'X' is an uninterpreted formula of a formal calculus, which has the form of a sentence but no content, then it makes no sense to say that it is true or false. Here, 'X is neither true nor false' means 'The words 'true' and 'false' do not apply to X', and though this is significant we cannot without contradiction derive from it the conclusion that 'true' is a significant description of X.[6] But it is not this use of 'true' which is exemplified by 'Ta'. We are not here saying that the uninterpreted letter 'a' is true, which would be non-significant, but instead we use the expression 'Ta' to abbreviate 'it is true that a' where 'a' is variable which takes as values actual sentences of ordinary language. Again, it would be non-significant to say that 'it is true that a' is true or false, since it contains a variable, but we are not, when we use 'T', talking about the expression 'Ta' any more than we are talking about the letter 'a'. We are using both 'T' and the variable to talk about sentences, and of all such sentences it is significant to say that they are true.

Now although we adopt criterion I very little progress can be made unless some decisions are made about the three-valued calculus which the sentential variables satisfy. And this is so even if we decide to restrict the significance logic to relations between the two-valued expressions 'Ta', 'Fa' and 'Sa'. For suppose we have a two-valued expression of the form '$T(a \,\&\, b)$; where '&' represents the three-valued 'and' and '$a \,\&\, b$' is itself three-valued. We may wish to replace '$a \,\&\, b$' by an equivalent expression in order to pass from, say, '$T(a \,\&\, b)$' to '$T\ulcorner(\ulcorner a \wedge \ulcorner b)$' where '$\ulcorner$' represents the three-valued 'not' and '$\wedge$' the three-valued 'or'. In ordinary two-valued logic, since we know that '$p.q$' is equivalent to '$\sim(\sim p \vee \sim q)$', where '$\sim$', '.', and '$\vee$' are respectively the two-valued connectives 'not', 'and' and 'or', we could make such a move under, say, one of the modal operators, *e.g.* 'N' (it is necessary that), and so pass from '$N(p.q)$' to '$N\sim(\sim p \vee \sim q)$'. But whether or not we can make such a move in significance logic will depend on whether or not '$a \,\&\, b$' is equivalent to '$\ulcorner(\ulcorner a \wedge \ulcorner b)$' and this in turn will depend on the particular three-valued logic chosen.

The question of which three-valued basis to adopt is crucial, therefore, and it can only be settled in terms of general criteria and the examination of particular cases which lead to unique definitions of the three-valued connectives. As one might expect, however, there are conflicting criteria; and because the number of possible definitions is so large some fairly arbitrary decisions have to be made. For even if we restrict our attention to definitions of the three-valued connectives which "contain" the standard two-valued connectives in the sense that those rows of a definitional table on which the value n does not occur are completed in the standard way, this still leaves 5 rows to be completed for each binary connective, and one row for the monadic connective '$\ulcorner$', and on each such row any one of the three values may be chosen. Hence there are $3^5 = 243$ possible definitions for each binary connective, and in general 3×243^4 possible combinations of definitions to be contemplated. It is clear, therefore, that at this point we have to step outside the logic and examine its intuitive basis.

2.1 *The System* **T1** The problem, then, is as follows:
We introduce the three-valued sentential variables,

$$a, b, c, etc.$$

and three-valued connectives,

$\ulcorner$(not); &(and); $\wedge$(or); $\rightarrow$(if . . . then . .); $\longleftrightarrow$(if, and only if)

and ask what criteria should be adopted in order that unique definitions of the connectives may be constructed. One such plausible criterion is the following:

II (a) Any compound expression with a non-significant component is non-significant
(b) Any compound expression in which all the components are significant is itself significant.

And if we now adopt the suggestion

III The definitions of the three-valued connectives should "contain" the classical connectives of the two-valued propositional calculus (**PC**), then we have an immediate solution of the general problem. For II(a) determines that on any row on which the value n occurs, the compound is n; II(b) determines that on all other rows the value of the compound is 1 or 0, and III determines exactly which value is to be chosen on each of these rows. Hence the definitions are as follows:

a	b	$\ulcorner a$	$a \& b$	$a \wedge b$	$a \rightarrow b$	$a \longleftrightarrow b$
1	1	0	1	1	1	1
0	1	1	0	1	1	0
n	1	n	n	n	n	n
1	0		0	1	0	0
0	0		0	0	1	1
n	0		n	n	n	n
1	n		n	n	n	n
0	n		n	n	n	n
n	n		n	n	n	n

It follows from this, however, that if we adopt the further criterion

IV A formula expresses a logical law if, and only if, it comes out true for all possible values of the variables

then there are no logical laws over sentences which may be true, false or non-significant. If, on the other hand, we weaken this requirement and accept instead

IV* A formula expresses a logical law, if, and only if, it does not come out false for any values of the variables (but may be either true or non-significant)

then all the usual laws of the classical PC are "laws" of the three-valued system so constructed but in many cases we shall validate arguments which have true premisses and non-significant conclusions: *e.g.* from 'cats are furry' using the law '$a \rightarrow (a \wedge b)$' we shall be able to derive the conclusion 'Cats are furry or rainbows are lazy' (which is non-significant if we take the standard sense of 'rainbow'), provided of course we retain *modus ponens* as a rule of inference.[7]

These particular difficulties, however, need not concern us if we are interested only in developing a two-valued logic which enables us effectively to talk about non-significant sentences without actually using them, for we are not then interested in asserting any three-valued laws. We need only decide under what conditions two three-valued expressions may be said to be equivalent, for this will enable us to replace one by the other under the operators 'T', 'F' and 'S'. And though it may be said that this presupposes that we assert three-valued equivalences, the assertion involved is not in any way objectionable because in such circumstances we do not permit inferences directly from the equivalences but only from two-valued expressions containing three-valued parts under operators. We can however take a further step which eliminates any suggestion that we may be asserting implicitly three-valued equivalence laws. For consider: since 'Ta', 'Fa' and 'Sa' are all two-valued, we can replace 'a' by 'b' without affecting the *truth-value* of 'Ta', 'Fa' and 'Sa' provided 'a' and 'b' have the same *significance-value*; that is to say, provided both are true, both false, or both non-significant. And since we are only interested in truth-value relationships between expressions such as 'Ta', 'Fa' and 'Sa', no laws of the two-valued significance logic will be affected by such a move. Thus, in terms of the definitional table for 'T', if 'a' and 'b' are both true, then 'Ta' and 'Tb' are both true; if 'a' and 'b' are both false, then 'Ta' and 'Tb' are both false; and if 'a' and 'b' are both non-significant, then 'Ta' and 'Tb' are, again, both false. Similarly, for the other operators; whenever 'a' and 'b' have the same significance-value, 'Za' and 'Zb', where 'Z' is some operator, have the same truth-value. We may therefore introduce a *two-valued* connective '$\equiv$' which relates three-valued formulae and expresses the relation 'has the same significance-value as'. That is, '$a \equiv b$' is true if both components are true, both false or both non-significant; otherwise false. Thus,

a	≣	*b*
1	1	1
0	0	1
n	0	1
1	0	0
0	1	0
n	0	0
1	0	*n*
0	0	*n*
n	1	*n*

The relation '≣' may be described as a ((3,3):2) mapping operator since it maps a pair of three-valued expressions into a two-valued compound.

We are now in a position to adopt criteria II, III and IV and yet by-pass Presley's difficulties if we adopt the general rule that if *A* ≣ *B* is a law then *A* may be replaced by *B*, and *vice versa*, under any one of the operators '*T*', '*F*' and '*S*'; for we do not at any point affirm three-valued laws. Thus, for example, we can affirm '(*a* & *b*) ≣ ⌐(⌐*a* ∧ ⌐*b*)' on the basis of these criteria since '*a* & *b*' and '⌐(⌐*a* ∧ ⌐*b*)' do have the same significance value, as the following table shows.

a	*b*	*a* & *b*	≣	⌐	(⌐*a*	∧	⌐*b*)
1	1	1	1	1	0	0	0
0	1	0	1	0	1	1	0
n	1	*n*	1	*n*	*n*	*n*	0
1	0	0	1	0	0	1	1
0	0	0	1	0	1	1	1
n	0	*n*	1	*n*	*n*	*n*	1
1	*n*	*n*	1	*n*	0	*n*	*n*
0	*n*	*n*	1	*n*	1	*n*	*n*
n	*n*	*n*	1	*n*	*n*	*n*	*n*

But in affirming this we are not affirming a three-valued law, even though the three-valued tables are used, and have to be used, in order to reach the conclusion. Hence it is not necessary to affirm a three-valued equivalence law in order to pass from, say, '*T*(*a* & *b*)' to '*T*⌐(⌐*a* ∧ ⌐*b*)'.

It is of course unnecessary to adopt the particular criteria II and III in order to proceed in this way. Any criterion which allows us to construct unique definitional tables for the three-valued connectives will in turn allow us to affirm laws of the sort '*a* ≣ *b*' and so enable us to make the necessary replacement moves under operators. Differences in criteria, however, will be reflected by the fact that different equivalence laws hold. Thus, for example, it is easy to choose criteria in terms of which '(*a* & *b*) ≣ ⌐(⌐*a* ∧ ⌐*b*)' is not a law. Variations of this sort, however, will be considered presently. For the moment we adopt criteria II, III, IV and the following rule V:

V Given that '$\overset{\equiv}{=}$' expresses the relation 'has the same significance value as' and '$\equiv$' the relation 'has the same truth-value as' (the classical **PC** equivalence) then:
if $A \overset{\equiv}{=} B$ is a law so are $TA \equiv TB$, $FA \equiv FB$ and $SA \equiv SB$. Hence, where A and B are any three-valued formulae satisfying these conditions, either one may be replaced by the other, under the operators 'T', 'F', 'S' and others definable in terms of these.

We are now in a position to define a system **T1** as a possible sentential basis for significance theory as follows:

(i) **T1** contains the classical two-valued variables, 'p', 'q', 'r', . . . *etc.* and connectives '$\sim$', '.', 'v', '$\supset$' and '$\equiv$', of **PC**. The well-formed formulae (wff) of **PC** are wff of **T1**, and the connectives are defined by the classical tables.

(ii) **T1** contains three-valued variables, 'a', 'b', 'c', *etc.* and connectives, '$\neg$' '&', '$\wedge$' '$\rightarrow$' and '$\leftrightarrow$' such that:
  (a) the interpretation of the three-valued connectives is the same as the interpretation of their two-valued analogues (*e.g.*, '$\neg$' reads 'not' as '$\sim$' does).
  (b) a *semi-wff* is a combination of three-valued variables and connectives constructed by applying analogues of the formation rules of the two-valued **PC** (*e.g.* 'a & b' is a semi-wff, but 'ab &' is not).
  (c) the connectives are defined in terms of criteria II and III (*i.e.* by the definitional tables which follow from these criteria).

(iii) **T1** contains the operators 'T', 'F' and 'S' such that:
  (a) TA, FA, SA are well-formed two-valued formulae and may be taken as values of the two-valued variables 'p', 'q', 'r', *etc.* provided A is either a wff or a semi-wff. (*i.e.* '$T(a \wedge b)$' is a wff since '$a \wedge b$' is a semi-wff; '$TT(a \wedge b)$' is a wff since '$T(a \wedge b)$' is a wff; 'Tp' is a wff since 'p' is a wff; *etc.*)
  (b) 'T', 'F' and 'S' are defined in accordance with criterion I (*i.e.* by the definitional tables as given).

(iv) **T1** contains the connective '$\overset{\equiv}{=}$' such that:
  (a) $A \overset{\equiv}{=} B$ is a wff provided both A and B are both semi-wff or both wff (In case both are wff, $A \overset{\equiv}{=} B$ reduces to $A \equiv B$).
  (b) '$\overset{\equiv}{=}$' is defined by the table as given and satisfies criterion V.

(v) A wff of **T1** is a logical law if, and only if, the final column of its significance table (or truth table in case it is a wff of **PC**) is 1 for all values of the variables. That is, a wff of **T1** is a law if, and only if, it satisfies criterion IV.

These conditions do not of course represent a minimal description of the logic so defined since, *e.g.*, 'F' and 'S' may defined in terms of 'T' and need not be taken as primitive. Again, '$\overset{\equiv}{=}$' can be defined in terms of 'T', 'F' and 'S', plus the two-valued connectives, and the rule V is then derivable. Thus, if we define 'F' and 'S' by,

$$Fa =_{df} T \ulcorner a$$

and

$$Sa =_{df} Ta \vee Fa,$$

as in *PRC*, it is easily seen that they have the required significance tables. Similarly, if we define '$\overset{=}{\equiv}$' by,

$$a \overset{=}{\equiv} b =_{df} (Ta.Tb) \vee (Fa.Fb) \vee ({\sim}Sa.{\sim}Sb)$$

then the significance table for '$\overset{=}{\equiv}$' follows and '$(a \overset{=}{\equiv} b) \supset (Ta \equiv Tb)$', '$(a \overset{=}{\equiv} b) \supset (Fa \equiv Fb)$' and '$(a \overset{=}{\equiv} b) \supset (Sa \equiv Sb)$' are laws.
Hence rule V may be derived.

The conditions (i)—(v) do, however, represent a complete description of the system **T1**. That is to say, the significance tables determine a systematic finite method for deciding for any wff whether or not it is a law. For, first distribute values over the three-valued variables; next use the definitional tables of (ii c) to calculate the values of semi-wff occurring in the formula; next use the definitional tables of (iii b) and (iv b) to calculate the values of wff involving operators or the connective '$\overset{=}{\equiv}$'; finally, calculate the value for the whole formula using the two-valued truth tables of **PC**. Thus, for example, we show that '$(a \overset{=}{\equiv} b) \supset (Ta \equiv Tb)$' is a law as follows:

$(a$	$\overset{=}{\equiv}$	$b)$	$\supset$	$(Ta$	$\equiv$	$Tb)$
1	1	1	1	1	1	1
0	0	1	1	0	0	1
n	0	1	1	0	0	1
1	0	0	1	1	0	0
0	1	0	1	0	1	0
n	0	0	1	0	1	0
1	0	n	1	1	0	0
0	0	n	1	0	1	0
n	1	n	1	0	1	0
1	5	2	7	3	6	4

The order of calculation is indicated by the numbers on the columns.

Following are some further laws of **T1**:

$Ta \supset Sa$;	$Ta \vee Fa \vee {\sim}Sa$;[9]
$Fa \supset Sa$;	STa;
$Ta \supset {\sim}Fa$;	SFa;
$Fa \supset {\sim}Ta$;	SSa;
${\sim}Sa \equiv {\sim}Ta.{\sim}Fa$;	$TTa \equiv Ta$;
$T(a \,\&\, b) \equiv Ta.Tb$;	$TFa \equiv Fa$;
$T(a \wedge b) \supset Ta \vee Tb$;[8]	$TSa \equiv Sa$.

One feature of the connective '$\overset{=}{\equiv}$' is worth noting: namely that in many cases, if $A \equiv B$ is a two-valued law of **PC** then $\dot{A} \overset{=}{\equiv} \dot{B}$ is a law of T1, where $\dot{A}$ and $\dot{B}$ are the three-valued analogues of A and B. Thus, for example, if

A is '$p.q$' and B is '$\sim(\sim p \vee \sim q)$' then we have the **PC** law '$p.q \equiv \sim(\sim p \vee \sim q)$' and, as we have seen, it is readily shown by an application of the significance tables that $\dot{A} \equiv \dot{B}$ where $\dot{A}$ is '$a \,\&\, b$' and $\dot{B}$ is '$\ulcorner(\ulcorner a \wedge \ulcorner b)$'. This is not generally so, however. In fact, a necessary and sufficient condition for this relationship to hold is that A and B contain exactly the same variables. Thus, although '$p \vee \sim p \equiv q \vee \sim q$' is a **PC**-law, we do not have '$a \wedge \ulcorner a \equiv b \wedge \ulcorner b$' as a **T1**-law. Hence, in general, $\dot{A} \equiv \dot{B}$ only if A and B are so related that values of them have the same truth-value because they are synonymous, in some sense of 'synonymous', that is, they are not merely extensionally equivalent but extensionally equivalent *because* some stronger relation holds between them. For example, in some sense of 'means the same as' we may say that 'it is raining and the sun is shining' means the same as 'it is not the case that either it is not raining or the sun is not shining', but in no sense of 'means the same as' may we say that 'the cat is on the mat or it is not' means the same as 'the dog is on the chair or it is not'. Hence we may take '$\equiv$' as picking out those cases of truth-functional equivalence which are synonymies, and we may say that it expresses the relation of extensional synonymy.[10]

The system **T1** can be modified in various ways without changing its essential two-valued nature. Thus, for example, since the two-valued connectives '$\sim$', '.', '$\vee$', *etc.* are definitionally "contained in" their three-valued analogues '$\ulcorner$', '&', '$\wedge$', *etc.*, so it is not logically necessary, though it is psychologically useful, to employ both kinds. Provided we keep the two different kinds of variables we may use the three-valued constants throughout since in case the components related by the constants are two-valued, n is not an admissible value in the tables and the definitions reduce to their two-valued cases. Indeed we might extend the definition of 'well formed formula' to include expressions such as '$Ta \rightarrow a$', or even '$p \rightarrow a$' where one component is two-valued, one three. The distribution of values over the components would thus be determined by the nature of the components rather than the nature of the connectives, but the one set of three-valued definitional tables would still serve as a criterion for the unique evaluation of compound expressions. Alternatively, we may recognise only one type of variable and retain two different kinds of connective, for though it is in general unnecessary to distinguish two sorts of variables and two sorts of connective it is essential to make one such distinction if criterion IV is adopted. If we adopt IV*, however, no distinction is necessary. These simplifications will be made in the next system **T2**.

A further, different, modification which would extend the set of laws arises if formulae at present classified as ill-formed (*e.g.* '$ab \rightarrow$') are admitted but affirmed to be non-significant for all values of the variables; that is, laws such as '$\sim S(ab \rightarrow)$' are asserted. With such an extension, however, though the formation rules can effectively be brought into the logic, the significance tables no longer provide an effective technique for distinguishing laws from non-laws since, *e.g.*, the tables cannot be applied to '$ab\rightarrow$' and hence not to $\sim S(ab\rightarrow)$. However, the tables can be extended to evaluate such expressions.

2.2 *The System* **T2** Such modifications as those suggested above do not change the essential structure of the logic, and in particular do not destroy its basis as a two-valued system. But it must now be asked whether it should be extended in other ways. For even if we permit expressions such as '$Ta \rightarrow a$' to be well-formed, none of them will be laws since there will always be at least one 'n' in the final column of the significance table. And yet, it might be said, some expressions of this kind ought to be laws, indeed, '$Ta \rightarrow a$' should be since it simply expresses in a quite general way that if an expression is true then it is assertable.

The question being raised here is the quite general one of whether we should permit as laws expressions in which a three-valued expression is used and not merely mentioned. So far, we have restricted the laws to statements about three-valued expressions and it might be said that this is unnecessarily restrictive. For it we are working on the assumption that the sentences used in ordinary language may be true, false or non-significant then, in ordinary language, we do all the time *use* three-valued expressions, and indeed very rarely mention them; and if this is so, then there must be laws which relate them when they do not occur associated with an operator such as 'T'. Yet if we accept this, then we have to accept that we do sometimes *use* non-significant sentences, in conjunction with other significant or non-significant sentences, in such a way as to make true statements. For if there are *laws* which relate three-valued expressions, *i.e.* formulae which are true for all values of the variables, then there will be cases of these laws in terms of actual sentences which express true statements but yet contain non-significant components, since n is one of the possible values of the variables. Thus, for example, if we affirm,

$$Ta \rightarrow a$$

then we shall be committed to saying that a special case of it is, *e.g.*, 'if it is true that Saturday is in bed then Saturday is in bed'; and that this expresses a true statement even though it contains a non-significant consequent in a context in which the words in it are taken to have their standard sense. And indeed it might be argued that this is acceptable. For since 'Saturday is in bed' is non-significant, 'it is true that Saturday is in bed' is false, and we simply have a case of a hypothetical with a false antecedent; but no harm comes of this since we can never affirm the antecedent and hence never detach the consequent and affirm it. That is, we are not thereby committed to the *affirmation* of non-significant sentences and so not involved in saying something that is nonsense even though we use nonsense in saying what we say. To accept this, however, is to repudiate criterion II(a) in terms of which any compound expression with a non-significant component is itself non-significant; and it is besides to accept that we may often assert *true* compound sentences in which *every* component is non-significant. For the case chosen to illustrate the point is favoured in that one component is two-valued. But if the general point is to be made then we may well have laws such as,

$$a \; \& \; b \rightarrow a;$$

and hence be committed to affirming cases like,

If Saturday is in bed and Sunday is having a bath then Saturday is in bed.

Now we can avoid these difficulties and at the same time retain criterion II(a) by adopting instead of condition IV, that a formula is a logical law if, and only if, it comes out true for all values of the variables, the alternative condition IV*, that a formula is a logical law if, and only if, it does not come out false for any values of the variables. This would allow us to say that, *e.g.* 'a & $b \rightarrow a$' is a law but would not commit us to saying that all cases of it are true; in particular, the case,

If Saturday is in bed and Sunday is having a bath then Saturday is in bed

is non-significant even though it is a special case of a law; and we can besides block objectionable inferences by stipulating that a law validates an argument only in cases where both the premisses and the implication are true. Thus we should not be able to infer from 'Cats are furry' to 'Cats are furry or Saturday is in bed' using '$a \rightarrow a \wedge b$' since under these substitutions, though we have true premisses, we have a non-significant implication in terms of criterion II(a).

Against this, however, it might be said that there are cases where we wish to validate arguments in which the implication is non-significant. Suppose, for example, we take a *reductio ad absurdum* argument in the literal sense of reduction to an absurdity. That is, we have an implication of the form $A \rightarrow B$; we affirm that B is non-significant and draw the conclusion that A is non-significant. If this is a valid argument then it takes the form,

$$[(a \rightarrow b) \ \& \ \ulcorner Sa] \rightarrow \ulcorner Sb;$$

and in fact, in terms of II(a) and IV*, it is a law. If, however, we restrict the application of this formula *qua* law to just those cases in which the premisses are true and the implication true then we exclude exactly those cases which the law is required to validate, *i.e.* the literal *reductio ad absurdum* arguments. Similarly, if we take a version of the familiar paradox-type argument: that if A implies a contradiction and $\ulcorner A$ also implies a contradiction then A is non-significant (neither true nor false), *i.e.*,

$$[(a \rightarrow (b \ \& \ \ulcorner b)) \ \& \ (\ulcorner a \rightarrow (b \ \& \ \ulcorner b))] \rightarrow \ulcorner Sa;$$

which is also a law in terms of II(a) and IV*, we again exclude exactly those cases which it is designed to cover if we restrict its application to arguments in which both the premisses and implication are true.

Considerations such as these, therefore, might seem to force us to the position of rejecting criterion II(a) though there is obviously some point in modifying the system **T1** in such a way that formulae such as those indicated above are laws in terms of IV* while leaving open the question of how exactly the logic so modified should be applied. Hence we define the system **T2** as follows:

(i) **T2** contains the three-valued variables 'a', 'b', 'c', *etc*. and three-valued connectives, '$\ulcorner$', '&', '$\wedge$' '$\rightarrow$', and '$\longleftrightarrow$', which are interpreted and defined as in **T1** (*i.e.* in accordance with criteria II and III which yield the definitional tables of **T1**).

(ii) **T2** contains the operator 'T' defined as in **T1** (*i.e.* in terms of criterion I)

(iii) The wff of **T2** are as follows:
(a) A variable is a wff.
(b) If A and B are wff so are $\ulcorner A$, A & B, $A \wedge B$, $A \rightarrow B$ and $A \longleftrightarrow B$.
(c) If A is a wff so is TA.
(d) Symbols introduced by definition as abbreviations of wff are wff.
(e) Wff enclosed in brackets are wff.

(iv) The initial definitions of **T2**, where A and B are wff, are as follows:
'FA' for '$T\ulcorner A$'
'SA' for '$TA \wedge FA$'
'$A \equiv B$' for '$(TA \;\&\; TB) \wedge (FA \;\&\; FB) \wedge (\ulcorner SA \;\&\; \ulcorner SB)$'

(v) A wff of **T2** expresses a logical law if, and only if, the final column of its significance table does not contain the value 0 for any distribution of values over its component parts (IV*).

Since, as we have seen, V is derivable, **T2** differs from **T1** in that it satisfies I, II, III, IV* and V instead of I, II, III, IV and V. It differs too, of course, in that no provision is made for two sorts of variable or two sorts of connective. This is now unnecessary since the definition of wff has been extended to include formulae such as '$a \;\&\; b \rightarrow a$', and since some of these turn out to be laws in terms of IV*, in fact exactly those which are three-valued versions of the PC laws, the two-valued **PC** laws are absorbed in their three-valued analogues.

The system **T2** thus contains **T1** since analogues of each of the laws of **T1** are laws of **T2**: *e.g.* $Ta \rightarrow Sa$; $Fa \rightarrow Sa$, *etc.*, together with the **PC**-type laws '$a \rightarrow a \wedge b$', *etc*; but it is wider since there are laws of **T2** which are not laws of **T1**. Many of these seem plausible. Thus, we have:

$$Ta \longleftrightarrow a;\; a \rightarrow Sa;\; ((a \rightarrow b) \;\&\; \ulcorner Sb) \rightarrow \ulcorner Sa \quad (\textit{reductio});$$
$$[(a \rightarrow (b \;\&\; \ulcorner b)) \;\&\; (\ulcorner a \rightarrow (b \;\&\; \ulcorner b))] \rightarrow \ulcorner Sa \quad (\textit{first paradox law});$$
$$[(a \rightarrow \ulcorner a) \;\&\; (\ulcorner a \rightarrow a)] \rightarrow \ulcorner Sa \quad (\textit{second paradox law}).$$

On the other hand, others are less plausible; for example,

$$[(a \rightarrow (b \;\&\; \ulcorner b)) \;\&\; (\ulcorner a \rightarrow (b \;\&\; \ulcorner b))] \rightarrow Sa;\; [(a \rightarrow \ulcorner a) \;\&\; (\ulcorner a \rightarrow a)] \rightarrow Sa.$$

In fact, we have,

$$[(a \rightarrow (b \;\&\; \ulcorner b)) \;\&\; (\ulcorner a \rightarrow (b \;\&\; \ulcorner b)] \rightarrow Sa \;\&\; \ulcorner Sa;$$
$$[(a \rightarrow \ulcorner a) \;\&\; (\ulcorner a \rightarrow a)] \rightarrow Sa \;\&\; \ulcorner Sa;$$

and this suggests that implication should be re-defined.

The laws $Ta \longleftrightarrow a$ and $a \rightarrow Sa$ are of special interest since they give rise to difficulties in the axiomatisation of **T2**. For either one, it seems, leads directly to the conclusion that every sentence is significant. Thus, suppose we have a formal system in which $a \rightarrow Sa$ is either an axiom or a theorem and the remaining laws of **T2** are also theorems. Now consider the following argument:

1.	$a \rightarrow Sa$	:
2.	$\neg Sa \rightarrow \neg a$	: using the equivalence law '$(a \rightarrow b) \leftrightarrow (\neg b \rightarrow \neg a)$'
3.	$\neg S \neg a \rightarrow \neg\neg a$	: putting '$\neg a$' for 'a' in 2.
4.	$\neg S \neg a \rightarrow a$	: using the equivalence law '$\neg\neg a \leftrightarrow a$'.
5.	$\neg S \neg a \rightarrow Sa$	: from 1, 4 by transitivity law for '$\rightarrow$' and detachment.
6.	$S \neg a \wedge Sa$	: using the equivalence laws '$(a \rightarrow b) \leftrightarrow (\neg a \wedge b)$' and '$\neg\neg a \leftrightarrow a$'.
7.	$Sa \wedge Sa$	: using the equivalence law '$Sa \leftrightarrow S \neg a$'.
8.	Sa	: using the equivalence law '$a \wedge a \leftrightarrow a$'.

Hence we derive a theorem which expresses the fact that every sentence is significant. Yet in terms of the significance tables, 'Sa' is not a law, as indeed it must not be if the logic is to have its intended application and not collapse into the two-valued **PC**. On the other hand, every step in the proof is made by reference to formulae which are laws in terms of the significance tables. So, it follows, if we wish to axiomatise **T2** in such a way that every theorem is a law in terms of the significance tables, and conversely every law is a theorem, all the laws used in the above "proof" must be theorems, yet the conclusion 8 must not be derivable. This means that one or other of the rules used in the derivation must be rejected. But only three rules are used, and each of them is a rule of standard logic: *viz.* replacement of equivalents (steps 2, 4, 6, 7, 8) uniform substitution (step 3) and detachment (step 5). It is clear, however, that detachment must be modified since 1, 2, 3 and 4 are laws in terms of the significance tables but 5 is not, and 5 arises by detachment.

Now we saw earlier that there is a case for saying that laws of **T2** do not validate arguments when the logic is applied unless both the premisses and the implication of the argument are true; and it is this fact which is now being reflected in the difficulties concerning the rule of detachment as it occurs in the formal system. For in permitting unrestricted detachment we are in effect permitting arguments from premisses and implications which may, for certain of the values given to their variables, turn out to be non-significant. Hence if we state the rule of detachment in the form

From $\vdash .TA$ and $\vdash T(A \rightarrow B)$ infer $\vdash .B$

instead of in the unrestricted form

From $\vdash .A$ and $\vdash (A \rightarrow B)$ infer $\vdash .B$

We shall mirror within the formal system that restriction which has to be applied at the level of interpretation and we shall block formal inferences within the system of the kind indicated in 1-8 above. Thus the step from 1 and 4 to 5 is blocked since neither

$$9'.\ T[((\ulcorner S \ulcorner a \rightarrow a) \,\&\, (a \rightarrow Sa)) \rightarrow (\ulcorner S \ulcorner a \rightarrow Sa)]$$

nor

$$10'.\ T[(\ulcorner S \ulcorner a \rightarrow a) \,\&\, (a \rightarrow Sa)]$$

are laws, though both

$$9.\ ((\ulcorner S \ulcorner a \rightarrow a) \,\&\, (a \rightarrow Sa)) \rightarrow (\ulcorner S \ulcorner a \rightarrow Sa)$$

and

$$10.\ (\ulcorner S \ulcorner a \rightarrow a) \,\&\, (a \rightarrow Sa)$$

are.

This restriction on detachment, however, will not succeed formally if we have the general rule that if A is a theorem so is TA, *i.e.*

From $\vdash .A$ infer $\vdash .TA$ (the T-rule);

a rule which is commonly accepted for 'N' (necessary) in modal logics. For given such a rule we should be able to pass from,

$$1.\ a \rightarrow Sa$$

to

$$1'.\ T(a \rightarrow Sa)$$

and similarly with the other steps, so eventually deriving 9' and 10' above and hence, in conjunction with restricted detachment,

$$T(\ulcorner S \ulcorner a \rightarrow Sa)\ TSa,$$

and finally,

$$Sa$$

It is clear, however, that the T-rule cannot be a rule of any system which stands as an axiomatisation of **T2**. For since many of the laws A contain an 'n' in the final column, in all such cases TA will not be a law (since it will contain a 0 in the final column). This fact simply reflects the weakened definition of a logical law in terms of IV* since we are counting as laws formulae which contain n's in their final column and hence cannot be said to be true for all cases. On the other hand an axiomatisation of **T1** can contain the T-rule since a formula of that system only expresses a law in case its final column consists of 1's only; and since **T2** contains **T1** (*i.e.* every theorem of **T1** is a theorem of **T2**) an axiomatisation of **T2** can contain the T-rule for just those laws of **T2** which are laws of **T1**, though it could also be extended to apply to the three-valued analogues of the **PC** laws without serious difficulties arising. These problems are essentially formal problems, however, since they concern the axiomatisation of **T2** and are irrelevant if we proceed entirely in terms of significance tables. Moreover they have all been solved satisfactorily.[11]

2.3 *Further considerations* I now want to look briefly at various reasons which could be used to support alternative sentential systems as a basis for significance theory. At least three kinds of reasons are relevant. It might be said, first, that there are general laws of significance logic which are not laws of either **T1** or **T2**, and hence that one or other or both of these should be extended to include them. Second, it might be argued that other systems which contain **T1** or **T2** have certain formal advantages in that they are easier to manipulate as axiomatic theories. Third, it might be claimed that the criteria in terms of which **T1** and **T2** are constructed are intuitively wrong. I shall limit the main discussion to this third point.

With regard to the first point, it must I think be accepted that **T1** is unnecessarily restrictive; for there are laws, such as the literal *reductio* and the paradox law, frequently used in philosophical literature which cannot be expressed in **T1**. As we have seen, however, many of these can be expressed in **T2**, and though this system presents difficulties of interpretation it satisfies the essential requirement that the required formulae are laws. Moreover, it is easy to introduce various technical tricks and construct systems which are law-equivalent to **T2** and which might be thought not to present the same interpretational problems. Thus, for example, if we introduce an operator 'K' defined by

a	Ka
1	1
0	0
n	1

which converts n to 1 and leaves 1 and 0 unchanged,[12] then if A is a law of **T2**, the final column of the significance table for KA will consist of 1's only. Moreover, since KA is equivalent to $\ulcorner FA$, and since we know for every law of **T2** that the values in its final column are always not false (true or non-significant), we could modify **T2** to admit the rule: if A is a **T2**-law then KA is a law. We then have a system **T3**, say, the laws of which satisfy IV rather than IV*, and are simply the laws of **T2** covered by 'K'. But this is a trivial move which does nothing towards explicating the basic problems of significance logic. It enables us to say that the laws of **T3** are true for all values of the variables and hence can be applied without restriction, but if we are prepared to accept this we may as well accept that the laws of **T2** have an unrestricted application. It would however be an important criticism of **T2**, as it is of **T1**, if it could be shown that laws which are generally accepted are not demonstrable in **T2**. Whether or not this is so, however, will be left as an open question.

The second type of criticism relates to various possible axiomatic developments and it may well be true that there are important formal advantages to be gained by creating systems which contain **T1** or **T2** together with other laws, whether or not these laws have an intuitive acceptance. Thus, for example, various possible three-valued operators are not definable in **T2** in terms of the primitive operators and connectives and it is not, therefore, functionally complete, though they can of course be introduced as further primitives in terms of the significance tables. But if our main concern is the intuitive basis of the theory rather than its formal development, these problems are not of immediate interest. The third criticism, however, is; and it must now be considered.

Any one of the criteria I–V might be regarded as intuitively unsatisfactory and we have already noted various difficulties concerning I and IV*. The main problem, however, arises in respect of II, for although it seems inherently plausible that a compound which contains a non-significant component is itself non-significant, otherwise significant, there are special cases which seem to run counter to it. It might, for example, be said that we can have true implications with non-significant components since we can derive absurdities from absurdities; or it might be said that an absurdity implies anything (by analogy with the two-valued case that a false sentence implies any) or nothing. Any one of these assumptions would entail a rejection of II. If absurdities imply absurdities then hypotheticals have the value 1 whenever both components have the value n; if absurdities imply anything then hypotheticals have the value 1 whenever their antecedents have the value n; if absurdities imply nothing, then hypotheticals have the value 0 whenever their antecedents have the value n.

To reject II is thus to re-open the problem of defining the sentential connectives and it might be thought, in view of what has been said so far, that the problem is limited to the examination of definitions which violate only one half of II, namely II(a), in order to allow for the possibility of significant compounds with non-significant components. In fact, however, II(b), hence III, has to be re-committed for these are cases where we seem to have non-significant compounds in which each component is significant. This is the problem of the zeugma. Ryle, for example, lists 'She came home in a sedan chair and a flood of tears' as a category mistake even though the component parts 'She came home in a sedan chair' and 'She came home in a flood of tears' are significant, and Sommers[13] takes this as a general problem for any formal theory of categories.

I shall therefore consider II(b) and II(a) in turn, and I shall try to show that for the limited objections which are examined, and especially as these arise in respect of conjunction, negation and implication, there are no strong grounds for rejecting II outright, though there is some case for a slightly modified definition of '$\rightarrow$'. Very few of the many possible objections are considered, however, and to that extend the discussion is seriously incomplete.

(a) *Conjunction and the problem of zeugmas* It is clear that if we simply accept the fact of the zeugma without trying to explain it, then we immediately destroy any possibility of developing a sentential significance logic on the basis of significance tables. For we obviously do not want to say that every conjunction which contains only significant components is non-significant, but only that some are. Hence we shall never be able to say in general what the value of a compound is. The significance table for 'a & b' will look something like

a	&	b
1	?	1
0	?	1
1	?	0
1	?	1

on the standard classical rows, and the question mark will only be removed when actual sentences are substituted for the variables. Alternatively, of course, we may simply deny that zeugmas exhibit category mistakes—a line which seems to have been taken by Quine.[14]

I now want to show however that it is possible to steer a middle course between these extremes since the zeugma presents no serious difficulty in sentential logic provided we have an adequate predicate logic.

First, it is clear, the problem is essentially a problem of predicate ambiguity. We take a predicate which is significant over two individuals x and y when considered separately but which is also such that its sense with respect to x is different, to a greater or lesser degree, from its sense with respect to y. Hence when we take the predicate as applying to both together the two senses clash and the ambiguity is emphasised. If either sense is suppressed in the conjunction, one conjunct becomes absurd; and if both are taken then the ambiguity makes it impossible to use the compound sentence significantly. Thus, for example, 'hard' has a different sense when used to refer to chairs (to mean 'physically firm') from the sense which it has when used to refer to questions (to mean 'difficult'), so 'The chair and the question are hard' has no single unambiguous sense.

In saying, however, that the predicate has two senses I do not wish to deny that there may be a common etymological source or a causal explanation which relates them. Because the colour blue makes people feel depressed so we can see the link between the two senses of 'blue', in 'blue socks' and 'blue thoughts', but though there is this link this does not change the fact that the two senses are different and hence does not in any way relieve us of the obligation to give some logical account of the absurdity 'His socks and his thoughts are blue'.

There are, therefore, two features of the zeugma which seem to be characteristic. First, the absurdity present in the zeugma is largely context independent, for the clash is between predicates, or the significance ranges of predicates, and will be present in every context. Second, the zeugma is so constructed grammatically that the ambiguity is emphasised. This arises because the single word is applied to both objects simultaneously. Thus, if we take 'H' to be 'hard', we do not have a simple conjunction of the form 'Hx & Hy' but rather '$H(x$ & $y)$' which forces one sense in circumstances in which it is not possible to take one significantly. Suppose, for example, we distinguish two senses of 'hard', H_1 (physically firm) and H_2 (difficult) then, if x is a chair and y a question, we have SH_1x, $\sim SH_1y$, $\sim SH_2x$ and SH_2y. Now the compound 'H_1x & H_2y', *i.e.* 'The chair is firm and the question difficult' is significant, and though this is a possible interpretation of the compound 'Hx & Hy' it is not a possible interpretation of the complex '$H(x$ & $y)$' for in so presenting the predicate that it applies to both together we force one sense and have to interpret it either as $H_1(x$ & $y)$ or $H_2(x$ & $y)$. But in general, if a predicate holds of two objects together, it holds of each separately, and we then have to accept either 'H_1x & H_1y' or 'H_2x & H_2y', each of which contains a non-significant component. In this case, however, each compound is non-significant in terms of the significance tables so far adopted; which is to say that the zeugma is.

Suppose, then, we accept that in the case of an ambiguous predicate its significant range of application can be partitioned into two or more mutually exclusive significance ranges: in the case of 'H', for example, we can partition its significance range into tasks and material objects. This being so, an arbitrary choice of value for 'x', say 'x_1' from the total significance range will not ensure that 'Hx_1' is significant unless we take that sense of 'H' which is appropriate to 'x_1'. If, for example, we choose 'x_1' from the material object part of the significance range and take 'H' in sense 'H_2' to mean 'difficult', then 'H_2x_1' is not significant. Ordinarily, of course, we automatically supply the appropriate sense since the context of use determines the reference of the value chosen for 'x' and this in turn forces upon us one sense rather than another in order to preserve significance. If, therefore, we take this to be a general principle which must be satisfied, namely that the choice of value for the individual variable determines the sense of the predicate, and disallow the possibility of taking both an arbitrary choice of value for the individual variable and an arbitrary choice of sense for the predicate, then we can deal with the zeugma in the following way:

We have,

The chair and the question are both hard

which we represent as,

(i) $H(x_1$ & $x_2)$, where x_1 is a chair and x_2 a question

Now if we specify that the choice of value for 'x' in 'Hx' determines the sense of 'H', then from the fact that x_1 is a chair we conclude that 'H' is taken in sense 'H_1' (to mean 'firm'); and from the fact that x_2 is a question we conclude that 'H' is taken in sense 'H_2' (to mean 'difficult'). Hence the conditions on the zeugma imply that it can only be significant if it is interpreted to mean

(ii) H_1x_1 & H_2x_2

But if we now suppose in general that '$P(x$ & $y)$' has the same significance value as 'Px & Py', provided the same sense is preserved throughout, then (i) is either,

$$\text{(ia)}\ H_1x_1\ \&\ H_1x_2 \quad \text{or} \quad \text{(ib)}\ H_2x_1\ \&\ H_2x_2$$

and hence is inconsistent with (ii), which is the condition for the zeugma to be significant. So we conclude that it is not significant; and we may go on to explain its non-significance by saying that it is equivalent to a conjunction with a non-significant component in whatever sense 'H' is taken.

It might perhaps be thought, however, that we beg the question by making the general assumption

$$P(x\ \&\ y) \equiv Px\ \&\ Py$$

since there seem to be many counter-examples. If, for example, 'P' is 'two' then 'John and Bill are two' does not have the same significance value as 'John is two and Bill is two' if 'two' refers to the number two in both sentences or to ages in both. But so to argue is not merely to make use of an ambiguous predicate but to take predication as itself ambiguous since the point can only be made if the predication in 'John and Bill are two' is taken of John and Bill *qua* pair, or a group, and yet taken of John and Bill individually in 'John is two and Bill is two'. We can overcome this difficulty however by insisting that '$P(x\ \&\ y)$' is to be interpreted as 'x & y' are both P', for then 'John is two and Bill is two' does have the same significance value as 'John and Bill are two'. If 'two' refers to the number two, both are non-significant; while if 'two' refers to the ages of John and Bill then both are significant. Significance only fails to be preserved if we allow 'two' to refer to ages in one sentence and a number in the other. That is why '$P(x\ \&\ y) \equiv Px\ \&\ Py$' can only be asserted on the condition that the same sense is taken for 'P' throughout; and on this condition, as we have seen, we can demonstrate the non-significance of the zeugma.

It seems, therefore, that in accepting the zeugma as non-significant we are in no way committed to the view that it is a non-significant compound of two (or more) significant components; or the contrary, it is non-significant just because one of its components is non-significant. Hence if we adopt this analysis we are not required to reject either II(b) or III, and indeed we have discovered at least partial confirmation for II(a) since the explanation offered depends crucially on the assumption that a conjunction with one non-significant component is itself non-significant.

This sort of account presents certain formal difficulties in the predicate logic. They are not insurmountable, however, and a full discussion will be given in Part II. Especially interesting is the extension to zeugmas involving relational terms since they are closely linked with heterogeneous relations. Thus, the relation 'R', 'came home in', if its domain term ranges over people (perhaps animals) has a converse domain term which ranges over vehicles, physical states of the person (a flood of tears, a hurry, a state of collapse), physical or temporal states of the world (a thunderstorm, Spring, February), mental states (a fit of depression, anger, fright) *etc.* These are, therefore, mutually exclusive partial significance ranges within the significant converse domain, and hence, an arbitrary choice of a value for 'y' from this total range does not ensure the significance of 'xRy', and this is characteristic of a heterogeneous relation (see *PRC*). On the other hand, if we adopt the solution proposed for monadic predicates and insist that a choice of value for 'y' determines the sense of 'R', then in effect we resolve 'R' into its univocal components, 'R_1', 'R_2', . . . *etc.* and remove the heterogeneity.[15] R differs from the usual heterogeneous relation, however, since only its significant converse domain can be partitioned (and there are other comparable relations for which only the significant domain can be partitioned). But a full heterogeneous relation is such that both ranges can be partitioned; and this being so the heterogeneity is not removed by the assumption that a choice of values for the individual variables determines the sense. For suppose we take 'ε' under type theory with significant domain consisting of items of all types and significant converse domain consisting of items of all types except the lowest, type 0. We may then distinguish the univocal (homogeneous) components of 'ε' as 'ε_0', 'ε_1', . . . *etc.* such that 'ε_0' takes type 0 items in the domain place and type 1 items in the converse-domain place, 'ε_1' takes type 1 items in the domain place and type 2 items in the converse domain place, *etc.* Suppose, then, we choose a value for 'x', say 'x_2' of type 2 for the significant domain then this determines the sense of 'ε' to be 'ε_2'; but if we now choose a value for 'y', say 'y_1', from the significant converse domain then this determines the sense of 'ε' to be 'ε_0', and we have non-significance arising from the clash of two senses in spite of the insistence that the choice of values determines the choice of sense.

These remarks, however, are intended only as an indication of the development which becomes necessary in the predicate logic if we accept in general that a zeugma is a non-significant conjunction with a non-significant component, since to accept this is to accept that predicates may be ambiguous and that their significance ranges can be partitioned. Given that these developments can be carried out, however, there seems to be no good reason why the definitions of '&' which has been accepted so far should be rejected. And indeed, on quite general grounds one would expect this definition to be a feature of the sentential basis of any significance theory since the assertion of a conjunction amounts to the assertion of its several components separately; hence if any one of these components is non-significant the compound assertion is, while if each is significant so is the compound assertion.

(b) *Negation* It has most commonly been taken for granted that the negation of a significant sentence is significant and the negation of a non-significant sentence, non-significant. In fact the definition of '$\urcorner$' which occurs in **T1** and **T2** is generally assumed without argument. No doubt the reason for this is the belief that if we insert a 'not' in a sentence such as 'Tom is happy' we do not thereby change the sense of what is said but only the truth-value. If happiness is significantly predicable of Tom then we may significantly say that he has it or that he does not. There are uses of 'not' which, at first glance, seem not to satisfy this requirement, but it can usually be shown that they are special cases which can be explained in terms of the standard use. Thus, for example, it might be thought that 'Tables do not talk' is significant, in fact true, while 'Tables talk' is non-significant. Here, however, the 'not' in 'Tables do not talk' is taken to mean 'cannot be significantly said to' and hence 'Tables talk' should be interpreted as 'Tables (can significantly be said to) talk'; in this case, both sentences are significant. I shall not therefore raise doubts about the definition of '$\neg$' which is characteristic of **T1** and **T2**.

(c) *Implication* The definition of implication presents the most difficulties, and there are various ways of approaching the problem. We might now, having settled the definitions of '&' and '$\neg$' decide which general laws we wish to affirm, for this will then force one definition of '$\rightarrow$' rather than another. This way of handling the problem is considered by Presley, following Dienes.[16] Thus, for example, if we insist that 'a & $b \rightarrow a$' should be a law and if we adopt criterion IV then we immediately fill three of the five blanks in the table for '$\rightarrow$', *viz*:

A	$\rightarrow$	B
n	1	1
n	1	0
n	1	n

Such a definition amounts to adopting the general criterion that a non-significant sentence implies any, and as suggested above one might try to justify this by analogy with the corresponding thesis in two-valued logic that a false sentence implies any. On the other hand, of course, if we adopt IV* instead of IV we have the choice of completing '$\rightarrow$' by either 1 or n on the rows indicated.

A similar, but different, approach would be to decide on some formal definition for '$\rightarrow$' in terms of the other connectives and then to derive the significance table from this definition. For if one sentence form is defined to mean the same as another then they should have the same significance values for all distributions of values over their component parts. Thus, suppose we adopt '$\neg(a$ & $\neg b)$' as the definitional equivalent of '$a \rightarrow b$' by analogy with the two-valued case. Then '$\neg(a$ & $\neg b) \equiv (a \rightarrow b)$' must be a law; and since we can determine a unique table for '$\neg(a$ & $\neg b)$' in terms of the criteria so far adopted, the significance table for '$a \rightarrow b$' follows. In fact, using this definition each blank has to be completed by n and we simply re-affirm the **T1** table for '$\rightarrow$'.

Either one of these approaches seems plausible, and they are not inconsistent with each other if we adopt IV* instead of IV, for we then simply re-affirm **T2** and dismiss the general problem of re-defining implication. But there are various other solutions which might be preferred and which are incompatible with those mentioned and with each other. I shall discuss only one of these without attempting to argue that it is a better solution than any other. For once moves are made beyond the criteria determing **T1** or **T2** it is difficult to give strong reasons for or against alternative sentential systems. The solution discussed is not unrelated to the solution suggested for the zeugma and requires a further brief excursion into predicate logic.

Consider, first, the usual identification of universal sentences of the form 'All P are Q' with universally quantified hypotheticals of the form 'for all x, if Px then Qx'. If we accept this, then it seems we may affirm that such sentences have the same significance value provided the predicates have the same sense in each sentence, *i.e.*,

$$(x)(Px \rightarrow Qx) \overset{\equiv}{=} \text{All } P \text{ are } Q$$

This identification is, however, misleading if a distinction is made between common noun and predicative sentences. Commonly, '$(x)(Px \rightarrow Qx)$' is interpreted ambiguously as '$(x)(x$ is $P \rightarrow x$ is $Q)$' a compound of two predicative forms, or '$(x)(x \varepsilon P \rightarrow x \varepsilon Q)$', a compound of two common-noun (membership) forms since these are taken to be equivalent in terms of the standard abstraction thesis '$x \varepsilon P \longleftrightarrow Px$'. The universal sentence 'All P are Q', on the other hand, is interpreted as the class inclusion sentence '$P \subset Q$' however '$(x)(Px \rightarrow Qx)$' is interpreted. But there are two distinct kinds of general sentence in ordinary language: the noun-noun form, *e.g.* 'All cats are mammals', and the noun-adjective form, *e.g.* 'All cats are furry' so that we ought to distinguish the two cases:

$$\text{(i)}\quad (x)(x\varepsilon\pi \rightarrow Qx) \qquad \text{All } \pi \text{ are } Q$$
$$\text{(ii)}\quad (x)(x\varepsilon\pi \rightarrow x\varepsilon\tau) \qquad \text{All } \pi \text{ are } \tau$$

Only the second of these is equivalent to a class inclusion statement, '$\pi \subset \tau$' or '$\pi \cap \tau = \pi$'; the first is a generalized form of attributive statement since confirming cases of it are such as '$x_1 \varepsilon Q\pi$', *e.g.* 'Tom is a furry cat'. Confirming cases of (ii), on the other hand, take the form '$x_1 \varepsilon(\pi \wedge \tau)$', *i.e.* 'Tom is both a cat and a mammal' (*cf.* the oddity 'Tom is a mammalian cat'; or similarly, as a confirming case of 'Cats are animals', 'Tom is an animal-cat'). So whereas we might represent (ii) as '$(x)(x \varepsilon \pi \cap \tau \longleftrightarrow x\varepsilon\pi)$' an equivalent of the class inclusion form, (i) has to be represented as '$(x)(x \varepsilon Q\pi \longleftrightarrow x \varepsilon \pi)$'.

If we make these distinctions and refuse also to identify sentences of the sort, (i)' (x) $(Px \rightarrow Qx)$, and (ii)' (x) $(Px \rightarrow x\varepsilon\tau)$ with universal sentences of the form 'All P are Q' we avoid one of the standard puzzles of classical logic. For sentences in the form (i)' and (ii)' are grammatically well-formed whereas the corresponding universal sentences are not. Thus, there is nothing grammatically odd about 'for all x if x is black then x is heavy' or 'for all x if x is black then x is a cat' but the corresponding universal sentences 'All black are heavy' and 'All black are cats' are grammatical deviants. Of course we are always advised parenthetically that we can convert these to good English sentences without losing or gaining anything of logical value by inserting the word 'things' after the subject term, or by treating the subject term as a plural noun 'blacks'. So to amend them, however, is automatically to put them into the form (i) or (ii) with a membership sentence as subject, and this amounts to saying that the forms (i)' and (ii)' are *not* proper equivalents of universal sentences.[17]

Now it was suggested in *PRC* that membership sentences are always two-valued, true or false, while predicative forms are three-valued, true, false or non-significant, and though this thesis is not essential to what follows, it is useful to adopt it in order to simplify the discussion. For in terms of it, the form (ii) '$(x)(x \varepsilon \pi \rightarrow x \varepsilon \tau)$' is a compound of two significant components, and hence, adopting II(b) and III, the compound is itself always significant. So, therefore, are general noun-noun sentences of the form 'All π are τ', *e.g.* ''Cats are mammals' (true), 'Cats are rainbows' (false), 'Cats are numbers' (false) *etc*. And that this should be so is clear in terms of the class-inclusion statements to which they are equivalent. For what is being said is, *e.g.* that the class of cats is contained in the class of mammals, and this is true; or that the class of cats is contained in the class of rainbows, and this is false; or that the class of cats is contained in the class of numbers, and this, too, is false. To say that the class of cats is contained in the class of numbers is to say that the predicates which are significant of numbers are significant of cats; and this, though false, is a significant statement. Indeed just because the predicates significant over numbers are not significant over cats, so cats are not numbers. We might, therefore, begin with these inclusion statements and argue from the significance of these to the significance of general statements of the form 'All π are τ' and thence to the significance of universally quantified sentences such as '$(x)(x \varepsilon \pi \rightarrow x \varepsilon \tau)$'. But if these are significant then, since they contain two significant components, we have to accept that a compound with two significant components is itself significant. That is, we again confirm II(b) and III.

But if sentences of the form (ii) are always significant, the case is quite different with sentences of the form (i). In this form we may make absurd utterances such as 'Rainbows are happy', 'Numbers are thoughtful', and so on. Here, we are applying a predicate—'happy', 'thoughtful'—to objects which, in terms of the classification effected by the subject-term, do not fall within its significant range. Taking the standard sense of 'rainbow' and 'happy', the significant range of the predicate 'happy' excludes the class of rainbows; and similarly, the significant range of 'thoughtful' excludes the class of numbers. The absurdity therefore arises, as in the case of the zeugma, from a clash of standard senses rather than from the context of use, and if we take the standard senses then we may say that the significance criteria for general sentences is independent of their actual use in a given context. And this conclusion is not altered if we take a non-standard use. Taking 'rainbows' to refer to trout, 'Rainbows are happy' is significant because the significant range of 'happy' does not exclude trout (or perhaps it does, since we may want to say that it makes no sense to affirm that trout are happy or unhappy; but the point can be made in terms of 'heavy' instead of 'happy'). What matters, however, is that the significance conditions, in whatever way we take 'rainbow', is not affected by the context of use.

If, therefore, we are to account for the absurd cases of sentences in the form (i), we must adopt a definition of '$\rightarrow$' in terms of which '$(x)(x \varepsilon \pi \rightarrow Qx)$' has the value n whenever 'All π are Q' has the value n. But 'All π are Q' will be n whenever the significance range of 'Q' excludes π, which is to say whenever 'Qx' is absurd for x belonging to π. This will be so, therefore, if all attributive sentences of the sort '$x \varepsilon Q\pi$' are absurd, and it can be shown (Part II) that these are absurd, if, and only if 'Qx' is absurd. Hence, we have '$(x)(x\varepsilon\pi \rightarrow Qx)$' is absurd, whenever '$Qx$' is absurd, whether the antecedent is true or false (since the antecedent is a membership form, it cannot itself be non-significant). So, it follows, any definition of '$\rightarrow$' which preserves (i) must satisfy the following partial table:

A	$\rightarrow$	B
1	1	1
1	0	0
1	n	n
0	1	1
0	1	0
0	n	n

In this way we define a three-valued implication which relates two values to three (a ((2,3):3) connective) suitable for the resolution of a universal sentence to a hypothetical form. It should be noted, however, that there appear to be counter-examples to the above suggestions. For let 'The world is flat' be false and 'The number 7 is thoughtful' be non-significant, and consider the following:

If the world is flat then the number 7 is thoughtful.

Such a remark might be made and taken as an emphatic affirmation that the antecedent is false: *i.e.* if the world is flat then anything goes. So to take it, however, is to accept that it is a true statement, since it is equivalent to the true statement 'it is false that the world is flat'; and this amounts to saying that the combination of values $(0,n)$ should yield 1 rather than n. But against this we should remember the difficulties which arise from the similar assumption in the two-valued case: that an implication with a false antecedent is always true, *e.g.*: 'If the world is flat then pigs fly (*i.e.* anything goes)'. We therefore neglect such cases, not because there is no point to them but rather because they seem to carry less weight, and perhaps lead to more oddities, than the argument in terms of universal sentences which leads to the partial table above.

It remains now to consider how we may extend the table to include hypotheticals with non-significant antecedents. That is, we have to examine the cases,

A	$\rightarrow$	B
n		1
n		0
n		n

Here there are various conflicting criteria which might be adopted. We might agree in general that anything whatever follows from a non-significant expression (which is to say that each row should be completed by 1); or that nothing does (which is to say that each row should be completed by 0 or n) or that only non-significant expressions follow from non-significant expressions (which is to say that only the last row should be completed by 1). If we adopt the second of these proposals and complete each row by n rather than 0 then again we simply re-affirm the definition of implication in **T1** and **T2**. And it might now begin to seem that **T1** or **T2** is the only reasonable basis for a theory of significance. But there is one fact which might cause us to prefer the third suggestion, that only non-significant sentences follow from non-significant sentences, namely that we wish to retain the literal *reductio ad absurdum*: whatever implies an absurdity is itself absurd. This sort of *reductio* seems to have been used by Ryle and others, and it obviously has some value. If we do adopt it we take the combination (n,n) to yield 1 and we have the option of completing the first two rows by either 0 or n. It seems clear, however, that neither of the following sentences could be used in any significant way if the general words in them are taken to have their standard sense:

If the number 7 is thoughtful then the world is round
If the number 7 is thoughtful then the world is flat;

and we therefore choose the value n to complete the remaining rows of the table while recognising that no argument whatever has been brought forward in support of this. This would lead to the following definition of a connective which we represent by '$\Rightarrow$' to distinguish it from '$\rightarrow$' of **T2**:

A	$\Rightarrow$	B
1	1	1
0	1	1
n	n	1
1	0	0
0	1	0
n	n	0
1	n	n
0	n	n
n	1	n

(d) *Disjunction and Equivalence* We now adopt quite arbitrary criteria for determing the definitional tables for '$\wedge$' and '$\longleftrightarrow$'. Thus, we stipulate that the two-valued *definitions* of '$\wedge$' in terms of '&', and '$\Longleftrightarrow$' in terms of '$\Longrightarrow$' should be preserved. This amounts to saying that '$\neg(\neg a \wedge \neg b)$' should have the same significance value as 'a & b'; and similarly, that '$a \Longleftrightarrow b$' should have the same significance value as '$(a \Longrightarrow b)$ & $(b \Longrightarrow a)$'. That is, the following should be laws:

$$a \ \& \ b \equiv \neg(\neg a \wedge \neg b)$$
$$a \Longleftrightarrow b \equiv (a \Longrightarrow b) \ \& \ (b \Longrightarrow a)$$

This determines the tables uniquely as,

a	b	$a \wedge b$	$a \Longleftrightarrow b$
1	1	1	1
0	1	1	0
n	1	n	n
1	0	1	0
0	0	0	1
n	0	n	n
1	n	n	n
0	n	n	n
n	n	n	1

Thus, the definition of '$\wedge$' is the same in **T1** and **T2** while the definition of '$\Longleftrightarrow$' like '$\Longrightarrow$' differs in one place only from the previous definition. It should be noted, however, that if we merely stipulate that the equivalence relation should hold between 'a & b' and '$\neg(\neg a \wedge \neg b)$', and between '$a \Longleftrightarrow b$' and '$(a \Longrightarrow b)$ & $(b \Longrightarrow a)$', rather than the relation of extensional synonymy, *i.e.*

$$a \ \& \ b \Longleftrightarrow \neg(\neg a \wedge \neg b)$$

and

$$(a \Longleftrightarrow b) \Longleftrightarrow (a \Longrightarrow b) \ \& \ (b \Longrightarrow a),$$

then many different definitions of '$\wedge$' and '$\Longleftrightarrow$' would satisfy such a requirement.

(e) *Conclusions* The arguments against II which have been considered are only few chosen from many, and the terms in which II has been supported are to some extent arbitrary. Accepting these limitations, however, the only modification which seems at all reasonable is the proposed definition of implication and the consequential treatment of equivalence. Now it can be shown that if we adopt the definitions of '$\neg$', '&', '$\wedge$', '$\Longrightarrow$' and '$\Longleftrightarrow$' and take these to be the primary sentential connectives, then analogues of all the **PC** laws are laws under criterion IV*, as in **T2**. It will be the case, however, that whereas no analogues of the **PC** laws are laws of **T2** under IV, certain of them are in terms of '$\Longrightarrow$' and '$\Longleftrightarrow$'. Thus, for example, although '$a \wedge b \rightarrow b \wedge a$' is a law of **T2** under IV* but not under IV,

'$a \wedge b \Rightarrow b \wedge a$' is a law under IV. This is so, of course, since the use of '$\Rightarrow$' in place of '$\rightarrow$' converts n to 1 on one row and hence affects the final column of **T2** laws in the form '$A \rightarrow B$' by converting n to 1 in at least one place, and often several places. Hence one should expect the same sort of thing to arise in respect of certain of the laws which contain operators; and this is so. Thus, '$a \rightarrow Sa \wedge a$' is a law of **T2** under IV* but not under IV, while '$a \Rightarrow Sa \wedge a$' is a law under IV.

Not only are certain of the **T2** laws strengthened, however, but others are excluded; and in particular a number of less intuitive **T2** laws are no longer demonstrable. Thus, although we still have the acceptable versions of the paradox-laws, *viz*.

$$[(a \Rightarrow (b \;\&\; \neg b)) \;\&\; (\neg a \Rightarrow (b \;\&\; \neg b))] \Rightarrow \neg Sa,$$
$$[(a \Rightarrow \neg a) \;\&\; (\neg a \Rightarrow a)] \Rightarrow \neg Sa,$$

the following, which are laws of **T2** using '$\rightarrow$', fail under both IV and IV* using '$\Rightarrow$':

$$[(a \Rightarrow (b \;\&\; \neg b)) \;\&\; (\neg a \Rightarrow (b \;\&\; \neg b))] \Rightarrow Sa$$
$$[(a \Rightarrow (b \;\&\; \neg b)) \;\&\; (\neg a \Rightarrow (b \;\&\; \neg b))] \Rightarrow Sa \;\&\; \neg Sa$$
$$[(a \Rightarrow \neg a) \;\&\; (\neg a \Rightarrow a)] \Rightarrow Sa$$
$$[(a \Rightarrow \neg a) \;\&\; (\neg a \Rightarrow a)] \Rightarrow Sa \;\&\; \neg Sa$$

There seems, therefore, to be a good deal of point in preferring '$\Rightarrow$' to '$\rightarrow$' in spite of the fact that it allows us to affirm that a compound in which every component is non-significant is itself significant. In order to use '$\Rightarrow$', however, it is unnecessary to develop a new system since we may simply introduce it by means of its significance table as a further constant of **T2**. For although in the two-valued **PC** no truth-table other than the classical one for '$\supset$' defines a constant which can reasonably be interpreted as 'if . . . then . . .', there is a wide choice of significance tables in a three-valued system which define constants any one of which, or even all of which, can be interpreted as 'if . . . then . . .'; and similarly for the other connectives.

Such alternative definitions will of course validate different laws. It seems not unreasonable, however, at least from a formal point of view, to take advantage of the richness of a three-valued system in order to define several different sets of connectives, to investigate the relations between them and to examine the differences between the laws which they validate. Thus, one might accept both '$\rightarrow$' and '$\Rightarrow$' and define yet a third, '$. . . >$', which is such that if $A \ldots > B$ and '$B \ldots > A$', A and B have the same significance value, *i.e.* $A \equiv B$; indeed several definitions will satisfy this requirement. Nor does it seem unreasonable to adopt such a policy in terms of the intuitive basis of the logic since it might well be argued that we do in ordinary language use the connective 'if . . . then . . .', perhaps indeed all the connectives, in different ways. If this is so, however, then what is being said is that certain arguments which are accepted as valid by X will be rejected as invalid by Y because different definitions of the constants are favoured. And how is one to decide which of them is right?

The compulsion of logical validity evaporates. One might way, perhaps, that X and Y are speaking different languages or playing different games within English, but why should they not? Both are using ordinary language and in some sense or other using it correctly, for the richness of a natural language permits these wide variations. Yet if it is the case that there is no one set of correct definitions for the logical constants, there is no unique criterion of validity, and argument is reduced to persuasion. This is not of course the sense of 'persuasion' in which one might say that argument is persuasion because the premisses are always open to objection; rather, what is being said, is that though both X and Y accept the premisses, one rationally and correctly denies the conclusion while the other, rationally and correctly, accepts it. For though both interpret the constants in the same way, *e.g.* as 'if . . . then . . .', they manipulate them in accordance with different definitions, which is to say in accordance with different logics. If the logical connectives are ambiguous in this sense, logic is a set of games, not the only possible one. So we are left with the question with which we began: are there correct definitions of the connectives?

NOTES

1. "Predicates, Relations and Categories" (*PRC*) *Australasian Journal of Philosophy*, 1966.

2. I shall not distinguish 'absurdity' and 'non-significance' and I shall most frequently use the latter. What I mean by it, however, is what Ryle means by 'absurdity'; that is, I do not intend to take into account expressions such as 'Brilling stu tonk', constructed from non-words, nor expressions such as 'Stamp the in what if' which are non-grammatical concatenations of words. Non-significant sentences, therefore, are grammatically well-formed like 'Saturday is in bed', but nevertheless fail to make sense.

3. See *PRC*.

4. *cf.* the use of subscript variables in "An Augmented Modal Logic" (*AML*), *Notre Dame Journal of Formal Logic*, vol. VI (1965), pp. 81-98; and F. R. Routley, "On a Significance Theory", § 3, *Australasian Journal of Philosophy*, 1966.

5. Generally, of course, we recognise the distinction between the use of a sentence and a statement about it by employing quote marks to signal the latter, but I take it that the use of operator locations such as 'it is true that . . . ' eliminates the need for quote marks or quotation functions, though one might wish to say that the operators absorb these signalling devices.

6. *cf.* *AML*, VII.

7. See Presley, "Arguments about Meaninglessness", *British Journal for the Philosophy of Science*, vol. XII (1961).

8. *cf.* the corresponding modal law for N(necessity) in which the implication goes the other way, *viz.* $Np \vee Nq \supset N(p \vee q)$.

9. Every sentence is either true, false or non-significant.

10. I am indebted to Mr. F. R. Routley for this description and the general point made in this paragraph.

11. Suitable axiomatic bases for both **T1** and **T2** which are consistent and complete over the significance tables have recently been developed by Mr. F. R. Routley and will be published separately. A system similar to **T2** (though it contains only the operator 'S' and not 'T' or 'F') in that three-valued formulae of the sort 'a & $b \rightarrow a$', '$a \rightarrow Sa$', *etc*. are admitted and criteria II, III and IV* are adopted, was developed earlier by Halldén (*The Logic of Nonsense*, Uppsala, 1949). Moreover $a \rightarrow Sa$ is taken as an axiom by Halldén and in consequence the rule of detachment is modified, though in a way rather different from that suggested above (Routley adopts Halldén's technique in his axiomatisation of **T2**). Another system similar in some respects to **T2** is the system **T**n which is presented axiomatically as a sub-system of a modal theory in *AML*. Here, the T-rule as well as unrestricted detachment are adopted but the theorems do not coincide with the laws of **T2**. In particular '$a \rightarrow Sa$' and '$a \leftrightarrow Ta$' are not theorems. The main reason for this is that in **T**n, '$Ta \wedge Tb \rightarrow T(a \wedge b)$' is a law, but its converse '$T(a \wedge b) \rightarrow Ta \wedge Tb$' is not, while in **T2** the reverse is the case. Given $T(a \wedge b) \rightarrow Ta \wedge Tb$ then 'Sa' could be derived immediately from '$a \wedge \neg a$' in **T**n. Thus, 1. $a \wedge \neg a$; 2. $T(a \wedge \neg a)$ using T-rule; 3. $Ta \wedge T\neg a$, using '$T(a \wedge b) \rightarrow Ta \wedge Tb$'; 4. Sa, using definitions of 'F' and 'S'.

12. 'K' is the operator '$\not{P}$' defined by Routley in § 4 *op. cit.*

13. "Types Ontology", *Philosophical Review*, vol. 72 (1963); See also Halldén, *op. cit.*

14. *Word and Object*, Cambridge, Mass., 1960.

15. This is not to say that R is the union relation of R_1, R_2, *etc*., *i.e*. that $R = R_1 \dot{\cup} R_2 \dot{\cup} \ldots \dot{\cup} R_n$, for that would be to suppose that the different senses can be combined into one. Rather, the condition is that $R = R_1$ or $R = R_2$ or . . . , in any significant context.

16. Presley, *op. cit.*; Dienes, "On ternary logic", *The Journal of Symbolic Logic*, 1949.

17. It will be shown in Part II that such conversion of the forms (i)' and (ii)' to (i) and (ii) is legitimate, given an analysis of attributive sentences, and hence that we need only consider the cases (i) and (ii).

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